

# Given Enough Eyeballs, All Bugs Are Shallow?

## Revisiting Eric Raymond with Bug Bounty Programs

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(Dated: May 30, 2017)

Bug bounty programs offer a modern platform for organizations to crowdsource their software security, and for security researchers to be fairly rewarded for the vulnerabilities they find. However, little is known on the incentives set by bug bounty programs – how they drive new bug discoveries, and how they may improve security through the progressive exhaustion of discoverable vulnerabilities. This article investigates the strategic interactions among the managers and participants of bug bounty programs. At the level of a single bug bounty program, security researchers face a mechanism reminiscent of the St-Petersburg paradox: the probability of finding additional bugs decays rapidly, and is difficult to compensate for with increasing monetary rewards. Furthermore, bug bounty program managers have an incentive to gather the largest possible crowd of experts, which increases competition among security researchers. As a result, we find that researchers have strong incentives to switch to newly launched programs, for which a reserve of *low-hanging fruit* vulnerabilities is still available. Our results shed light on the technical and economic mechanisms underlying the dynamics of bug bounty program contributions, and

help inform the mechanism design of bug bounty programs as they get increasingly adopted by cybersecurity-savvy organizations.

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## I. INTRODUCTION

On March 2nd, 2016, the Pentagon announced the launch of its first *bug bounty* program [1]. From this point on, one of the most paranoid organizations in the world would offer incentives to hackers to break into its systems and report found vulnerabilities for a reward. Although bug bounty programs have mushroomed in the last few years, this audacious announcement by a prominent defense administration may set a precedent, if not a standard, for the future of cybersecurity practice.

Software security has long been recognized as a challenging computational problem [2] that requires human intelligence as part of the solution. However, given the complexity of modern computer systems, human intelligence at the individual level is no longer sufficient. Instead, organizations are turning to tap the wisdom of crowds [3] to improve their security. Software security is not alone. Other disciplines have similarly turned to mobilizing people at scale to tackle their hard problems, such as sorting galaxies in astronomy [4], folding proteins in biology [5], recognizing words from low quality book scans [6], and addressing outstanding mathematics problems [7, 8].

These examples involve different aspects of human intelligence, ranging from pattern recognition (e.g., Captcha [6]) to higher abstraction levels (e.g., mathematical conjectures [7, 8]). It is not clear what kind of intelligence is necessary to find bugs and vulnerabilities in software, but it generally requires a high level of programming proficiency coupled with *hacking skills* – out-of-the-box thinking to find unintended uses for a software.

From hedonist pleasure to reputation building, to activism, motivations and incentives for hacking have evolved over time [9]. Among these, reputation and monetary incentives are increasingly put in place to entice security researchers to hunt for bugs. Bug bounty programs and the online platforms help set such incentives [10–13], while facilitating communication and transactions between security researchers and software editors. It however remains unclear how current mechanism designs and incentive structures will influence the long-term success of bounty programs. A better understanding of bug discovery mechanisms [14–16], and a better characterization of the utility functions of security researchers, organizations launching bug bounty programs and bug bounty platforms, will help shape

the way bug bounty programs evolve in the future.

In this study, we investigate a public data set of 35 public bug bounty programs from the *HackerOne* website. We find that with each vulnerability discovered within a bounty program, the probability of finding the next vulnerability in the program decreases more rapidly than the corresponding increase in payoff. Therefore, security researchers rationally switch to newly launched bounty programs at the expense of existing ones. This switching phenomenon has already been found in [13]. Here, we characterize it further, by quantifying how incentives evolve as more vulnerabilities get discovered in a program, and how researchers benefit in the long term by switching to newly launched programs.

[say something about the reflection between our results and Brady et al.]

This article is organized as follows. Related research is presented in Section II. Important features of the data set used here are detailed in Section III. We introduce the main mechanism driving vulnerability discovery in Section IV. Results are presented and discussed in Sections V and VI, respectively. We offer concluding remarks in Section VII.

## II. BACKGROUND

Software reliability is an age-old problem [2, 17, 18]. Early empirical work on software bug discovery dates back to the time of UNIX systems [19], and over the years, numerous models for vulnerability discovery have been developed (see [16, 20] for some contemporary approaches). As early as in 1989, it was recognized that the time to achieve a given level of software reliability is inversely proportional to the desired failure frequency level [2]. For example, in order to achieve a  $10^{-9}$  probability of failure, a software routine should be tested  $10^9$  times. Actually, the random variable  $P(T > t) = 1/t$  corresponds to the Zipf's law [21, 22], which diverges as the random variable sample increases (i.e., no statistical moment is defined). Thus, there will always be software vulnerabilities to be discovered as long as enough resources can be provided to find them.

Taking an evolutionary perspective brings additional insights. Finding bugs is compara-

ble to the survival process involved in the selection of species: Defects are like genes, which get expressed under the pressure of environment changes. Brady et al. [15] have shown that software testing follows the principle of entropy maximization, which preserves *genetic variability* and thus, removes only the minimum possible number of bugs, following the exploration of use cases (the software environment). With her out-of-the-box thinking, the *hacker* security researcher is precisely good about envisioning a broad range of possible use cases, which may reveal a software defect (i.e., program crash) or an unintended behavior.

Software solutions have been developed to systematically detect software inconsistencies and thus potential bugs (e.g., Coverity, FindBugs, SLAM, Astree, to name a few). However, to date, no systematic algorithmic approach has been found to detect and remove bugs at a speed that would keep pace with software evolution and expansion. Thus, human intelligence is still considered as one of the most efficient ways to explore novel use case scenarios – by manual code inspection or with the help of bug testing software – in which a software may not behave in the intended way.

