

## Property Game

Author<sup>1</sup>, Author<sup>2</sup>, Author<sup>3,\*</sup>

**1** Author<sup>1</sup> Dept/Program/Center, Institution Name, City, State, Country

**2** Author<sup>2</sup> Dept/Program/Center, Institution Name, City, State, Country

**3** Author<sup>3</sup> Dept/Program/Center, Institution Name, City, State, Country

\* E-mail: Corresponding author@institute.edu

## Abstract

According to John Locke (1689) “The reason why men enter into society, is the preservation of their property [...] to limit the power, and moderate the dominion, of every part and member of the society.” [1]. In many ways, the level of trust people place in a society is often measured by its ability to defend their individual *property* rights [ ]. The decision to join and keep faith into a society is thus a social dilemma, controlled by the enforcement of individual rights. Here, we report the sudden phase-transition from cooperation to defection, in a world where property violations are possible when individuals imitate superior prisoner’s dilemma strategies and show success-driven migration with the possibility to expel other individuals from their location. In our model, individuals are unrelated, and do not inherit behavioral traits. They defect, cooperate and migrate (including by expelling neighbors in the migration range) selfishly when their expected pay-off is increased, and they do not know how often they will interact with another individual in their neighborhood. The threshold and the nature of the phase-transition between sustained cooperation and invasion of defector are controlled by the migration range and the population density. Our results suggest that societies can manage high levels of property violation, provided that individuals expelled by property violators find a *good enough* relocation. This relocation is made easy with large migration ranges. In the latter conditions, a minority of cooperative individuals can survive by forming clusters in a world surrounded by defectors.

## Introduction

[say something striking about mobility and property violation]

[tie back to some empirical driven research on the topic, if available]

We explore how violations of *individual rights*, such as private property, undermine cooperation in nature and society. Living organisms develop regulation and defense strategies to minimize aggression from their environment [], but also against inner threats such as cancer cells or parasites [], while primitive and advanced human societies have developed laws and judicial institutions to enforce private property [2,3].

Below, we model cooperation in a game-theoretical way [], and integrate a model of individual *property* right violations. This is motivated by the observation that individuals are subjected to violations of individual rights, such as their private properties [], digital privacy [], or even their human rights []. These events make the violator better off, taking a part of the victims assets, while the latter is left in with less favorable options. In the case of a real-estate property violation, the incumbent is forced out of her property, and needs to find another empty location.

In our model, individuals consider a location in which they would be better off within a migration range. How favorable a new neighborhoods expected to be is determined by test interactions with individuals in that area (neighborhood testing). If they select an already occupied location, the incumbent is expelled with some probability, and forced to find another empty location within her own migration range.

So far, the role of migration has received little attention in game theory [], probably because it has been found that mobility can undermine cooperation by supporting defector invasion []. However, this primarily applies to cases, where individuals choose their new location in a random way. In the case of success-driven migration, cooperation has been found to strive and even to spontaneously emerge in the presence of randomness in actions taken by individuals [4].

As we will show, *property violations* undermine cooperation in surprising fashions, depending on the migration range and population density. Yet if the migration range is large enough, a minority of cooperative individuals can form dense clusters and resist to defectors.

## Model

Our study is carried out for the prisoner's dilemma game (PD), which is often used to model selfish behavior of individuals when cooperation is risky and defection is tempting, but where the outcome of mutual defection is inferior to cooperation on both sides [5, 6]. Formally, the so-called reward  $R$  represents the payoff for mutual cooperation, while the payoff for defection of both sides is the *punishment*  $P$ .  $T$  represents the *temptation* to unilaterally defect, which results in the *sucker's payoff*  $S$  for the cooperating individual. The inequalities  $T > R > P > S$  and  $2R > T + S$  define the classical prisoner's dilemma, in which it is more profitable to defect, no matter what strategy the other individual selects. Therefore, rationally behaving individuals would be expected to defect when they meet once. However, defection by everyone is implied as well by the game-dynamical replicator equation [7], which takes into account imitation of superior strategies, or payoff-driven birth-and-death processes. In contrast, a coexistence of cooperators and defectors is predicted for the snowdrift game (SD). Although it is also used to study social cooperation, its payoffs are characterized by  $T > R > S > P$  (i.e.,  $S > P$  rather than  $P > S$ ).

