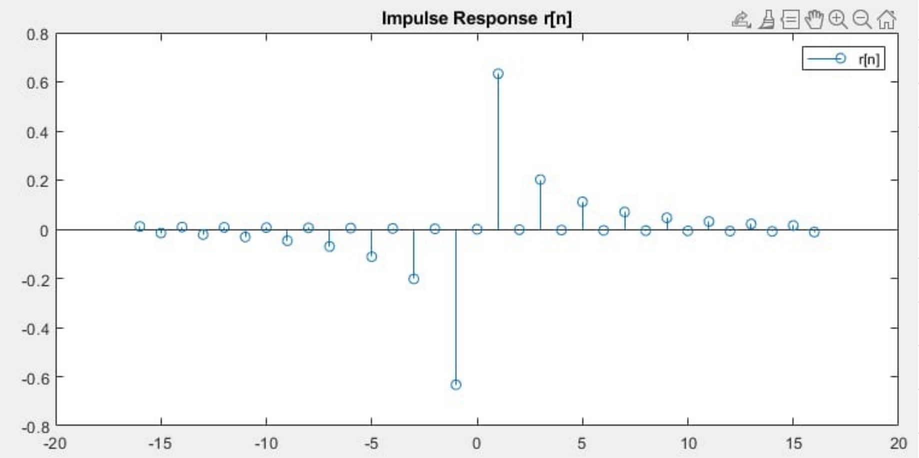
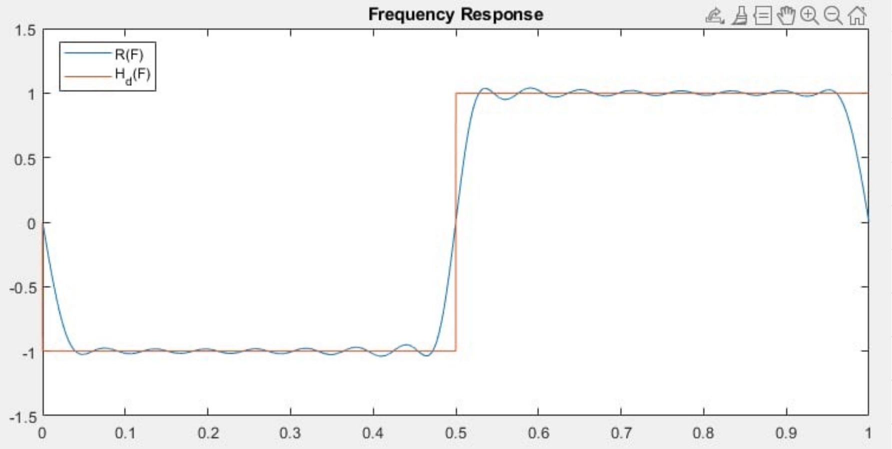
(1) Write a Matlab or Python code that uses the frequency sampling method to design a (2k+1)-point discrete Hilbert transform filter (k is an input parameter and can be any integer). (25 scores)

Set k = 16

(i) Impulse response



(ii) Frequency response



(2) Estimate the length of the digital filter if both the passband ripple and the stopband ripple are smaller than 0.01, the sampling interval $\Delta t = 0.00005$, and the transition band is from 5000Hz to 6000Hz. (10 scores)

$\delta_1, \delta_2 = 0.01 \quad f_s = 20000 \qquad F = \frac{f}{f_s} \qquad \Delta F = \frac{6000}{20000} - \frac{5000}{20000}$

$$= \frac{1000}{20000} = \frac{1}{20}$$

$$N = \frac{2}{3} \frac{1}{\frac{1}{20}} \log_{10}\left(\frac{1}{10 \times 0.01 \times 0.01}\right)$$

$$= \frac{40}{3} \log_{10}(1000) = \underline{40} \ \#$$

(3) Why it is improper to use the method of $y[n] = \text{IDFT}(\text{DFT}(x[n])H[m])$ for FIR filter design? (5 scores)

large computation loading.

