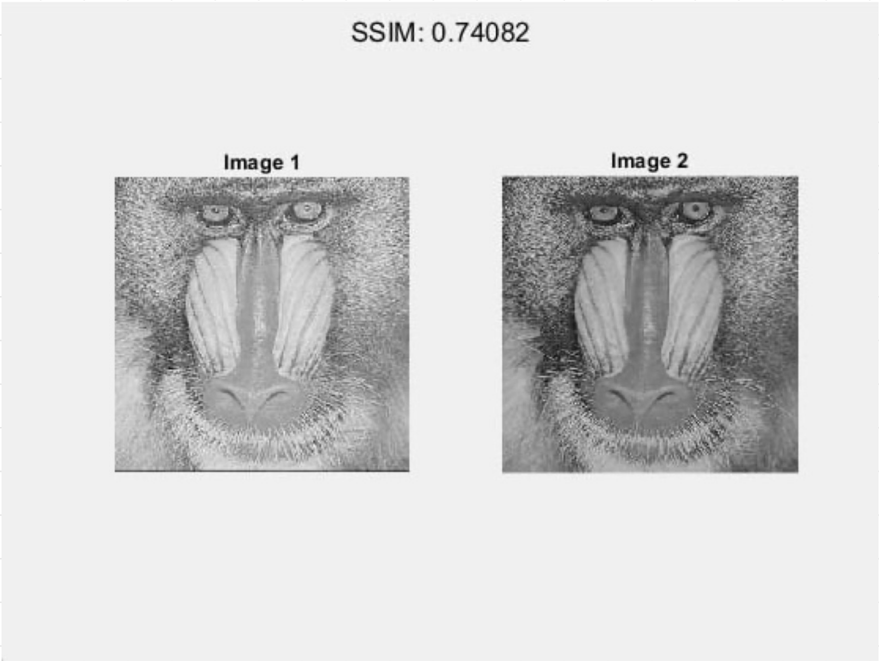
(1) Write a Matlab or Python program to measure the structural similarity (SSIM) of two images A and B. The sizes of A and B are equivalent.

SSIM(A, B, c1, c2)

where c1 and c2 are some adjust constants.

The Matlab or Python code should be handed out by NTUCool. (20 scores)



(2) Suppose that the probabilities of Chinese characters can be modeled as

$$P[n] = (\exp(0.002)-1)\exp(-0.002n) \qquad n = 1,2,3,\cdots\cdots,80000$$

(a) Determine the entropy of the Chinese characters. (b) Estimate the range of the coding length if we use the <u>Huffman code</u> to encode 10^5 Chinese characters using binary numbers. (c) Estimate the range of the coding length if we use the <u>arithmetic code</u> to encode 10^5 Chinese characters using binary numbers.

(15 scores)

(a) $\text{entropy} = \sum_{n=1}^{80000} (\exp(0.002)-1)\exp(-0.002n) \cdot \ln \frac{1}{(\exp(0.002)-1)\exp(-0.002n)}$

$= 7.2146$ #

(b)

$\text{ceil}\left(10^5 \frac{7.2146}{\ln 2}\right) \le b \le \text{floor}\left(10^5 \frac{7.2146}{\ln 2} + 10^5\right)$

$1040847 \le b \le 1140846$ #

(c)

$\text{ceil}\left(10^5 \cdot \frac{7.2146}{\ln 2}\right) \le b \le \text{floor}\left(N \frac{7.2146}{\ln k} + \log_k^2 + 1\right)$

$1040847 \le b \le 1040848$

#

(3) Suppose that x is a complex number. What are the constraints of θ such that the multiplication of x and $\exp(j\,\theta)$ required only 2 real multiplications?

(10 scores)

if $c + jd = e^{j\theta} = \cos\theta + j\sin\theta$ $\quad c = \cos\theta \quad d = \sin\theta$

trivial multiplications $\boxed{\begin{matrix} 乘 \pm 2^k \\ 乘 \pm j 2^k. \end{matrix}}$

$$\begin{bmatrix} c & -d \\ d & c \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

when $\cos\theta = 0$, $\sin\theta = 1$ $\begin{bmatrix} e \\ f \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$ $\quad e = -b$, $f = a$

if $|c| = |d|$

$MUL = 2.$ $\Rightarrow$ $\theta = \pm \frac{n\pi}{4}$ $\quad n \in \mathbb{Z}$ ①

use trivial multiplications properties

$\cos\theta = 2^{-1}, 2^{-2}, \ldots 2^{-k}$ $\quad k \in \mathbb{Z}$ $\Rightarrow$ $\theta = \pm\cos^{-1} 2^{-k} + 2n\pi$, $n \in \mathbb{Z}$ ②

$\sin\theta = 2^{-1}, 2^{-2} \ldots 2^{-k}$ $\quad k \in \mathbb{Z}$ $\Rightarrow \theta = \pm\sin^{-1} 2^{-k} + 2n\pi, n \in \mathbb{Z}$ ③

(4) What is the complexity of the $M\times N\times P$-point 3D DFT? The deriving process should be given.

