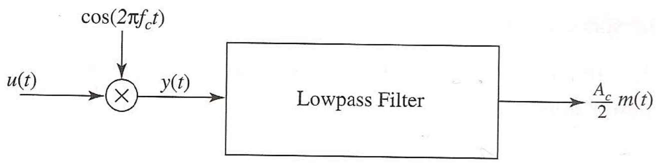

Figure 3.9 Demodulation of DSB-AM signals

consists of multiplying (mixing) the modulated signal by a sinusoidal with the same frequency and phase of the carrier and then passing the product through a lowpass filter. The oscillator that generates the required sinusoidal at the receiver is called the *local oscillator.*

3.3.1 DSB-AM Demodulation

In the DSB case the modulated signal is given by $A_c m(t)\cos(2\pi f_c t)$, which when multiplied by $\cos(2\pi f_c t)$, or mixed with $\cos(2\pi f_c t)$, results in

$$y(t) = A_c m(t)\cos(2\pi f_c t)\cos(2\pi f_c t) = \frac{A_c}{2}m(t) + \frac{A_c}{2}m(t)\cos(4\pi f_c t) \tag{3.3.1}$$

where $y(t)$ denotes the mixer output, and its Fourier transform is given by

$$Y(f) = \frac{A_c}{2}M(f) + \frac{A_c}{4}M(f-2f_c) + \frac{A_c}{4}M(f+2f_c) \tag{3.3.2}$$

As it is seen, the mixer output has a lowpass component of $(A_c/2)M(f)$ and high-frequency components in the neighborhood of $\pm 2f_c$. When $y(t)$ passes through a lowpass filter with bandwidth W, the high-frequency components will be filtered out and the lowpass component, $(A_c/2)m(t)$, which is proportional to the message signal, will be demodulated. This process is shown in Figure 3.9.

ILLUSTRATIVE PROBLEM

Illustrative Problem 3.5 [DSB-AM demodulation] The message signal $m(t)$ is defined as

$$m(t) = \begin{cases} 1, & 0 \le t < \frac{t_0}{3} \\ -2, & \frac{t_0}{3} \le t < \frac{2t_0}{3} \\ 0, & \text{otherwise} \end{cases}$$

This message DSB-AM modulates the carrier $c(t) = \cos 2\pi f_c t$, and the resulting modulated signal is denoted by $u(t)$. It is assumed that $t_0 = 0.15$ s and $f_c = 250$ Hz.

1. Obtain the expression for $u(t)$.

2. Derive the spectra of $m(t)$ and $u(t)$.
3. Demodulate the modulated signal $u(t)$ and recover $m(t)$. Plot the results in the time and frequency domains.

SOLUTION

1, 2. The first two parts of this problem are the same as the first two parts of Illustrative Problem 3.1, and we repeat only those results here:

$$u(t) = \left[\Pi\left(\frac{t-0.025}{0.05}\right) - 2\Pi\left(\frac{t-0.075}{0.05}\right)\right]\cos(500\pi t)$$

and

$$\begin{aligned}
\mathcal{F}[m(t)] &= \frac{t_0}{3}e^{-j\pi f t_0/3}\operatorname{sinc}\left(\frac{t_0 f}{3}\right) - \frac{2t_0}{3}e^{-j\pi f t_0}\operatorname{sinc}\left(\frac{t_0 f}{3}\right)\\
&= \frac{t_0}{3}e^{-j\pi f t_0/3}\operatorname{sinc}\left(\frac{t_0 f}{3}\right)\left(1 - 2e^{-j2\pi t_0 f/3}\right)\\
&= 0.05e^{-0.05j\pi f}\operatorname{sinc}(0.05f)\left(1-2e^{-0.01j\pi f}\right)
\end{aligned}$$

Therefore,

$$\begin{aligned}
U(f) &= 0.025e^{-0.05j\pi(f-250)}\operatorname{sinc}(0.05(f-250))\left(1-2e^{-0.1j\pi(f-250)}\right)\\
&\quad + 0.025e^{-0.05j\pi(f+250)}\operatorname{sinc}(0.05(f+250))\left(1-2e^{-0.1j\pi(f+250)}\right)
\end{aligned}$$

