ILLUSTRATIVE PROBLEM

Illustrative Problem 3.1 [DSB-AM modulation] The message signal $m(t)$ is defined as

$$m(t) = \begin{cases} 1, & 0 \le t \le \frac{t_0}{3} \\ -2, & \frac{t_0}{3} < t \le \frac{2t_0}{3} \\ 0, & \text{otherwise} \end{cases}$$

This message DSB-AM modulates the carrier $c(t) = \cos 2\pi f_c t$, and the resulting modulated signal is denoted by $u(t)$. It is assumed that $t_0 = 0.15$ s and $f_c = 250$ Hz.

1. Obtain the expression for $u(t)$.
2. Derive the spectra of $m(t)$ and $u(t)$.
3. Assuming that the message signal is periodic with period $T_0 = t_0$, determine the power in the modulated signal.
4. If a noise is added to the modulated signal in part 3 such that the resulting SNR is 10 dB, find the noise power.

SOLUTION

1. $m(t)$ can be written as

$$m(t) = \Pi\left(\frac{t - t_0/6}{t_0/3}\right) - 2\Pi\left(\frac{t - t_0/2}{t_0/3}\right)$$

   Therefore,

$$u(t) = \left[\Pi\left(\frac{t - 0.025}{0.05}\right) - 2\Pi\left(\frac{t - 0.075}{0.05}\right)\right]\cos(500\pi t) \tag{3.2.9}$$

2. Using the standard Fourier transform relation $\mathcal{F}[\Pi(t)] = \text{sinc}(t)$ together with the shifting and the scaling theorems of the Fourier transform, we obtain

$$\begin{aligned}\mathcal{F}[m(t)] &= \frac{t_0}{3}e^{-j\pi f t_0/3}\text{sinc}\left(\frac{t_0 f}{3}\right) - 2\frac{t_0}{3}e^{-j\pi f t_0}\text{sinc}\left(\frac{t_0 f}{3}\right) \\ &= \frac{t_0}{3}e^{-j\pi f t_0/3}\text{sinc}\left(\frac{t_0 f}{3}\right)\left(1 - 2e^{-j2\pi t_0 f/3}\right) \qquad (3.2.10)\end{aligned}$$

   Substituting $t_0 = 0.15$ s gives

$$\mathcal{F}[m(t)] = 0.05e^{-0.05j\pi f}\text{sinc}(0.05f)\left(1 - 2e^{-0.1j\pi f}\right) \tag{3.2.11}$$

For the modulated signal $u(t)$, we have

$$U(f) = 0.025e^{-0.05j\pi(f-f_c)}\text{sinc}\,(0.05(f-f_c))\left(1-2e^{-0.1j\pi(f-f_c)}\right)$$
$$+\,0.025e^{-0.05j\pi(f+f_c)}\text{sinc}\,(0.05(f+f_c))\left(1-2e^{-0.1j\pi(f+f_c)}\right)$$

Plots of the magnitude spectra of the message and the modulated signals are shown in Figure 3.2.