As is well-known [6], cooperation can, for example, be supported by repeated interactions [5], by intergroup competition with or without altruistic punishment [8–10], and by network reciprocity based on the clustering of cooperators [11–13]. In the latter case, the level of cooperation in 2-dimensional spatial

games is further enhanced by “disordered environments” (10% inaccessible empty locations) [14], and by diffusive mobility, provided that the mobility parameter is in a suitable range [15]. Usually, strategy mutations, random relocations, and other sources of stochasticity can significantly challenge the formation and survival of cooperative clusters [1]. *Success-driven* migration, in contrast, is a robust mechanism: By leaving unfavorable neighborhoods, seeking more favorable ones, and remaining in cooperative neighborhoods, it supports cooperative clusters very efficiently against the destructive effects of noise, thus preventing defector invasion in a large area of payoff parameters [4].

However, *success-driven* migration excludes that an individual can take the place of another individual. The *private property game* however is a success-driven game, which implies the possibility to expel players from locations with higher pay-off. We assume  $N$  individuals on a two dimension square with periodic boundary conditions and  $L \times L$  sites, which are either empty or occupied by one individual. Individuals are updated asynchronously, in a random sequential order, and each individual gets updated on average  $N$  iterated (i.e., the number of Monte Carlo Steps  $MCS = N \times L^2$ ). At each step, the randomly selected individual performs simultaneous interactions with the 4 direct neighbors and compares the overall payoff with that of these neighbors. If one neighbor has a better payoff, the individual updates her strategy with the one of her best performing neighbor. In absence of noise ( $r = 0$ ), this update is sure, the individual cannot spontaneously start to cooperate ( $q = 0$ ).

The *success-driven migration* step is implemented as follows [4]: before the imitation step, an individual explores the expected payoffs for *all* sites in the Moore neighborhood  $(2M+1) \times (2M+1)$  of range  $M$ . If the fictitious payoff is higher than in the current location, the individual is assumed to move to the site with the highest payoff with probability  $m$  (in absence of migration noise,  $m = 1$ ). If the site with highest payoff is already occupied by another individual, the *property-violation migration* occurs with probability  $s$  this neighbor is expelled to the best empty site in her own Moore neighborhood (**her strategy is not changed → check this**). With probability  $1 - s$ , the property violation tentative is aborted and the individual migrates to the empty site with the highest payoff. Besides property violation, which is a random variable, the individual is fully rational about her choices given information available in her Moore’s neighborhood [**bounded rationality → Herbert Simon**]. As such, our model encompasses the unsure nature of choices regarding property violation, such as the probability to get caught by law enforcement [1], or limited by some physical [1] or technical constraints [1]. [**Note that by design the private property game requires success-driven migration**].

## Results

Computer simulations of the model presented above show that the probability of property violation  $s^*$  beyond which cooperation cannot survive is highly dependent on the migration range  $M$  and the population density  $d$ . At time  $t = 0$ , the simulation starts with an equiprobable number of cooperators and defectors scattered uniformly across occupied sites.

When there is no migration ( $M = 0$ ), and by definition no property violation ( $s = 0$ ), evolution occurs only through replication of direct neighbor strategies. Cooperators form clusters to prevent invasion by defectors (see Figure ??A). In absence of property violation, any migration range  $M \geq 1$  unambiguously promotes cooperation (see for instance Figures ??B and ??E, and defectors rapidly disappear (the migration range has no effect on the speed at which cooperators invade the population. See Figure SI ??)).

However, considering the evolution of cooperation when individuals have small migration range  $M \leq 5$ , even little property violation can completely destroy cooperation, past an abrupt critical (phase tran-

sition) point  $s^*$ . For instance, for a unit Moore's migration range, a 8% probability property violation is sufficient to destroy cooperation (see Figure ??D, as well as Figure ??G for  $M = 5$ ). For  $s < s^*$ , a majority of cooperators ( $c > 0.5$ ), along with a minority of defectors, which cluster together for  $M = 1$  (see Figure ??C) or scatter around clusters of cooperators for  $M \geq 5$  (see e.g., Figure ??F).

