

**NoelKDF:  
A Compute Time and Sequential Memory Hard  
Key Derivation Function**



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For Consideration in the Password Hashing Competition

Chapel Hill, NC, January 2014

***Abstract.** Scrypt demonstrated a new way to thwart off-line brute-force password guessing attacks, as the first successful sequential memory hard key derivation function (KDF). While Scrypt forces an attacker to use significant amounts of memory, custom ASICs can speed up Scrypt's inner loop substantially. Attackers also gain up to a 4X benefit in time\*memory cost through a time-memory trade-off (TMTO). With cache-timing information, an attacker can attack Scrypt on a massively parallel scale. NoelKDF is a new “hybrid” sequential-memory-hard KDF which is compute time hardened through sequential multiply operations in its inner loop, limiting a custom ASIC core speed advantage to about 2X. NoelKDF greatly increases the time\*memory cost per core of custom ASIC attacks, while greatly reducing any benefit from time-memory trade-offs. NoelKDF's “hybrid” design also provides good defense against cache-timing attacks, without sacrificing time\*memory performance.*

## 1 Introduction

NoelKDF is a sequential-memory and compute-time hard key derivation function (KDF) that maximizes an attackers time\*memory cost for guessing passwords. NoelKDF:

- Insures high memory cost for an attacker by filling memory rapidly
- Insures high runtime for an attacker through serial multiply operations that are difficult to speed up on an ASIC
- Mostly eliminates time-memory trade-off benefits to attackers
- Provides effective defense against cache-timing attacks
- Can use parallelism present on the machine for improved protection
- Is suitable for desktop PCs, web servers, embedded applications, and high end graphics cards
- Supports client independent hash updates
- Performs well hashing many GiB of DRAM, or just a few MiB of cache
- Is named after a cat

In this paper, memory-hard KDFs which do no password derived memory addressing are called “pure”, while those that do so early in hashing are called “dirty”, and combinations of the two are called “hybrid”. “Pure” memory-hard KDFs are 100% immune to cache-timing attacks, while “dirty” KDFs can be attacked on a massively parallel scale once an attacker knows what memory addresses are accessed when hashing a correct password. “Hybrid” KDFs can have good resistance against cache-timing attacks while at the same time can be as effective against brute-force guessing attacks as dirty KDFs. Scrypt[2] is a “hybrid” KDF, and suffers far less from cache-timing attacks than if it were “dirty”, but since it was not

designed specifically to thwart cache-timing attacks or TMTO attacks, its resistance can be improved.

Like Scrypt, NoelKDF uses a 2-loop architecture, where in the first loop is a “pure” KDF. No password derived memory addresses are accessed in the first loop. The second loop is a “dirty” KDF, which significantly improves defense against offline brute-force guessing attacks as well as limiting TMTO options for an attacker.

Against cache-timing-attacks, NoelKDF has a time\*memory cost about 16X lower than when no addressing information is known since the second loop can be aborted early, and the first loop can be computed efficiently with about 1/8th of the total memory. However, NoelKDF's “dirty” second loop provides about a 3-4X improved time\*memory cost than “pure” KDFs. As a result, NoelKDF should have only about a 5-to-1 disadvantage against cache-timing attacks relative to “pure” KDFs. NoelKDF trades off some cache-timing attack resistance in order to retain full resistance against brute-force guessing attacks.

While I deserve credit for this poor document, my fairly awesome code, and the inspiration for naming a key derivation function after my cat<sup>1</sup>, most of the good ideas found here come from the authors of Escrypt and Catena[1], as well as several other members of the PHC email list who also contributed ideas and feedback. The cache timing defense strategy is only possible due to the excellent work and generosity of ideas from Christian Forler, while many of the other good ideas came from Alexander Peslyak (aka Solar Designer). I consider them both to be the unofficial primary authors of this work until such time as they prefer to be listed in the top spots officially. Since they have generously offered their ideas to the other PHC entrants, I accept the help they offered in the spirit intended, and hope that in some small ways I have helped the eventual winning entry to be better. In particular, any PHC author should feel free to borrow any ideas in NoelKDF to improve their own entry.

## 2 Algorithm Specification

### 2.1 Simplified “Dirty” NoelKDF

NoelKDF was originally a “dirty” KDF, until Christian Forler demonstrated an effective strategy against cache-timing attacks, which sent the other PHC authors scrambling. A simplified version of “dirty” NoelKDF is:

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<sup>1</sup> My wife informs me that naming NoelKDF after our cat was actually her idea.