(4) Derive the way to use the algorithm on page 58-61 to implement an odd symmetric filter with even length (i.e., type 4 on page 90). (10 scores)

$$-\sin(2\pi(n-\tfrac{1}{2})F) + \sin(2\pi(n+\tfrac{1}{2})F) = 2\sin(\pi F)\cos(2\pi n F)$$

$$R(F) = \sin(\pi F) \sum_{n=0}^{k-1} s_1[n] \cos(2\pi n F)$$

$$= \sum_{n=0}^{k-1} s_1[n] \sin(\pi F)\cos(2\pi n F)$$

$$= \frac{1}{2} \sum_{n=0}^{k-1} s_1[n] \sin(2\pi(n+\tfrac{1}{2})F) + \frac{1}{2} \sum_{n=0}^{k-1} s_1[n] \sin(2\pi(\tfrac{1}{2}-n)F)$$

$$= -\frac{1}{2} \sum_{n=0}^{k-1} s_1[n] \sin(2\pi(n-\tfrac{1}{2})F) + \frac{1}{2} \sum_{n=0}^{k-1} s_1[n] \sin(2\pi(n+\tfrac{1}{2})F)$$

$$= -\sum_{n=0}^{k-1} \frac{1}{2} s_1[n] \sin(2\pi(n-\tfrac{1}{2})F) + \sum_{n=1}^{k+1} \frac{1}{2} s_1[n-1] \sin(2\pi(n-\tfrac{1}{2})F)$$

$\sin(-\pi F) \qquad \sin(\pi F)$

$$\Rightarrow R(F) = -\frac{1}{2} S_1[0] \sin(-\pi F) + \sum_{n=1}^{k_1} \frac{1}{2}\left(-S_1[n] + S_1[n-1]\right) \sin\left(2\pi\left(n-\frac{1}{2}\right)F\right)$$
$$+ \frac{1}{2} S_1[k_1] \sin\left(2\pi\left(k_1+\frac{1}{2}\right)F\right)$$

$$= \left[S_1[0] - \frac{1}{2} S_1[1]\right] \sin(\pi F) + \sum_{n=2}^{k-\frac{1}{2}} \frac{1}{2}\left(-S_1[n] + S_1[n-1]\right) \sin\left(2\pi\left(n-\frac{1}{2}\right)F\right)$$
$$+ \frac{1}{2} S_1\left[k-\frac{1}{2}\right] \sin(2\pi k F)$$

Let, $k_1 + \frac{1}{2} = k$, $k_1 = k - \frac{1}{2}$

比較係數 $\begin{cases} S[1] = S_1[0] - \frac{1}{2} S_1[1] \\ S[n] = \frac{1}{2}\left(-S_1[n] + S_1[n-1]\right) \quad \text{for } n = 2, 3, \ldots k-\frac{1}{2} \\ S\left[k+\frac{1}{2}\right] = \frac{1}{2} S_1\left[k-\frac{1}{2}\right] \end{cases}$

$$err(F) = \left[R(F) - H_d(F)\right] W(F)$$
$$= \left[\sin(\pi F) \sum_{n=0}^{k-\frac{1}{2}} S_1[n] \cos(2\pi n F) - H_d(F)\right] W(F)$$
$$= \left[\sum_{n=0}^{k-\frac{1}{2}} S_1[n] \cos(2\pi n F) - \csc(\pi F) H_d(F)\right] \sin(\pi F) W(F)$$

$H_d(F) \rightarrow \csc(\pi F) H_d(F)$

$W(F) \rightarrow \sin(\pi F) W(F)$

$k \rightarrow k - \frac{1}{2} = \frac{N}{2} - 1$

$k = \frac{N-1}{2}$

$x[n] = \int_0^1 X(F)\, e^{j2\pi Fn}\, dF$

(5) Suppose that $x[n] = 1+\sin(n)$. (a) What is the Hilbert transform of $x[n]$?
(b) What is the analytic function corresponding to $x[n]$? (10 scores)

(a) $x_1[n] = 1 \quad X_1(F) = \delta(F) \quad X_{H_1}(F) = 0 \times \delta(F-0) = 0$ $\quad \frac{1}{2\pi} \doteq 0.1591\ldots$

$x_{H_1}[n] = 0$

$x_2[n] = \sin(n) \quad X_2(F) = \frac{1}{2j}\delta(f-\frac{1}{2\pi}) - \frac{1}{2j}\delta(f+\frac{1}{2\pi})$

$X_{H_2}(F) = -\frac{1}{2}\delta(f-\frac{1}{2\pi}) - \frac{1}{2}\delta(f+\frac{1}{2\pi})$

$x_{H_2}[n] = -\frac{1}{2}e^{jn} - \frac{1}{2}e^{-jn} = -\frac{1}{2}(\cos n + j\sin n)$

