Spectral estimation techniques for the spectral calibration of a color image scanner

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Several methods of spectral sensitivity estimation for color image scanners are developed and evaluated. The estimation procedure involves response measurements of the scanner through a set of spectrally selective filters. These measurements form the observations for generalized inverse, smoothing, and Wiener estimation processes. Wiener estimation is found to provide the best results.

I. Introduction

The spectral sensitivity of a color scanner must be determined to calibrate its response. Direct spectral measurements over the continuum of the spectral band are often difficult to obtain. However, responsivity measurements can be made through spectrally selective filters to estimate the continuous spectral sensitivity of the color scanner.

II. Spectral Radiance Estimation

Many tasks in color and multispectral image restoration involve the estimation of the spectral radiance function $c(\lambda)$ from a series of observations of the form

$$x_i = \int c(\lambda)s_i(\lambda)d\lambda + n_i, \tag{1}$$

where $s_i(\lambda)$ is the spectral sensitivity of the spectral measurement filter for $i = 1,2, \dots P$ observations. The term n_i represents additive noise or uncertainty in the measurement. Discrete estimation techniques can be applied to this problem solution.[1] The first step is to discretize the continuous integral to form the vector equation

$$x_i = \mathbf{s}_i{}^T\mathbf{c} + n_i, \tag{2}$$

where $\mathbf{s}_i$ and $\mathbf{c}$ are $Q \times 1$ vectors of quadrature samples of $s_i(\lambda)$ and $c(\lambda)$, respectively. Then, the set of P observations may be arranged into the $P \times 1$ vector

$$\mathbf{x} = \mathbf{S}\mathbf{c} + \mathbf{n}, \tag{3}$$

where the vector $\mathbf{s}_i{}^T$ occupies the ith row of the matrix $\mathbf{S}$. The system of equations represented by Eq. (3) is normally highly underdetermined if sufficient quadrature mesh points are taken to reduce the quadrature error to reasonable bounds.

An estimate $\hat{\mathbf{c}}$ of the true spectral energy distribution $\mathbf{c}$ can be obtained by the generalized inverse estimate[2]

$$\hat{\mathbf{c}} = \mathbf{S}^{-}\mathbf{x} = \mathbf{S}^T(\mathbf{S}\mathbf{S}^T)^{-1}\mathbf{x}. \tag{4}$$

Although the generalized inverse provides a minimum mean-square error, minimum norm estimate of $\mathbf{c}$, ill-conditioning of $\mathbf{S}$ coupled with observational errors can lead to oscillatory estimates. Since $\mathbf{c}$ is generally smooth, it is reasonable to impose some smoothing constraints on the solution. A common type of smoothing estimate is given by[3]

$$\hat{\mathbf{c}} = \mathbf{M}^{-1}\mathbf{S}^T(\mathbf{S}\mathbf{M}^{-1}\mathbf{S}^T)^{-1}\mathbf{x}, \tag{5}$$

where $\mathbf{M}$ is a smoothing matrix of the typical form

$$\mathbf{M} = \begin{bmatrix} 1 & -2 & 1 & 0 & 0 & 0 & 0 & \cdots & & & & 0 \\ -2 & 5 & -4 & 1 & 0 & 0 & 0 & & & & & \vdots \\ 1 & -4 & 6 & -4 & 1 & 0 & 0 & & & & & \vdots \\ 0 & 1 & -4 & 6 & -4 & 1 & 0 & & & & & \vdots \\ 0 & 0 & 1 & -4 & 6 & -4 & 1 & & & & & \vdots \\ \vdots & & & & \ddots & & & & & & & \vdots \\ \vdots & & & & & & 1 & -4 & 6 & -4 & 1 & 0 \\ \vdots & & & & & & 0 & 1 & -4 & 6 & -4 & 1 \\ \vdots & & & & & & 0 & 0 & 1 & -4 & 5 & -2 \\ 0 & \cdots & & & & & 0 & 0 & 0 & 1 & -2 & 1 \end{bmatrix}. \tag{6}$$

A third alternative is to apply Wiener estimation methods.[4] With Wiener estimation, the vector $\mathbf{c}$ to be estimated is assumed to be a sample of a vector random process with a known mean and covariance matrix $\mathbf{K}_c$. The Wiener estimate is given by

$$\hat{\mathbf{c}} = \mathbf{K}_c\mathbf{S}^T(\mathbf{S}\mathbf{K}_c\mathbf{S}^T + \mathbf{K}_n)^{-1}\mathbf{x}, \tag{7}$$

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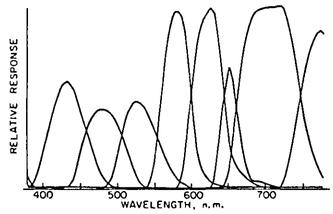

Fig. 1. Spectral shapes of absorption filters.

