**2014 Numerical Analysis Midterm Exam, 100 points**

1. Draw a figure to show the IEEE 754 representation of 32-bit floating-point numbers. You have to explain each part of the word and the method of excess-127 representation for the exponent part. (10%)
2. Assume x and y are 2 floating-point numbers. They are not exactly stored in our computer. The arithmetic operators of the computer also introduce arithmetic errors. Please deduce (推算) the round-off error of the following expression:

***2\*x + y/x.*** (10%)

1. The Taylor expansion of ex is . Now, we want to approximate *ex* by using a Taylor series with a limited number of terms. How many terms are required such that the truncation error of e1 is less than or equal to 0.001? You can expand *ex* at x=0 and use Taylor’s theorem to calculated the solution. (10%)
2.  (a) Compute the root by using bi-section method. Assume that the initial interval is [0, 0.9]. Please iterate the computation 3 times. (b) Compute the root by using Newton’s method, assuming the initial guess x0 = -1. Please repeat the iteration by 3 times. (15%)
3. Prove that the converge rate of the Newton root-finding method is quadratic (二次收斂)。 (10%)

Table 1

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| x | -0.5 | -0.25 | 0 | 0.25 | 0.5 | 0.75 |
| f(x) | 8 | 2 | 1 | 0 | -1 | -2 |

1. Assume that we have 6 sample points of (x, f(x)) as shown in Table 1. (a) Deduce the Lagrange polynomial for interpolation. Just write the polynomial is good enough for this problem. (b) Deduce Newton polynomial by using the divided difference method. Please draw the table to show the computational steps. (c) Interpolate f(0.2) by using the Newton polynomial and Horner’s algorithm. (15%)
2. (a) What is the upper bound of the truncation error of the Newton polynomial method? (b) Use the error bound to prove that the truncation error for x in [-0.5, -0.25] is larger than the truncation for x in [0, 0.25]. (10%)
3. (a) Compute the numerical integral of by using Trapezoid method, assuming the sample points of (x, f(x)) are listed in Table 1. (b) Compute the integral by using Simpson’s method. (10%)
4. Assume *|f”(x)|≤M* in [-0.5,0.75]. Deduce the error bound of the Trapezoid method. (10%)