*Simplified Dirty NoelKDF:*

Inputs: hashlen, password, salt, memlen

Output: hash[hashlen]

```
mem[0 .. hashlen-1] = H(hashlen, password, salt)
value = 0
prevAddr = 0
destAddr = hashlen
for i = 1 .. memlen/hashlen:
    randAddr = value % i
    for j = 0 .. hashlen-1:
        value = value*(mem[prevAddr++] | 3) + mem[randAddr++]
        mem[destAddr++] = value
    output H(hashlen, mem[memlen-hashlen .. memlen-1], salt)
```

The function H is a cryptographically strong hash function such as PBKDF-SHA256, which is used in the reference implementation.

## 2.2 Multiplication-Hard KDFs

Speeding up multiplication on custom ASICs versus modern CPUs is difficult, and unlikely to result in a significant speed-up without exotic technologies such as liquid nitrogen cooling. In comparison, the Salsa20/8 hash function used in Scrypt[2] is likely to run at about 1ns per Salsa20/8 round per core, generating 64GiB/second of data on a well optimized advanced technology custom ASIC. In comparison, Scrypt on my development machine hashes 0.5GiB twice in 1.8 seconds, which is about 100X slower. This is because it has only 16 levels of ADD/XOR logic per 32-bit register, of which there are 16, making Salsa20/8 it a bit less complex than 16 Booth-encoded 32x32 multipliers running in parallel. On the development machine, 32x32 bit multiplies have a 0.9ns latency.

NoelKDF is hardened against such attacks through the use of a multiplication-hard hash function, which cannot be significantly sped up on custom AISCs compared to advanced CPUs. Each iteration depends on the previous, and must be computed with one sequential multiply followed by one sequential addition. A lower bound on the runtime is:

$$T(memlen) \geq memlen \times multTime$$

On my development machine, a 32-bit multiply operation has latency  $multTime = 0.9ns$ . Hashing 2GiB of data this way would require a minimum of  $0.9ns \times 2^{31}/4 = 0.48$  seconds. The reference version of NoelKDF performs this calculation in 0.69 seconds, or only 44% longer than an optimally multiplication bound loop.

### **Definition: Multiplication-Hard Hash Function**

*A multiplication-hard hash function is a hash function that sequentially computes values using no more than a 1-to-3 ratio of sequential multiplication operations to sequential simple*

*operations, where simple operations are the usual single-cycle ALU operations: add, sub, XOR, AND, OR, complement, increment, decrement, and shift/rotate.*

The reason for the choice of 1-to-3 is that current advanced Intel, AMD, and ARM processors have either 3 or 4 clock cycle latencies to compute a 32x32 multiplication, while all of the simple operations are computed in 1. This means in a multiplication-hard hash function, it may be possible to spend at least 50% of the compute time on multiplications, assuming other tasks can execute in parallel. NoelKDF has a multiplication to simple operation ratio of 1-to-1. It does one multiply and one addition sequentially in every loop. The other operations, including increments, OR, and memory read and write can be done in parallel.

NoelKDF is sequential-memory-hard in the sense described in Catena[1]. Additional CPUs cannot significantly speed up its computations, since the random address computed in the second loop cannot be known until the prior loop iteration has completed. If an attacker decides to cheat and use less memory, he will suffer a penalty when he has to recompute the missing data, just as in Script[2]. Even stronger, NoelKDF memory blocks written in the second loop depend on not just the previous, as in Script, but also on another block earlier in memory. Attackers will have to recompute any missing input blocks that are accessed. With NoelKDF's 2-fanouts per node, this can grow exponentially through the computation DAG if an attacker has too little coverage of computed results in memory.

## **2.3 Algorithm Enhancements**

By far, the most extensive enhancement over the original NoelKDF algorithm is the addition of a “pure” first loop. Several other enhancements were needed as well.

### **2.3.1 Speed Enhancements**

For small hashlen, cache miss delays will dominate the runtime. Therefore NoelKDF has a separate blocklen parameter to enable high speed with short hash lengths.

The reference implementation, when run with 2 threads, achieves 12GiB/s memory bandwidth on a machine I estimate to have a 25GiB/s maximum. A SIMD optimized version with temporal cache hints is likely to achieve 22GiB/s, as running 2 copies of 2-thread NoelKDF at the same time increases memory bandwidth to 22GiB/s.

While the original NoelKDF algorithm was optimized for external DRAM hashing, the current version is equally suitable for running in small amounts of memory entirely from CPU cache. This is enabled by choosing a few MiB rather than a GiB or more, setting blockSize to a small value such as 64, which is well matched to cache line sizes, and setting “repetitions” to high enough of a value to make the runtime long enough.