Management techniques and governance approaches have been developed to help software developers and security researchers in their review tasks, starting with pair programming [23]. To protect against cyber-criminals, it is also fashionable to hire *ethical hackers*, who have a mindset similar to potential attackers, to probe the security of computer systems [24–26]. The policy of full disclosure, originating from the hacking and open source communities, have played a significant role in software security by forcing software owners to acknowledge and fix vulnerabilities discovered and published by independent researchers [27]. The full-disclosure model has evolved into responsible disclosure, a standard practice where the security researcher agrees to allow a period of time for the vulnerability to be patched before publishing the details of the uncovered flaw. In most of these successful human-driven approaches, there is a knowledge-sharing component, either between two programmers sitting together in front of a screen, ethical hackers hired to probe the weaknesses of a computer system, or the broader community being exposed to open source code and publicly disclosed software vulnerabilities [28]. Thus, Eric Raymond’s famous quote “Given enough eyeballs, all bugs are shallow” [29] tends to hold, even though in practice things are often slightly more complicated [30].

One way to gather enough eyeballs is to resort to the larger crowd of security researchers. For this purpose, bug bounty programs and vulnerability *markets* have emerged in recent years to facilitate the trading of bugs and vulnerabilities. These two-sided markets provide economic incentives to support the transfer of knowledge from security researchers to software organizations [31], as they help simultaneously harness the wisdom of crowds and to reveal the security level of organizations through a competitive incentive mechanism [32]. Nonetheless, the efficiency of bug bounty programs has been questioned on both theoretical [33, 34] and empirical grounds [35, 36].

Building on previous work by Schechter [32], Andy Ozment [37] theorized that the most efficient mechanisms are not markets *per se*, but rather auction systems [38]. In a nutshell, the proposed (monopsonistic) auction mechanism implies an initial reward  $R(t = t_0) = R_0$ , which increases linearly with time. If a bug is reported more than once, only the first reporter receives the reward. Therefore, security researchers have an incentive to submit a vulnerability early (before other researchers submit the same bug), but not too early, so that they can maximize their payoff  $R(t) = R_0 + \epsilon \times t$  with  $\epsilon$  the linear growth factor, which is meant to compensate for the increasing difficulty of finding each new bug. However, setting the right incentive structure  $\{R_0, \epsilon\}$  is non-trivial given uncertainties in the amount of work needed, the level of competition (e.g., the number of researchers enrolled) in the bug bounty program [39], or the nature and likelihood of overlap between two submissions by different researchers. Nevertheless, bug bounty programs have emerged as a tool used by many software organizations, with a range of heterogeneous incentive schemes [11]. For instance, some bug bounty programs include no monetary rewards [12]. Meanwhile, dedicated platforms have been launched to act as trusted third parties in charge of clearing transactions between organizations and security researchers. These platforms also assist organizations in the design and deployment of their own program. One of the leading platforms is HackerOne which runs public and private programs for organizations across a wide range of business sectors. A subset of the private and public programs award bounties, while other bug bounty programs capitalize on incentives associated with reputation building, which is an important motivation driver in the hacker community [40]. These programs report bounty awards on their company program pages on the HackerOne website. Previous research has investi-

gated vulnerability trends, response and resolve behaviors, as well as reward structures of participating organizations [12, 13]. In particular, it was found that a considerable number of organizations experienced diminishing trends for the number of reported vulnerabilities, even as the monetary incentives exhibit a significantly positive correlation with the number of vulnerabilities reported [13].

### III. DATA

The data were collected from the public part of the HackerOne website. From 35 public bounty programs, we collected the rewards received by security researchers (in US dollars), with their timestamps (45 other public bounty programs do not disclose detailed information on rewards, and the number of private programs is not disclosed). Since HackerOne started its platform in December 2013, new public programs have been launched roughly every two months, following an essentially memoryless Poisson process ( $\lambda = 57$  days,  $p < 0.001$  and  $R^2 > 0.99$ ). Figure 1A shows the timeline of the 9 most active programs with at least 90 valid (i.e., rewarded) bug discoveries, as of February 15, 2016. When a new program is launched, we observe an initial peak within weeks after launch, which accounts for the majority of discoveries. Following the initial surge of vulnerability discoveries, bounty awards become less frequent following a decay function with long-memory, following a robust power law decay  $\sim t^\alpha$  with  $\alpha = -0.40(4)$  ( $p < 0.001$  and  $R^2 = 0.79$ ) at the aggregate level and over all 35 bounty programs (see Figure 1B). Some programs depart from this averaged trend: For instance Twitter exhibits a steady, almost constant, bug discovery rate and VKontakte exhibits its peak activity months after the initial launch. These peculiar behaviors may be attributed to program tuning and marketing, to sudden change of media exposure or even to fundamental differences of program comparative fitness, for which we do not have specific information.

The long-memory process of bug discovery following the launch of a bug bounty program (c.f., Figure 1B) is compatible with the long-established  $\sim 1/\tau$  law of bug discovery in software testing [2]. The difference of exponent  $\alpha \approx -1$  instead of  $-0.4$  in the  $\sim 1/\tau$  law of software reliability,  $\tau$  refers to testing time, i.e., the number of tests to perform, in order to reach a given reliability level. The long-memory process associated with the decay of bug

