The University Of Victoria
Department Of Computer Science

CSC 305

Mid-Term EXAM Summer 2009

Time Allowed 70 mins. June 22, 2009

Closed Book, calculators are permitted. The Mid-term is marked out of 16 (15% of course total + one bonus mark) and each questions is worth 4 marks. Marks will be given for answers to four out of five questions. Show all of your working. One single page of notes allowed (both sides).

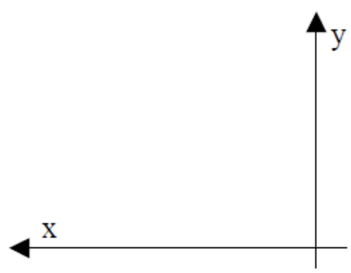
1. Mathematical Foundations

   (a) Let v =[3,5,2] What is the length of v?

   (b) What angle does v make with the y axis = [0, 1, 0]

   (c) Use the parametric representation to build an equation of a line from $p1 = [1,\ 2]$ to $p2 = [3,\ -2]$

   (d) Provided the definition of the positive x and y axes below, would the z axis of a right hand coordinate system come out of the page towards you or point behind the page?



2. (Parametric Curves)

   (a) A Hermite curve interpolates the points $P_1,\ P_4$, and has the corresponding tangent vectors $R_1,\ R_4$. A point on the curve is given by the following basis functions:

   $$Q(t) = P_1(2t^3 - 3t^2 + 1) + P_4(-2t^3 + 3t^2) + R_1(t^3 - 2t^2 + t) + R_4(t^3 - t^2)$$

   If $R_1 = 3(P_2 - P_1)$ and $R_2 = 3(P_4 - P_3)$, derive an expression for a point on the curve in terms of the control points, $(P_1,\ P_2,\ P_3,\ P_4)$
   Compare these basis functions to the Bezier curve basis functions?

   (b) Given a Bezier curve segment defined by the control points: $(-1, -2), (-5, 4), (11, 2), (17, -16)$. Calculate the 8 control points that define two Bezier segments found by applying de Castlejeau's algorithm to subdivide the original curve.

3. (Scene Graph)

   (a) (Graph Structures) A simple 2D model of a robot consists of a circle for a head, a rectangle for the body and two rectangles to represent legs and 2 for arms. A master robot has a helmet made from 4 rectangles and a circle, which fits nicely on its head. A scene consists of 8 robots arragned in a circle and 2 master robots placed near the centre.

   Draw a diagram and describe the data structure and transformations you use, (scale, rotations, translations). Use rectangles, circles or lines to represent the parts. You can assume the traversal algorithm given in class will be used on the data structure.

... Computer Science 305 cont'd

4. (Viewing Systems)

   (a) The perspective transformation matrix M_p is given by:

$$\begin{pmatrix} n & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ 0 & 0 & n+f & -fn \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

   where n is the distance to the near plane and f to the far.

   When M_p is multiplied by some $(x, y, z, 1)$ it produces a new coordinate (x_v, y_v, z_v, w). Find an expression for z_v in terms of z,n,f (don't forget to divide by W).

   What are the limits of z and z_v?

   (b) Sketch a graph of z against z_v. Use the graph to explain why the depth of objects are sometime not resolved when they are far from the eye. (i.e. the wrong object appears to be in front at a closer z-value).

5. (General) Answer any four of the following:

   (a) Write down the cartesian coordinates, corresponding to the following homogeneous coordinates : $(12, 6, 3)$ and $(15, 10, 5)$.

   (b) For a parametric curve, distinguish between parameter space and modeling (object) space. How is a point on a point on the curve, $Q(t)$, calculated from a set of basis functions $B_i(t)$?

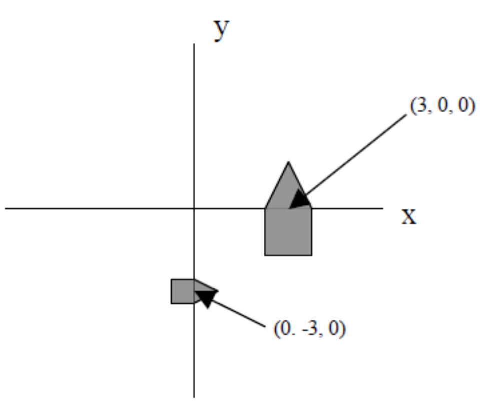
   (c) For the eye position e=[0, 2, 0], a gaze vector g=[0, -1, 0], and a view-up vector t = [1, 1, 0], what is the camera transformation matrix?

   (d) What does it means if two vectors are orthogonal? How can you determine if two vectors are orthogonal?

   (e) How do you show that a matrix is special orthogonal? What is the inverse of a special orthogonal matrix?

   (f) Briefly describe how the implicit form of a circle equation is used in the derivation of the circle scan coversion algorithm.

   (g) Calculate a chain of 4 x 4 matrices that, when post-multiplied by the vertices of the house (below), will translate and rotate the house from (3, 0, 0) to (0, -3, 0). The transformation must also scale the size of the house by half.



Selected Answers of CSC 305
Mid-term Exam Summer 2009

Question 1.(a). The length of $v = \sqrt{3^2 + 5^2 + 2^2} = \sqrt{38} = 6.164414$.

Question 1.(b). The inner product, or dot product of two vectors $\vec{a}$ and $\vec{b}$ has the property of $\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cos\theta$, where $|\vec{a}|$ denotes the length of $\vec{a}$.