### **2.3.2 Improvements Over Pepper**

Pepper is a small number of secret bits of the salt, which are sent as 0's to the user. When authenticating, he must try all the possible salt guesses until he guesses the correct salt. This

has the impact of increasing the compute effort for both an attacker and defender. If pepper is chosen well, the user can try all the salt guesses in parallel, feeling little increase in latency, while an attacker feels the full impact.

This scheme has some problems. First, it gives an attacker a free TMTO, and eliminating these trade-offs is a major goal of NoelKDF. Second, if we use pepper in a server-relief system, it is likely that the user will attempt to authenticate with the server multiple times, increasing network traffic. Finally, it seems to be naturally applied externally to the password KDF, and won't be able to take full advantage of SIMD instructions when used that way, especially when the inner loop hash function is very simple as is the case with NoelKDF.

To overcome these limitations, NoelKDF executes with a user-specified level of parallelism. A pthread version of NoelKDF has been developed to demonstrate the use of this parameter, while the reference version simply computes the parallel tasks in series.

### *2.3.3 Garlic and Data*

NoelKDF “borrows” the concept of “garlic” from Catena[1], along with other goodies such as “client independent update”, and of course a cache-timing resistance strategy. I also copied `catena_test_vectors.c` to generate vectors for NoelKDF. Please read the Catena paper for a more in-depth description of these features. Client independent updates are where a sysadmin can increase the difficulty of computing a password hash without having to know the user's password.

A data input has also been added, following Catena's naming convention, which allows a software developer to provide application specific data such as secondary authentication keys to the hash. For example, when a server has a password specific key that is decrypted using the server's master key during password authentication, that password specific key can be added as data to the hash. This renders brute force attacks impractical to attackers who do not have the master key. Similarly, if a user provides a key file, a hash of that file can be used as data, again making brute-force attacks impractical. A similar concept appears in Escrypt (local parameters) and Blakrypt (session keys).

### *2.3.4 Additional Enhancements*

In the second “dirty” loop, the prior memory block to hash is no longer chosen uniformly randomly. Instead it follows a cube-law, reducing the average edge length from  $\frac{1}{2}$  to  $\frac{1}{4}$ , significantly improving TMTO resistance. Having short edges causes the computation DAG depth to be considerably higher, with typical sub-DAGs of attackers using only 1/8th of memory being on average around 1/8th the size of the entire DAG, even with decent strategies for pebble placement, such as evenly spacing them, covering high degree nodes, and covering nodes with short incoming edges. With a uniform distribution for selecting the prior memory block to hash, only 1 in 100 edges are shorter than 1% of the node number,

while with the cubed distribution, 21.5% are this short. The average edge length from node  $i$  is computed as:

$$l = i \int_0^1 x^3 dx = \frac{1}{4} i$$

One intended use for NoelKDF is specifically in applications like TrueCrypt where only parameters that are indistinguishable from true random data can be used. Salt is such a parameter, but garlic, memory-size, and such are not. In these cases, the application can choose default settings for most values, and select an appropriate garlic level. After hashing the user's password, the resulting derived key is hashed one more time with a strong cryptographic hash and stored along with the salt in the encrypted volume. When verifying a password later, the application compares the resulting hash after each level of garlic is applied, and stops if it matches. This can run as a Password Halting Puzzle[3].

## 2.4 The NoelKDF Algorithm

All integers are unsigned 32-bit, unless they specify a memory index, in which case they are 64-bits to allow for hashing more than 4GiB of memory. They may be 32-bits instead on 32-bit platforms. Implementers need to be careful to insure that 32-bit values are cast to 64-bits during computations resulting in a 64-bit results. Hashsize and blocksize must be multiples of 4. Ranges are kept low enough to allow signed integers to be used rather than unsigned. Total memory, which is  $2^{20} * \text{memsize} * 2^{\text{garlic}}$  should be  $\leq 2^{50}$ . Memlen, specified in MiB, must be long enough for at least 2 blocks per parallel task. If ranges are invalid, or memory cannot be allocated, the algorithm returns null for versions that return byte arrays such as Python, and false for versions that write to a user-supplied buffers such as the C reference version.

