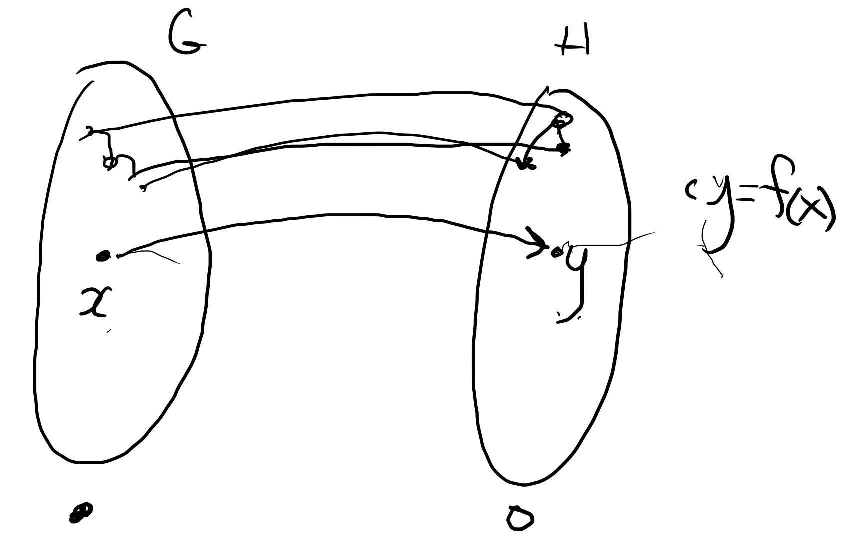
Problem: Let $G \to H$ be an isomorphism:



$$f(a \cdot b) = f(a) \cdot f(b)$$

(1) Identity is mapped to identity.

$$f(e_G) = e_H$$

• let $y = f(x) = f(e_G x) = f(e_G) \cdot f(x)$

$y = f(x) = f(x e_G) = f(x) \cdot f(e_G)$

$$f(e_G) \cdot f(x) = f(x) \cdot f(e_G) = f(x) = y$$

$$\varepsilon y = y \varepsilon = y$$

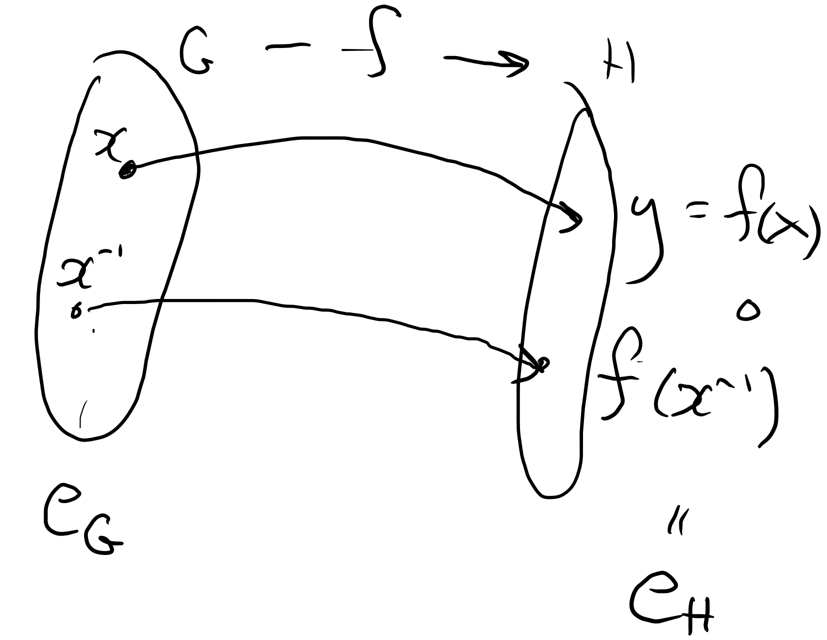
$$\therefore \boxed{f(e_G) = e_H}$$

② In general we have $f(xy) = f(x)f(y)$

$$f(xx^{-1}) = f(x)f(x^{-1})$$
$$\shortparallel$$
$$f(e_G) = e_H$$

$\therefore f(x)f(x^{-1}) = e_H$

$\therefore f(x^{-1})$ is the inverse of $f(x)$.

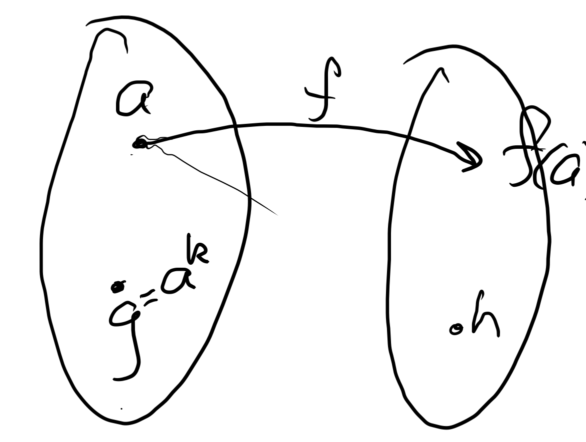


$f(ab) = f(a)f(b)$.