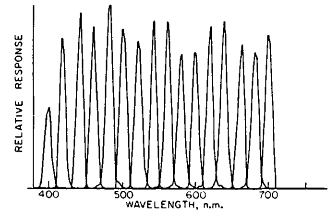

Fig. 2. Spectral shapes of interference filters.

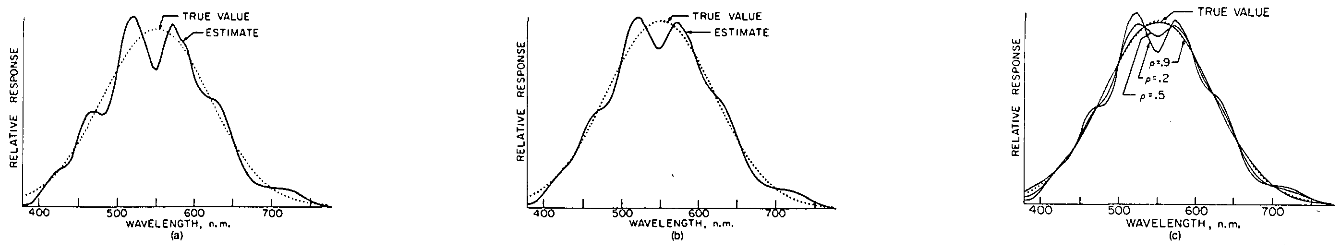

Fig. 3. Comparison of actual and estimated spectral response for absorption filters obtained by computer simulation: (a) pseudoinverse estimate; (b) smoothing estimate; (c) Wiener estimate, SNR = 1000.

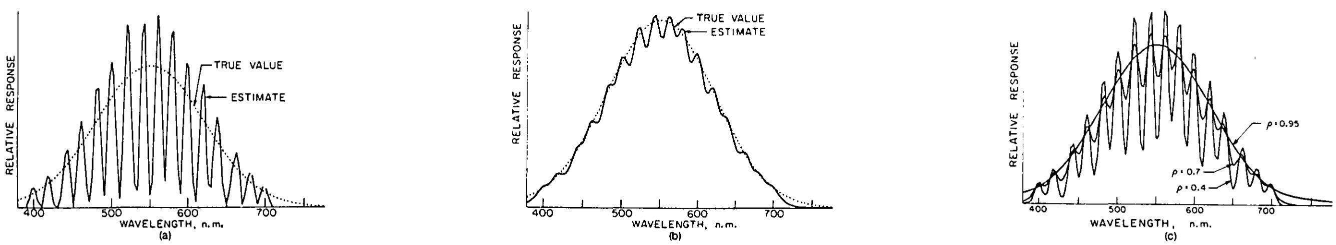

Fig. 4. Comparison of actual and estimated spectral response for interference filters obtained by computer simulation: (a) pseudoinverse estimate; (b) smoothing estimate; (c) Wiener estimate, SNR = 1000.

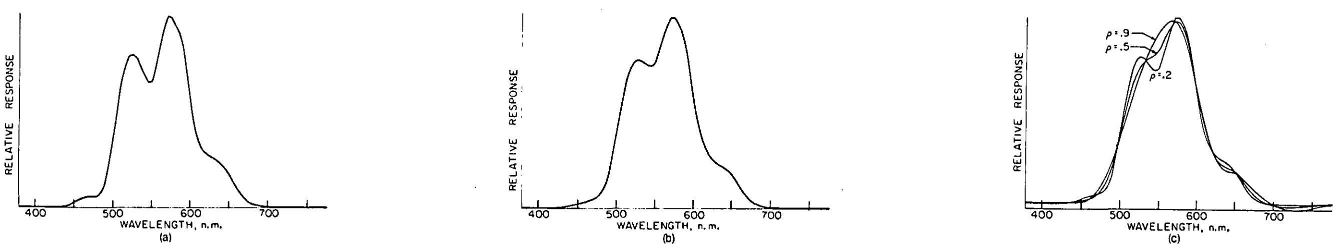

Fig. 5. Estimated spectral response for absorption filters for microdensitometer color scanner: (a) pseudoinverse estimate; (b) smoothing estimate; (c) Wiener estimate, SNR = 1000.