Therefore, the angle θ between the two vectors can be calculated as

$$\theta = \arccos \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|} = \arccos \frac{5}{6.164414} = 51.3^\circ.$$

Question 1.(c). For a straight line segment between two given points, we can use the linear interpolation equation:

$$\vec{p} = t(\vec{p}_2 - \vec{p}_1) + \vec{p}_1,\ t \in [0,\ 1].$$

In our question $\vec{p}_2 - \vec{p}_1 = (2,\ -4)$. If we write the above equation in Cartesian coordinates instead of the vector form, it will be

$$x = 2t + 1,\ y = -4t + 2,\ t \in [0,\ 1].$$

Question 1.(d). Behind the page.

Question 2.(a). Substituting $R_1 = 3(P_2 - P_1)$ and $R_2 = 3(P_4 - P_3)$ in the given equation yields

$$\begin{aligned} Q(t) = P_1(2t^3 - 3t^2 + 1) + P_4(-2t^3 + 3t^2) + \\ (3P_2 - 3P_1)(t^3 - 2t^2 + t) + (3P_4 - 3P_3)(t^3 - t^2). \end{aligned}$$

Collecting the t terms by P_1, P_2, P_3, P_4 gives

$$\begin{aligned} Q(t) &= P_1(-t^3 + 3t^2 - 3t + 1) + 3(t^3 - 2t^2 + t)P_2 - 3(t^3 - t^2)P_3 + t^3 P_4 \\ &= (1-t)^3 P_1 + 3t(1-t)^2 P_2 + 3t^2(1-t)P_3 + t^3 P_4, \end{aligned}$$

which is exactly the basis functions for the Bezier curve.

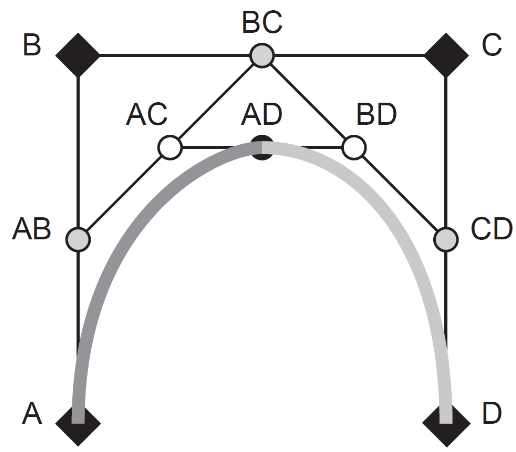


Figure 1: The de Casteljau algorithm as shown in the Figure 15.17 in the textbook, section 15.6.1, *"Bézier Curves"*.

Question 2.(b). As shown in Figure 1:

$$\begin{aligned}
A &= (-1,\ -2),\ B = (-5,\ 4),\\
C &= (11,\ 2),\ D = (17,\ -16),\\
AB &= \frac{A+B}{2} = (-3,\ 1),\\
BC &= \frac{B+C}{2} = (3,\ 3),\\
CD &= \frac{C+D}{2} = (14,\ -7),\\
AC &= \frac{AB+BC}{2} = (0,\ 2),\\
BD &= \frac{BC+CD}{2} = (\frac{17}{2},\ -2) = (8.5,\ -2),\\
AD &= \frac{AC+BD}{2} = (\frac{17}{4},\ 0) = (4.25,\ 0).
\end{aligned}$$

The first Bezier segment (dark grey in the figure) is defined by the control points $(A,\ AB,\ AC,\ AD)$. The second segment (light grey in the figure) is defined by $(AD,\ BD,\ CD,\ D)$.

Question 5.(a). The homogeneous coordinate $(12,\ 6,\ 3)$ corresponds to a two dimensional Cartesian coordinate $(4,\ 2)$. $(15,\ 10,\ 5)$ corresponds to $(3,\ 2)$.

Question 5.(b). The parameter space of a parametric curve is the space from which the parameters of the curve draw their values. For a parametric curve with only one parameter, such as $Q(t)$, its parameter space is a one dimensional space $t \in \mathbb{R}^1$. The object space is where the curve itself resides. For a planar curve it is the two dimensional space $Q \in \mathbb{R}^2$.

For a given set of control points P_1, ..., P_n, a point on the parametric curve $Q(t)$ is calculated as a weighted summation of all the control points, taking the basis functions as the weights.

$$Q(t) = \sum_{i=1}^{n} B_i(t) P_i.$$

Question 5.(c). As explained in the textbook, section 7.1.3, *"The Camera Transformation"*:

$$\begin{aligned}
\mathbf{w} &= -\frac{\mathbf{g}}{|\mathbf{g}|} = (0,\ 1,\ 0), \\
\mathbf{v} &= \mathbf{w} \times \mathbf{u} = (1,\ 0,\ 0), \\
\mathbf{u} &= \frac{\mathbf{t} \times \mathbf{w}}{|\mathbf{t} \times \mathbf{w}|} = (0,\ 0,\ -1).
\end{aligned}$$

$$\mathbf{M}_{\text{cam}} = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Question 5.(d). Two vectors are orthogonal if they are perpendicular to each other, have a $90°$ angle between them, and their inner product (dot product) is zero. The last property can be used to determine if two vectors are orthogonal

Question 5.(e). A special orthogonal matrix has a determinant of 1. The inverse of a special orthogonal matrix is its transpose.

Question 5.(g). The movement of the house shown in the picture consists of a translation of the house to the origin point, a scaling by a half, a counter-clockwise rotation by 90° and a translation to $(0,\ -3,\ 0)$. Therefore,

$$
\begin{aligned}
\mathbf{M} &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.5 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} 0 & 0.5 & 0 & 0 \\ -0.5 & 0 & 0 & -1.5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned}
$$

Please refer to textbook section 6.3 *"Translation and Affine Transformations"* for how to construct these matrices.

answers by lucky Feb 2016.