CSC 225 SPRING 2008
ALGORITHMS AND DATA STRUCTURES I
FINAL EXAMINATION
UNIVERSITY OF VICTORIA

1. Student ID: ______________________
2. Name: ______________________
3. DATE: 14 APRIL 2008
   DURATION: THREE HOURS
   INSTRUCTOR: V. SRINIVASAN
4. THIS QUESTION PAPER HAS FOURTEEN PAGES (INCLUDING THE COVER PAGE).
5. THIS QUESTION PAPER HAS EIGHT QUESTIONS.
6. ALL ANSWERS TO BE WRITTEN ON THIS EXAMINATION PAPER.
7. THIS IS A CLOSED BOOK EXAM. NO AIDS ARE ALLOWED.
8. KEEP YOUR ANSWERS SHORT AND PRECISE.

Q1 (15)	
Q2 (15)	
Q3 (10)	
Q4 (10)	
Q5 (15)	
Q6 (10)	
Q7 (10)	
Q8 (15)	
TOTAL (100) =	

1. [15 Marks] For each of the following, give the correct answer by circling one of the two choices, TRUE or FALSE.

(a) 2^{100} is $O(1)$. TRUE FALSE

(b) $3 \log n + 2 \log \log n$ is $\omega((\log n)^2)$. TRUE FALSE

(c) $\log_b(a^c) \neq c \log_b a$. TRUE FALSE

(d) Stacks are used to implement method invocation in run-time environment. TRUE FALSE

(e) A Doubly-linked list of size n can support *insertAtRank* operation in worst-case $O(\log n)$ time. TRUE FALSE

(f) A sorted sequence is an implementation of the ADT priority queue. TRUE FALSE

(g) A binary search tree with n nodes always has height $O(\log n)$. TRUE FALSE

(h) The worst-case running time of Quick-Sort and Selection-Sort are equal in the asymptotic sense. TRUE FALSE

(i) The heap is a special case of a binary search tree. TRUE FALSE

(j) For any simple, undirected graph G on n vertices and m edges, $m \leq n(n-1)/2$. TRUE FALSE

(k) Baruvka's Algorithm for computing minimum-spanning-tee is an example of a greedy algorithm. TRUE FALSE

(l) The adjacency-list data structure for graphs can support the *areAdjacent(u,v)* operation in $O(1)$ time. TRUE FALSE

(m) Given any graph G on n nodes and m edges, it is possible to check if G has a cycle or not in worst-case running time $O(m)$. TRUE FALSE

(n) If G is a tree on n nodes with m edges, $m = n - 1$. TRUE FALSE

(o) Dynamic programming technique can be used to give an algorithm for the All-Pairs-Shortest-Path problem. TRUE FALSE

2. (i)[5 Marks] Order the following functions by increasing growth rates:

   $1^{n \log n}$, $n^{\log n}$, 2^5, $\sqrt{\log n}$, $2^{n!}$, $\frac{1}{n}$, n^2, $2^{\log n}$, $n!$, 100^n.

   (ii) Consider the following recursive algorithm:

   QUIBBLE(n)
   if $n = 0$
   then return 1
   else return **QUIBBLE**($n-1$)+**QUIBBLE**($n-1$)

   Let $T(n)$ count the number of addition operations in this algorithm on input n.

   (a)[5 Marks] Write down the recurrence equation for $T(n)$.

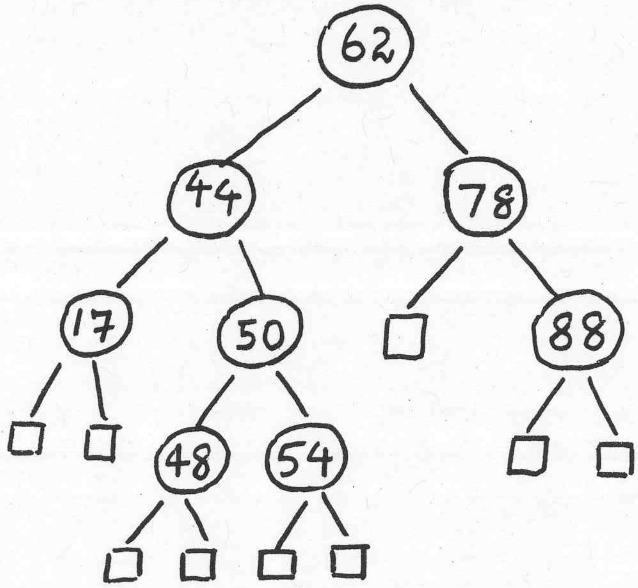
(b)[5 Marks] Solve the recurrence equation and get a closed form for $T(n)$.

3. Suppose that the alphabets A, B, C, ..., Z are encoded using the numbers 0, 1, 2,..., 25. That is, A is encoded as 0, B as 1 and so on.

   Suppose the keys E A S Y Q U E S T I O N are inserted in that order into an initially empty 13-item hash table using the hash function $h(k) = k \bmod 13$. Show the contents of the final hash table if (i)[5 Marks] Open chaining was used for collision resolution.