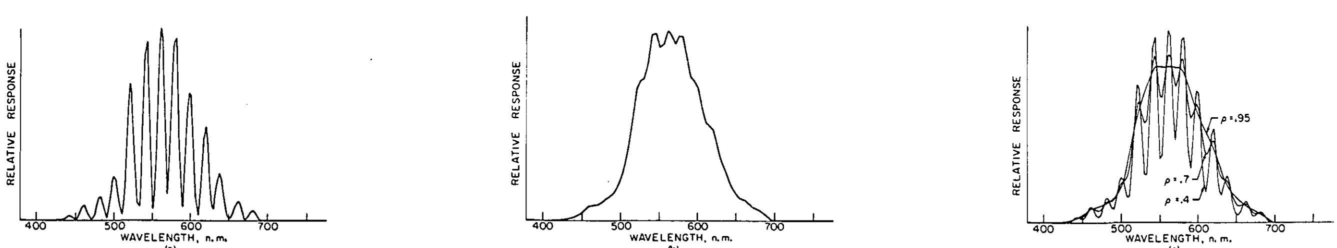

Fig. 6. Estimated spectral response for interference filters for microdensitometer color scanner: (a) pseudoinverse estimate; (b) smoothing estimate; (c) Wiener estimate, SNR = 1000.

where $\mathbf{K}_n$ is the covariance matrix of the additive observational noise assumed independent of $\mathbf{c}$. As a convenient approximation, the covariance matrix can be modeled as a first-order Markov process covariance matrix of the form

$$K_c = \frac{\sigma_c^2}{Q}\begin{bmatrix} 1 & \rho & \rho^2 & \cdots & \rho^{Q-1} \\ \rho & 1 & \rho & \cdots & \rho^{Q-2} \\ \vdots & & & & \vdots \\ \rho^{Q-1} & \cdots & & & 1 \end{bmatrix}, \tag{8}$$

where $0 \leq \rho \leq 1$ is the adjacent element correlation factor and $\sigma_c{}^2$ represents the energy of $\mathbf{c}$. Observation noise is commonly modeled as a white noise process with covariance equal to

$$K_n = \frac{\sigma_n^2}{Q} I, \tag{9}$$

where $\sigma_n{}^2$ is the noise energy and $\mathbf{I}$ is an identity matrix.

III. Color Image Scanner Calibration

A common problem in the evaluation and calibration of color image scanners is to determine the total spectral response of the scanner taking into account the spectral radiance of the illumination source, spectral absorption and scattering of the optics, and spectral sensitivity of the photodetector. Direct measurements are often not feasible. Referring to Eq. (1), let $c(\lambda)$ be redefined to represent the spectral sensitivity response of the scanner and $s_i(\lambda)$ be one of P spectral test functions. The measurement procedure then proceeds as follows. An optical filter of known spectral characteristics, such as an absorption filter or narrowband interference filter is introduced into the scanner; and an output reading is obtained. The process is repeated for a number of filters whose peak transmissivities span the spectral region of interest. The measurements form the vector of observations, and an estimation operation is then invoked to obtain an estimate of the scanner spectral response.

To evaluate the estimation procedure, a computer simulation experiment was performed in which simulated measurements were taken of a Gaussian-shaped spectral function through simulated absorption and interference filters. Figures 1 and 2 contain plots of the spectral shapes of the filters. The simulated measurements were then utilized as spectral observations for estimation of $c(\lambda)$. Figures 3 and 4 illustrate the performance of the three estimation methods for simulated measurements through the two types of filters. In these experiments the mean-square fit between the actual spectral function and its estimate was least for the simulated interference filter measurements using a Wiener estimate with $\rho = 0.9$ and an SNR of 1000.

The spectral estimation procedures have also been applied to the estimation of the spectral response of an Optronics model S 2000 flat bed scanning microdensitometer. Figures 5 and 6 show the estimate obtained with absorption and interference filters for the three estimation methods. No direct measurements are available for the scanner, so no ground truth can be established. But, on the basis of the simulation experiments, it is concluded that the Wiener estimate obtained with the set of interference filters is a reasonable estimate of the actual spectral response.

IV. Summary

Several techniques of spectral sensitivity estimation for color image scanners have been evaluated. Simulation and experimental results indicate that the Wiener estimate provides a good estimate of spectral response.

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