With larger Moore migration ranges ( $M \geq 5$ ), cooperative populations can sustain much more property violation. For instance, for population density  $d = 0.5$ , cooperators account for on average 80% of the population after 200 iterations, with property violation as large as  $s_-^* = 0.45$  with migration range  $M = 11$  (see Figure ??B), as well as for  $M = 13$  and nearly half chance for property violation (see Figure ??F).

Also for large migration ranges ( $M \geq 5$ ), an intermediary state  $s_-^* \leq s \leq s_+^*$  appears, in which cooperative populations can survive while on average in minority (see Figures ??C and ??G). For  $s > s_+^*$ , cooperative populations do not strive (see Figures ??D and ??H).

As individuals search their *best-shot* – maximizing expected pay-off at the selected location (in their Moore's migration range  $M$ ) – they only try to expel another individual, with property violation probability  $s$ , if this *best-shot* location is already occupied. The effects on cooperation, are not only a function of property violation, but also of the migration range as well as the population density: The lower the density  $d$  and the larger the migration range  $M$ , the more opportunities to find a *best-shot* location, which is empty. On the contrary, the higher the population density and the smaller the migration range, the more individuals must rely on property violation to make a successful move. As shown on Figure 1 for  $M = \{5, 7, 11, 18\}$ , the level of property violation  $s^*$  beyond which cooperation cannot survive, depends on both the migration range  $M$  and the population density  $s$ . For large  $M \geq 7$ , the intermediary state  $s_-^* \leq s \leq s_+^*$ , where cooperative populations can survive while on average in minority, appears in green. We also find that for high population densities  $d > 0.9$ , cooperation cannot survive for  $s > 0$ ; Even with no property violation  $s = 0$ , there is a probability that cooperators cannot invade the whole population, in particular for large migration ranges (see SI Section ).

## Going Further

To further understand the properties of the *property game* as a special *success-driven* migration game, we shall consider how individuals exploit the degrees of freedom offered to them, such as empty sites available when  $d < 1$  and their capabilities to reach these sites according to their mobility  $M > 0$ , how they achieve even greater opportunities by expelling individuals from their location. We finally investigate how cooperation is affected by the relocation of expelled individuals. In our model, individuals are fully rational as they will imitate they imitate their neighbors and migrate with probability 1, if the find a strategy and a location respectively, with higher payoff. Only the probability of property violation is uncertain and controlled by  $0 \leq s \leq 1$ .

We consider the actual mobility if individuals  $m = \sqrt{|x^2 + y^2|} \leq M$  bounded by the migration range  $M$ , versus average distance of sites (weighted by their payoff and whether they are free or not). This has something to do with the structure of clusters → a proxy could be density of cooperators vs. defectors in the migration range.

We shall also who uses property violation? defectors or cooperators?

Note that the larger the migration range, the more likely to find a number of sites for which the payoff is the same. In that case, the migration site is chosen in the following way: randomly among empty sites, and if there is no empty site, randomly among non

empty site with some probability of property violation  $s$ . Note that calculating payoff from 8 neighbors instead of 4 may help alleviate this intrinsic randomness.

Note that if the migration range is large enough, agents can “jump” from one cluster to another → think of hubs (technology, finance, politics)

## Discussion

### Limitations

- What the resource is not exclusive (e.g. personal information)?

## Conclusion

The *property game* is a mobility game, which implies that with some probability  $s$  a player can be expelled from her site. In general, property violation heavily undermines cooperation and the collapse of cooperation is associated with a sharp phase transition (Figure ??), which becomes smoother as the migration range  $M$  gets larger.

But in some circumstances, cooperation may be maintained as a majority strategy even for large  $s$ . The main condition is the migration range of players, which is equal to the migration range of property violators: As the migration range  $M$  gets large enough  $M > ??$  cooperation remains the dominant strategy (Figure ??) [yet with some increased volatility?].

The function  $c(M, d, s, i = N)$  governing the probability of cooperation remains unclear, but we could study the edge of the phase transition for  $M > 7$  (see Figure ?? g.)