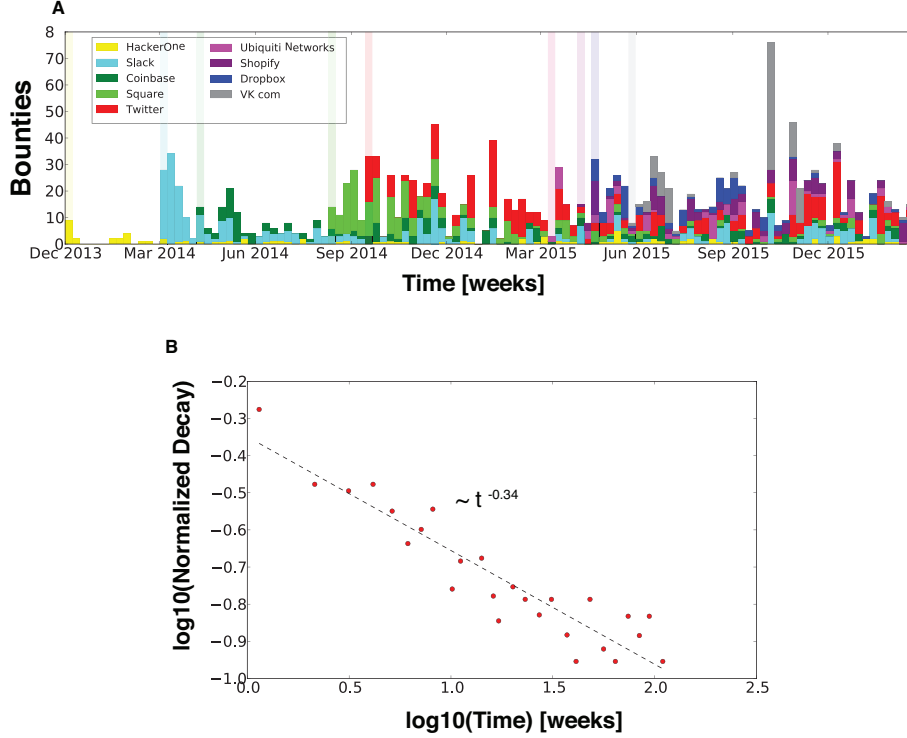


FIG. 1. **A.** Weekly vulnerability discoveries for the 9 most active programs (with at least 90 bug discoveries as of February 15, 2016). The light colored vertical bars represent the start of the program, occurring when the first bounty is awarded. Most programs exhibit an initial shock, followed by a decay of discoveries, which is characterized at the aggregate level by a long-memory process (panel **B**) characterized by a power law decay  $\sim t^\alpha$  with  $\alpha = -0.40(4)$  ( $p < 0.001$  and  $R^2 = 0.79$ ). Each data point in the figure is the median of normalized vulnerability numbers of all 35 programs considered in this study.

discovery (c.f., Figure 1B) is presented in absolute time. Long-memory processes associated with human behaviors are often associated with human timing effects, which correspond to task priority queueing and a rationalize use of time as non-storable scarce resource [41]. Long-memory processes observed in human behaviors are also associated with critical cascades of events. **Self-excited Hawkes Process with power law kernel (following human timing)[42].** Here, the rationale for cascades builds on 2 ingredients : (i) each security researcher generates a cascade of bug discoveries, and (ii) each bug discovery attracts new security researchers, who in turn trigger their own bug discovery cascade. Cascades are typically slightly heavy-tailed, however bounded, as we'll later : we will dissect these cascades in conjunction with in-



centives provided by the accumulation of bounties (reputation, slight chance to find less obvious bugs bring more reputation, and increased bounties rewards).

: When the program launches, it takes some time first for the researcher to be exposed to the new program (through the media and social media), second for the researcher to find and submit bugs, and third for the organization managing the bug bounty program to assess the quality of each submission, and assign a proper reward. To account for all these delays, one may resort to priority queueing applied to humans: First, competing attention prevents immediate exposure to the news of a new program; Second, when security researchers get interested in a new program, they may still be actively searching bugs on other programs or performing other tasks (such as e.g., their regular job, leisure, family matters); Third, when subjected to a flow of bug submissions, security teams at organizations leading bug bounty programs assign priorities among submissions, and resolve them with human resources available at the time of submission.

These delays are best rationalized by human timing contingencies, and moreover, by an economy of time as a scarce, non-storable resource, which is known to generate long-memory responses of the form  $\sim t^{-1.5}$  between the arrival and the execution of a task [41]. The observed much slower decay may result from the compound effect of multiple delays, such as those mentioned above. The initial burst of discoveries, followed by a long-memory decay may also result from the increasing difficulty associated with finding new bugs for each bounty program, as the most obvious vulnerabilities get uncovered first. Since we consider only the *time of discovery* as the moment when the validity of the bug submitted is acknowledged by the program manager, we are mostly blind to the human timing effects associated with the long-memory process observed on Figure 1B, including when submissions are made, but don't lead to a discovery associated with a monetary reward.

Here, we don't consider the timing effects encompassing delays, effort, processing time, but rather the incremental valid bug discovery and reporting by security researchers [refine here].

#### IV. METHOD

Bug bounty programs work on the premise that humans as a crowd are efficient at searching and finding bugs. Their mere existence is a *de facto* recognition that market approaches for bug and security vulnerabilities discovery involving large pools of security researchers bring efficiency, beyond in-house security. Bug bounty programs signal that organizations are ready to complement their vertical cost-effective security operations with market approaches, which are traditionally perceived as less cost-effective, yet more adaptive [43]. Brady et al. [15] have early on offered a hint for the existence of such markets for bugs: According to their proposed theory, each researcher has slightly different skills and mindset. When a security researcher tests a software piece, by choosing the inputs, she offers a unique operational environment. This environment is prone to the discovery of new bugs, which may have not be seen by others. The proposed theory by Brady et al. [15] intrinsically justifies the existence of bug bounty programs structures as markets, which provide the necessary diversity to account for the highly uncertain risk horizon of bug discovery. Here, we develop a quantitative method to formalize a mechanism and to test the theory proposed in [15]. This validation step shall help provide organizational design insights for bug bounty programs.

For that, we investigate the interplay between the vulnerability exhaustion process, and the cumulative reward distributed to security researchers within and across 35 public bounty programs hosted on HackerOne. When a bug bounty program starts, it attracts a number of security researchers, who in turn submit bugs. Subsequent bug discoveries get increasingly difficult. The difficulties faced by security researchers can be technical. It can also be the result of insufficient or conflicting incentives. Here, we develop and test a model, which accounts for technical difficulties and insufficient incentives together. We further address conflicting incentives by measuring the effect of newly launched bug bounty program on incumbent programs.

