As before, there are two terms present in the mixer output. The bandpass term can be filtered out by a lowpass filter. The lowpass term, $(A_c/2)m(t)\cos(\phi)$, depends on ϕ, however. The power in the lowpass term is given by

$$P_{\text{dem}} = \frac{A_c^2}{4} P_m \cos^2\phi \tag{3.3.5}$$

where P_m denotes the power in the message signal. We can see, therefore, that in this case we can recover the message signal with essentially no distortion, but we will suffer a power loss of $\cos^2\phi$. For $\phi = \pi/4$ this power loss is 3 dB, and for $\phi = \pi/2$ nothing is recovered in the demodulation process.

3.3.2 SSB-AM Demodulation

The demodulation process of SSB-AM signals is basically the same as the demodulation process for DSB-AM signals—that is, mixing followed by lowpass filtering. In this case,

$$u(t) = \frac{A_c}{2} m(t)\cos(2\pi f_c t) \mp \frac{A_c}{2}\hat{m}(t)\sin(2\pi f_c t) \tag{3.3.6}$$

where the minus sign corresponds to the USSB and the plus sign corresponds to the LSSB. Mixing $u(t)$ with the local oscillator output, we obtain

$$\begin{aligned} y(t) &= \frac{A_c}{2} m(t)\cos^2(2\pi f_c t) \mp \frac{A_c}{2}\hat{m}(t)\sin(2\pi f_c t)\cos(2\pi f_c t) \\ &= \frac{A_c}{4} m(t) + \frac{A_c}{4} m(t)\cos(4\pi f_c t) \mp \frac{A_c}{4}\hat{m}(t)\sin(4\pi f_c t) \end{aligned} \tag{3.3.7}$$

which contains bandpass components at $\pm 2f_c$ and a lowpass component proportional to the message signal. The lowpass component can be filtered out using a lowpass filter to recover the message signal. This process for the USSB-AM case is depicted in Figure 3.13.

ILLUSTRATIVE PROBLEM

Illustrative Problem 3.7 [LSSB-AM example] In a USSB-AM modulation system, if the message signal is

$$m(t) = \begin{cases} 1, & 0 \le t < \frac{t_0}{3} \\ -2, & \frac{t_0}{3} \le t < \frac{2t_0}{3} \\ 0, & \text{otherwise} \end{cases}$$

with $t_0 = 0.15$ s, and the carrier has a frequency of 250 Hz, find $U(f)$ and $Y(f)$ and compare the demodulated signal with the message signal.

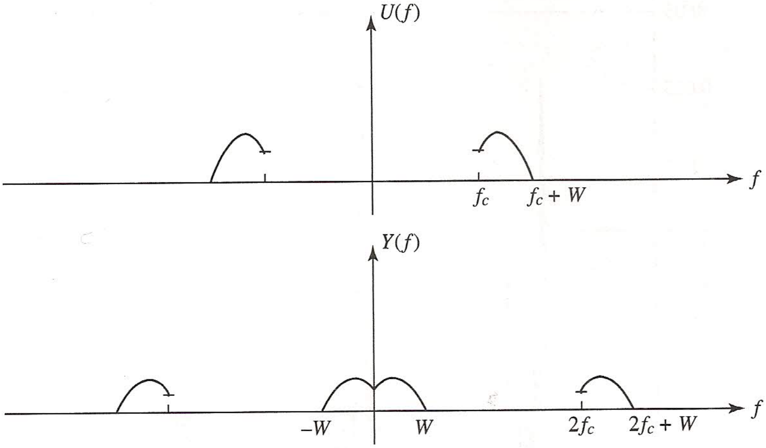

Figure 3.13 Demodulation of USSB-AM signals

SOLUTION

The modulated signal and its spectrum are given in Illustrative Problem 3.4. The expression for $U(f)$ is given by

