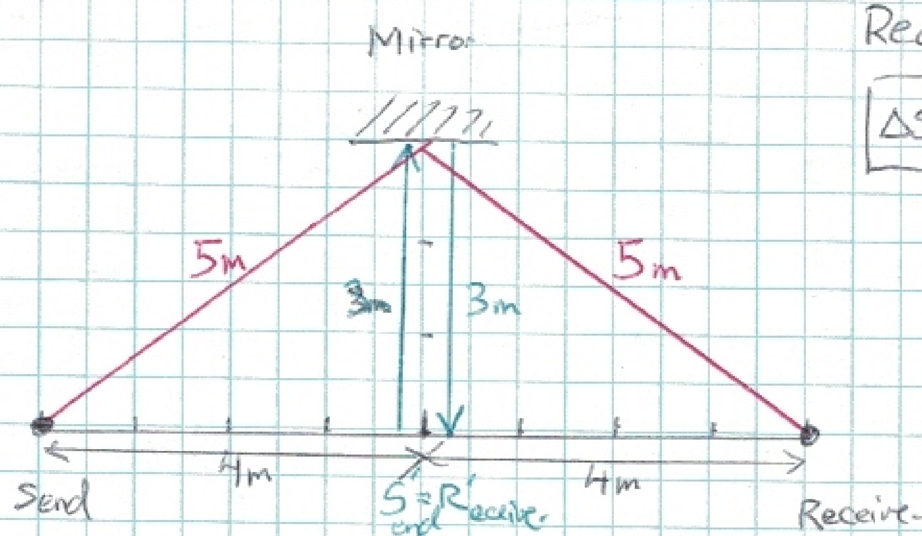


Recall:

$$\boxed{\Delta S^2 = c^2 \Delta t^2 - \Delta X^2}$$

Invariant!

Frame:	F (stationary)	F' (moving)
	$\Delta X = 8\text{ m}$	$\Delta X' = 0\text{ m}$
	$c\Delta t = 10\text{ m}$	$c\Delta t' = 6\text{ m}$

$\Delta S^2 = 10^2 - 8^2$

$= 100 - 64$

$\Delta S^2 = 36\text{ m}^2$

$\boxed{\Delta S = 6\text{m}}$

$\Delta S'^2 = 6^2 - 0^2$

$= 36 - 0$

$\Delta S'^2 = 36$

$\boxed{\Delta S' = 6\text{ m}}$

Same!

Also: the velocity of the moving frame is just

$$v = \frac{\Delta X}{\Delta t}$$

$$\Rightarrow \boxed{v = \frac{8}{10} c}$$

• In algebra equating ΔS & $\Delta S'$ for this case:

$$c^2 \Delta t'^2 = c^2 \Delta t^2 - \Delta X^2$$

$$\Delta t'^2 = \Delta t^2 - \frac{1}{c^2} \Delta X^2$$

$$= \Delta t^2 \left(1 - \frac{\Delta X^2}{\Delta t^2} \frac{1}{c^2}\right)$$

$$\Rightarrow \boxed{\Delta t' = \Delta t \sqrt{1 - v^2/c^2}}$$

Time Dilation

- the time passing on a moving object is called <u>proper time</u> and denoted τ.

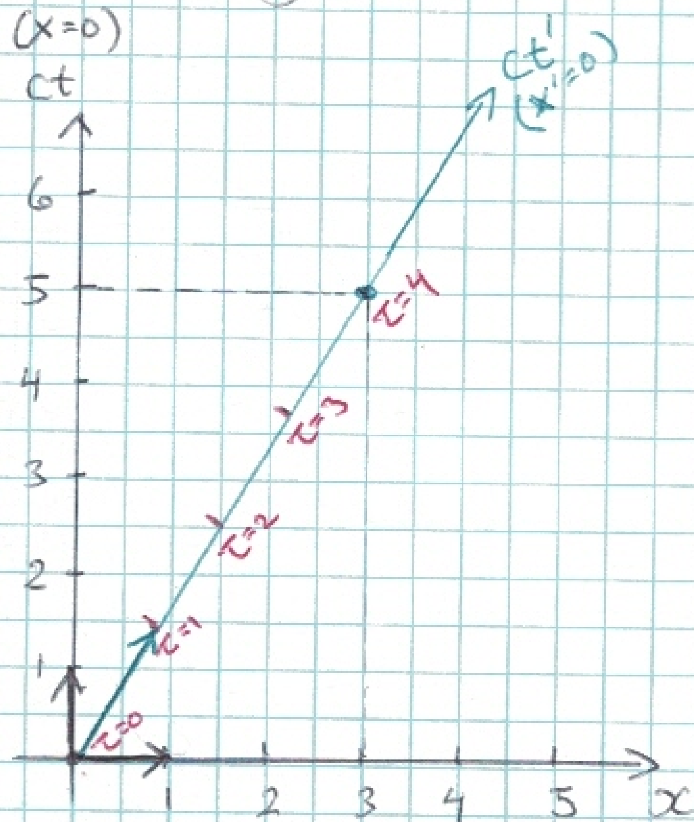
- infinitesimally we have $\boxed{d\tau = dt\sqrt{1-\frac{v^2}{c^2}}}$