The algorithm for NoelKDF is:

*NoelKDF Algorithm:*

Inputs: hashsize, an integer from 1 .. 1024 specifying byte length of hash result in bytes

password: byte array of length 1 .. 1024

salt: byte array of length 1 .. 1024

memsize: 32-bit int 1 ..  $2^{30}$  specified in MiB

garlic: 32-bit integer 0 .. 30 specifies multiplier on memsize of  $2^{\text{garlic}}$

data: byte array of length 0 .. 1024, may be null

blocksize: integer 4 ..  $2^{30}$  specifies memory hashed at once in bytes

parallelism: integer 1 ..  $2^{20}$

repetitions: integer 1 ..  $2^{30}$

Output: hash result in byte array of length hashsize, or null if there is an error

function NoelKDF\_HashPassword(hashsize, password, salt, memsize, garlic, data,  
blocksize, parallelism, repetitions):

```

if input ranges are invalid:
    return null
if data != null:
    derivedSalt = H(hashsize, data, salt)
    hash = H(hashsize, password, derivedSalt)
else:
    hash = H(hashsize, password, salt)
wordHash = toUint32Array(hash)
memlen = memsize/4
blocklen = blocksize/4
numblocks = memlen/(2*parallelism*blocklen)
mem = array of 2*numblocks*blocklen*parallelism*2^garlic 32-bit integers
if mem allocation failed:
    return null
for i = startGarlic .. stopGarlic:
    value = 0
    for p = 0 .. parallelism-1:
        hashWithoutPassword(p, wordHash, mem, blocklen, numblocks, repetitions)
        // This truncates to 32 bits:
        value += mem[2*p*numblocks*blocklen+blocklen-1]
    for p = 0 .. parallelism-1:
        hashWithPassword(p, mem, blocklen, numblocks, parallelism, repetitions, value)
    xorIntoHash(wordHash, mem, blocklen, numblocks, parallelism)
    numblocks *= 2
return toUint8Array(wordHash)

function hashWithoutPassword(p, wordHash, mem, blocklen, numblocks, repetitions):
    uint64 start = 2*p*numblocks*blocklen
    mem[start .. start+blocklen-1] = toUint32Array(H(blocklen*4,
        toUint8Array(wordHash), toUint8Array([p])))
    value = 1
    mask = 1
    uint64 toAddr = start + blocklen
    for i = 1 .. numblocks-1:
        if mask << 1 <= i:
            mask = mask << 1
        reversePos = bitReverse(i, mask)
        if reversePos + mask < i:
            reversePos += mask
        uint64 fromAddr = start + blocklen*reversePos
        value = hashBlocks(value, mem, blocklen, fromAddr, toAddr, repetitions)
        toAddr += blocklen

function hashWithPassword(p, mem, blocklen, numblocks, parallelism, repetitions, value):
    startBlock = 2*p*numblocks + numblocks
    // start, toAddr, v, v2, v3, and distance are 64-bit ints
    uint64 start = startBlock*blocklen
    uint64 toAddr = start

```



```

for i = 0 .. numblocks-1:
    uint64 v = value
    uint64 v2 = v*v >> 32
    uint64 v3 = v*v2 >> 32
    uint64 distance = (i + numblocks - 1)*v3 >> 32
    if distance < i:
        uint64 fromAddr = start + (i - 1 - distance)*blocklen
    else:
        q = (p + i) % parallelism
        b = numblocks - 1 - (distance - i)
        uint64 fromAddr = (2*numblocks*q + b)*blocklen
    value = hashBlocks(value, mem, blocklen, fromAddr, toAddr, repetitions)
    toAddr += blocklen

function hashBlocks(value, mem, blocklen, fromAddr, prevAddr, toAddr, repetitions):
    uint64 prevAddr = toAddr - blocklen
    for r = 0 .. repetitions-1:
        for j = 0 .. blocklen-1:
            // This truncates to 32-bits
            value = value*(mem[prevAddr + j] | 3) + mem[fromAddr + j]
            mem[toAddr + j] = value
    return value

function xorIntoHash(wordHash, mem, blocklen, numblocks, parallelism):
    for p = 0 .. parallelism-1:
        uint64 pos = 2*(p+1)*numblocks*blocklen - length(wordHash)
        for i = 0 .. length(wordHash)-1:
            wordHash[i] ^= mem[pos+i]

function bitReverse(value, mask):
    result = 0
    while mask != 1:
        result = (result << 1) | (value & 1)
        value >>= 1
        mask >>= 1
    return result

```

The first “pure” loop uses a power-of-two sliding window with a bit-reversal function to select the target within this window. Alexander Peslyak suggested the power-of-two sliding window to provide a more uniform distribution of incoming edges, while Christian Forler invented using a bit-reversal distribution of edges to thwart cache-timing attacks.

The second “dirty” loop is the NoelKDF cubed-distribution. When running on multiple threads, each thread computes its results from all of the threads results computed in the first loop. This forces an attacker to keep all of the first loop results in memory while the second loops run. While an attacker can run the second loop sequentially, he will still be forced to

have most of memory loaded at once (or suffer a substantial recomputation penalty), greatly increasing his time\*memory cost.