$G \xrightarrow{f} H$

③ Cyclic groups $\longrightarrow$ generators $\longrightarrow$ generators.

$G = \langle a \rangle = \{a^0 a^1 a^2 \cdots a^{n-1}\}$



Note: $\boxed{f(a^k)} = f(\underbrace{a \cdot a \cdots a}_{k\text{-times}}) = \underbrace{f(a)f(a)\cdots f(a)}_{k\text{-times}} = \boxed{f^k(a)}$

Know: $g = a^k$

Pick arb. elt $h \in H \longrightarrow \left.\begin{array}{l} h = f(g) \\ h = f(a^k) = f^k(a) \end{array}\right\} \therefore \boxed{h = f^k(a)}$ $\therefore$ $f(a)$ is a generator of H.

Example: Pinter p. 90

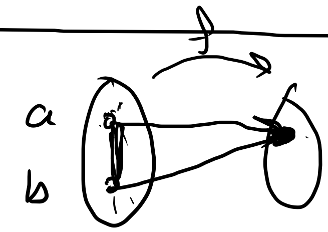
$\{\mathbb{R}, \times\} \not\cong \{\mathbb{C}, \times\}$

G: set of real no.'s under addition $\{\mathbb{R}, +\}$

H: set of pos. real's under multn $\{\mathbb{R}^+, \times\}$

Claim: these two groups are isomorphic.

Choose: $\boxed{f(x) = e^x}$



$\mathbb{C} \sim \mathbb{R} \times \mathbb{R}$

Bijective?: 1. Injective

If $f(a) = f(b)$ Show $a = b$.

$\rightarrow e^a = e^b$

$\log(e^a) = \log(e^b)$

$\rightarrow a = b$.

Surjective: Yes, by inspection.

Compatible: Does $f(a \circ_G b) = f(a) \circ_H f(b)$

$\downarrow \qquad \downarrow$

$+ \qquad \times$

$f(a+b) = f(a) \cdot f(b)$?

Does $e^{a+b} = e^a \cdot e^b$? YES

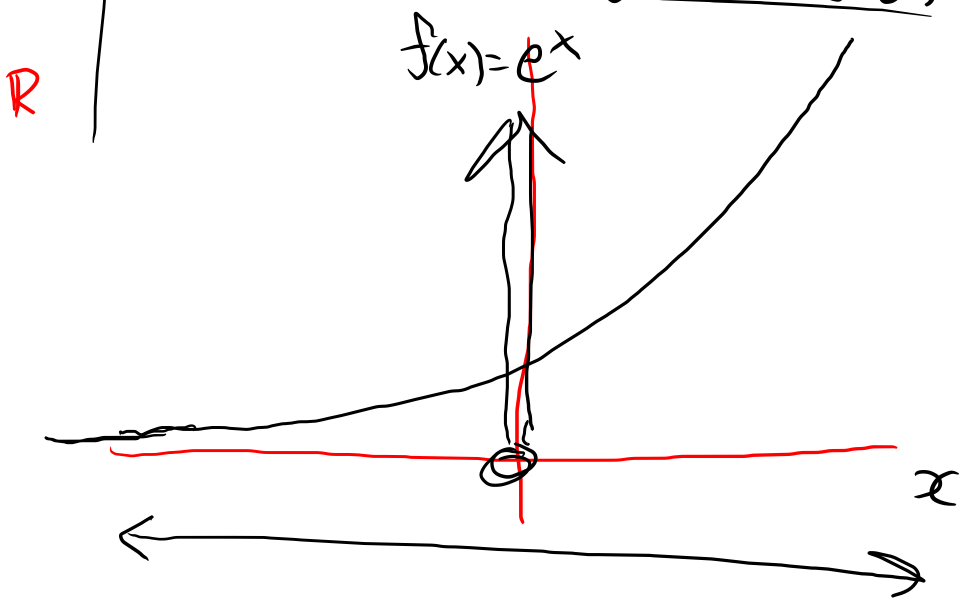
Isomorphism:

① bijective map

② compatability

$f(a \circ b) = f(a) \cdot f(b)$.

$f(a \circ_G b) = f(a) \circ_H f(b)$



Aside:

$e^{a+b} = e^a \cdot e^b$

$\boxed{f(x) = e^x}$

$f(a+b) = f(a) \cdot f(b)$

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