$\boxed{\begin{array}{c}\text{OR//} \\ dt = \gamma\, d\tau \\ \gamma = \frac{1}{\sqrt{1-v^2/c^2}}\end{array}}$

~~note $\Delta\tau$:~~ <u>Note:</u> $d\tau \leq dt$ always

$\rightarrow$ moving clocks run slower!

<u>Example:</u> Moving frame is travelling at $v = \frac{3}{5}c$



So $\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$

$= \frac{1}{\sqrt{1-\frac{9}{25}}}$

$= \frac{1}{16/25}$

$\gamma = \frac{5}{4}$

$\therefore \boxed{\Delta\tau = \frac{4}{5}\Delta t}$

$\rightarrow$ <u>So:</u> In 5 time steps of t in the stationary frame, τ only passes 4 time steps in the moving frame.

$\therefore \dfrac{A(v_1)}{A(v_2)} = A(v_{12})$. $\leftarrow$ notice: RHS depends on v_1, v_2 and the relative angle btwn the two, θ.

LHS does not depend on the angle.

$\therefore \boxed{A = \text{const}}$

(cf separation of variables.

$$\frac{X'(x)}{X(x)} = \frac{Y'(y)}{Y(y)} \Rightarrow \text{LHS=RHS = const}$$

$\therefore$ LHS $= 1 = A$ (on RHS).

$\therefore \boxed{ds^2 = ds'^2}$ for infinitesimal intervals.

$\therefore S^2 = S'^2$ for finite intervals $\rightarrow$ $\boxed{\text{AN INVARIANT}}$

i.e. $c^2\Delta t^2 - \Delta\vec{x}^2 = c^2\Delta t'^2 - \Delta\vec{x}'^2$ for all inertial frames.

Proper Time (Dilation).

- suppose a clock in the S'-co-ords is moving relative to S.
- in infinitesimal time dt in the S-system it moves distance.

$$\sqrt{dx^2+dy^2+dz^2} = |d\vec{x}|$$

- in S' it hasn't moved at all in time dt'

$$\therefore ds^2 = c^2dt'^2 = c^2dt^2 - |d\vec{x}|^2$$

$$\therefore dt'^2 = \cancel{\frac{1}{c^2}(c^2 dt} \; dt^2 - \frac{|d\vec{x}|^2}{c^2}$$

$$dt' = dt\sqrt{1 - \frac{1}{c^2}\frac{|d\vec{x}|^2}{dt^2}} \Rightarrow \boxed{dt' = dt\sqrt{1-\frac{v^2}{c^2}}} \quad (*)$$

- integrating $t_2' - t_1' = \int_{t=t_1}^{t_2} \sqrt{1-\frac{v^2}{c^2}}\,dt$ (This holds for non-uniform motion too.)

This is called proper time, τ.

The time on a clock moving wrt a stationary frame

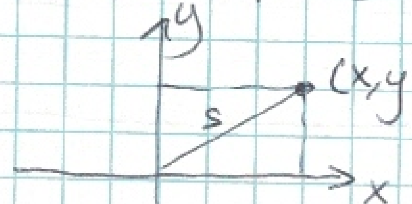
- note $\Delta\tau \leq \Delta t$. $\rightarrow$ moving clocks run slow.
- it's also $\boxed{\text{invariant}}$ $\because$ $ds^2 = c^2 dt'^2 = c^2 d\tau^2$ in the moving frame

Relation Between (x,t) & (x',t') given v.

Recall Euclidean case:

$$ds^2 = dx^2 + dy^2$$

- look at distance from origin



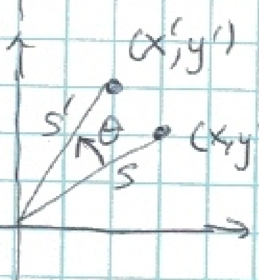
$$\boxed{s^2 = x^2 + y^2}$$

- linear transformations that preserve this distance are rotations.

Why Linear?

- an object moving $\bar{c}$ constant velocity in one IF must appear the same in all others.
- $\rightarrow$ map lines in $(ct, \vec{x})$ to lines in $(ct', \vec{x}')$.