(10 scores)

$M \times N \times P$ DFT Complexity:

① along m $\Rightarrow$ $N \times P$ DFTs, complexity $\Rightarrow$ $MNP \log M$

② along n $\Rightarrow$ $M \times P$ DFTs, complexity $\Rightarrow$ $MNP \log N$

③ along p $\Rightarrow$ $M \times N$ DFTs, complexity $\Rightarrow$ $MNP \log P$

Total complexity $\Rightarrow$ $MNP(\log M + \log N + \log P) = MNP \log(MNP)$ #

(5) How do we implement the 4-point DST-I with the least number of nontrivial multiplications? The number of real multiplications should also be shown.

$$X[m] = \sum_{n=1}^{4} \sin\left(\frac{\pi}{5}mn\right)x[n] \qquad \begin{matrix} m = 1, 2, 3, 4 \\ n = 1, 2, 3, 4 \end{matrix}$$

(15 scores)

$$\begin{bmatrix} X[1] \\ X[2] \\ X[3] \\ X[4] \end{bmatrix} = \begin{bmatrix} a & b & b & a \\ b & a & -a & -b \\ b & -a & -a & b \\ a & -b & b & -a \end{bmatrix} \begin{bmatrix} x[1] \\ x[2] \\ x[3] \\ x[4] \end{bmatrix} \qquad a = 0.5878, \quad b = 0.9511$$

(Hint: we can convert it into two 2x2 matrices.)

$$\begin{pmatrix} X[2] \\ X[4] \end{pmatrix} = \begin{bmatrix} b & a \\ a & -b \end{bmatrix} \begin{bmatrix} x[1] - x[4] \\ x[2] - x[3] \end{bmatrix}$$ 3 MUL

(by p.346, case (4))

$$\begin{pmatrix} X[1] \\ X[3] \end{pmatrix} = \begin{bmatrix} a & b \\ b & -a \end{bmatrix} \begin{bmatrix} x[1] + x[4] \\ x[2] + x[3] \end{bmatrix}$$ 3 MUL

total 6 MUL #

(6) Determining the numbers of real multiplications for the (a) 143-point DFT, (b) 195-point DFT, and the (c) 196-point DFT. (15 scores)

(a) $MUL_{143} = 13\,MUL_{11} + 11\,MUL_{13}$

$= 13 \times 40 + 11 \times 52$

$= \underline{1092}$ #

(b) $MUL_{195} = 13\,MUL_{15} + 15\,MUL_{13}$

$= 13 \times 40 + 15 \times 52 = \underline{1300}$ #

(c) $MUL_{196} = 4\,MUL_{49} + 49\,MUL_{4}$

$= 4 \times 332 + 49 \times 0$

$= \underline{1328}$ #

(7) Derive the transform matrices of the (a) forward and (b) inverse 5-point NTTs where the prime number M is 11 and the value of α should be as small as possible. (15 scores)

root of unity of order N

$M=11 \quad N=5$

smallest $\alpha = 3$

$\because 3^5 \equiv 1 \pmod{11}$

$3^k \not\equiv 1 \pmod{11}$

$3^0 \equiv 1 \pmod{11}$
$3^1 \equiv 3 \pmod{11}$
$3^2 \equiv 9 \pmod{11}$
$3^3 \equiv 5 \pmod{11}$
$3^4 \equiv 81 \equiv 4 \pmod{11}$

(a)

NTT matrix $W = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 3 & 9 & 5 & 4 \\ 1 & 9 & 4 & 3 & 5 \\ 1 & 5 & 3 & 4 & 9 \\ 1 & 4 & 5 & 9 & 3 \end{pmatrix}$ #

position (i,j) in the matrix is $\alpha^{ij} \bmod M$ for $i,j = 0,1,2,3,4$

$N=5$, $5 \times 9 \equiv 1 \pmod{11}$ $\quad N^{-1} = 9$ $\quad \alpha = 3$, $3 \times 4 \equiv 1 \pmod{11}$

$\alpha^{-1} = 4$

(b)

$$W^{-1} = \underset{\substack{\| \\ 9}}{N^{-1}} \cdot \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 4 & 5 & 9 & 3 \\ 1 & 5 & 3 & 4 & 9 \\ 1 & 9 & 4 & 3 & 5 \\ 1 & 3 & 9 & 5 & 4 \end{pmatrix} = \begin{pmatrix} 9 & 9 & 9 & 9 & 9 \\ 9 & 3 & 1 & 4 & 5 \\ 9 & 1 & 5 & 3 & 4 \\ 9 & 4 & 3 & 5 & 1 \\ 9 & 5 & 4 & 1 & 3 \end{pmatrix}$$

#

$4^0 \equiv 1 \pmod{11}$
$4^1 \equiv 4 \pmod{11}$
$4^2 \equiv 5 \pmod{11}$
$4^3 \equiv 9 \pmod{11}$
$4^4 \equiv 3 \pmod{11}$

加分題：學號尾數 9

$$MUL_{231} = 11 \cdot MUL_{21} + 21\, MUL_{11}$$

$$= 11 \cdot 62 + 21 \cdot 40 = \underline{1522} \ \#$$