Starting from an initial probability of discovering the first vulnerability  $P(k = 0) = 1$ , the probability to find a second bug, is a fraction of the former probability:  $P_{k+1} = \beta * P_k$  with  $\beta$  a constant strictly smaller than one. The probability that no more discovery will be made after  $k$  steps is given by  $P_k = \beta^k(1 - \beta)$ . Conversely, starting from the initial

reward  $R_0 = R(k=0)$ , the subsequent reward  $R_1 = \Lambda_1 \cdot R_0$ , and further additional reward  $R_2 = \Lambda_2 \Lambda_1 \cdot R_0$ . After  $n$  steps, the total reward is the sum of all past rewards:

$$R_n = R_0 \sum_{k=1}^n \Lambda_1 \dots \Lambda_k. \quad (1)$$

Thus,  $R_n$  is the recurrence solution of the Kesten map ( $R_n = \Lambda_n R_{n-1} + R_0$ ) [44, 45]: As soon as amplification occurs (technically, some of the factors  $\Lambda_k$  are larger than 1), the distribution of rewards is a power law, whose exponent  $\mu$  is a function of  $\beta$  and of the distribution of the factors  $\Lambda_k$ . In the case where all factors are equal to  $\Lambda$ , this model predicts three possible regimes for the distribution of rewards (for a given program): thinner than exponential for  $\Lambda < 1$ , exponential for  $\Lambda = 1$ , and power law for  $\Lambda > 1$  with exponent  $\mu = |\ln \beta| / \ln \Lambda$  (see Appendix). The expected payoff of vulnerability discovery is thus given by,

$$U_k = P_k \times R_k, \quad (2)$$

with both  $P_k$  and  $R_k$  random variables respectively determined by  $\beta$  and  $\Lambda$ . Because  $U$  is a multiplication of two diverging components, its nature is reminiscent of the St. Petersburg paradox (or St. Petersburg lottery), proposed first by the Swiss Mathematician Nicolas Bernoulli in 1713, and later formalized by his brother Daniel in 1738 [46]. The St. Petersburg paradox states the problem of decision-making when both the probability and the reward are diverging when  $k \rightarrow \infty$ : A player has a chance to toss a fair coin at each stage of the game. The pot starts at two and is doubled every time a head appears. The first time a tail appears, the game ends and the player wins whatever is in the pot. Thus the player wins two if a tail appears on the first toss, four if a head appears on the first toss and a tail on the second, eight if a head appears on the first two tosses and a tail on the third, and so on. The main interest of Bernoulli was to determine how much a player would be ready to pay for this game, and he found that very few people would like to play this game even though the expected utility increases (in the simplest case proposed by Bernoulli,  $U_n = \sum_{k=0}^n U_k = n$ ) [46]. The situation of a security researcher differs from the St. Petersburg lottery as bug search costs are incurred at every step. Since these costs influence the probability to find an additional bug, we could assume that they are integrated into the utility as  $U_k^* = U_k - c_k$ . The security researcher may also decide

to stop searching for bugs in a program, at any time  $k$ . This is equivalent to setting  $P_{k+1} = 0$ .

The expected payoff  $U_k$  therefore determines the incentive structure for security researchers, given that the bounty program manager can tune  $R_0$  and in some cases, the manager may also set  $\Lambda$ . The utility function may also incorporate non-monetary incentives, such as reputation: Finding a long series of bugs may signal some fitness for a bug bounty program and as thus, create a permanent job opportunity [47]. Similarly, discovery of a rare bug that no other researcher has found before has a strong signaling effect, which can help build a career. However, these strategies are high-risk high-return, therefore they carry additional fame. In the next section, we will calibrate our model to the bug discovery process associated with 35 bounty programs publicly documented on the HackerOne platform.

~~In principle, the manager could set  $R_0$  to influence  $P_0$  and indirectly  $P_k$ . Mapping the discovery rank  $k$  into the rate of discovery may also help considering discounting aspects in presence of competing opportunities and inter-temporal choices under uncertainty [48]. A new public bounty program is launched at a Poisson rate, approximately every 2 months, and each launch brings its windfall effect, leaving the researcher with the choice to either keep digging increasingly harder vulnerabilities (rare but with higher reward), or turning to the low hanging fruit (frequent but with low reward) of a new program. We shall therefore verify whether newly launched programs actually influence security researchers.~~

## V. RESULTS

The discovery process in a bug bounty program is driven by the probability to find an additional bug given that  $k$  bugs have already been discovered (i.e., the exhaustion process), and program managers aim to maximize  $B_c$ , the total number of bugs found. Our results show that the number of bugs discovered is a super-linear function of security researchers who have enrolled in the program (see Figure 2A). While bug bounty programs benefit from the work of a large number of researchers, researchers overall benefit from diversifying their efforts across programs (see Figure 3C). This benefit is particularly tangible regarding the cumulative reward they can extract from their bug hunting activity. In particular, we illustrate how researchers take the strategic decision to enroll in a newly launched program,

at the expense of existing ones they have formerly been involved in.

#### A. Security researcher enrollment determines the success of a bug bounty program

As presented on Figure 2A, we find that the number  $B_c$  of bug discovered scales as  $B_c \sim h^\alpha$  with  $\alpha = 1.10(3)$  and  $h$  the number of security researchers enrolled in a program. Since  $\alpha > 1$ , a bounty program benefits in a super-linear fashion from the enrollment of more researchers. This result is reminiscent of productive bursts and critical cascades of contributions in open source software development [42]: Each enrollment (i.e., *mother* event) initiates a cascade of bug discoveries (i.e., *daughter* events). Here, each cascade stems from a single security researcher (with some exceptions researchers mostly search bugs alone) and the nature of these cascades is captured at the aggregate level by their size as a random variable. As shown on Figure 2B, the distribution of bounty discoveries per researcher and per program follows a power law tail  $P(X > x) \sim 1/x^\gamma$  with  $\gamma = 1.63(7)$ . The first moment of the distribution (i.e., the mean) is however well-defined (as a result of  $\gamma > 1$ ). Moreover, we observe an upper cut-off of the tail with  $x_{max} \approx 400$  bounties. Thus, each enrollment of a security researcher in a program provides a statistically bounded amount of new bug discoveries.