Note: $s'^2 = s^2$



- use the rot^n matrix

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$\hookrightarrow$ $\boxed{\begin{aligned} x' &= cx - sy \\ y' &= sx + cy \end{aligned}}$

Trig. Identity:

$$\boxed{\cos^2\theta + \sin^2\theta = 1}$$

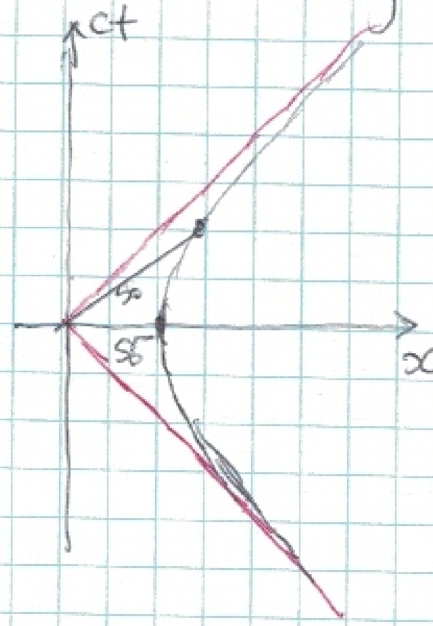
$$\boxed{ch^2\theta - sh^2\theta = 1}$$

(Hyperbolic)

Check invariance:

$$x'^2 + y'^2 = c^2x^2 - \cancel{2xy \cdot cs} + s^2y^2$$
$$+ s^2x^2 + \cancel{2xy \cdot cs} + c^2y^2$$
$$= (c^2+s^2)x^2 + (s^2+c^2)y^2 = x^2+y^2.$$

In Minkowski space we look for something similar, but the minus sign changes things a bit.



- now we look for a linear transformation that preserves $\boxed{s^2 = c^2t^2 - x^2}$
- note for a given $s = s_0$ this describes a hyperbola.
- in the Eucledian case we had a nice cancellation of the middle terms. + $\boxed{\sin^2 + \cos^2 = 1}$ identity
- that won't work here because of the $(-)$ in the metric
- but hyperbolic $\sinh = sh$ & $\cosh = ch$ have the identity

$$\boxed{ch^2\theta - sh^2\theta = 1} \longleftarrow \text{a } \underline{\text{minus}}!$$

- try it...

$$\begin{bmatrix} x' \\ ct' \end{bmatrix} = \begin{bmatrix} ch\theta & sh\theta \\ sh\theta & ch\theta \end{bmatrix} \begin{bmatrix} x \\ t \end{bmatrix} \Rightarrow \boxed{\begin{aligned} x' &= ch\cdot x + sh\cdot ct \\ ct' &= sh\, x + ch\cdot ct \end{aligned}} \quad (*)$$

$$\begin{aligned}
\Rightarrow c^2t'^2 - x'^2 &= (sh\, x + ch\cdot ct)^2 - (ch\cdot x + sh\cdot ct)^2 \\
&= sh^2x^2 + \cancel{2sh\cdot ch\cdot xct}^{\,0} + ch^2c^2t^2 \\
&\quad - ch^2x^2 - \cancel{2ch\, sh\cdot x\cdot ct}^{\,0} - sh^2c^2t^2 \\
&= \cancel{(sh^2 - ch^2)}^{\,-1}x^2 + \cancel{(ch^2 - sh^2)}^{\,1}c^2t^2 \\
&= c^2t^2 - x^2 \quad \checkmark
\end{aligned}$$

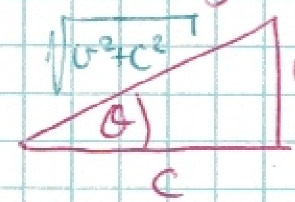
- the angle θ can only depend on v, the rel velocity of S & S'.
- consider the motion of the origin of S' as it moves through S.
- it has co-ord $x' = 0$. What does it look like in S?

$$x' = 0 \Rightarrow \quad x = ct' \cdot sh(\theta)$$
$$ct = ct' \cdot ch(\theta).$$

- dividing: $\dfrac{x}{ct} = \cancel{th}\ \tanh(\theta)$.

or $\boxed{th(\theta) = \dfrac{v}{c}}$

Recall trig.:



- hypotenus $= \sqrt{v^2+c^2}$

$\therefore \sin\theta = \dfrac{v}{\sqrt{v^2+c^2}} = \dfrac{v/c}{\sqrt{1+v^2/c^2}}$

$\cos\theta = \dfrac{c}{\sqrt{v^2+c^2}} = \dfrac{1}{\sqrt{1+v^2/c^2}}$

- using analogous trig identities:

$$\sinh(\theta) = \frac{v/c}{\sqrt{1-v^2/c^2}}$$

$$\cosh(\theta) = \frac{1}{\sqrt{1-v^2/c^2}}$$

Sub'ing into (*) pg. ~~9~~ 14 we get the Lorentz transformations:

$$\boxed{x' = \frac{x - v \cdot t}{\sqrt{1-v^2/c^2}}} \qquad \boxed{t' = \frac{t - v/c^2 \cdot x}{\sqrt{1-v^2/c^2}}}$$

- do a check using proper time, t'.