### 3 Security Analysis

#### 3.1 Results are Indistinguishable from Random

The resulting hash from NoelKDF will appear indistinguishable from true random data, and no change in input will produce a predictable change in the output. This is because it starts by creating an intermediate derived key using PBKDF2, and in the reference implementation, the hash function is SHA-256. The intermediate derived key, stored in “hash”, thus has these properties, to the extent that we believe PBKDF2-SHA256 has them. Once the intermediate derived key is computed, the password and input data are never accessed again, and can (and should) be cleared from memory. The derived key is hashed by PBKDF2 again to create the first block of data for each thread, using the thread ID,  $p$ , as the salt, and the intermediate derived key as the password. Therefore, we can be confident that the seed data for each thread is no more correlated with the derived key than PBKDF2-SHA256 allows. To compute the final hash, the results from each thread are XOR-ed into the intermediate derived key, and since the seed for each thread is uncorrelated with the derived key, the results from each thread also are uncorrelated with the derived key. No matter how apparently non-random the output of each thread may be, XOR-ing these outputs into the derived key makes it no less indistinguishable from true random data.

Because of this architecture, the inner loop hash function used to fill memory need not be cryptographically strong. NoelKDF takes advantage of this for improved speed and compute-time hardness.

#### 3.2 The NoelKDF Hash Function

NoelKDF introduces a new non-cryptographic hash function that is meant to be compute time hardened and fast rather than producing data that is undetectably non-random. There is no need for this hash function in the inner loop to produce cryptographically pseudo-random data, as the output is cryptographically pseudo-random regardless. For the inner loop hash function, there are two primary concerns:

1. Entropy loss needs to be insignificant
2. To compute  $\text{mem}[i]$ , an attacker must first compute  $\text{mem}[0 \dots i-1]$

NoelKDF uses the following inner loop hash function:

$$\text{value} = \text{value} * (\text{mem}[\text{prevAddr}] \mid 3) + \text{mem}[\text{randAddr}]$$

This is motivated from a desire for the hash function to be similar to a permutation. With permutations, no entropy is lost. If  $\text{mem}[\text{prevAddr}]$  and  $\text{mem}[\text{randAddr}]$  were replaced with a PRNG stream seeded only from the password and salt, then this would be a true

permutation, and would be reversible. Being reversible is undesirable, as it gives attackers more TMTO options.

For this hash function to work well and not lose significant entropy, `mem[prevAddr]` needs to be highly scrambled relative to value. With a default page size of 4096, value is hashed 4096 times since the last time `mem[prevAddr]` was hashed into value. Page sizes of 256 passed the dieharder tests, indicating that even 256 iterations of the hash is far enough away. A side benefit of choosing Alexander's sliding-power-of-two window in the first loop is that the 1<sup>st</sup> block, which is generated using PBKDF-SHA256, is rehashed into the stream at every node in the DAG that is a power of two, potentially reintroducing lost entropy in case there were any.

Besides making one input odd, so that the multiplication is reversible, the OR seems to reduce the possibility that an attacker can compute `mem[i]` in a closed form, enabling him to compute `mem[i]` without first computing `mem[0 .. i-1]`.

Passing the dieharder tests is not a goal in itself, but simply additional evidence that NoelKDF does not leak entropy quickly. Dieharder tests are run with a blocklen of 256 rather than 4096. This is because the 32-bit value use to mix between values in a block is an awfully thin pipe for mixing 4KB quickly. The dieharder tests seem to detect correlations between sequential memory blocks if they are too long. However, this does not mean the hash function is losing entropy. It simply means that the dieharder tests can detect the low amount of mixing between large adjacent pages. The effect is similar to what would happen if we wrote out every value as it is computed in Salsa20/8. Writing out data every 8 rounds easily passes the dieharder tests, but if we wrote out all 64 computed values for every round of Salsa20/2, there is no way this stream would pass tests for randomness. However, it would generate considerably more data much more quickly while not losing entropy, and attackers would still be forced to compute it all. This would be a very effective strategy for a fast memory-hard KDF hash function, except that Salsa20/8, as well as most cryptographically strong hashes, use CPU operations like ADD, SHIFT, and XOR which can be sped up in custom ASICs by a large factor.

### **3.3 Weaknesses**

The single largest weakness in NoelKDF seems to be shared among all memory-hard KDFs. Writing password derived data to large amounts memory increases the odds that password derived data will be leaked to an attacker, through swap, hibernation, memory recycling without reinitialization, or core dumps. With this data, an attacker can abort incorrect passwords and mount a massively parallel attack with little CPU time or memory per core.