#### B. Security researchers are incentivized to diversify their contributions across bug bounty programs

For security researchers, the main metric is the expected cumulative payoff earned from the accumulation of bounty awards over all programs. This expected payoff is governed by the probability to find a given number of bugs and their associated payoff, as discussed in Section IV. To fully understand the incentive mechanisms at work, we consider 3 perspectives: (i) the expected cumulative down-payment made by bug bounty program managers (see Figure 3A), the expected cumulative payoff from the viewpoint of a security researcher for (ii) one program (see Figure 3B), and for (iii) all programs (see Figure 3C).

The average cumulative down-payment per program exhibits a super-linear scaling as

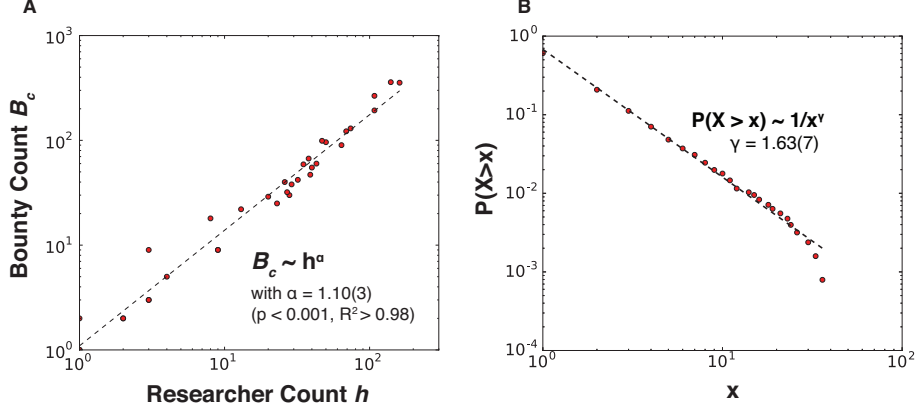


FIG. 2. **A.** The number of bounty discoveries per program  $B_c$  scales as  $h^\alpha$  with  $\alpha = 1.10(3)$  and  $h$  the number of security researchers enrolled in a program (fit and confidence interval were obtained by ordinary least squares of the logarithm of researcher and bounty counts). Since  $\alpha > 1$ , a bounty program benefits in a super-linear fashion to the enrollment of more researchers. **B.** The tail distribution of bounty discoveries per researcher per program follows a power law distribution  $P(X > x) \sim 1/x^\gamma$  with  $1 < \gamma = 1.63(7) < 2$  (obtained by maximum likelihood estimation, and confidence interval bootstrapping, following [21, 49]). The distribution is therefore relatively well bounded (with the first moment being well-defined). Furthermore, we observe an upper cut-off of the tail with  $x_{max} \approx 400$  bounties. Thus, from **A.** and **B.** combined, we find that the number of vulnerabilities is mainly driven by the number of researchers enrolled in programs.

$\sim k^{1.27}$  ( $p < 0.001$ ,  $R^2 > 0.99$ ), while the frequency of bugs  $P_k$  is only slightly upwards trended, increasing as  $\sim k^{0.13}$  ( $p < 0.001$ ,  $R^2 = 0.40$ ). The expected down-payment by bug bounty program managers therefore scales as  $\sim k^{1.40}$ . This is a considerable super-linear increase (as  $k \rightarrow \infty$ ), which casts questions on the long-term sustainability of bug bounty programs.

From the security researcher's viewpoint and her expected payoff from a single bug bounty program, the increase of average cumulative reward ( $R_k \sim k^{1.40}$ ) does not offset the fast decay of probability ( $P_k \sim k^{-1.85}$ ) to find a vulnerability of rank  $k$ . The expected payoff therefore follows  $U_k \sim k^{-0.45}$ , which clearly does not bring high incentives to explore in depth a bug bounty program. It is important to note however, that the bug bounty manager cannot fix  $P_k$ , which is a genuine feature of bug discovery by humans. To maintain positive individual incentives, the manager should set an incremental reward such that  $R_k \sim k^\alpha$

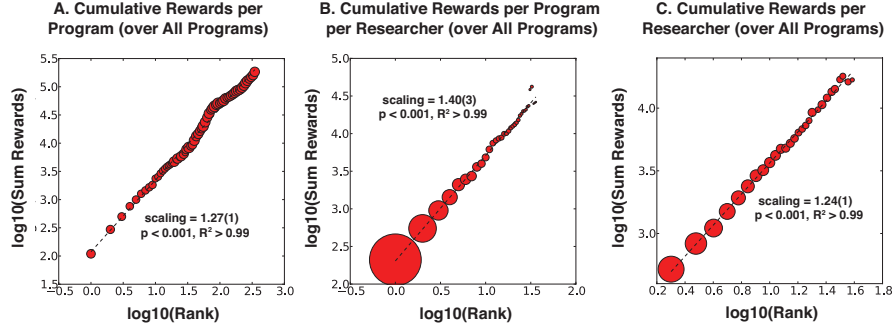


FIG. 3. **A.** (Log-)binned cumulative down-payment per program over all public programs on the HackerOne platform, scales as  $R_k \sim k^{1.27}$  ( $p < 0.001$ ,  $R^2 > 0.99$ ), with  $k$  the rank. Each log-bin shows the mean value and the circle sizes depict the number of values in each bin (i.e., the rank frequency). The super-linear *scaling* relationship between the cumulative reward and the rank shows that reward increases as a function of  $k$ . However, the frequency of vulnerabilities  $P_k$  is only slightly upwards trended increasing as  $\sim k^{0.13}$  ( $p < 0.001$ ,  $R^2 = 0.40$ ). **B.** Considering the opposite figure from the viewpoint of researcher's expected payoff when enrolling in a single bug bounty program, the super-linear effect is much stronger ( $R_k \sim k^{1.40}$  with  $p < 0.001$  and  $R^2 > 0.99$ ). However, the frequency decays following a power law of the form  $P_k \sim k^{-1.85}$  ( $p < 0.001$ ,  $R^2 = 0.97$ ). **C.** Over all bug bounty programs, security researchers have another expected payoff: the reward scaling is smaller ( $R_k \sim k^{1.24}$  with  $p < 0.001$ ,  $R^2 > 0.99$ ), yet the frequency of bug discoveries decays much slower as a function of rank  $P_k \sim k^{-1.05}$  ( $p < 0.001$ ,  $R^2 = 0.85$ ).