→ a clock stationary in the S' system sitting at the pt → x_0'

→ setting $v \longrightarrow -v$ to get t in terms of t':

$$t = \frac{t' + v/c^2 \, x_0'}{\sqrt{1 - v^2/c^2}}$$

- consider two time instances of the clock, t_1 & t_2.

$$\Rightarrow \quad t_1 = \frac{t_1' + v/c^2 \, x_0'}{\sqrt{1 - v^2/c^2}} \qquad t_2 = \frac{t_2' + v/c^2 \, x_0}{\sqrt{1 - v^2/c^2}}.$$

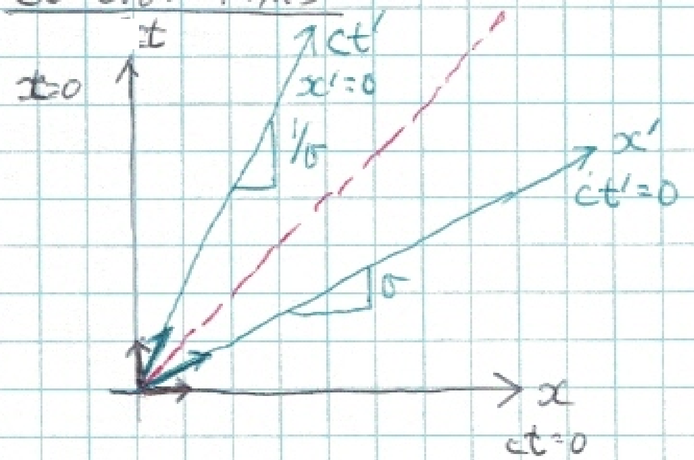
$$\therefore \quad t_2 - t_1 = \boxed{\Delta t = \frac{\Delta t'}{\sqrt{1 - v^2/c^2}}}$$

⇒ agrees $\bar{c}$ (✱) on pg 12.

Recall coord transforms:

$$\boxed{\begin{aligned} x &= x' \cdot ch(\theta) + ct' sh(\theta) \\ ct' &= x' sh(\theta) + ct' ch(\theta) \end{aligned}}.$$

Co-ord Axis



① ct'-axis ⇒ $x' = 0$

$$\therefore \quad x = ct' sh(\theta)$$
$$ct = ct' ch(\theta).$$

$$\text{i.e.} \rightarrow \frac{ct}{x} = \frac{ch(\theta)}{sh(\theta)} = \frac{1}{th(\theta)} = \frac{c}{v}$$

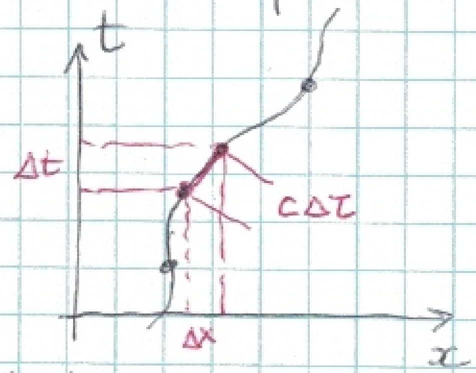
② x'-axis ⇒ $ct' = 0$.

$$\left.\begin{aligned} x &= x' ch(\theta) \\ ct &= ct' sh(\theta) \end{aligned}\right\} \quad \frac{ct}{x} = \frac{sh(\theta)}{ch(\theta)} = th(\theta) = v/c$$

4-Vectors

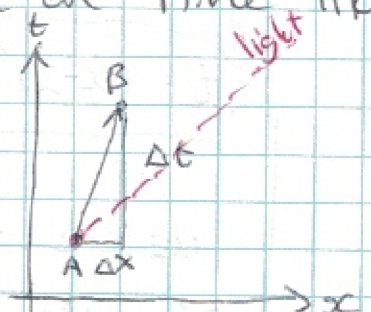
- an event is specified by a time & 3-dim pos^{n} which we've packaged in a 4-dim vector:

- idea of the path of a particle moving thru space-time



- particles move on time-like trajectories.