With cache timing information, an attacker gains an estimated 16X lower time\*memory cost. However, this relies on the assumption that the sliding-reverse window DAGs cannot be efficiently pebbled with little recomputation with less than 1/4 memory versus the first-loop DAG size. The NoelKDF first loop uses the DAG architecture that tested most resistant in

this respect, but only upper bounds on recomputation penalties have been established. There is some risk that lower upper bounds can be found, reducing the effectiveness of defending against cache-timing attacks.

Against the simple brute-force attacks, NoelKDF is resistant to TMTOs. The best TMTO found so far reduces the time\*memory cost by only 3%. There is some risk that better TMTOs can be found with more clever memory coverage of computed values.

## 4 Efficiency Analysis

NoelKDF introduces an efficient compute-time hardened hash function, in addition to being sequential-memory-hard. This hash function on most machines will take from 4 to 8 clock cycles per 32-bit result written to memory, with one thread, because the multiply operation takes 3-4 cycles, the addition takes 1, and the rest (memory read/write, OR, and increments) can often be done in parallel. With multiple threads or SIMD optimization, throughput can be increased to fill the memory bandwidth.

While other hash functions exist that can produce data to fill memory faster, none of them will have significantly stronger compute-time hardness against ASIC attacks, as the multiply operation is the most difficult common CPU operation to accelerate in custom hardware, and it's latency dominates the inner loop execution time. In this regard, NoelKDF's hash function has nearly optimal latency, having one multiply and one simple instruction per iteration of the inner loop. Another choice would be to use 64-bit multiplication, but this would be less optimal on 32-bit CPUs, and on high-end CPUs with SIMD instructions, twice as many 32-bit multiplications can typically be done in parallel as 64-bit multiplications.

### 4.1 The Cost of Pure Cache Timing Resistance

The “pure” KDFs proposed so far seem to allow an attacker to use about  $\frac{1}{4}$  of the memory compared to the computed DAG size, with little or no recomputation penalty. There are good reasons for this. First, there is a recomputation-free  $\frac{1}{2}$  memory attack against all “pure” KDFs which have computation DAGs with max fanout degree 2. When pebbling a pure KDF's DAG, simply pick up one of the pebbles used to compute the next node, or if those pebbles will be needed in future computations, pick a pebble which is not pointed to by any node beyond the node being pebbled. This always works for DAGs with max fan-out degree  $\leq 2$ .

For DAG pebbling with  $\frac{1}{3}$  the pebbles compared to the DAG size, we can always pebble to the  $\frac{2}{3}$  mark with no recomputation. For every new node pebbled after that, we gain a pebble that is not needed to cover some node that is still pointed to by the remaining unpebbled nodes. This makes it very hard to design a hard-to-pebble DAG where an attacker has  $\frac{1}{3}$  pebbles. This results in pure KDFs having a 3-4X lower memory cost than dirty or hybrid KDFs.

When CPU limited, run-times should be approximately the same for pure, hybrid, and dirty KDFs when using the same hashing algorithm, though some KDFs do not write to memory corresponding to DAG nodes that have in-degree 1, saving on memory bandwidth. This can lead to a lower performance penalty than my estimated 3-4X, and this is in fact the case for Catena-2, which has between a 2-3X reduction in the time\*memory cost compared to dirty KDFs.

I have written a pebbling application that attempts to pebble NoelKDF, Catena-3, and what I believe is similar to Escrypt's sliding power-of-two window. In the pebbling algorithm, I assume an attacker knows every detail about the computation DAG ahead of time, and can plan his memory usage strategy carefully. Automated pebbling confirms that all DAG types tested are easily pebbled with  $\frac{1}{4}$  pebbles compared to number of nodes.

In summary, the cost for pure KDFs versus dirty KDFs is typically about a 3-4X penalty in time\*memory cost, though in some cases it may be in the 2-3X range.

## 4.2 SIMD Optimization

Having a parallelism parameter enables NoelKDF to be SIMD optimized. In particular, if parallelism is a multiple of 2 or 4, the inner hashing loop should be executable as 2 or 4 32-bit loops in parallel. Memory for first loop can be interleaved, enabling efficient 128-bit parallel operations. The second loop has each parallel task reading from a pseudo-random memory location (fromAddr), and will require more memory bandwidth as a result, but interleaving memory still enables reading from prevAddr and writing to toAddr 128-bits at a time for 4 parallel tasks.