with  $\alpha > 1.40$ , which in turn would worsen the down-payment function both in terms of incremental expenditures and in exploration of bugs with higher ranks. This approach does not consider possible individual human hard limits preventing finding additional bugs, which would drive iterative costs  $c_k$  sufficiently high as  $k$  gets large. In that latter case, setting higher reward incentives would have no effect.

Security researchers tend to switch from one bounty program to another program [12, 13]. The strategy can be assimilated to portfolio diversification [50]. Over all bug bounty programs, security researchers have another much more favorable expected payoff: The reward scaling is smaller ( $R_k \sim k^{1.24}$  with  $p < 0.001$ ,  $R^2 > 0.99$ ), yet the frequency of bug discoveries decays much slower as a function of rank  $P_k \sim k^{-1.05}$  ( $p < 0.001$ ,  $R^2 = 0.85$ ).

Therefore, over all bounty programs, security researchers have an increasing, yet marginally decreasing, incentive to explore higher ranks as  $U_k \sim k^{0.19}$ . In a nutshell, security researchers have an incentive to keep searching for bugs on a large variety of bug bounty programs.

### C. Influence of newly launched programs on researcher behaviors

As security researchers weigh their strategic choice to switch their attention from one program to another, the time factor is determinant because the expected payoff is dependent on the current vulnerability rank, which maps into the time dimension (i.e, the duration between two discoveries is drawn from a characteristic random variable, which is not considered here). While a researcher may decide to switch at any time, the most obvious moment is when a new program is being launched: Incentives shift suddenly, and security researchers may decide to abandon older programs at the expense of the new program with fresh bug discovery opportunities. However, a number of factors may influence their decision: The reputation of the organization launching the program (it brings more fame to submit a bug to e.g., Twitter compared to a less famous organization), the amount of reward, and the relative time between an old program and the newest one. To encompass the effects of new public bug bounty programs on incumbent programs, we aim to test 3 hypotheses:

- **H1:** An existing bounty program will receive less reports when more new programs are launched,
- **H2:** An existing bounty program will receive less reports when bounty rewards provided by newly launched programs is higher,
- **H3:** The number of newly launched programs has a larger impact on the contribution to older programs.

We specify a simple ordinary least square (OLS) regression model as follows:

$$V_{it} = \beta_0 + \beta_1 dP_t + \beta_2 T_{it} + \beta_3 A_{it} + \beta_4 B_{it} + \epsilon_{it}. \quad (3)$$

$V_{it}$  is the number of vulnerability reports received by bounty program  $i$  in the month  $t$ .  $dP_t$  is the number of new programs launched in month  $t$ . Hypothesis **H1** predicts that its coefficient ( $\beta_1$ ) is negative.  $T_{it}$  is the number of months since bounty program  $i$  launched.



We consider two control variables that could influence researcher’s decision [13]. We first incorporate  $A_i$  the log of the Alexa rank, which measures web traffic as a proxy of popularity for organization  $i$ .  $B_i$  is the log of the average amount of bounty paid per bug by bounty program  $i$ . Both  $A_i$  and  $B_i$  are assumed to remain constant over time. Finally,  $\epsilon_{it}$  is the unobservable error term. In models **2-4**, we extend the basic model (model **1**) to further study competition occurring between bounty programs. These alternative specifications include:

- **Average bounty of newly launched programs:** Intuitively, if new programs offer higher rewards, they should attract more researchers from existing programs. We calculate the average bounty for all new programs in month  $t$  as  $NB_t$  in models **2-4**.
- **Interaction between  $dP_t$  and  $T_{it}$ :** Conceivably, the effect of new programs on existing programs depends on certain characteristics of the latter, such as age. In particular, we ask if a new entrant has more negative effects on older programs compared to younger programs? To examine this, we consider an interaction term between the number of new programs ( $dP_t$ ) and the age of the program ( $T_{it}$ ) in models **3-4**. Hypothesis H3 predicts that its coefficient is negative.
- **Program fixed effect:** To better control for program-specific, time-invariant characteristics, e.g., the reputation among researchers, we add program fixed effect in model **4**. The addition of this fixed effect allows us to examine how bug discovery changes over time within each program  $i$ .

The regression results are shown in Table I. Consistent with our prediction in Hypothesis **H1**, the coefficient of  $dP_t$  is negative and statistically significant in all 4 specifications. *Ceteris paribus*, the launch of new programs reduces the number of vulnerabilities reported to existing programs. In other words, the entry of new programs indeed attracts researcher’s attention away from existing programs, which is consistent with the fast decreasing expected payoff for individuals searching bugs for a specific program. Also, the average bounty paid by new programs ( $B_{new,t}$ ) has a negative effect on existing programs as well, but the coefficient is only significant in model **4**. Again this result is consistent with the theory and Hypothesis **H2**, as researchers have a higher incentive to switch to new programs, if they

TABLE I. Regression results.