$$U(f) = \begin{cases} 0.025e^{-0.05j\pi(f-250)}\text{sinc}(0.05(f-250))\left(1-2e^{-0.1j\pi(f-250)}\right) & \\ +0.025e^{-0.05j\pi(f+250)}\text{sinc}(0.05(f+250))\left(1-2e^{-0.1j\pi(f+250)}\right), & |f| \le f_c \\ 0, & \text{otherwise} \end{cases}$$

and

$$\begin{aligned} Y(f) &= \frac{1}{2}U(f-f_c) + \frac{1}{2}U(f+f_c) \\ &\approx \begin{cases} 0.0125e^{-0.05j\pi f}\text{sinc}(0.05f)\left(1-2e^{-0.01j\pi f}\right), & |f| \le f_c \\ 0.0125e^{-0.05j\pi(f-500)}\text{sinc}(0.05(f-500))\left(1-2e^{-0.01j\pi(f-500)}\right), & f_c \le f \le 2f_c \\ 0.0125e^{-0.05j\pi(f+500)}\text{sinc}(0.05(f+500))\left(1-2e^{-0.01j\pi(f+500)}\right), & -2f_c \le f \le -f_c \\ 0, & \text{otherwise} \end{cases} \end{aligned}$$

A plot of $Y(f)$ is shown in Figure 3.14. The signal $y(t)$ is filtered by a lowpass filter with a cutoff frequency of 150 Hz; the spectrum of the output is shown in Figure 3.15. In Figure 3.16 the original message signal is compared with the demodulated signal.

