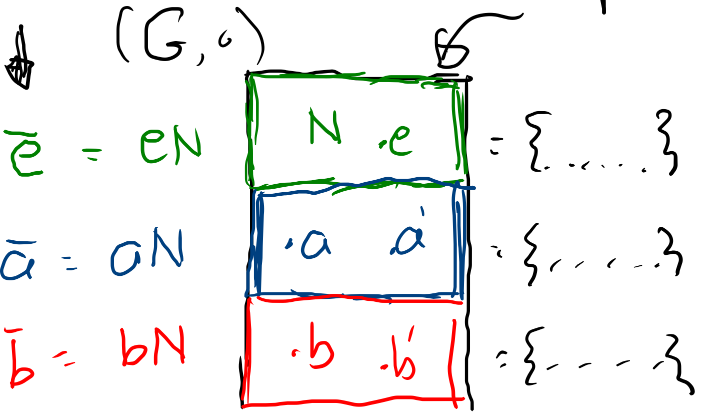
Quotient Groups: Choose a normal subgroup, N, of G.

$(G, \circ)$



$N \trianglelefteq G$

Form a group from the cosets of N in G.

Had shown for normal subgroups, left & right cosets are equal

$aN = Na$,

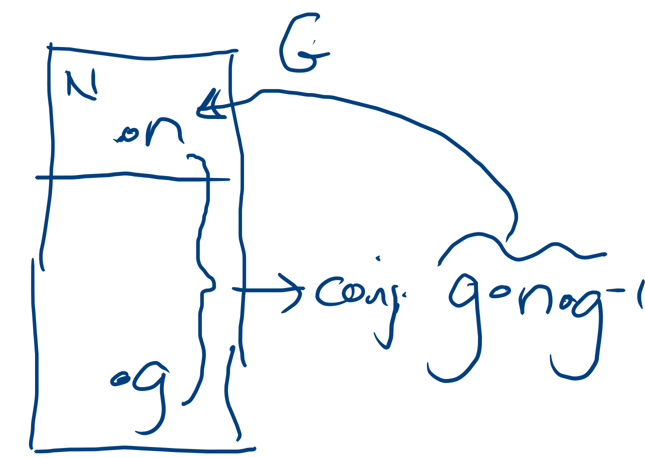
$\{\bar{e}, \bar{a}, \bar{b}\}$ form a group under the operation defined as:

$\rightarrow \bar{a} \circ_c \bar{b} = \overline{a \circ_G b}$ $\leftarrow$ Induced operation.

This is the quotient group: $\boxed{G/N}$ = {cosets of N in G, induced operation above}.

Example 2 of Quotient Groups:

- start $\bar{c}$ $\boxed{G=(\mathbb{Z},+)}=\{0,+1,-1,+2,-2,\ldots\}$
  with $x\circ y \equiv x+y$.
- notice G is Abelian $\rightarrow$ commutative. $\therefore$ $x+y=y+x$ $\quad 3+2=2+3=5$.
- $\therefore$ all subgroups of G are normal. Why?



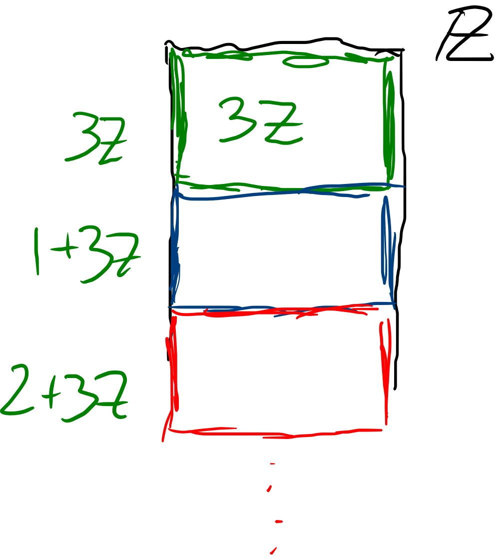
Choose $N<G$.