## 4.3 Computation DAGs

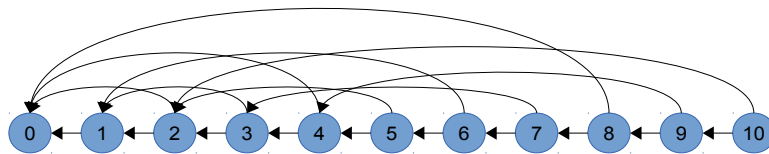
The original Script algorithm writes a linear chain of hashed blocks to memory, each hashed block depending sequentially on the data computed for the previous. Its directed acyclic computation graph is a linear chain:

*Script Computation DAG:*



Script is vulnerability to TMTO attacks. An attacker covering nodes 3 and 7 sees an average recomputation of 1.5 nodes + second loop hashing, for a 2.5 computations per node in the second loop. Adding first loop computations gets the total to 3.5 computations per node, compared with 2 computations per node when keeping all nodes in memory. That's a 1.75X computation penalty, but the attacker only covered 1 in 4 nodes, so his time\*memory is down to 0.44X of his original cost. As an attacker reduces his memory coverage, his time\*memory cost converges to  $\frac{1}{4}$  of the original.

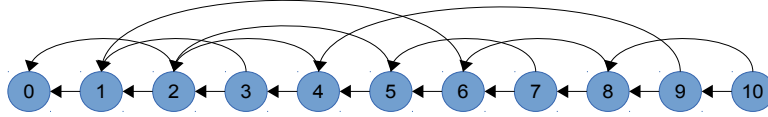
*NoelKDF's "Pure" Computation DAG:*



One visual give-away that this is a sliding-power-of-two bit-reversal DAG is that every power of 2 points to node 0. To compute the destination of an edge, take the binary representation of the source node, remove the leading 1, and reverse the bits. For example, node 6 is 110, which becomes 10 after removing the leading 1, and then 01 with bit-reversal, so node 6 points to node 1. If an edge length is 2 greater than largest power of 2 less than the source node number, then add that power of two. So, for example, if the graph were larger, we'd see node 11000 (node 24) would point to node 0001 (node 1), but since  $1 + 16 + 2 = 19 < 24$ , we add 16. Therefore, node 24 points to node 17. This causes the destination node to always fall within the "sliding" power-of-two window preceding a node (actually it follows 2 pebbles behind to avoid 1-long edges).

This DAG architecture was chosen after it demonstrated the strongest resistance of all DAG architectures tested to my automated pebbling algorithm. An attacker attempting to pebble such graphs with a combination of fixed-spaced pebbles, fixing pebbles on high degree nodes, and fixing pebbles on destinations of short edges will find this graph requires more pebbles and recomputation than the other DAGs tested.

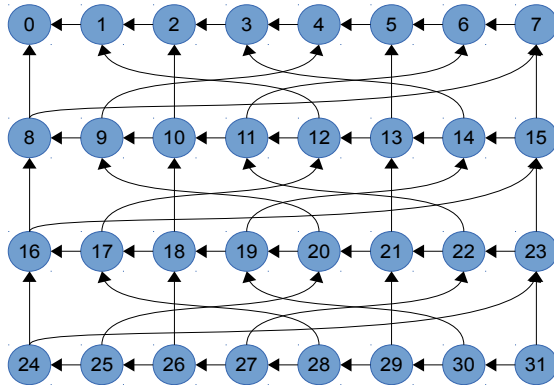
NoelKDF's "Dirty" Computation DAG:



NoelKDF "dirty" computation DAGs have the same linear chain, but instead of waiting until all memory is written before doing password dependent pseudo-random reads, it reads and hashes while writing. This creates a random-ish looking graph where edges on average point back  $\frac{1}{4}$  from their position, and there are lots of short edges. This keeps the midpoint cut size around 17% of the number of nodes. In this case, the cut size to the right of node 5 is 4.

If an attacker has covered only nodes 3 and 7 only, then to compute node 8 requires recomputation of every single missing node because 8 points to 6, which points to 5 which points to 4, which points to 2 and then onto 0. Similarly, computing 9 and 10 also require full recomputation of every missing node, since they point to 8. In general, a NoelKDF graph recomputation penalty grows to a substantial portion of the entire graph for each missing node requiring recomputation by the time an attacker has only  $\frac{1}{8}$ th of the nodes in memory, resulting in a runtime proportional to the square of the graph size. While a  $\frac{1}{2}$  memory TMTO with every other memory block kept results in no significant change in memory\*time cost, anything lower than  $\frac{1}{2}$  rapidly becomes too expensive to compute. An attacker keeping every 4<sup>th</sup> node in memory suffers an additional computation factor of over 2000X for a 1M node graph. A reasonable attack against NoelKDF would be to save  $\frac{1}{2}$  of the last  $\frac{1}{2}$  of memory. For 1,000,000 node graphs the gain is < 2% in the time\*memory cost. If we increase spacing to 3 in the last  $\frac{1}{4}$ , then to 4 in the last  $\frac{1}{8}$ , and so on, the memory\*time cost drops 3% versus the normal algorithm.