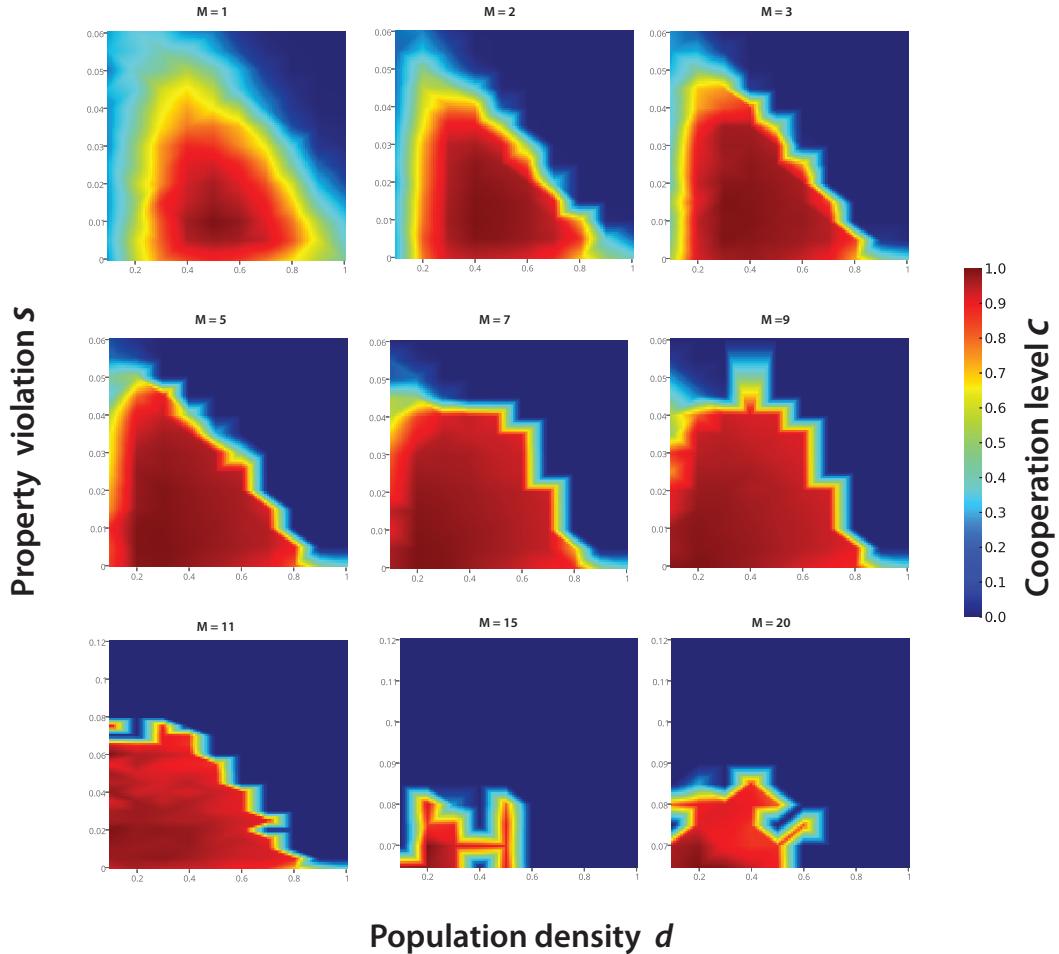
## Acknowledgments

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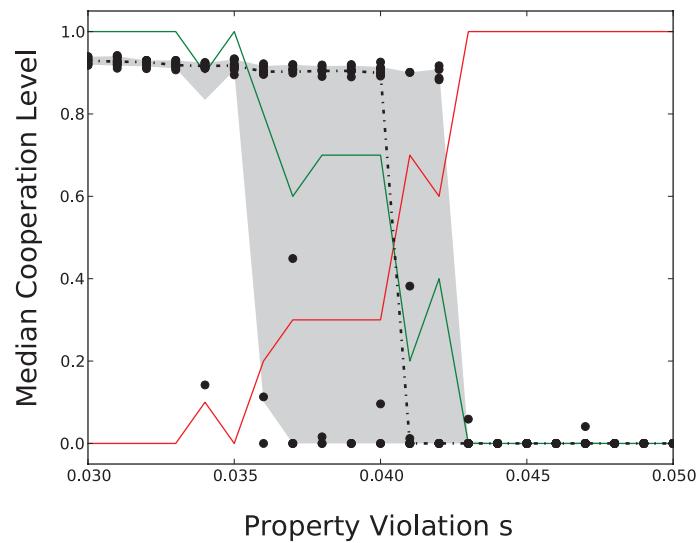
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## Figures