That is,



- the (invariant) proper time elapse in moving from A→B is:

$$\Delta\tau^2 = \Delta t^2 - \Delta x^2$$

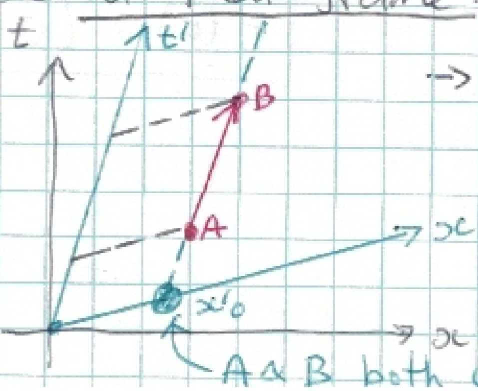
Note ~~Δt~~: $\Delta\tau^2 > 0$ if $\Delta t^2 > \Delta x^2$, i.e., the slope of $\overrightarrow{AB}$ is greater than 1 $\Rightarrow$ $\frac{\Delta t}{\Delta x} > 1$

Or $\frac{\Delta x}{\Delta t} < 1$ *

$|v_{AB}| < 1$ is less than the speed of light.

Significance of time-like sep^n?

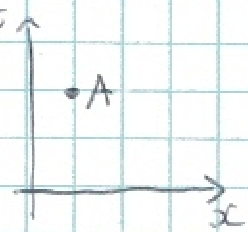
$\Rightarrow$ There always exists a moving frame s.t. they are at the same point & in space for that frame?



→ Note there is always a first, A, then B.

A & B both occur at pos^n x'_0!

- the motion of a particle can be thought of as a sequence of small time-like intervals.



- we can write $\underset{\sim}{A} = a^t \underset{\sim}{e}_t + a^x \underset{\sim}{e}_x + a^y \underset{\sim}{e}_y + a^z \underset{\sim}{e}_z$

basis vectors

- so $\underset{\sim}{A}$ is rep'd by the 4-tuple (a^t, a^x, a^y, a^z),

<u>OR</u> (a^0, a^1, a^2, a^3) <u>OR</u> $(a^0, \vec{a})$.

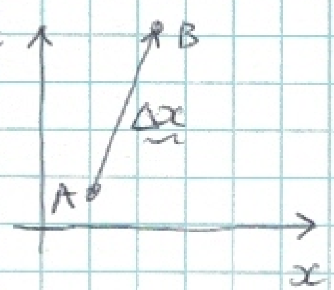
spatial 3-dim part

scalar 1-dim part.

- so $\underset{\sim}{A} = \sum_{\alpha=0}^{3} a^\alpha \underset{\sim}{e}_\alpha$

or $= a^\alpha \underset{\sim}{e}_\alpha$ where we leave the sum^n implied by duplicated upper & lower indices.

- we can have displacement 4-vectors.



$\Delta \underset{\sim}{x} = (t_B - t_A, x_B - x_A, y_B - y_A, z_B - z_A)$

OR, write as: $\boxed{\Delta x^\alpha = x^\alpha_B - x^\alpha_A}$

- scalar (inner product).

$\underset{\sim}{a} \cdot \underset{\sim}{b} = (a^\alpha \underset{\sim}{e}_\alpha) \cdot (b^\beta \underset{\sim}{e}_\beta)$

$= a^\alpha b^\beta \underbrace{(\underset{\sim}{e}_\alpha \cdot \underset{\sim}{e}_\beta)}$ ← double sum.

metric

Define this quantity as $\boxed{\eta_{\alpha\beta} = \underset{\sim}{e}_\alpha \cdot \underset{\sim}{e}_\beta}$

So $\boxed{\underset{\sim}{a} \cdot \underset{\sim}{b} = \eta_{\alpha\beta}\, a^\alpha b^\beta}$

• recall $\Delta s^2 = \Delta t^2 - \Delta \vec{x}^2$

$= \Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2.$

or $\Delta s^2 = \Delta \underset{\sim}{x} \cdot \Delta \underset{\sim}{x}$

$$\Delta s^2 = \eta_{\alpha\beta} \Delta x^\alpha \Delta x^\beta$$

where $\eta_{\alpha\beta} = \begin{pmatrix} 1 & & & \\ & -1 & \emptyset & \\ & & -1 & \\ \emptyset & & & -1 \end{pmatrix}$ $\longrightarrow$ the Minkowski metric

• infinitessimally, $\boxed{ds^2 = \eta_{\alpha\beta}\, dx^\alpha dx^\beta}$

Recall the Lorentz transforms:

$$t' = \frac{t - xv}{\sqrt{1-v^2}} \qquad x' = \frac{x - vt}{\sqrt{1-v^2}} \qquad y' = y \qquad z' = z$$

$$\boxed{\begin{aligned} t' &= \gamma(t - vx) \\ x' &= \gamma(x - vt) \\ y' &= y \\ z' &= z. \end{aligned}}$$

– setting $c = 1$

So a general 4-vector $\underset{\sim}{A} = (a^t\, a^x\, a^y\, a^z)$ transforms to

$\underset{\sim}{A} = (a^{t'}\, a^{x'}\, a^{y'}\, a^{z'})$ in the new basis

(same $\underset{\sim}{A}$!)

when $a^{t'} = \gamma(a^t - va^x)$

$a^{x'} = \gamma(a^x - va^t)$

$a^{y'} = a^y$

$a^{z'} = a^z.$

Example of Invariance of Inner Product.

