

Overlap again:

ex. 1s 1p

$$S = \begin{bmatrix} s_{11} & s_{12} & s_{13} & s_{14} \\ s_{21} & 1 & 0 & 0 \\ s_{31} & 0 & 1 & 0 \\ s_{41} & 0 & 0 & 1 \end{bmatrix} \quad \text{normalization}$$

1. step: build each of the primitive Gaussians
w/ the normalization constant

your code
from hw 2
 $\vec{r} = \{x, y, z\}$

H-1s function $w_1 = N_1 (x-x)^0 (y-y)^0 (z-z)^0 \exp(-3.425 \cdot (\vec{r}-\vec{R})^2)$

$w_2 = N_2 (x-x)^0 (y-y)^0 (z-z)^0 \exp(-0.6239 \cdot (\vec{r}-\vec{R})^2)$

$w_3 = N_3 (x-x)^0 (y-y)^0 (z-z)^0 \exp(-0.1688 \cdot (\vec{r}-\vec{R})^2)$

2. step get N_1, N_2, N_3

$$S = 1 = \int \int \int w_1 w_1 d\vec{r} = \underbrace{N_1 \cdot N_1}_{N_1^2} \cdot \underbrace{\int \int \int w_1 w_1 d\vec{r}}_{\text{from hw 2}}$$

ex. $s_{11} \approx 1.9$

$$N_1 = \frac{1}{\sqrt{s_{w_1, w_1}}}$$

now know primitives!

$$S^{HH} = S^{SS}$$

$$S = \begin{bmatrix} s_{H1} & s_{H2} \\ s_{H1} & s_{H2} \end{bmatrix} \quad \begin{bmatrix} 1 & s_{12} \\ s_{21} & 1 \end{bmatrix}$$

$$s_{12} = s_{21} = \sum_k \sum_L d_{kS_{H1}} d_{LS_{H2}} N_{kS_{H1}} N_{LS_{H2}} S^{w_k w_L}$$

ex. 1st sum $k=1, L=1 \rightarrow 0.1543 \cdot 0.1543 \cdot N_1 \cdot N_1 \cdot S^{w_1 w_1}$

2nd sum $k=1, L=2 \rightarrow 0.1543 \cdot 0.5353 \cdot N_1 \cdot N_2 \cdot S^{w_1 w_2}$

3rd sum $k=1, L=3 \rightarrow 0.1543 \cdot 0.4446 \cdot N_1 \cdot N_3 \cdot S^{w_1 w_3}$

\vdots
 $k=2, L=1$

\vdots
 $k=2, L=2$

Final S in AO basis $S = \begin{bmatrix} 1 & 0.6599 \\ 0.6599 & 1 \end{bmatrix}$ \leftarrow I'll send this

Build Hamiltonian $H = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} = \begin{bmatrix} -13.6 & -15.705 \\ -15.705 & -13.6 \end{bmatrix}$ 0.6599

$$H_{12} = H_{21} = \frac{1}{2} \cdot 1.75 (-13.6 - 13.6) \cdot S^{H_{12} H_{21}}$$

we look for

$$= -15.705 \quad \leftarrow \text{unit is eV}$$

$\downarrow \downarrow \downarrow$
 $Hc = SCE \rightarrow$ standard eigenvalue problem $Hc = CE$

\rightarrow orthogonalize $\rightarrow S = \underline{1}$

symmetric orthogonalization (could canonical)

1. diagonalize Overlap matrix \rightarrow eigenvalues, eigenvectors

$$S^{-\frac{1}{2}} = \begin{pmatrix} S_1^{-\frac{1}{2}} & 0 \\ 0 & S_2^{-\frac{1}{2}} \end{pmatrix} \quad \begin{matrix} S \\ \text{vector} \end{matrix} \quad \begin{matrix} U \\ \text{matrix} \end{matrix}$$

S_1, S_2 are eigenvalues

transformation matrix $X = U S^{-\frac{1}{2}} U^T$ check if your X
is correct

\hookrightarrow transform Hamiltonian $H' = X^T H X$, and $X^T S X = \mathbb{1}$

$\hookrightarrow H' C' = C' E$
 $\xrightarrow{\text{in orthonormal basis}}$

solve $H' C' = C' E$

C' matrix \rightarrow coeff. \rightarrow transform back to AO $C = X C'$ coeff to build MO

E vector \rightarrow list of energies of molecular orbitals

$$E = \begin{bmatrix} -17.65 \\ 6.19 \end{bmatrix} \quad E = \sum_{i=1}^{occ} 2 \cdot \epsilon_i = 2 \cdot (-17.65 \text{ eV}) = -35.3 \text{ eV}$$