**Figure 1.** Cooperation levels after  $N = 200$  iterations [i.e.,  $200 \times 49^2 = 480,200$  Monte Carlo Steps (MCS)] for migration Moore's distances  $M = \{1, 2, 3, 5, 7, 9, 11, 15, 20\}$ , as a function of population density  $d$  and probability of property violation  $s$ . At initialization ( $t = 0$ ), there is a 50% chance that a player will cooperate (resp. defect). The uneven landscapes reflect the statistical fluctuations of simulations. For all values of  $M$ , the cooperation exhibits a sharp drop for  $d > d^*$  and  $s > s^*$  with  $(d^*, s^*)$  being a function of  $M$ . For high grid density ( $d > 0.9$ ), cooperation cannot be sustained even with low property violation. As the migration range gets large  $M > 9$ , the area of sustainable cooperation shrinks drastically.



**Figure 2.** Phase transition of cooperation levels after  $N = 200$  iterations [i.e.,  $200 \times 100^2 = 2M$  Monte Carlo Steps (MCS)] for migration Moore's distance  $M = 5$  and population density  $d = 0.5$ . The cooperation level (dotted black line) exhibits a sharp drop around  $0.035 < s^* < 0.044$ , with an equal probability that cooperative players will strive or disappear around  $s^* \approx 4.01 \times 10^{-2}$ : Probabilities that cooperative players invade the grid ( $> 0.8$ ) or disappears ( $< 0.2$ ) are shown (green and red lines). The cooperation is stochastic and does not depend on the initial grid configuration

## Supplementary Material

### SI Text

#### Motivation and Explanation of the Property Violation Game

At local, regional and global scales, migrations increasingly occur increasingly in uncontrolled ways, with number of unintended contingencies and challenges for destinations of migration.

Likewise, crime and robbery, which aim to destroy or to steal the property of others, relies on fast mobility in neighborhoods, or cities (c.f., gangs in the USA).

In the cyberspace, all neighbors are only a few steps away from each others, physically (think of the Internet Protocol (IP) addressing system) or logically through social networks [?].

With unprecedented mobility, invading, destroying or stealing, abusing the assets of people or communities has never been so accessible, leading to concrete consequences for the stability of physical and online societies. According to repeated United Nation reports, mass migration, resulting from wars and climate change, may threaten the stability of long-established societies across the world. Also, for instance, a high crime rate undermines trust among citizen, reduces the enjoyment, and thus often the economic vigor of a city. Similarly, cyber crime and large-scale cyber attacks undermine trust on the cyber space, and thus, its economic potential.

Our *private-property* model operationalizes the intricate relationship between success-driven migration – involving an one-sided move perceived as a violation of property by the other side located at a migration distance – and how it may undermine trust. Trust is characterized by the capability of individuals to engage into cooperation, in a situation where the Nash equilibrium is 0 ( in other words, when both individuals have an individual incentive to defect). The prisoner’s dilemma is such a situation where cooperators trust each others.

We have tested our model with a variety of parameters, such as population density, migration range, and the probability of property violation, i.e., the probability that an individual may migrate to an occupied site, hence expelling the incumbent individual to another empty location.

The rationale for varying the population density stems from the intuition that densely populated areas are more resource intensive (i.e., available resource per individual is more scarce), and thus, in this case, property violation has more negative effects, such as making relocation more difficult for the expelled individual. We indeed found that beyond a given population density (in general when  $d > 0.8$ ), cooperation cannot survive in presence of even the slightest level of property violation.

The large span of Moore’s migration range, from very local mobility ( $M = 1$ ) to levels close to full mobility on the grid (i.e.,  $M = 24$  for a grid of  $49 \times 49$ ), reflects large inequalities in mobility at local, regional and global scales, also depending on a given environment. For instance, for those having access to the Internet, mobility is unlimited, at least in theory. Also, populations of wealthy social classes can afford traveling nearly all over the world. On the contrary, some individuals have only very limited mobility within a considered environment. The migration range we used shall be viewed as relative to the world under scrutiny: for  $M = 1$  the individual can only move by 1/24 of the world size (defined by a grid of size  $49 \times 49$ ) at each step; for  $M = 12$ , the individual may move within half the size of the world, which is assumed to be uniformly populated. Clusters may form locally but since only one individual can occupy a grid site the density remains overall well distributed, unlike other types of models such as

AB models [?] in which multiple agents can accumulate on a single site. The migration range applies to all individuals, regardless whether they just migrate to an empty site or if they attempt to expel another player. There is no designated property violator in our model. All individuals may turn property violators based on their payoff opportunity, as well as following their chance to overcome property enforcement.

