# Homework 1

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## Summations

1)

(a)

$$\frac{\sum_{i=1}^{n} a_i x_i}{\sum_{i=1}^{n} a_i} = \sum_{i=1}^{n} x_i$$

$$\text{Suppose } a_1 = 2, a_2 = 4, x_1 = 5, x_2 = 7$$

$$\frac{\sum_{i=1}^{2} a_i x_i}{\sum_{i=1}^{2} a_i} = \frac{2(5) + 4(7)}{2+4} = \frac{38}{6}$$

$$\sum_{i=1}^{2} x_i = 5 + 7 = 12$$

$$\frac{38}{6} \neq 12 \text{ (counterexample to original equation)}$$

$$\frac{\sum_{i=1}^{n} a_i x_i}{\sum_{i=1}^{n} a_i} \neq \sum_{i=1}^{n} x_i \text{ (original equation is false)}$$

(b)

$$\sum_{i=1}^{n} a_3 x_i = n a_3 \bar{x}$$

$$n a_3 \bar{x} = n a_3 \left(\frac{1}{n} \sum_{i=1}^{n} x_i\right)$$

$$n a_3 \left(\frac{1}{n} \sum_{i=1}^{n} x_i\right) = a_3 \sum_{i=1}^{n} x_i$$

$$a_3 \sum_{i=1}^{n} x_i = \sum_{i=1}^{n} a_3 x_i \text{ (since } a_3 \text{ is constant)}$$

$$n a_3 \bar{x} = \sum_{i=1}^{n} a_3 x_i \text{ (so original equation is true)}$$

(c)

$$\sum_{i=1}^{n} a_i x_i = n\bar{a}\bar{x}$$

Suppose $a_1 = 2, a_2 = 4, x_1 = 5, x_2 = 7$

$$\sum_{i=1}^{n} a_i x_i = 2(5) + 4(7) = 38$$

$$n\bar{a}\bar{x} = 2(\frac{2+4}{2})(\frac{5+7}{2})$$

$$n\bar{a}\bar{x} = 2(3)(6) = 36$$

$36 \neq 38$ (counterexample to original equation)

$$\sum_{i=1}^{n} a_i x_i \neq n\bar{a}\bar{x} \text{ (original equation is false)}$$

# Calculus

2)

(a)

$$\sigma(x) = \frac{1}{1+e^{-x}}$$
$$1-\sigma(x) = 1-\frac{1}{1+e^{-x}}$$
$$1-\frac{1}{1+e^{-x}} = \frac{1+e^{-x}}{1+e^{-x}} - \frac{1}{1+e^{-x}}$$
$$\frac{1+e^{-x}}{1+e^{-x}} - \frac{1}{1+e^{-x}} = \frac{e^{-x}}{1+e^{-x}}$$
$$(\text{Note } e^{-x} = \frac{1}{e^x})$$
$$\frac{e^{-x}}{1+e^{-x}} = \frac{1}{e^x(1+e^{-x})} = \frac{1}{e^x+e^0} = \frac{1}{e^x+1}$$
$$\sigma(-x) = \frac{1}{1+e^{-(-x)}} = \frac{1}{1+e^x}$$
$$1-\sigma(x) = \frac{1}{1+e^x} = \sigma(-x)$$
$$1-\sigma(x) = \sigma(-x) \text{ (original equation is true)}$$

(b)

$$\frac{d}{dx}\sigma(x) = \sigma(x)(1-\sigma(x))$$
$$\frac{d}{dx}\sigma(x) = \frac{d}{dx}(\frac{1}{1+e^{-x}})$$
Use Quotient Rule for Derivatives
$$\frac{d}{dx}(\frac{1}{1+e^{-x}}) = \frac{(1+e^{-x})(0)-1(-e^{-x})}{(1+e^{-x})^2}$$
$$= \frac{e^{-x}}{(1+e^{-x})^2} = (\frac{1}{e^x(1+e^{-x})})(\frac{1}{1+e^{-x}})$$
$$= (\frac{1}{1+e^{-x}})(\frac{1}{e^0+e^x}) = (\frac{1}{1+e^{-x}})(\frac{1}{1+e^x})$$
We know that $\sigma(x) = \frac{1}{1+e^{-x}}$ and from part (a) that $1-\sigma(x) = \frac{1}{1+e^x} = \sigma(-x)$
$$(\frac{1}{1+e^{-x}})(\frac{1}{1+e^x}) = \sigma(x)(1-\sigma(x))$$
Therefore, original equation is true:
$$\frac{d}{dx}\sigma(x) = \sigma(x)(1-\sigma(x))$$

