University of Victoria
Department of Computer Science
CSC 355 Digital Logic and Computer Design
ASSIGNMENT 1 DUE Thursday September 24 AT BEGINNING OF CLASS

Neatness Counts!

It is expected that answers to assignments are either typed or written *extremely* neatly. In all cases, the Karnaugh Maps formats below *must* be used, either copied and edited to add the required bits and circles or printed and written on.

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1. Determine the radix r given that $(257)_r = (11010110)_2$ (show all your work).

$(11010110)_2 = 128+64+16+4+2 = 214_{10}$
Note: It is clear that r is less than 10 because 214 is smaller than 257.
Using a trial and error method, guess 9:
$2\text{x}9^2+5\text{x}9+7 = 162+45+7 = 214_{10}$ $\therefore$ base 9 is the correct answer!

2. Simplify the following expression, using Boolean algebra, into minimum sum of products form:
$(B+C)(\overline{B}+D)(\overline{A}+C)(\overline{A}+\overline{D})$
Show each Boolean algebra rule used in the simplification.

$(B+C)(\overline{B}+D)(\overline{A}+C)(\overline{A}+\overline{D})$
$= (B\overline{B}+BD+\overline{B}C+CD)(\overline{A}\,\overline{A}+\overline{A}C+\overline{A}\,\overline{D}+C\overline{D})$,distributive
$= (BD+\overline{B}C+CD)(\overline{A}+\overline{A}C+\overline{A}\,\overline{D}+C\overline{D})$, complement, idempotent
$= (BD+\overline{B}C)(\overline{A}+C\overline{D})$, consensus, absorption
$= \overline{A}BD+\overline{A}\,\overline{B}C+BCD\overline{D}+\overline{B}CC\overline{D}$, distributive
$= \overline{A}BD+\overline{A}\,\overline{B}C+\overline{B}C\overline{D}$, complement, idempotent

3. Convert the following expression into minimum product of sums form :
$\overline{A}\,\overline{C}\,\overline{D}+A\overline{B}\,\overline{D}+ACD+\overline{A}BD$

F = $\overline{A}\,\overline{C}\,\overline{D}+A\overline{B}\,\overline{D}+ACD+\overline{A}BD$

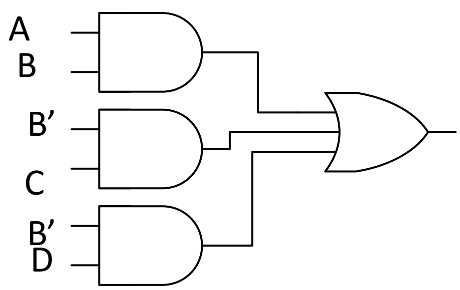
AB \ CD	00	01	11	10
00	1	0	0	0
01	1	1	1	0
11	0	0	1	0
10	1	0	1	1

F' = B'C'D+ABC'+A'B'C+BCD'
$\therefore$ F=(B+C+D')(A'+B'+C)(A+B+C')(B'+C'+D)

4. Design circuits as specified below. In each case assume that all variables are available in both true and inverted form. Gates may have any fan-in up to 4. Marks will be awarded for keeping the total number of gates as small as possible.

   a. a two level circuit for a(b + a') + b' (c + d) +ab using only AND, OR gates;

F = a(b + a') + b' (c + d) +ab = ab + aa' + b'c + b'd + ab = ab + b'c + b'd

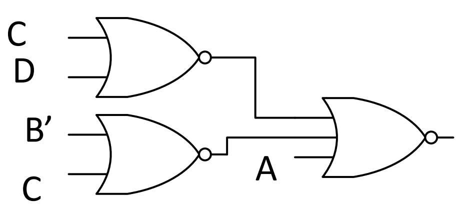


AB \ CD	00	01	11	10
00		1	1	1
01				
11	1	1	1	1
10		1	1	1

Its already minimal. . . .I did not need the K-map!

   b. a circuit for a'bc + a'b'd + a'c using NOR gates only

F = a'bc + a'b'd + a'c

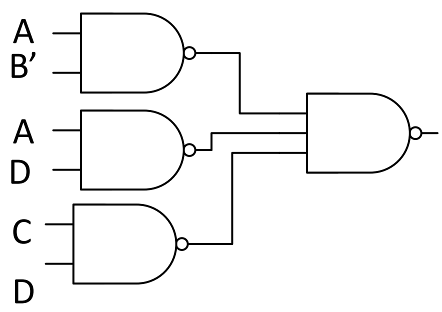


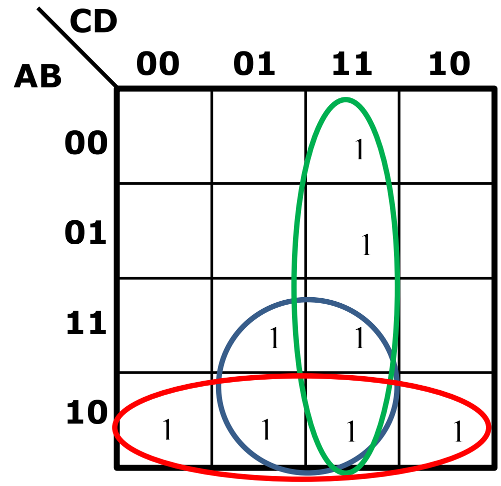
AB \ CD	00	01	11	10
00	0	1	1	1
01	0	0	1	1
11	0	0	0	0
10	0	0	0	0

F' = c'd'+bc'+a   ∴F=(c+d)(b'+c)a'

c. a circuit for ab'c' + a'cd + ab'c + abd using NAND gates only.