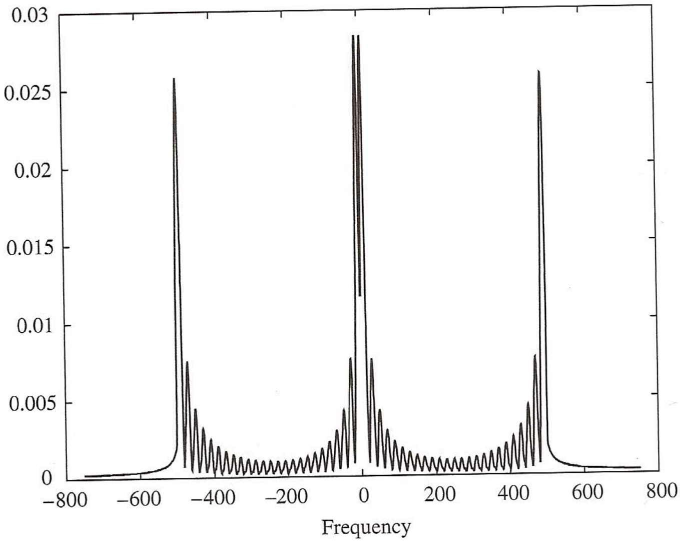

Figure 3.14 Magnitude spectrum of the mixer output in Illustrative Problem 3.7

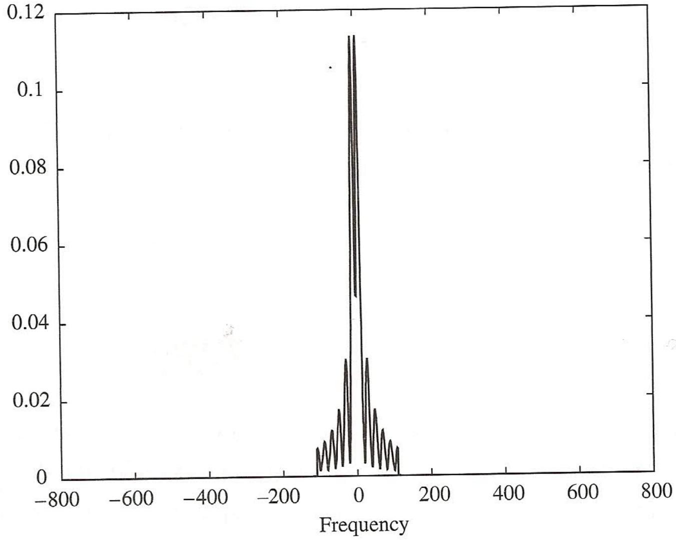

Figure 3.15 The demodulator output in Illustrative Problem 3.7

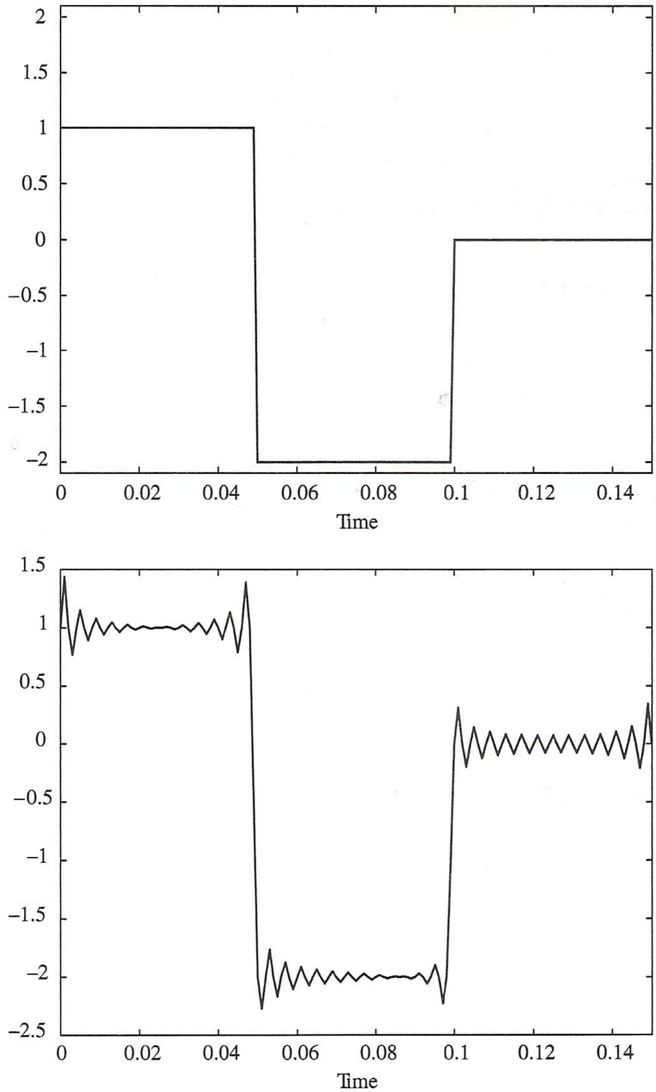

Figure 3.16 The message signal and the demodulator output in Illustrative Problem 3.7

The MATLAB script for this problem follows.