Being normal by def^n: $\forall n\in N, \forall g\in G \Rightarrow g\circ n\circ g^{-1}\in N$ ✓

If G is Abelian, then $g\circ \underbrace{n\circ g^{-1}} = \cancel{g\circ g^{-1}}^{\,e}\circ n$
$= n$

Choose subgroup $\{3\mathbb{Z},+\}=\{0,3,-3,6,-6,9,-9,\ldots\}$
It's normal in $\mathbb{Z}$.

$3n+3m$ $\quad , n,m\in\mathbb{Z}$.
$=3(n+m)$



$0+3\mathbb{Z} = \{0, 3, -3, 6, -6, \ldots\} = 0 \circ N$

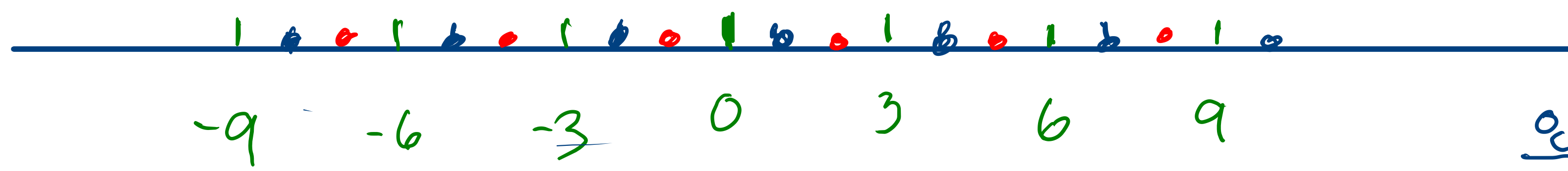
$1+3\mathbb{Z} = \{1, 4, -2, 7, -5, \ldots\} = 1 \circ N$

$2+3\mathbb{Z} = \{2, 5, -1, 8, -4, \ldots\} = 2 \circ N$

$\underline{3}+3\mathbb{Z} = \underline{0}+3\mathbb{Z} = 3\mathbb{Z}.$

$aH = a \circ H$

$$\boxed{\mathbb{Z}/3\mathbb{Z} \simeq \mathbb{Z}_3 \simeq C_3}$$



$\bar{0} = 0+3\mathbb{Z} = \{ \ldots \}$

$\bar{1} = 1+3\mathbb{Z} = \{ \ldots \}$

$\bar{2} = 2+3\mathbb{Z} = \{ \ldots \}$

$\bar{a}+\bar{b} \equiv \overline{a+b}$

$\circ$	$\bar{0}$	$\bar{1}$	$\bar{2}$
$\bar{0}$	$\bar{0}$	$\bar{1}$	$\bar{2}$
$\bar{1}$	$\bar{1}$	$\bar{2}$	$\bar{0}$
$\bar{2}$	$\bar{2}$	$\bar{0}$	$\bar{1}$

$\bar{1}+\bar{1} \overset{?}{=} \bar{4}+\bar{7} = \overline{11} = \bar{2}$

$\bar{0}+\bar{0} = \overline{0+0} = \bar{0}$

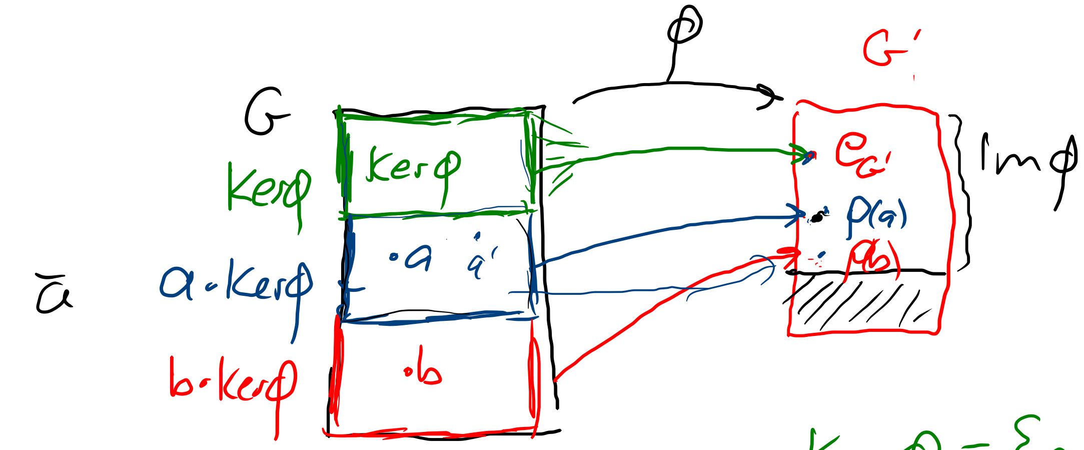
$\bar{0}+\bar{1} = \overline{0+1} = \bar{1}$