Say $a^\alpha = (1, 0, 0, 0)$

$b^\beta = (0, 1, 0, 0)$.

Then $\underline{a} \cdot \underline{b} = \eta_{\alpha\beta} a^\alpha b^\beta = 0 \rightarrow$ they're orthogonal!

In a primed system we have.

$a^{\alpha'} = (\gamma, -v\gamma, 0, 0)$.

$b^{\alpha'} = (-v\gamma, \gamma, 0, 0)$.

Recall:

$t' = \gamma(t - vx)$

$x' = \gamma(x - vt)$

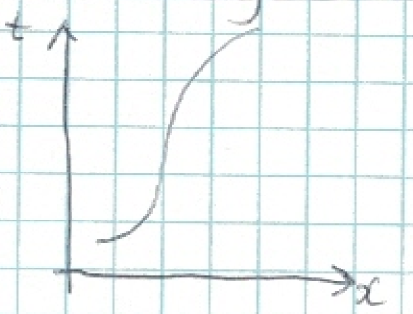
So for 4-vectors:

$$\boxed{\begin{aligned} a^{t'} &= \gamma(a^{t} - v a^{x}) \\ a^{x'} &= \gamma(a^{x} - v a^{t}) \end{aligned}}$$

$\eta_{\alpha\beta} a^{\alpha'} b^{\beta'} = \gamma(-v\gamma) - (-v\gamma)\gamma + 0 + 0$

$= 0$. ✓ $\Rightarrow$ orthogonal in ' system.

Parameterizing Motion In Space-Time.



- we could write a particle's path as

$\underset{\sim}{x}(t) : \mathbb{R} \longrightarrow \mathbb{R}^3$

$t \longmapsto (x(t), y(t), z(t))$
$\quad\quad\quad \overset{\shortparallel}{\underset{\sim}{x}(t)}$.

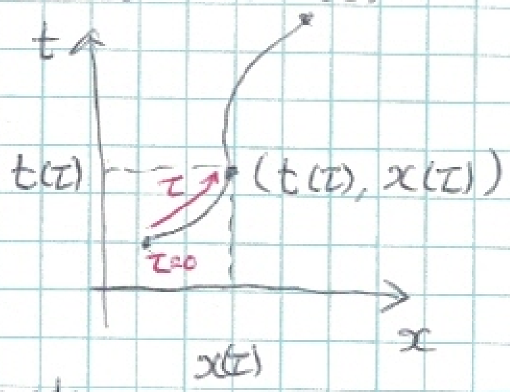
- but t is different from frame to frame as is $\underset{\sim}{x}$.
- better to parameterize the path using something that is invariant between frames

$\Rightarrow$ use <u>proper time</u> – the time of a clock moving $\bar{c}$ the particle.

– this is just the arc length $\rightarrow$ Recall $\boxed{d\tau^2 = dt^2 - dx^2}$

$\Rightarrow$ clocks are devices that measure distance along time-like

• so $x^\alpha = x^\alpha(\tau)$. $\longrightarrow$ $x^\alpha = (x^0(\tau), x^1(\tau))$ in 2d
$= (t(\tau), x(\tau))$



So we define the 4-velocity as

$$\frac{d}{d\tau} x^\alpha = u^\alpha$$

Components:

• now $\dfrac{d\cancel{x}^0}{d\tau} = \dfrac{d}{d\tau} t$

$\boxed{u^0 = \gamma}$ $\longrightarrow$ recall $\Delta t = \gamma \Delta \tau$
and $\gamma = \gamma(|\vec{v}|)$

• $\dfrac{dx^1}{d\tau} = \dfrac{dx}{dt}\dfrac{dt}{d\tau}$
$= \gamma v_x$ $\rightarrow$ ordinary 3-velocity

• similarly $\dfrac{dx^2}{d\tau} = \dfrac{dy}{dt}\dfrac{dt}{d\tau} = \gamma v_y$

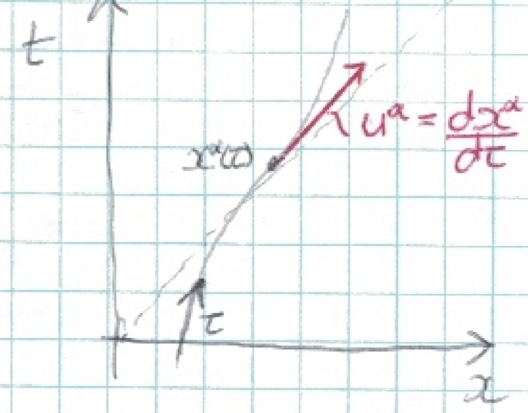
$\dfrac{dx^3}{d\tau} = \gamma v_z$.

