Homework 1 (Due: March 20th) /

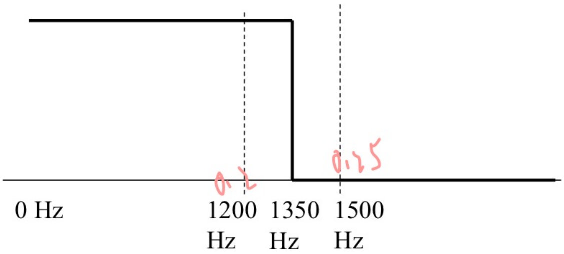
(1) Design a Mini-max **lowpass** FIR filter such that (40 scor

① Filter length = 17, ② Sampling frequency f_s = 6000Hz,

③ Pass Band 0~1200Hz ④ Transition band: 1200~1500 Hz,

⑤ Weighting function: $W(F)$ = 1 for passband, $W(F)$ = 0.6 for stop band .

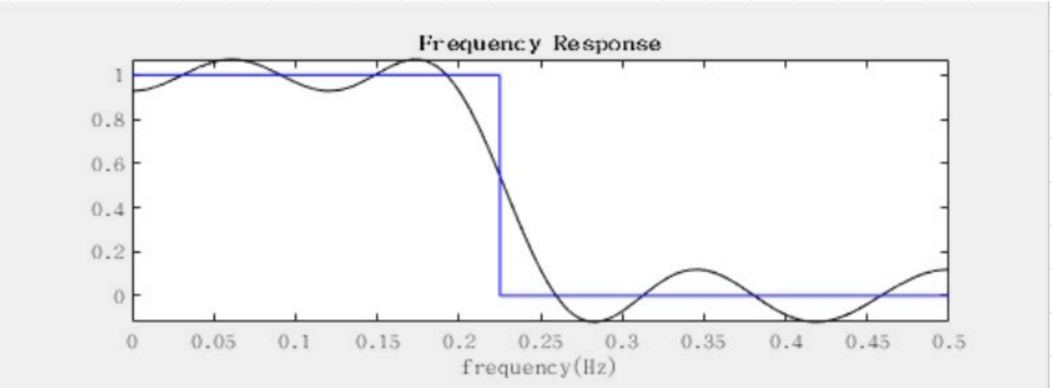
⑥ Set Δ = 0.0001 in Step 5.



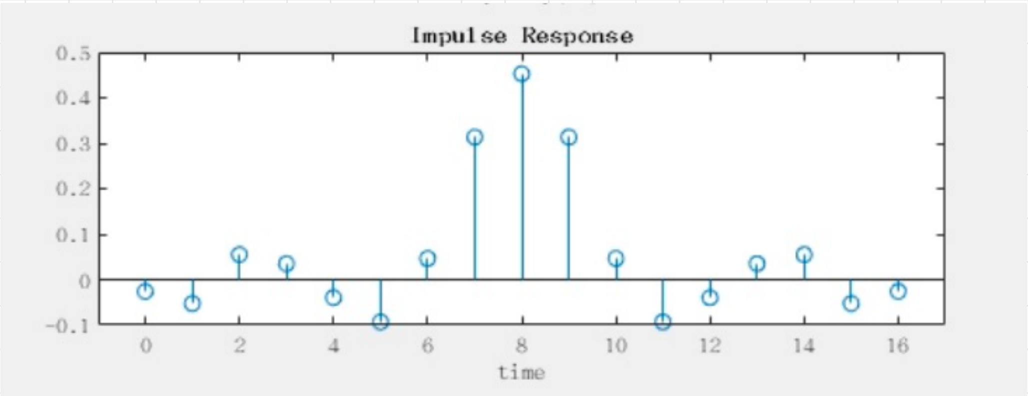
※ The code should be handed out by NTUCool, too.

Show (a) the frequency response, (b) the impulse response $h[n]$, and (c) the maximal error for each iteration.

(a)



(b)



(C)

Iteration	1	2	3	4	5
Maximal Error	0.1832	0.1340	0.0794	0.0712	0.0712

(2) How do we implement $y[n] = x[n] *(0.8^n u[n] + 0.5^n u[n])$ efficiently where * means convolution and $u[n]$ is the unit step function? (10 scores)

if $n<0$ $y[n]=0$

if $n \geq 0$, $u[n]=1$, $y[n] = x[n] * \underbrace{(0.8^n + 0.5^n)}_{h[n]}$

$Y(z) = X(z)H(z)$, $H(z) = \sum_{n=-\infty}^{\infty} h[n] z^{-n} = \sum_{n=0}^{\infty} (0.8^n + 0.5^n) z^{-n}$

$= \sum_{n=0}^{\infty} \underbrace{0.8^n z^{-n}}_{(0.8 z^{-1})^n} + \sum_{n=0}^{\infty} 0.5^n z^{-n} = \frac{1}{1-0.8z^{-1}} + \frac{1}{1-0.5z^{-1}}$