F = ab'c' + a'cd + ab'c + abd





F = ab’+ad+cd

5. Simplify the following Boolean expressions (to the simplest sum of products form) using three variable maps.

a. F(A,B,C) = $(\bar{A}\,\bar{C} + \bar{A}\,BC + \bar{B}C)$

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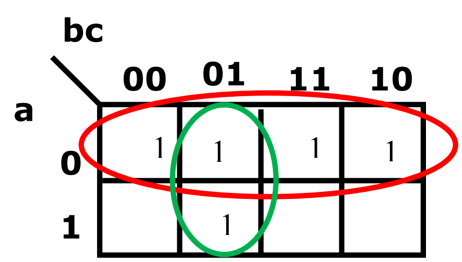
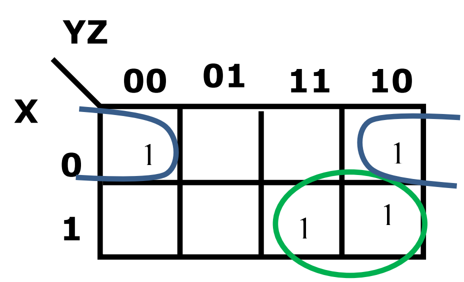


F =a+b’c

b. F(X,Y,Z) = $\sum m(0,2,6,7)$

F(A,B,C) = $\sum m(0,2,6,7)$



F =x’z’+xy

6. Simplify the following Boolean expressions (to the simplest sum of products form) using four variable maps:

a. F(A,B,C,D) = $\left(A+B+\overline{C}\right)\left(\overline{A}+\overline{B}+D\right)\left(\overline{A}+C+\overline{D}\right)\left(A+\overline{C}+D\right)$

F(A,B,C,D) = $\left(A+B+\overline{C}\right)\left(\overline{A}+\overline{B}+D\right)\left(\overline{A}+C+\overline{D}\right)\left(A+\overline{C}+D\right)$

AB \ CD	00	01	11	10
00	1	1	0	0
01	1	1	1	0
11	0	0	1	0
10	1	0	1	1

F = A’C’+BCD+AB’D’+AB’C

b. F(w,x,y,z) = $\sum m(0,1,5,8,10,12,13,14)$ (3 terms)

F(A,B,C,D) = $\sum m(0,1,5,8,10,12,13,14)$

WX \ YZ	00	01	11	10
00	1	1		
01		1		
11	1	1		1
10	1			1

F = W’X’Y’+XY’Z+WZ’

c. F(w,x,y,z) = $\sum m(0, 1, 2, 8, 9, 10, 12, 14)$ (3 terms)

F(A,B,C,D) = $\sum m(0, 1, 2, 8, 9,10, 12, 14)$

AB \ CD	00	01	11	10
00	1	1		1
01				
11	1			1
10	1	1		1

F = B'C'+AD'+B'D'

7. Given the function F(w,x,y,z) = $\sum m(1, 3, 7, 13)$ + d(4, 5, 9, 14). Find:
   a. the minimum sum of products form for F

F(w,x,y,z) = $\sum m(1, 3, 7, 13)$ + d(4, 5, 9, 14).

WX \ YZ	00	01	11	10
00		1	1	
01	x	x	1	
11		1		x
10		x		

F = W'Z+Y'Z

b. the minimum product of sums form for F.

F(w,x,y,z) = $\sum m(1, 3, 7, 13) + d(4, 5, 9, 14)$.

WX \ YZ	00	01	11	10
00	0	1	1	0
01	x	x	1	0
11	0	1	0	x
10	0	x	0	0

F' = Z'+WY $\quad\therefore$ F = Z(W'+Y')

8. A switching network has 3 inputs and 2 outputs. The output variables, a and b, represent the first and second bits, respectively of a binary number, N. N equals the number of inputs which are "1". For example, if w=1, x=0. y=1, then a=1, b=0.
   a. Find the minterm expansion for a and b.
   b. Find the maxterm expansion for a and for b.

   Express each answer in abbreviated notation (with Σ or π).

w	x	y	a	b
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

a. a = Σm(3, 5, 6, 7) b = Σm(1, 2, 4, 7)

b. a = πM(0, 1, 2, 4) b = π M(0, 3, 5, 6)