So 4-velocity is given by $\boxed{u^\alpha = (\gamma, \gamma\vec{v}) = (\gamma, \gamma v_x, \gamma v_y, \gamma v_z)}$ $= \boxed{\gamma(1, \vec{v})}$

• note when stationary in $(x,y,z) \rightarrow \vec{v} = 0$ & $\gamma = 1$
$\boxed{u^\alpha = (1, 0, 0, 0)}$

$\rightarrow$ note: this is the rest frame of the moving particle

So the 4-velocity in that frame is the time axis for a momentary co-moving frame.



- what is the magnitude of u^α? $\Rightarrow$ $\underset{\sim}{u} \cdot \underset{\sim}{u} = |\underset{\sim}{u}|^2$.
- recall inner product $\underset{\sim}{a} \cdot \underset{\sim}{b} = \eta_{\alpha\beta}\, a^\alpha b^\beta$
- So $\underset{\sim}{u} \cdot \underset{\sim}{u} = \eta_{\alpha\beta}\, u^\alpha u^\beta$

$$\boxed{|\underset{\sim}{u}|^2 = \eta_{\alpha\beta} \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau}}$$

But we have the invariant $d\tau^2 = -\eta_{\alpha\beta}\, dx^\alpha dx^\beta$

So $\dfrac{d\tau^2}{d\tau^2} = -\eta_{\alpha\beta} \dfrac{dx^\alpha dx^\beta}{d\tau^2}$

$$\boxed{-1 = \eta_{\alpha\beta} \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau}}$$

$$\therefore \boxed{|\underset{\sim}{u}|^2 = -1}$$

Always time-like.

You can also see this by $u^\alpha = (\gamma, \gamma v_x)$ (e.g. 1-1 space)

$$\eta_{\alpha\beta} U^\alpha U^\beta = -\gamma^2 + \gamma^2 v_x^2$$

Need

$U^\alpha = (\gamma, \gamma v_x)$

$\eta = \begin{pmatrix} -1 & & & \emptyset \\ & 1 & & \\ & & 1 & \\ \emptyset & & & 1 \end{pmatrix}$

$$= \frac{-1}{1-V_x^2} + \frac{1}{1-V_x^2} \cdot V_x^2$$

$$= \frac{-(1-V_x^2)}{1-V_x^2}$$

$$= -1 \checkmark$$

4-Acc^n

Similarly we can define the 4-acc^n.

$$\underset{\sim}{a}(\tau) = \frac{du}{d\tau}$$

Note that since $\underset{\sim}{u} \cdot \underset{\sim}{u} = -1$ and diff'ing both sides:

$$\frac{d}{d\tau}(\underset{\sim}{u} \cdot \underset{\sim}{u}) = \frac{d}{d\tau}(-1)$$

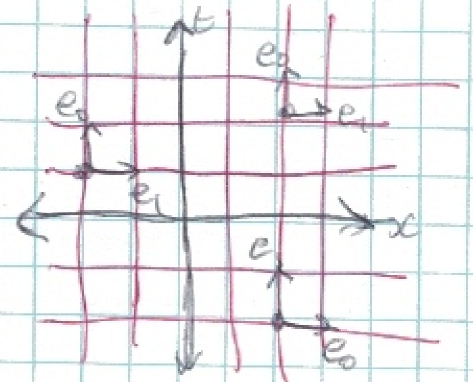
$$\underset{\sim}{u} \cdot \frac{du}{d\tau} + \frac{du}{d\tau} \cdot u = 0$$

equal by symm. of i.p.

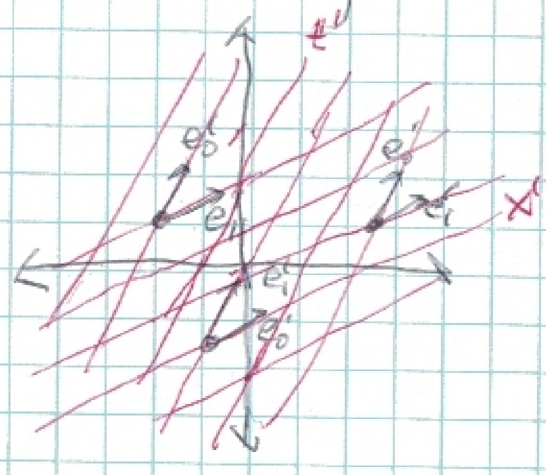
$$2\underset{\sim}{u} \cdot \frac{du}{d\tau} = 0$$

$\therefore \boxed{\underset{\sim}{u} \cdot \underset{\sim}{a} = 0} \longrightarrow$ the 4-vel. & 4-acc^n are always orthogonal.

