**2017 Numerical Analysis Midterm Examination**

100 points**過程要列出，否則不計分**

1. 畫圖解釋IEEE 754 floating point representation (32 bits). 包含下列3項目的長度(Numbers of bit)，儲存方式(例如Normalization and excessive-xx encoding)
   1. Sign, exponent, and mantissa (10%) 請忽略特殊情況，如0 和denormalized numbers.
2. Define the following terms (15%)
   1. Round-off error, Truncation error of Taylor theorem, Absolute error, Relative error, Arithmetic error
3. (20%)
   1. Assuming that x0 = 0, compute x1 and x2 by using Newton’s method.
   2. Assuming that x0=0 and x1=1, compute x2 and x3 by using secant method.
   3. Let a=0 and b=1, compute the root by using Bisection method for 2 iterations.
4. We are given a set of sample points as listed below: (20%)

Table 1, Sample Data

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| x | 0 | 1 | 2 | 3 | 4 |
| y | 1 | 2 | 2 | 1 | 1 |

Please complete the following computation:

* 1. 寫出Lagrange polynomial和Newton polynomial，。(5+10=15 points)
  2. Assume p(x) is the Newton polynomial,使用Horner’s algorithm計算p(1.5). (5 points)

1. Assuming , compute by using the following methods. (N=6, number of sample points=7, compute the sample point first.) (15%)
   1. Trapezoid rule
   2. Simpson’s method
   3. Gaussian quadrature (using 4 Gaussian points)
2. Deduce the linear system step by step for solving the following non-linear system by using 2D Newton’s root-finding method: (10%)
3. Let sets **A**={1, x, x2, x3} and **B**={x2-1, x2+1, x3, x3-x}. Prove the following property (10%)
   1. **A** and **B** span the same functional space.