Designing An Unsupervised Segmentation

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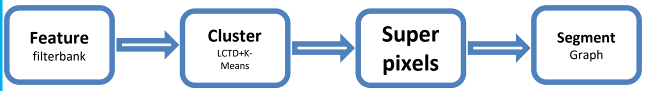
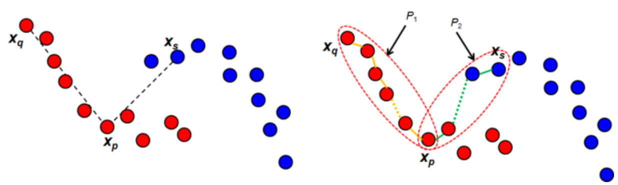
I. Flowchart



II. Transitive Distance



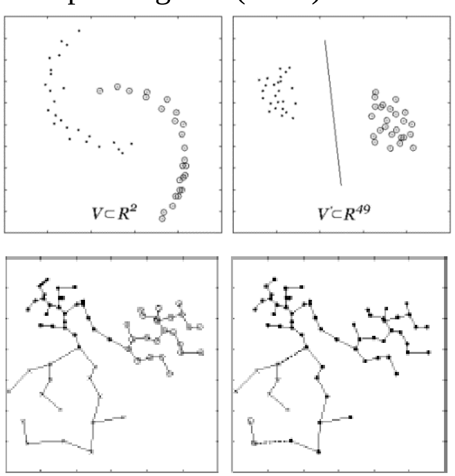
Euclidean Distance **Transitive Distance**

Math Definition:

$$D_T(x_p, x_q) = \min_{\mathcal{P}\in\mathbb{P}} \max_{e\in\mathcal{P}} \{d(e)\}$$

Proposition 1: Every finite ultrametric space with n distinct points can be isometrically embedded into an n−1 dim Euclidean space.

Proposition 2: Given a weighted graph with edge weights, each transitive edge lies on the minimum spanning tree (MST).



III. LCTD Clustering

The Locality Constrained Transitive Distance Defination:

LCTD-1:

$$D_{LCTD-1}(i,j) = D_{td}(i,j) * D_{eu}(i,j),$$

LCTD-2:

$$D_{LCTD-2}(i,j) = \alpha D_{td}(i,j) + (1-\alpha) D_{eu}(i,j),$$

where $D_{td}(i,j)$ is the transitive distance and $D_{eu}(i,j)$ is the Euclidean distance. α is a weight parameter to balance locality

Algorithm 1 Computing the transitive distance matrix

1: Construct a total graph $G = (V, E)$ from data where edge weights in E are the Euclidean distances.
2: Sort E based on edge weights. Initialize $G_{MST} = (V, E')$ where $E' = \varnothing$.
3: Take an edge $e_{i,j}$ from E.
4: If G_{MST} and $e_{i,j}$ form a circle, discard $e_{i,j}$.
5: Otherwise, add $e_{i,j}$ to E'. Find sets of nodes V_i and V_j currently connected to edge nodes i and j respectively in G_{MST}. Update the pairwise distances of all possible combinations with $|e_{i,j}|$.
6: Repeat 3-5 sequentially for all edges in E.

IV. A New Top-Down Clustering Framework

LCTD matrix ~ a block diagonal matrix

$$D_{LCDT} = D_{block} + E$$

Top-down clustering: k-means over the rows of rotated and normalized distance matrix.

$$D' \triangleq D_{LCDT} V \Lambda^{-1} = U$$

where Λ is the eigenvalue matrix of D_{LCDT} and V is the initial rotation basis:

$$D_{LCDT} = U \Lambda V^*$$

V. Results

Dataset:

-- Quantitative segmentation evaluation

	PRI	VoI	GCE	BDE
MGD	0.7559	2.4701	0.1925	15.10
NTP	0.7521	2.4954	0.2373	16.30
Ncut	0.7853	2.1031	0.1947	**12.9703**
PRIF	**0.8006**	–	–	–
Ours	0.7926	**2.0871**	**0.1835**	13.1707