**2018 Fall Semester, Numerical Analysis, Homework #4**

Direct Methods and Least Square Method (Two Weeks)

1. In this homework, we are going to solve over-constraint problems by using the Least Square Methods employing the Gaussian elimination and QR-decomposition solvers.
2. The data comes from Example 4.5 in page 197 of our textbook.
   1. Define
   2. Generate a data set
   3. Thus, there are 11 sample points.
3. Please fit the data by using a polynomial of degree 7: . At first form the following system where

, , . This is a 11 by 8 system.

1. Implement the Gaussian-elimination method and the QR-decomposition method.
2. Re-model the system as follows:
   1. Form the a new system: , which is theoretically solvable. Call it , which is an 8 by 8 system.
3. Result 1: Solve the new system by using the Gaussian elimination solver.
4. Result 2: Solve the new system by using the QR-decomposition solver.
5. Result 3: Solve the original system by using the QR-decomposition solver.
6. Requirements:
   1. Use Horner’s algorithm to generate the test data. Print out the test data. (10%)
   2. Compute Results 1-3 by using your solvers and print out the results. (50%)
   3. Compare the precisions of the three results and compute their errors (in 2-norms and ∞-norm). You have to draw some conslusions. (15%)
   4. Analyze the results and discuss the stabilities of these three procedures. (15%)
   5. Repeating the procedure for polynomial of degrees 8, 9, 10, … In each case, the number of sample points is always greater than the degree of the polynomial by at least 4. When will these methods fail to compute any meaningful results? (10%).
   6. Further test cases and analysis are always welcome. (10%)

You can modify the sample programs given in our classes to construct the solver.