3. To demodulate, we multiply $u(t)$ by $\cos(2\pi f_c t)$ to obtain the mixer output $y(t)$:

$$\begin{aligned}
y(t) &= u(t)\cos(2\pi f_c t)\\
&= \left[\Pi\left(\frac{t-0.025}{0.05}\right) - 2\Pi\left(\frac{t-0.075}{0.05}\right)\right]\cos^2(500\pi t)\\
&= \frac{1}{2}\left[\Pi\left(\frac{t-0.025}{0.05}\right) - 2\Pi\left(\frac{t-0.075}{0.05}\right)\right]\\
&\quad + \frac{1}{2}\left[\Pi\left(\frac{t-0.025}{0.05}\right) - 2\Pi\left(\frac{t-0.075}{0.05}\right)\right]\cos(1000\pi t)
\end{aligned}$$

whose Fourier transform is given by

$$\begin{aligned}
Y(f) &= 0.025e^{-0.05j\pi f}\operatorname{sinc}(0.05f)\left(1-2e^{-0.01j\pi f}\right)\\
&\quad + 0.0125e^{-0.05j\pi(f-500)}\operatorname{sinc}\left(0.05(f-500)\right)\left(1-2e^{-0.1j\pi(f-500)}\right)\\
&\quad + 0.0125e^{-0.05j\pi(f+500)}\operatorname{sinc}\left(0.05(f+500)\right)\left(1-2e^{-0.1j\pi(f+500)}\right)
\end{aligned}$$

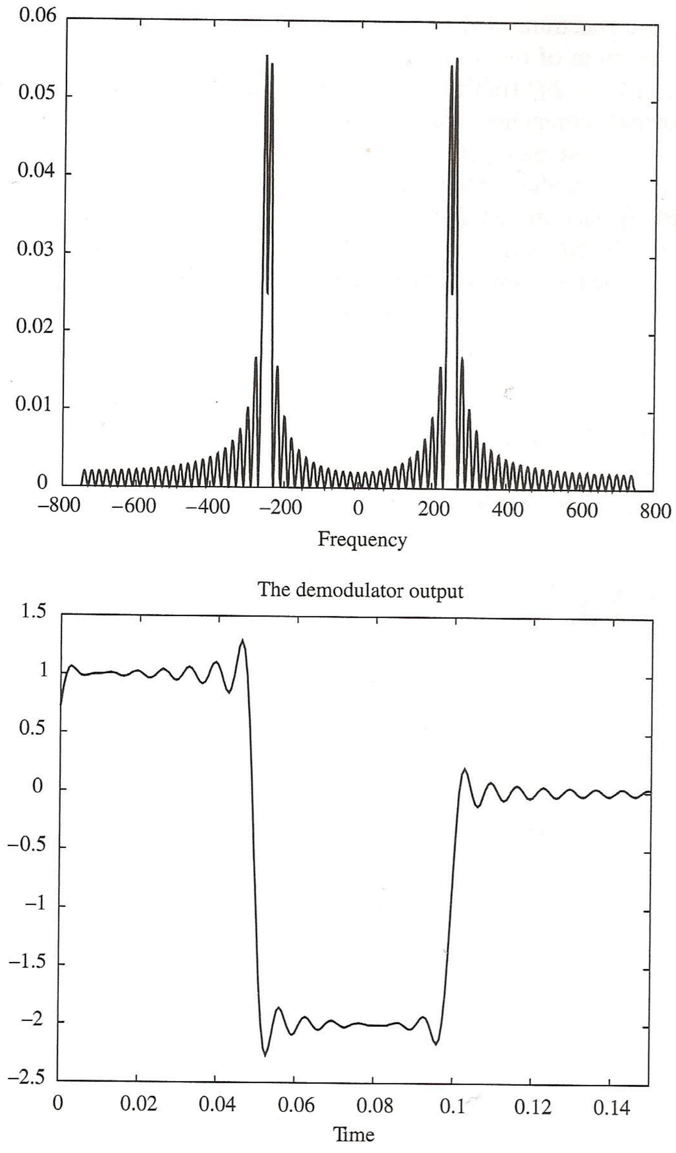

Figure 3.10 Spectra of the modulated signal and the mixer output in Illustrative Problem 3.5