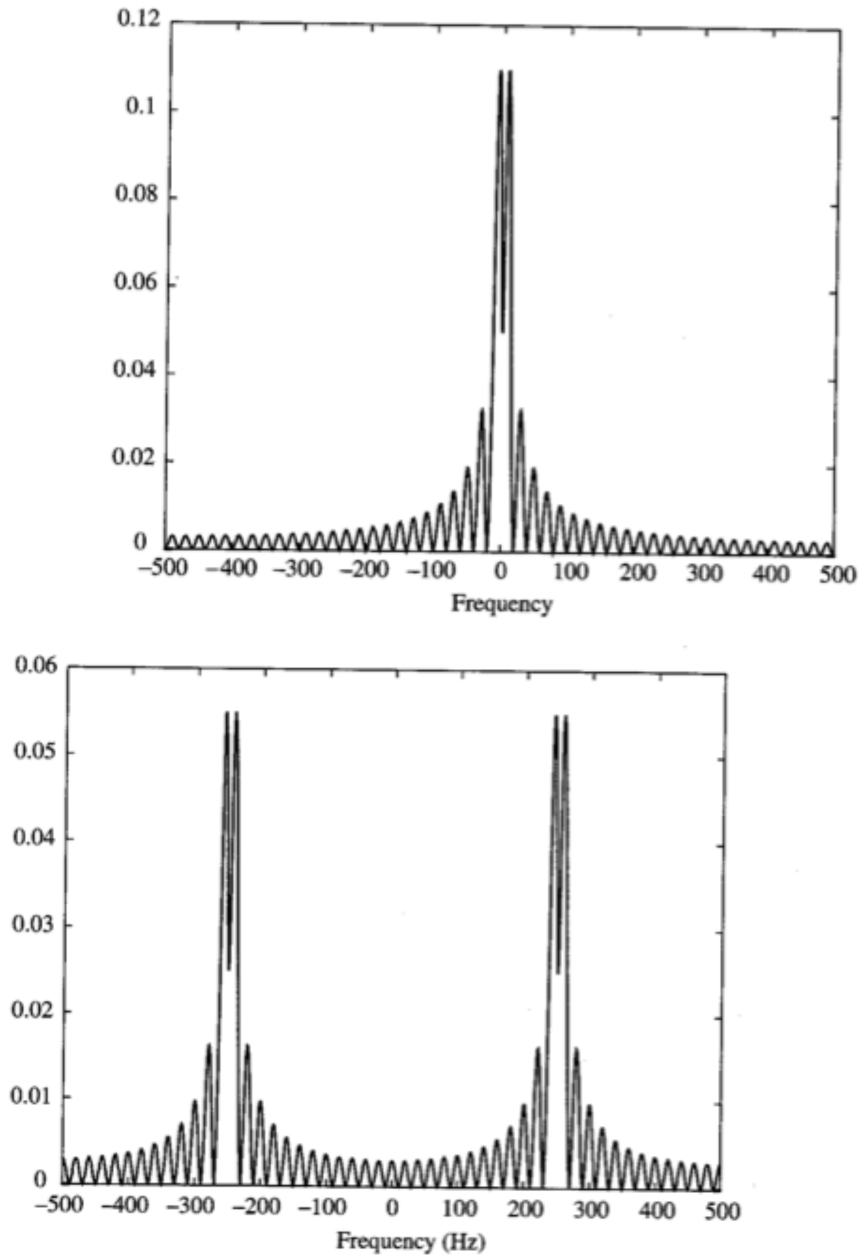

Figure 3.2 Magnitude spectra of the message and the modulated signals in Illustrative Problem 3.1

3. The power in the modulated signal is given by

$$P_u = \frac{A_c^2}{2} P_m = \frac{1}{2} P_m$$

where P_m is the power in the message signal;

$$P_m = \frac{1}{t_0} \int_0^{2t_0/3} m^2(t)\,dt = \frac{1}{t_0}\left(\frac{t_0}{3} + \frac{4t_0}{3}\right) = \frac{5}{3} = 1.666$$

and

$$P_u = \frac{1.666}{2} = 0.833$$

4. Here

$$10 \log_{10}\left(\frac{P_R}{P_n}\right) = 10$$

or $P_R = P_u = 10P_n$, which results in $P_n = P_u/10 = 0.0833$.

The MATLAB script for the preceding problem follows.

M-FILE

```
% Matlab script for Illustrative Problem 3.1.
% Matlab demonstration script for DSB-AM modulation. The message signal
% is +1 for 0 < t < t0/3, -2 for t0/3 < t < 2t0/3, and zero otherwise.
echo on
t0=.15;                                  % signal duration
ts=0.001;                                % sampling interval
fc=250;                                  % carrier frequency
snr=20;                                  % SNR in dB (logarithmic)
fs=1/ts;                                 % sampling frequency
df=0.3;                                  % desired freq. resolution
t=[0:ts:t0];                             % time vector
snr_lin=10^(snr/10);                     % linear SNR
% message signal
m=[ones(1,t0/(3*ts)),-2*ones(1,t0/(3*ts)),zeros(1,t0/(3*ts)+1)];
c=cos(2*pi*fc.*t);                       % carrier signal
u=m.*c;                                  % modulated signal
[M,m,df1]=fftseq(m,ts,df);               % Fourier transform
M=M/fs;                                  % scaling
[U,u,df1]=fftseq(u,ts,df);               % Fourier transform
U=U/fs;                                  % scaling
[C,c,df1]=fftseq(c,ts,df);               % Fourier transform
f=[0:df1:df1*(length(m)-1)]-fs/2;        % freq. vector
signal_power=spower(u(1:length(t)));     % power in modulated signal
noise_power=signal_power/snr_lin;        % compute noise power
noise_std=sqrt(noise_power);             % compute noise standard deviation
```

```
noise=noise_std*randn(1,length(u));          % generate noise
r=u+noise;                                   % add noise to the modulated signal
[R,r,df1]=fftseq(r,ts,df);                   % spectrum of the signal+noise
R=R/fs;                                      % scaling
pause  % Press a key to show the modulated signal power
signal_power
pause  % Press any key to see plot of the message
clf
subplot(2,2,1)
plot(t,m(1:length(t)))
xlabel('Time')
title('The message signal')
pause  % Press any key to see a plot of the carrier
subplot(2,2,2)
plot(t,c(1:length(t)))
xlabel('Time')
title('The carrier')
pause  % Press any key to see a plot of the modulated signal
subplot(2,2,3)
plot(t,u(1:length(t)))
xlabel('Time')
title('The modulated signal')
pause    % Press any key to see plots of the magnitude of the message and the
         % modulated signal in the frequency domain.
subplot(2,1,1)
plot(f,abs(fftshift(M)))
xlabel('Frequency')
title('Spectrum of the message signal')
subplot(2,1,2)
plot(f,abs(fftshift(U)))
title('Spectrum of the modulated signal')
xlabel('Frequency')
pause  % Press a key to see a noise sample
subplot(2,1,1)
plot(t,noise(1:length(t)))
title('Noise sample')
xlabel('Time')
pause  % Press a key to see the modulated signal and noise
subplot(2,1,2)
plot(t,r(1:length(t)))
title('Signal and noise')
xlabel('Time')
pause  % Press a key to see the modulated signal and noise in freq. domain
subplot(2,1,1)
plot(f,abs(fftshift(U)))
title('Signal spectrum')
xlabel('Frequency')
subplot(2,1,2)
plot(f,abs(fftshift(R)))
title('Signal and noise spectrum')
xlabel('Frequency')
```