*Catena-3 Computation DAG:*



The Catena-3 DAG is computed using  $\frac{1}{4}$  of the memory requirement compared to the size of its computation DAG. However, an attacker is unlikely to succeed at improving his time\*memory cost while using even one less memory location than  $\frac{1}{4}$ . Our algorithm pebbles Catena with 0 recomputation penalty for a  $\frac{1}{4}$  pebble coverage just like NoelKDF and Escript. With one fewer pebbles, the Catena pebbling penalty jumps to 1.8X for a 1024 node graphs, using fixed pebbles every 4. The penalty seems independent of graph size. When a Catena-3 sub-DAG is embedded in the first row, the penalty jumps to 3X, when using fixed pebbles every 4 and pebbling nodes pointed to by short edges, which improves the situation on the first row.

Code for the pebbling application can be found at:

<https://github.com/waywardgeek/noelkdf/tree/master/predict>

## 5 Results

The following benchmarks were run on our quad-core i7 CPU development machine running Manjaro (Arch) Linux, using the default install package for Scrypt, and the version of Catena available on 1/24/14 from github, using the waywardgeek branch I created. All were run in single-threaded mode.

	Memory	CPU Time (seconds)	Time*Memory
Scrypt	500MiB	1.8	1.0X
Catena-3	1GiB	1.37	2.6X
Catena-2	1GiB	1.06	3.4X
NoelKDF	1GiB	0.4	9X



The Catena-3 code used the same exact hash function as NoelKDF, as I literally copied and pasted the hashing code from noelkdf-ref.c into catena-multihash.c.

Next, I compared resistance to cache timing attacks with my automated pebbler. Catena-3 seems to be the clear winner for a “pure” KDF, as even reducing 1 pebble punishes an attacker. However “hybrid” KDFs fill memory with all the computed results, so users do not benefit from the reduced memory consumption the way they do with Catena. For NoelKDF, I tested Catena-2 and Catena-3, with and without an embedded Catena-3 graph in the first row (“Enhanced Catena”), and various other DAG styles, including the combination of Alexander's power-of-two sliding window and Christian's bit-reversal. As I enhanced the algorithm, the recomputation penalties decreased, but the relative order of results has not changed. To improve pebbling, I manually tuned my 3 parameters to minimize the recomputation penalty. Fixed pebbles at fixed spacing had the greatest impact. Enhanced Catena needs a heuristic to cover nodes pointed to by short edges, and the sliding-reverse DAGs needed a heuristic for fixing pebbles on nodes of high in degree.

	Spacing	Max Degree	Min Edge Length	Recomputation Penalty
Catena-3	7	0	0	10X
Catena-2	5	0	0	2.6X
Enhanced Catena-3	8	0	25	89X
Enhanced Catena-2	5	0	25	6.5X
Sliding-Reverse	16	3	0	1176X

All of these runs pebbled graphs with 128 pebbles. The Catena-3 and sliding-reverse graphs had 1024 nodes, while the Catena-2 graphs had 768 nodes.

Not too much should be read into these numbers as they are simply upper bounds. However, sliding-reverse was chosen for hybrid-NoelKDF based in part on these results. Also, the sliding-reverse DAG does a better job in the second loop at forcing attackers to keep results in memory.

## 6 Conclusion

A natural choice for memory-hard KDFs seems to exist between the extremes of “pure” and “dirty” KDFs. “Hybrid” KDFs, while maybe 5X-ish less resistant to cache-timing attacks are 3-4X-ish more resistant against the more common brute-force guessing attacks, all else being equal.

Further, substantial resistance to ASIC attacks can be had through multiplication-time hardened hashing functions.

NoelKDF is just such a hybrid KDF.

## 7 Intellectual Property Statement

I, Bill Cox, place noelkdf-ref.c, the algorithm it contains, and all other files and intellectual property associated with this project into the public domain. I will file no patents on any idea used in this project. NoelKDF includes sha.c and sha.h which I copied from the scrypt source code, and which is released under the BSD license, and noelkdf-test.c was copied from Catena's catena\_test\_vectors.c and is released under the MIT license.

NoelKDF is and will remain available worldwide on a royalty free basis, and I am unaware of any patent or patent application that covers the use or implementation of the NoelKDF algorithm.

## 8 No Hidden Weaknesses

I, Bill Cox, assert that NoelKDF has no deliberately hidden weakness such as back doors, and I know of no weaknesses other than those discussed in this document. No unusual constants are used in the code.

## 9 Bibliography

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