M-FILE

```
% lssb_dem.m
% Matlab demonstration script for LSSB-AM demodulation. The message signal
% is +1 for 0 < t < t0/3, -2 for t0/3 < t < 2t0/3, and zero otherwise.
echo on
```

```
t0=.15;                                      % signal duration
ts=1/1500;                                   % sampling interval
fc=250;                                      % carrier frequency
fs=1/ts;                                     % sampling frequency
df=0.25;                                     % desired freq.resolution
t=[0:ts:t0];                                 % time vector
% the message vector
m=[ones(1,t0/(3*ts)),-2*ones(1,t0/(3*ts)),zeros(1,t0/(3*ts)+1)];
c=cos(2*pi*fc.*t);                           % carrier vector
udsb=m.*c;                                   % DSB modulated signal
[UDSB,udsb,df1]=fftseq(udsb,ts,df);          % Fourier transform
UDSB=UDSB/fs;                                % scaling
n2=ceil(fc/df1);                             % location of carrier in freq. vector
% remove the upper sideband from DSB
UDSB(n2:length(UDSB)-n2)=zeros(size(UDSB(n2:length(UDSB)-n2)));
ULSSB=UDSB;                                  % generate LSSB-AM spectrum
[M,m,df1]=fftseq(m,ts,df);                   % spectrum of the message signal
M=M/fs;                                      % scaling
f=[0:df1:df1*(length(M)-1)]-fs/2;            % frequency vector
u=real(ifft(ULSSB))*fs;                      % generate LSSB signal from spectrum
% mixing
y=u.*cos(2*pi*fc*[0:ts:ts*(length(u)-1)]);
[Y,y,df1]=fftseq(y,ts,df);                   % spectrum of the output of the mixer
Y=Y/fs;                                      % scaling
f_cutoff=150;                                % choose the cutoff freq. of the filter
n_cutoff=floor(150/df);                      % design the filter
H=zeros(size(f));
H(1:n_cutoff)=4*ones(1,n_cutoff);
% spectrum of the filter output
H(length(f)-n_cutoff+1:length(f))=4*ones(1,n_cutoff);
DEM=H.*Y;                                    % spectrum of the filter output
dem=real(ifft(DEM))*fs;                      % filter output
pause % Press a key to see the effect of mixing
clf
subplot(3,1,1)
plot(f,fftshift(abs(M)))
title('Spectrum of the Message Signal')
xlabel('Frequency')
subplot(3,1,2)
plot(f,fftshift(abs(ULSSB)))
title('Spectrum of the Modulated Signal')
xlabel('Frequency')
subplot(3,1,3)
plot(f,fftshift(abs(Y)))
title('Spectrum of the Mixer Output')
xlabel('Frequency')
pause % Press a key to see the effect of filtering on the mixer output
clf
subplot(3,1,1)
plot(f,fftshift(abs(Y)))
title('Spectrum of the Mixer Output')
xlabel('Frequency')
subplot(3,1,2)
plot(f,fftshift(abs(H)))
```

```
title('Lowpass Filter Characteristics')
xlabel('Frequency')
subplot(3,1,3)
plot(f,fftshift(abs(DEM)))
title('Spectrum of the Demodulator Output')
xlabel('Frequency')
pause % Press a key to see the message and the demodulator output signals
subplot(2,1,1)
plot(t,m(1:length(t)))
title('The Message Signal')
xlabel('Time')
subplot(2,1,2)
plot(t,dem(1:length(t)))
title('The Demodulator Output')
xlabel('Time')
```

ILLUSTRATIVE PROBLEM

Illustrative Problem 3.8 [Effect of phase error on SSB-AM] What is the effect of phase error on SSB-AM?

SOLUTION

Assuming that the local oscillator generates a sinusoidal with a phase offset of ϕ with respect to the carrier, we have

$$\begin{aligned} y(t) &= u(t)\cos(2\pi f_c t + \phi) \\ &= \left[\frac{A_c}{2} m(t)\cos(2\pi f_c t) \mp \frac{A_c}{2}\hat{m}(t)\sin(2\pi f_c t)\right]\cos(2\pi f_c t + \phi) \\ &= \frac{A_c}{4} m(t)\cos\phi \pm \frac{A_c}{4}\hat{m}(t)\sin\phi + \text{high-frequency terms} \end{aligned} \tag{3.3.8}$$

As seen, unlike the DSB-AM case, the effect of the phase offset here is not simply attenuating the demodulated signal. Here the demodulated signal is attenuated by a factor of $\cos\phi$ as well as distorted by addition of the $\pm(A_c/4)\hat{m}(t)\sin\phi$ term. In the special case of $\phi = \pi/2$, the Hilbert transform of the signal will be demodulated instead of the signal itself.

3.3.3 Conventional AM Demodulation

We have already seen that conventional AM is inferior to DSB-AM and SSB-AM when power and SNR are considered. The reason is that a usually large part of the modulated signal power is in the carrier component that does not carry information. The role of the