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General Topology

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1 Metric Spaces

Def 1. *A **metric** d defined on a set X is a mapping $d : X \times X \to \mathbb{R}$ which has the following properties.*

(m_1) $\forall x, y \in X : d(x,y) \geq 0$, $d(x,y) = 0 \iff x = y$

(m_2) $\forall x, y \in X : d(x,y) = d(y,x)$

(m_3) $\forall x, y, z \in X : d(x,y) + d(y,z) \geq d(x,z)$

If such map is well defined on a certain set X, we can now introduce the notion of metric spaces, which is merely a set equipped with a well defined metric.

Def 2. *A **metric space** (X, d) is a set X equipped with a metric $d : X \times X \to \mathbb{R}$.*

Ex 1. *These are some simple examples of metric spaces.*

1. $(\mathbb{R}^n, d)$ *where* $d(x,y) =$
2. (X, d) *where* $d(x,y) = 0$ *if* $x = y$ *and* $d(x,y) = 1$ *if* $x \neq y$ ***(discrete metric)***

Now it is natural to think of how the metric would be affected under the subsets, and the following theorem states that the restriction of such metric is still a metric on the subsets.

Thm 1. *Let (X,d) be a metric space and let $Y \subset X$. Then $d \restriction_{Y\times Y}: Y \times Y \to \mathbb{R}$ is a metric on Y.*

Proof. left as an exercise $\square$

Def 3. *Let (X,d) be a metric space and $x \in X$, $r \in \mathbb{R}$ where $r > 0$. The* ***open ball*** *$B(x;r)$ is defined as $\{y \in X : d(x,y) < r\}$ where x is called the* ***center*** *of $B(x;r)$, and r is called the* ***radius*** *of $B(x;r)$.*

Ex 2. *Some basic examples of open balls on different metric spaces.*

1. *On $(\mathbb{R}, d)$ where d is the usual metric, $B(x;r)= (x-r, x+r)$.*
2. *On $(\mathbb{R}, d)$ where d is the discrete metric, $B(x;r)= \{x\}$ if $r \in (0,1]$ and $B(x;r)= \mathbb{R}$ if $r > 1$*

Lem 1. *Let (X,d) be a metric space, where $x \in X$ and $r > 0$. Then,*

1. $\bigcap\limits_{r>0}$