$\bar{1}+\bar{1} = \overline{1+1} = \bar{2}$

$\bar{1}+\bar{2} = \overline{1+2} = \bar{3} = \bar{0}$

First Isomorphism Thm : $\boxed{G/\ker\phi \cong \operatorname{Im}\phi}$

- it connects quotients with homomorphisms.



$\ker\phi \equiv \{g \in G \mid \phi(g) = e_{G'}\}$

<u>Comp. Cond^n of ϕ:</u>

$\phi(a \circ_G b) = \phi(a) \circ_{G'} \phi(b)$

Piece ① <u>Claim:</u> all elt's in the same coset are mapped to the elt in G'.

i) pick any coset, and call it $\bar{a}$.
It has an elt a in it.

- any other elt in the coset $\bar{a}$ has the form $a' = a \circ n$ by the def^n of cosets. for some $n \in N := \ker\phi$.

- where does a' get mapped to?

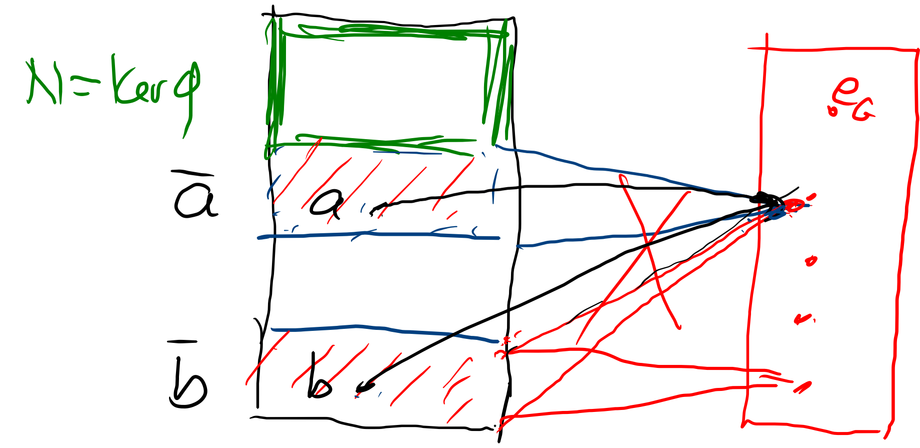
$$\phi(a') = \phi(a \circ n)$$

$$\overset{c.c.}{=} \phi(a) \circ \phi(n)$$

$$= \phi(a) \circ e_{G'}$$

$$\boxed{\phi(a') = \phi(a).}$$

② Claim: If we have elts in different coset they have to mapped to different elt's in the codomain ✓



Choose $a \in \bar{a}$, $b \in \bar{b}$.

Assume $\boxed{\phi(a) = \phi(b)}$

$= \phi(b)^{-1}$

Then $\phi(a)\circ\phi(b^{-1}) = \phi(b)\circ\phi(b^{-1})$

$\phi(a)\circ\phi(b^{-1}) = e_{G'}$

c.c. $\phi(a\circ b^{-1}) = e_{G'}$

$\therefore\ a\circ b^{-1} \in N$, ker ϕ.

$a\circ b^{-1} = n$ for some $n \in N$

$\boxed{a = n\circ b}$ → just say a & b are in the same coset.

$\boxed{a \in \bar{b}}$

Coset $\bar{c} = cN$

$\Rightarrow\ = \{c\circ n \mid n \in N\}$.