$= \frac{1-0.5z^{-1} + 1-0.8z^{-1}}{(1-0.8z^{-1})(1-0.5z^{-1})}$

$= \frac{2-1.3z^{-1}}{(1-0.8z^{-1})(1-0.5z^{-1})}$

$\Rightarrow \frac{(1-0.8z^{-1})(1-0.5z^{-1})}{2-1.3z^{-1}} Y(z) = X(z)$

$(1-0.8z^{-1})(1-0.5z^{-1})Y(z) = (2-1.3z^{-1})X(z)$

$(1-1.3z^{-1}+0.4z^{-2})Y(z) = (2-1.3z^{-1})X(z)$

$\Downarrow$ inverse z

$y[n] - 1.3y[n-1] + 0.4y[n-2] = 2x[n] - 1.3x[n-1]$

$\Rightarrow y[n] = 2x[n] - 1.3x[n-1] + 1.3y[n-1] - 0.4y[n-2]$ #

(3) (a) What are the two main advantages of the Fourier transform (FT)? (b) What are the two main problems to implement the FT? (10 scores)

(a) ① spectrum analysis ② convolution → multiplication can be useful in Linear system Analysis

(b) ① not real operation

② irrational number multiplication

(4) Suppose that $x[n] = y(0.002n)$ and the length of $x[n]$ is 2000. If $X[m]$ is the FFT of $x[n]$, which frequencies do (a) $X[200]$ and (b) $X[1600]$ correspond to? (10 scores)

$\Delta t = 0.002$ $\quad f_s = \frac{1}{\Delta t} = \frac{1}{0.002} = 500Hz$

$N = 2000$

$f = m\frac{f_s}{N} \Rightarrow$

(a) $f = 200 \times \frac{500}{2000} = 50Hz$ #

(b) $f = 1600 \times \frac{500}{2000} = 400Hz$ (over) $400 - 500 = -100Hz$ #

#

(5) Why (a) the step invariance method and (b) the bilinear transform can reduce or avoid the aliasing effect in IIR filter design? (10 scores)

(a) Decrease the high frequency part using integration.

(b) Allowing the bandwidth of an analog filter with an infinite bandwidth to be converted into a finite bandwidth through mapping, and then sampling it.

$-\infty < f_c < \infty \Rightarrow -\frac{f_s}{2} < f_{c,1} < \frac{f_s}{2}$

(6) (a) Which of the following filters are usually even? (b) Which of the following filters are usually odd? (i) Notch filter; even (ii) highpass filter; even (iii) edge detector; odd (iv) integral; odd (v) differentiation 4 times; even (vi) particle filter; (vii) matched filter. (10 scores)

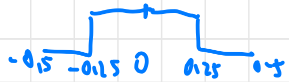
(a) (i), (ii), (v),

(b) (iii) (iv)

(7) Use the MSE method to design the 7-point FIR filter that approximates the lowpass filter of $H_d(F) = 1$ for $|F| < 0.25$ and $H_d(F) = 0$ for $0.25 < |F| < 0.5$.

(15 scores)

$$K = \frac{(7-1)}{2} = 3.$$



$$S[0] = \int_{-\frac{1}{2}}^{\frac{1}{2}} H_d(F)\, dF = 1 \times \frac{1}{2} = 0.5$$

$$S[n] = 2\int_{-\frac{1}{4}}^{\frac{1}{4}} \cos(2\pi n F)\, dF = \frac{1}{\pi n} \sin(2\pi n F)\Big|_{-\frac{1}{4}}^{\frac{1}{4}}$$

$$= \frac{2\sin\left(\frac{\pi n}{2}\right)}{\pi n}$$

$$\Rightarrow S[n] = \begin{cases} \dfrac{2\sin\left(\frac{\pi n}{2}\right)}{\pi n}, & n = 1, 3, 5, 7, \ldots \\ 0, & n = 0, 2, 4, 6, \ldots \end{cases}$$

Set $h(K) = S[0] = 0.5$

$h[K+n] = \frac{S[n]}{2}$, $h[K-n] = \frac{S[n]}{2}$, for $n = 1, 2, 3, \ldots K$

$$h[0] = \frac{S[3]}{2} = -0.1061$$

$$h[1] = \frac{S[2]}{2} = 0$$

$$h[2] = \frac{S[1]}{2} = 0.3183$$

$$h[3] = S[0] = 0.5$$

$$h[4] = \frac{S[1]}{2} = 0.3183$$

$$h[5] = \frac{S[2]}{2} = 0$$

$$h[6] = \frac{S[3]}{2} = -0.1061$$

$h[n] = 0$ for $n < 0$ and $n \geq N$

Then, $h[n]$ is the impulse response