(ii)[5 Marks] Linear probing was used for collision resolution.

4. Consider the following AVL Tree:



(ii)[5 Marks] Draw the AVL tree after removal of the key 62 from the AVL Tree shown above. Show all the steps of the restructuring procedure, if any.

(i)[5 Marks] Draw the AVL Tree resulting from the insertion of the key 51. into the AVL Tree shown on the previous page. Show all the steps of the restructuring procedure, if any.

5. (i)[5 Marks] Show all the steps in the execution of Bubble-Sort on the following example:

12 62 51 25 47 9 38 92 84 70

(ii)[3 Marks] What is a stable sorting algorithm? Give an example.

(iii)[7 Marks] Explain why any comparison-based sorting algorithm has a lower bound of $\Omega(n \log n)$ on its worst-case running time.

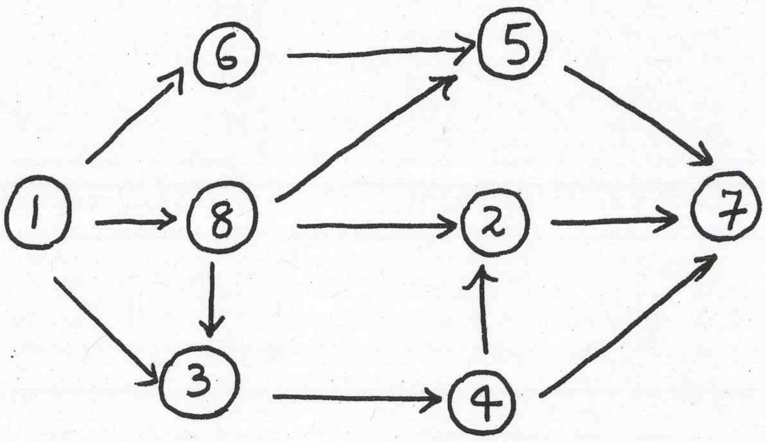
6. (i) [6 Marks] Describe in pseudo-code the algorithm *quickSelect* that outputs the kth element of a sequence of n elements. You can assume that the algorithm picks the last element of S as the pivot.

(ii) [4 Marks] Use the Master Theorem to solve the following recurrences:

$$\begin{aligned} T(n) &= c \text{ for } n < d \\ &= 4T(n/2) + \log n \text{ for } n \geq d \end{aligned}$$

$$\begin{aligned} T(n) &= c \text{ for } n < d \\ &= 2T(n/2) + n^2 \text{ for } n \geq d \end{aligned}$$

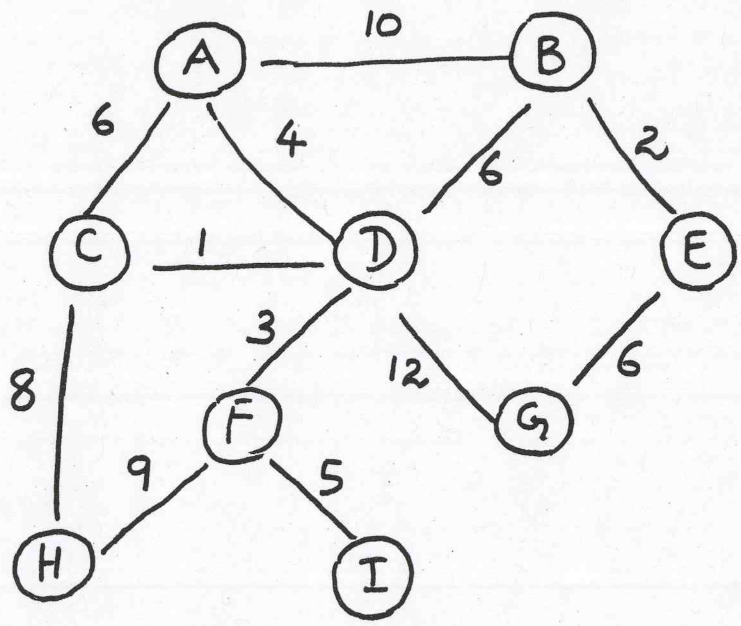
7. Consider the following directed graph G:



(i)[4 Marks] Perform a DFS traversal on the graph above starting at vertex 1. Assume that, in the traversal, the adjacent vertices of a given vertex are returned in increasing order of the vertex names. Use the DFS ordering to check if a topological ordering of the vertices of G exists. Show one, if any exists.

(ii)[6 Marks] Compute its transitive closure and clearly list all the edges that are not in G but belong to its transitive closure in the order in which they are discovered.

8. Consider the following undirected, weighted graph G:



(i)[9 Marks] Find a minimum spanning tree of G above. Clearly, list all the edges in the order in which they are discovered. What is the weight of the minimum spanning tree?

(ii)[6 Marks] Give a proof of correctness that explains why Kruskal's algorithm always outputs a minimum spanning tree.

END OF EXAM