$- \frac{1}{2}(\cos n - j\sin n)$

$= -\frac{1}{2}\cos n \cancel{- \frac{1}{2}j\sin n}$

$- \frac{1}{2}\cos n \cancel{+ \frac{1}{2}j\sin n}$

$= -\cos n$

$\underline{x_H[n] = x_{H_1}[n] + x_{H_2}[n] = -\cos(n)}$ #

(b) $\underline{x_a[n] = 1 + \sin(n) - j\cos(n)}$ #

(6) Among the following filters: (i) the Notch filter (ii) the Hilbert transform, (iii)
the matched filter, (iv) the difference, (v) the Kalman filter, (vi) the particle filter, and (vii) the Wiener filter,

(a) Which filters are suitable for edge detection? (b) Which filters are suitable for prediction? (10 scores)

(a) (ii) (iv) (iii)

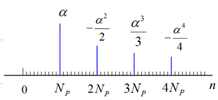
(b) (v) (vi)

(7) (a) What are the <u>two main advantages</u> of the minimum phase filter? (b) Compared to the equalizer, what are the <u>two main advantages</u> of the cepstrum to deal with the multipath problem? (10 scores)

(a) 1, Let the energy concentrating on the region near to $n=0$

2, Let both the forward and the inverse transforms stable

(b) 1. $x[n] \rightarrow y[n]$



When dealing with Echo problems, the cepstrum can be used with a lifter to remove each Echo(N_p) without needing to know α or other parameters.

2, The $H(z)$ of Equalizer may not be stable, but Cepstrum is stable.

Equalizer: $H(z) = \dfrac{1}{\alpha z^{-n_1} + \alpha_2 z^{-n_2} + \alpha_3 z^{-n_3} + \ldots}$

(8) If the z-transform of $h[n]$ is $H(z) = \dfrac{1+z^{-1}-1.5z^{-2}+z^{-3}}{1-0.3z^{-1}-0.4z^{-2}}$

(a) Determine the cepstrum of $h[n]$. P.185-187.

(b) Convert the IIR filter into the minimum phase filter. (20 scores)

$$(a)\quad H(z) = \frac{1+z^{-1}-1.5z^{-2}+z^{-3}}{1-0.3z^{-1}-0.4z^{-2}} = \frac{z^{-3}(z^3+z^2-1.5z+1)}{z^{-2}(z^2-0.3z-0.4)}$$

$$= \frac{(z+2)\cancel{z^{-1}}\left[z-\left(\tfrac{1}{2}+\tfrac{1}{2}j\right)\right]\left[z-\left(\tfrac{1}{2}-\tfrac{1}{2}j\right)\right]}{(z-0.8)(z+0.5)}$$

$$= \frac{2\left(1-\left(-\tfrac{1}{2}\right)z\right)\left[1-\left(\tfrac{1+j}{2}\right)z^{-1}\right]\left[1-\left(\tfrac{1-j}{2}\right)z^{-1}\right]}{(1-0.8z^{-1})(z+0.5z^{-1})}$$

$$\hat{x}[n] = \begin{cases} \log(2), & n=0 \\ -\dfrac{(0.5+0.5j)^n}{n} + \dfrac{(0.5-0.5j)^n}{n} + \dfrac{(0.8)^n}{n} + \dfrac{(0.5)^n}{n}, & n>0 \\ \dfrac{(-0.5)^{-n}}{n}, & n<0 \end{cases}$$

#

(b)

outside unit

$$H(z) = \frac{\underbrace{(z+2)}_{=z-(-2)}\left[z-\left(\frac{1}{2}+\frac{1}{2}j\right)\right]\left(z-\left(\frac{1}{2}-\frac{1}{2}j\right)\right]}{z(z-0.8)(z+0.5)}$$

allpass filter $H_{ap}(z) = -2\,\dfrac{z+\frac{1}{2}}{z+2}$

$$H_{mp}(z) = H(z)\,H_{ap}(z) = \frac{-2\left(z+\frac{1}{2}\right)\left[z-\left(\frac{1}{2}+\frac{1}{2}j\right)\right]\left[z-\left(\frac{1}{2}-\frac{1}{2}j\right)\right]}{z(z-0.8)(z+0.5)}$$

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加分題 學號尾數(9)

Phenomenon 是指 Gibbs phenomenon

訊號擷取系統只能擷取一定範圍內的訊號導致出現頻率截斷,引起 time domain 產生 Ringing effect。