	(1)	(2)	(3)	(4)
VARIABLES	$V_{it}$	$V_{it}$	$V_{it}$	$V_{it}$
$dP_t$	-1.235*** (0.305)	-1.350*** (0.327)	-2.310*** (0.603)	-1.236** (0.515)
$A_i$	-23.61*** (2.140)	-23.72*** (2.156)	-23.72*** (2.152)	-7.188** (3.473)
$B_i$	16.64*** (1.311)	16.56*** (1.315)	16.75*** (1.339)	-7.414 (5.698)
$T_{it}$	-0.690 (0.426)	-0.658 (0.427)	-3.312*** (1.239)	-3.758*** (1.128)
$B_{new,t}$		-0.0445 (0.0280)	-0.0312 (0.0277)	-0.0321* (0.0184)
$T_{it} \times dP_t$			0.106** (0.0431)	0.0755* (0.0406)
Constant	160.2*** (16.12)	170.4*** (18.80)	190.3*** (23.17)	136.5*** (26.17)
Observations	1,212	1,212	1,212	1,212
R-squared	0.314	0.316	0.319	0.647
Program FE	No	No	No	Yes

Robust standard errors in parentheses

\*\*\* p&lt;0.01, \*\* p&lt;0.05, \* p&lt;0.1

have more low-hanging fruits and higher bounty.

The interaction coefficients for term  $T_{it} \times dP_t$  in models **3** and **4** are positive and statistically significant, so they do not support Hypothesis **H3**. The result shows that the impact of newly launched programs depends on the age of the existing programs: Compared to younger programs, the negative impact of  $dP_t$  is smaller for programs with a longer history,

i.e., those with larger  $T_{it}$ . At first sight, this results may look at odds with the fact that individual expected payoff from a specific program decreases as a function of rank  $k$ , and presumably the older a program the more likely it has a high rank. Thus, the switching effect should be stronger. Perhaps our OLS regression model is limited in the sense that it does account for the absolute activity (which decreases very slowly as  $t \rightarrow \infty$ , as shown on Figure 1B), instead of the variation rate.

Consistent with previous research [13], the regression results also show that a program with higher reputation ( $A_i$ ) or higher bounty ( $B_i$ ) is associated with more bugs received in a month. The regression results also show that a program with longer age ( $T_{it}$ ) is associated with less bugs found. This observation corresponds to the power law decay of bug submission observed following an initial shock (c.f., Figure 1B).

## VI. DISCUSSION AND RECOMMENDATIONS

### A. Discussion

Finding bugs in software code is one of the oldest and toughest problems in software engineering. While algorithm based approaches have been developed over the years, human verification has remained a prime way to debugging and vulnerability hunting. Moreover, the idea of getting “enough eyeballs” to inspect the code has been a cornerstone argument for the open source software movement [29] along with the full-disclosure and responsible disclosure arguments, as well as for vulnerability markets [10]. Bug bounty programs are perhaps the latest successful incarnation of markets for trading bugs and vulnerabilities [10], which set incentives to disclose early, combined with an increase of payoff for more rare and difficult bugs.

Here, we have found that the number of discovered bugs and vulnerabilities in a bounty program is super-linearly associated with the number of security researchers. However, the distribution of bugs found per researcher per program is skewed, but not extreme. In other words, each researcher enrolled in a bug bounty program may contribute her fair share of valid bugs, and no researcher is found to contribute orders of magnitude more than the average. This result is rather surprising, as the current common wisdom is rather to use

bug bounty programs for selecting most talented security researchers [47]. In some way, there is a conceptual flaw in this common wisdom reasoning: If selecting one (resp. a few) particularly talented security researcher(s), then the Coase theorem would apply and it would be more interesting for an organization to internalize the resource by hiring security consultants, or having a in-house auditing team, both of which are already commonly done by organizations [43]. And because bug bounty programs exist and develop, it somehow demonstrates that the former security practices are probably insufficient. On the contrary to this common wisdom, we posit that bug bounty programs reach a large and diverse population of security researchers, who can independently look at the focal software from as many different perspectives.

This observation is reminiscent of an early proposition on the topic: Brady et al. [15] took an evolutionary theory perspective to the problem of software reliability, and basically argued that software is sensitive to environmental changes (i.e., it is not evolutionary fit) because it is usually designed for one purpose. The purpose however changes over time (think e.g., of software packages in Linux, how they are surprisingly linked together [21], and how they are often used in unintended ways). On the contrary to species who adapt by the way of selection (only the fittest portion of the population survives), according to Brady et al., this feature is essentially absent in software. Here, the focal point is a software piece, or more precisely a set of complementary software pieces, which define the service offered by the focal organization. Software fitness is assessed by security researchers, internally (internal audit), externally (ethical hackers), or by resorting to the crowd (bug bounty programs). The software runs in a well-defined environment, and it would be hard, if not impossible, to deploy it in other environments (i.e., for a different use, e.g., by a different population), in order to test its robustness. Note that some very large companies by a matter of fact extensively test their software in a variety of environments given the pervasive nature of their service. One may think of Facebook with currently more 1.5 billion users worldwide.

Because they all carry their own unique experience, security researchers offer a form of confrontation with environments, which are particularly adverse on purpose. Additionally, the more remote from the focal organization, the more original the view on the software piece (without the hassle of deployment). Our results offer a similar conceptual view, as

well as with the quote by Eric Raymond “Given enough eyeballs, all bugs are shallow”. In sum, diversity of views prevails over accumulated expertise, although we make no claim that expertise is not required. We simply observe that its individual effects are just bounded. These results also cast questions on learning curves and incentives that keep the researcher dig for bugs in a program she is already familiar with. We observe that overall these incentives become quickly insufficient in comparison with the increasing difficulty for a researcher to find additional bugs.

Moreover, we find that the larger the population of enrolled researchers, the even more bugs are found. In that process, the initial windfall effect of a newly launched program is critical and determines an important portion of the bug discovery timeline, and accordingly researchers are ready to switch their attention towards newly launched programs, at the expense of older ones. These results have critical implications for software security: If we consider an arbitrary focal bug to be discovered, the chance that it will be discovered increases with the number of researchers. If half researchers interested in the security of the focal software are black hats, there is roughly 50% chance that the focal bug will be discovered by a black hat. If the proportion of population types (white and black hats) compounded over time is uneven, then the probability of discovery falling in one of both categories changes accordingly. In other words, in order to be effective (statistically speaking), a bug bounty program must reach much more white hats, compared to the estimated amount of black hats interested in finding holes in the focal software piece.