The probability of property violation reflects the limits of property enforcement, whatever the type of enforcement applied, which can be of legal, police and military nature, or as a result of capabilities by individual to protect their own assets.

Configurational Analysis of surprises:

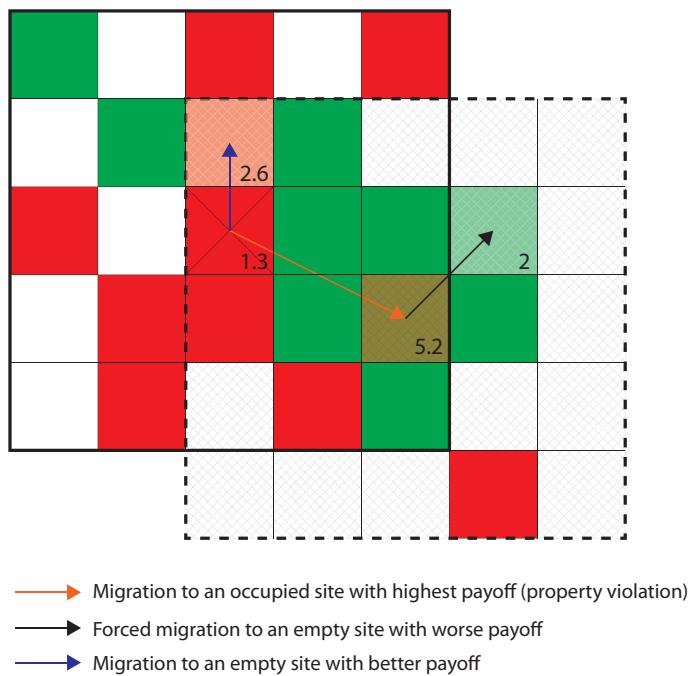
1. phase transition as a function of  $d, M, s \rightarrow$  study actual migration ranges!
2. intermediary state with cooperator level steadily  $< 0.5$
3. can we “boot” a world with less than 50% cooperators with or without property violation? (in particular, show that the game cannot go to a stable state with  $c \approx 0.45$  with such initial level?).
4. high density populations with  $s = 0$  and  $s > 0$

In previous research, Helbing and Yu [] have found that We have searched for counter-intuitive situations where property violations would have positive implications for cooperation. So far, we have not found such a result. The intuition for this hypothesis would be that cooperators forced to migrate would create “unforeseen” situations, which would help spreading cooperation.

**Property game step by step:**

1. **Player selection:** At each Monte-Carlo Step ( $MCS = N \times l \times h$  with  $N$  the number of iterations, and  $l$  the grid length and  $h$  the grid height) a site is selected. If the site is occupied, the **focal player** is selected. Each player is thus expected to be selected  $d \times N$  times with  $d$  the grid density.
2. **Success driven migration (exploration) :** With probability  $1 - m$ , the focal player strategically explores its own migration (Moore) neighborhood  $(2M + 1) \times (2M + 1)$  of range  $M$ , searching for a site with better payoff with her current strategy (either cooperate or defect). To assess for sites with best payoff, the individual plays the prisoner’s dilemma with her own strategy and with neighbors for each site within the migration range. For each site assessed, the “virtual” payoff is computed as the sum of outcomes from playing the prisoner’s dilemma with all neighbors.
3. **Success driven migration (best empty location) :** If among the sites with the highest payoff, some are empty, the player moves to the closest empty site.
4. **Success driven migration (property violation) :** If there is no empty site among those with highest payoff, the focal player expels the target player with probability  $s$ . The expelled target player is forced to move to the empty location with highest payoff within her own migration range. The expelled player may find a new empty site with either higher or lower payoff. For both the focal and the target player, in case multiple sites with highest payoff are available, the closest one is automatically selected. If they are at the same distance, one site is randomly chosen among best sites with smallest migration distance.
5. **Success driven migration (better empty location) :** If there is no empty site among those with highest payoff and property violation did not occur, the focal player is forced to move to the empty location with higher – yet not highest – payoff, with probability  $1 - s$ .