where the first term corresponds to the message signal and the last two terms correspond to the high-frequency terms at twice the carrier frequency. We see that filtering the first term yields the original message signal (up to a proportionality constant). A plot of the magnitudes of $U(f)$ and $Y(f)$ is shown in Figure 3.10.

As shown, the spectrum of the mixer output has a lowpass component that is quite similar to the spectrum of the message signal, except for a factor of $\frac{1}{2}$, and a bandpass component located at $\pm 2f_c$ (in this case, 500 Hz). Using a lowpass filter, we can simply separate the lowpass component from the bandpass component. In order to recover the message signal $m(t)$, we pass $y(t)$ through a lowpass filter with a bandwidth of 150 Hz. The choice of the bandwidth of the filter is more or less arbitrary here because the message signal is not strictly bandlimited. For a strictly bandlimited message signal, the appropriate choice for the bandwidth of the lowpass filter would be W, the bandwidth of the message signal. Therefore, the ideal lowpass filter employed here has a characteristic

$$H(f) = \begin{cases} 1, & |f| \leq 150 \\ 0, & \text{otherwise} \end{cases}$$

A comparison of the spectra of $m(t)$ and the demodulator output is shown in Figure 3.11, and a comparison in the time domain is shown in Figure 3.12.

The MATLAB script for this problem follows.

M-FILE

```
% dsb_dem.m
% Matlab demonstration script for DSB-AM demodulation. The message signal
% is +1 for 0 < t < t0/3, -2 for t0/3 < t < 2t0/3, and zero otherwise.
echo on
t0=.15;                                  % signal duration
ts=1/1500;                               % sampling interval
fc=250;                                  % carrier frequency
fs=1/ts;                                 % sampling frequency
t=[0:ts:t0];                             % time vector
df=0.3;                                  % desired frequency resolution
% message signal
m=[ones(1,t0/(3*ts)),-2*ones(1,t0/(3*ts)),zeros(1,t0/(3*ts)+1)];
c=cos(2*pi*fc.*t);                       % carrier signal
u=m.*c;                                  % modulated signal
y=u.*c;                                  % mixing
[M,m,df1]=fftseq(m,ts,df);               % Fourier transform
M=M/fs;                                  % scaling
[U,u,df1]=fftseq(u,ts,df);               % Fourier transform
U=U/fs;                                  % scaling
[Y,y,df1]=fftseq(y,ts,df);               % Fourier transform
Y=Y/fs;                                  % scaling
f_cutoff=150;                            % cutoff freq. of the filter
n_cutoff=floor(150/df1);                 % design the filter
f=[0:df1:df1*(length(y)-1)]-fs/2;
H=zeros(size(f));
H(1:n_cutoff)=2*ones(1,n_cutoff);
H(length(f)-n_cutoff+1:length(f))=2*ones(1,n_cutoff);
DEM=H.*Y;                                % spectrum of the filter output
dem=real(ifft(DEM))*fs;                  % filter output
pause % Press a key to see the effect of mixing
```

```
clf
subplot(3,1,1)
plot(f,fftshift(abs(M)))
title('Spectrum of the Message Signal')
xlabel('Frequency')
subplot(3,1,2)
plot(f,fftshift(abs(U)))
title('Spectrum of the Modulated Signal')
xlabel('Frequency')
subplot(3,1,3)
plot(f,fftshift(abs(Y)))
title('Spectrum of the Mixer Output')
xlabel('Frequency')
pause % Press a key to see the effect of filtering on the mixer output
clf
subplot(3,1,1)
plot(f,fftshift(abs(Y)))
title('Spectrum of the Mixer Output')
xlabel('Frequency')
subplot(3,1,2)
plot(f,fftshift(abs(H)))
title('Lowpass Filter Characteristics')
xlabel('Frequency')
subplot(3,1,3)
plot(f,fftshift(abs(DEM)))
title('Spectrum of the Demodulator Output')
xlabel('Frequency')
pause % Press a key to compare the spectra of the message and the received signal
clf
subplot(2,1,1)
plot(f,fftshift(abs(M)))
title('Spectrum of the Message Signal')
xlabel('Frequency')
subplot(2,1,2)
plot(f,fftshift(abs(DEM)))
title('Spectrum of the Demodulator Output')
xlabel('Frequency')
pause % Press a key to see the message and the demodulator output signals
subplot(2,1,1)
plot(t,m(1:length(t)))
title('The Message Signal')
xlabel('Time')
subplot(2,1,2)
plot(t,dem(1:length(t)))
title('The Demodulator Output')
xlabel('Time')
```