Most security researchers however participate in multiple bug bounty programs, and when a new program is launched, they face the strategic choice of switching program. Our results show that researchers have a decreasing incentive to explore higher bug ranks within the same program (Figure 3B), while they have an increasing, yet marginally decreasing, incentive to explore multiple programs (Figure 3C). We further confirm these results with a simple regression model that researchers tend to switch when new programs are launched. This is a strong signal that researchers make rational choices in the bug hunting environment: They have high incentives to switch quickly to a new program and harvest as fast as possible many frequent bugs with little reward, rather than less frequent yet more endowed bugs, even though the reward structure of bug bounty programs seems to incentivize gen-

erously the reward of high rank (i.e., less probable) vulnerabilities (Figure 3A). This result raises questions on some hard limits associated with incentives associated with bug discovery by humans: The disincentive clearly stems from the difficulty for individual researchers to reach high ranks (i.e.,  $P_k \rightarrow 0$  when  $k \rightarrow \infty$ ), not from the reward scheme. While at first sight this turnover of security researchers may look bad to a bug bounty program manager, it may in the end be beneficial provided that a renewal of security researchers occurs. The bug bounty platform must be designed carefully to ensure that a sufficient inflow of new security researchers are enrolled and scattered among all programs hosted on the platform, including older ones, in order to compensate for researchers switching to new programs.

The migration of researchers from one program to another is also related to researchers' specialization. Since vulnerability discovery is a difficult and also competitive activity, researchers typically need to have specialized knowledge and skills. Previous research revealed that there are at least two types of specialization in bug bounty: program-specific and vulnerability-specific [12, 13]. *Program-specific* specialization contains the knowledge, experience and skill of finding vulnerabilities in websites and software products under one particular program. Since specialization is relatively unique to the program, a specialized researcher has less incentive to switch. *Vulnerability-specific* specialization focuses on knowledge and skills of a particular type of vulnerability, which can exist in many different products. Therefore, for researchers with vulnerability-specific specialization, they have stronger incentive to try out their skills on different bug bounty programs. Incorporating researcher specialization into the models discussed in this work would be an interesting future work.

Using *rankings* provides handy insights on the processes governing the vulnerability discovery process, and to some extent, associated incentives. However, the rank is an arbitrary measure of time, which hardly accounts for the effort spent on researching bugs, as well as for discounting effects. For instance, if the time required to find a vulnerability increases with the rank, then the expected payoff shall be discounted accordingly. Other aspects enter the equation: While most submissions occur early on after the program launch, this is also the moment when an organization might be less prepared to respond to a large flow of tasks, which in turn may trigger priority queueing and contingent delays [41]. While some workaround may be envisioned, publicly available data currently limit some desirable

investigations, involving timing and discounting effects.

## B. Recommendations

Based on the results of this work, we make the following recommendations to today’s bug bounty programs, and bug bounty platforms like HackerOne and BugCrowd.

Currently, bug bounty programs rarely increase their bounty level overtime. However, we recommend that a bug bounty program shall consider increasing rewards constantly to offset the fast decay of probability to find a vulnerability of higher rank, as we have discussed in Section VB. In addition, increasing bounty can also offset the impact of newly launched programs on attracting researchers away, as we observed in Section VC. Therefore, the adjustment of bounty level should take both the vulnerabilities found, and also statistics of newly launched programs into accounts. Constructing a pricing model based on empirical results obtained in this work would be an interesting and important future work.

We also suggests that both individual bug bounty programs and bug bounty platforms shall aim at constantly attracting more researchers, in light of the results in Section VA. Previous work [13] has made several suggestions, such as a first time bonus, to achieve this goal. We further suggest that current private bounty programs should gradually increase their enrollment and eventually go public. Receiving no bugs in the private stage does not indicate that the system is secure. Rather, it is more likely that the system has not been fully tested by the crowd. In addition, bug bounty platforms should improve their researcher invitation/allocation mechanisms to encourage the flow of diverse researchers to different programs [51].

## C. Summary

In this study, we have considered the incentive mechanisms at the aggregate level. Managers however organize enrollment, set incentives and tackle the operational pipeline, involving submission reviews and payroll processing. All these aspects, which are unique to each program, may crowd in, or on the contrary crowd out, security researchers from bug bounty programs. In particular their willingness to participate will be affected, but also the amount

of effort they are ready to throw in the search of vulnerabilities. It is the hope of the authors to get increasingly fine-grained insights in the future, to compare bug bounty programs, and thus establish benchmarks of most performing programs, in an environment driven by large deviation statistics, and incentives structures, which resemble the St-Petersburg paradox, a well-known puzzle for decision making in behavioral economics.

## VII. CONCLUSION

In this paper, we have investigated how crowds of security researchers hunt software bugs and vulnerabilities on the public part of a bug bounty platform. We have found that it is essential for managers to design their program in order to attract and enroll the largest possible number of security researchers. Studying the incentive structure of 35 public bug bounty programs launched at a rate of one per month over 2 years, we have found that security researchers have high incentives to rush to newly launched programs, in order to scoop rewards from numerous easy bugs, and as the program ages (and therefore, the probability of finding a vulnerability decreases), switch to newer “easier” programs. Our results suggest that yet incentives are generously set by program managers, it is quickly getting harder for a researcher to find vulnerabilities, once she has discovered the obvious ones. This windfall effect is positive as it allows security researchers provide their unique perspective in many bug bounty programs. However, the loss of researchers by older bug bounty programs should be compensated with new security researchers in order to ensure renewal of perspectives.

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