## Minimization

3)

$$f(c) = \frac{1}{n}\sum_{i=1}^{n}(x_i - c)^2$$

Find where the derivative is zero to get the min/max

$$f'(c) = \frac{1}{n}\frac{d}{dc}(\sum_{i=1}^{n}(x_i - c)^2) = 0$$

$$= \frac{1}{n}(\sum_{i=1}^{n}2(x_i - c)(-1))$$

$$= \frac{-2}{n}\sum_{i=1}^{n}x_i - c$$

$$= \frac{-2}{n}\sum_{i=1}^{n}x_i + \frac{2}{n}\sum_{i=1}^{n}c = 0$$

$$\frac{2}{n}\sum_{i=1}^{n}c = \frac{2}{n}\sum_{i=1}^{n}x_i$$

$$\sum_{i=1}^{n}c = \sum_{i=1}^{n}x_i$$

$$nc = \sum_{i=1}^{n}x_i$$

$$c = \frac{1}{n}\sum_{i=1}^{n}x_i$$

The function $f(c)$ is a parabola whose leading coefficient is positive (all the $c^2$ variables in each term of the summation are positive because it gets squared), so it is a parabola that opens upward. In a parabola that opens upward, the one critical point is always a minimum, so the value we found for $f'(c) = 0$ must be a minimum.

## Probability and Statistics

4)

(a) 2. cannot be found with the information in the article. We have the percentage of adults who had a great deal of trust in scientists and those who had a great deal of trust in religious leaders separately. Based on this information alone, we cannot determine the percentage of those who simultaneously had a great deal of trust in both religious leaders and scientists.

(b) I would guess that the car is a Toyota. Assuming the car is still within Alameda county, the number of Toyotas (one of the most common makes of cars in the county) would be far greater than Lamborghinis (which is a rare, expensive sports car). Since there are more Toyotas than Lamborghinis, the probability of the car being a Toyota is much higher. Based on this information, I guessed that the car would be a Toyota.

5)

Use Baye's Rule to find $\mathbb{P}[\text{has cancer} \mid \text{test is positive}]$

$$\mathbb{P}[\text{has cancer} \mid \text{test is positive}] = \frac{\mathbb{P}[\text{test is positive} \mid \text{has cancer}] * \mathbb{P}[\text{has cancer}]}{\mathbb{P}[\text{test positive}]}$$

$$\mathbb{P}[\text{has cancer}] = 0.01$$

$$\mathbb{P}[\text{test is positive} \mid \text{has cancer}] = 0.8$$

Using Total Probability, we can calculate the probability of a test being positive

$$\mathbb{P}[\text{test positive}] = \mathbb{P}[\text{test positive}|\text{has cancer}]\mathbb{P}[\text{has cancer}] + \mathbb{P}[\text{test positive}|\text{has no cancer}]\mathbb{P}[\text{has no cancer}]$$

$$\mathbb{P}[\text{test positive}] = (0.8)(0.01) + (0.096)(0.99) = 0.10304$$

$$\mathbb{P}[\text{has cancer} \mid \text{test is positive}] = \frac{0.8 * 0.01}{0.10304}$$

$$\mathbb{P}[\text{has cancer} \mid \text{test is positive}] = 0.0776 \text{ (7.76\% chance)}$$

6)

B. 6.1. The histogram has a bell-shaped curve (much like a normal distribution), therefore, finding the standard deviation is finding the distance of the inflection points from the mean. The inflection points appear to be roughly around 143-144 and 155-156. So, the standard deviation should be some value close to 6.