Recall the co-ord frames of a stationary & interial frame

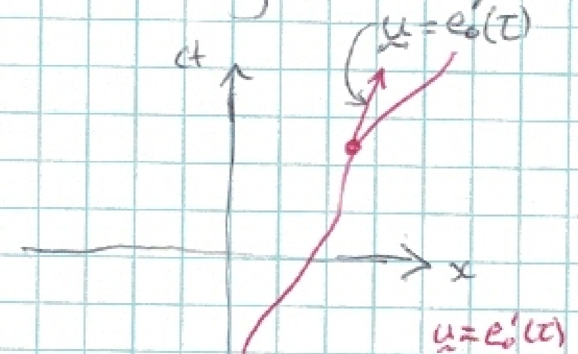


S

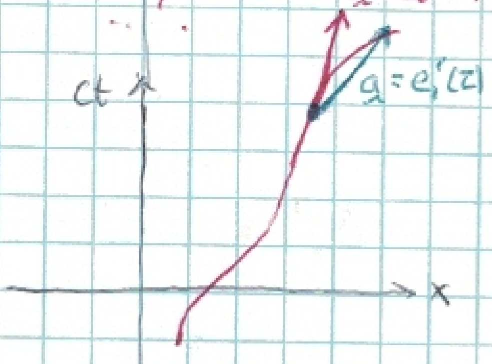


S'

- the co-ord system's basis are fixed for const. veloc
- when the velocity varies under acc^n define the instantaneous rest frame S' for which the moving observer is momentarily at rest.



- so the velocity vector in general defines a local time axis.
- so $\boxed{\underset{\sim}{e}_0'(\tau) = \hat{\underset{\sim}{u}}(\tau)}$.



- in (t, x) 1-1 space we can the the orthogonal $\underset{\sim}{a}(\tau)$ to be the spatial basis vector at that pt

$$\boxed{\underset{\sim}{e}_1'(\tau) = \hat{\underset{\sim}{a}}(\tau)}$$

* This forms a (momentarily inertial) co-moving frame along the moving observer's world line *

- note that at a particular point the observer's 4-velocity is $(1, \vec{0}) = \underset{\sim}{u}' (= u'^{\mu})$
- since the ~~ee~~ 4-acc^n is $\perp$ it must have the form

$$a'^{\mu} = \underset{\sim}{a} = (0, \vec{a})$$ → $\vec{a}$ is the proper (particle's momentary rest frame) acc^n.

where $\vec{a} = \left(\frac{d^2x}{dt^2}, \frac{d^2y}{dt^2}, \frac{d^2z}{dt^2}\right)$ in the comoving frame

$$= \frac{d}{dt}(\vec{v}_x, \vec{v}_y, \vec{v}_z)\Big|_{\text{in c.m.f}}$$

$$= \frac{d\vec{v}}{dt}\Big|_{\text{in c.m.f.}}$$

Relativist Addition of Velocities.

- consider a moving frame K' at velocity V w.r.t K.
- a particle is moving at velocity $\boxed{w = \frac{dx'}{dt'} \text{ in } K'}$
- what is its velocity $\boxed{u = \frac{dx}{dt} \text{ in } K\,?}$

- take differentials of the Lorentz transforms:

$$\boxed{\begin{aligned} x &= \gamma(x' + Vt') \\ t &= \gamma\left(t' + \frac{Vx'}{c^2}\right) \end{aligned}}$$

where V is a const.

$\Rightarrow$ recall $dx = \frac{\partial}{\partial x'}(\text{rhs})\,dx' + \frac{\partial}{\partial t'}(\text{rhs})\,dt'$

So

$$\boxed{\begin{aligned} dx &= \gamma(dx' + V dt') \\ dt &= \gamma\left(dt' + \frac{V}{c^2} dx'\right) \end{aligned}}$$

$$\begin{cases} dx = dt' \cdot \gamma\left(\overbrace{\frac{dx'}{dt'}}^{=w} + V\right) \\ dt = dt' \cdot \gamma\left(1 + \frac{V}{c^2} \overbrace{\frac{dx'}{dt'}}^{=w}\right) \end{cases}$$

So, $\begin{cases} dx = dt'\gamma(\omega + V) \\ dt = dt'\gamma\left(1 + \frac{V\omega}{c^2}\right) \end{cases}$

divide

in K frame

in K' frame

$$\therefore \boxed{u = \frac{dx}{dt} = \frac{\omega + V}{1 + \frac{\omega V}{c^2}}}$$

$\Rightarrow$ Note: as $c \to \infty$

$\boxed{u = \omega + V}$ – Galilean result.