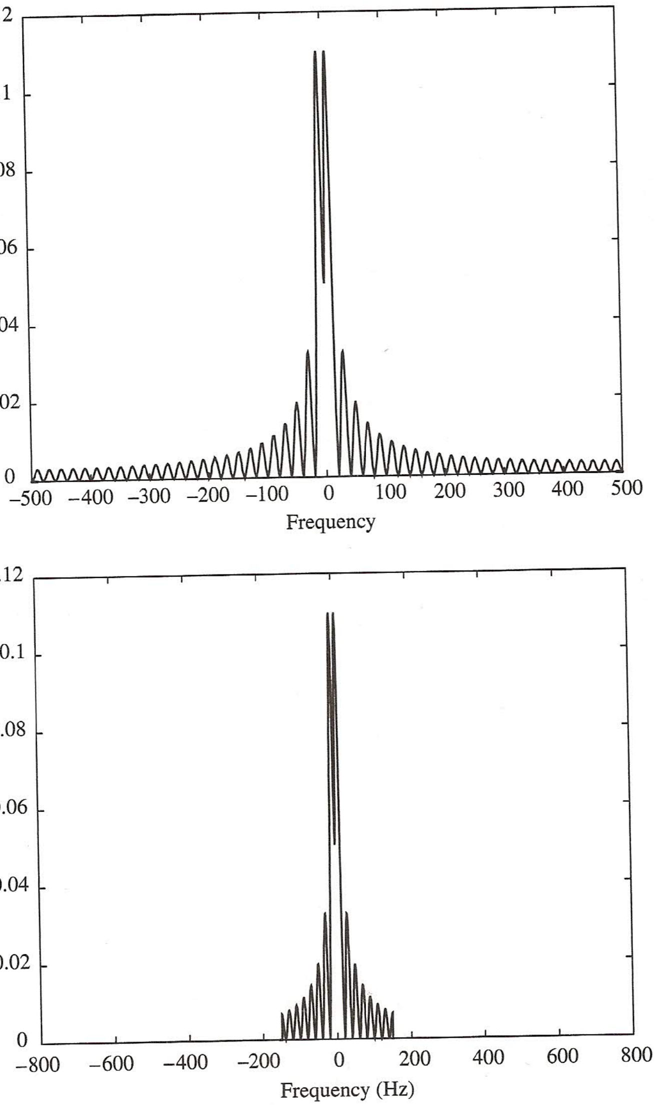

Figure 3.11 Spectra of the message and the demodulated signals in Illustrative Problem 3.5

ILLUSTRATIVE PROBLEM

Illustrative Problem 3.6 [Effect of phase error on DSB-AM demodulation] In the demodulation of DSB-AM signals we assumed that the phase of the local oscillator is equal to the phase of the carrier. If that is not the case—that is, if there exists a phase shift ϕ between the local oscillator and the carrier—how would the demodulation process change?