6. **Success driven migration (no migration)** : If all empty sites in the migration range have a payoff worse than the incumbent payoff on the focal site, the player does not move.
7. **Random relocation**: With probability  $m$ , the focal player relocates to a random location either occupied with probability  $s$ , or to an empty location with probability  $1-s$ . There is no consideration of random distance in the case of random relocation.
8. **Imitation step** : Whether it has moved or not, the player is allowed to update her strategy (cooperate or defect), by playing the prisoner's dilemma with her neighbors. If one neighbor's strategy leads to a better payoff, the focal player may imitate this strategy with probability  $1-r$ . With probability  $r$ , however, her strategy is reset as follows: the player will cooperate with probability  $q$  and defect with probability  $1-q$ . If the (target) player is forced to move after a property violation, she is not allowed to update her strategy (because it is not her turn to play).



**Figure 3.** Migration diagram for focal player on site with payoff 1.3 (defector on crossed site). The best site in the migration range has highest payoff ( $1.3 \times 4 = 5.2$ ) for a defector, which is the strategy of the focal player. The site with highest payoff is occupied by a cooperative player and may be expelled with probability  $s$  (orange arrow). In this case, the player on the target site is forced to move to the nearest empty site with payoff 2 (black arrow). However, with probability  $1-s$ , the target player moves to the empty site with payoff (2.6) than the payoff at the focal site.

#### Biological and social coevolution (pasted from Helbing and Yu 2009)

. We certainly believe that there could be a coevolution of social behavior and genetically determined (hard-wired) behavior, as is discussed, for example, by Gintis and Bowles in their articles on strong reciprocity and human sociality (2, 3). However, it is still worthwhile to ask how and why strong

reciprocators (who cooperate and punish noncooperators even if the probability of future interactions is low) would be born in sufficient numbers to induce the cooperation of self-interested agents. The aforementioned coevolution can be represented by models assuming group selection. For example, a group containing a sufficient number of strong reciprocators would have larger chances of survival in a world characterized by frequent crises than a group containing no or a few strong reciprocators. Although the connections between group selection and success-driven migration are rather limited, one could argue that individuals with a stronger tendency to cluster have an advantage over individuals who have a weak tendency to cluster. Moreover, one could think that there is something like a coevolution between the strategies individuals choose and their spatial organization. However, none of this is hard-wired or genetically inherited in our model. All individuals in our model are assumed to be of the same kind, and they do not learn to behave differently in the course of time (i.e., their behavioral rules or strategy choices are not history-dependent).

### **Costly punishment (paste from Helbing and Yu 2009)**

In our model, individuals do not impose a cost on defectors, but they evade future interactions with them. In comparison with staying in the neighborhood of defectors, this reduces the average payoff of defectors and may be considered as a particular kind of punishment. Therefore, our model may eventually contribute to a better understanding of emergent norms, which increase the probability of individuals to show certain behaviors, thereby making social interactions more predictable and successful on average. Note that punishment by evasion is not costly for cooperators in our model. However, evasion from defectors would be costly, if we would introduce transaction or relocation costs in our model. These costs, however, would not be restricted to cooperators. Whenever defectors move into the neighborhood of cooperators, they have to pay relocation costs as well. As the relocation costs of cooperators for evading defectors and the relocation costs of defectors following cooperators are basically the same, introducing relocation costs does not change the conclusions of our study.

→ Actually the migration range is an implicit cost of relocation: if  $M$  is small relocation costs are high and the larger  $M$  the less costly migration.

### **Network reciprocity, partner selection, and reputation. (paste from Helbing and Yu 2009)**

Our model introduces a mechanism for the self-organization of cooperative clusters (assortment), which differs from the clustering mechanism observed in spatial games without migration. Assortment is known to support cooperation, and it is sometimes referred to by the terms network reciprocity or graph selection. In fact, moving to cooperative neighborhoods reminds of partner selection (i.e., the formation of a friendship network with cooperative individuals, while discontinuing interactions with defecting individuals). However, forming links with friends requires a reputation mechanism and, with this, higher cognitive abilities. In our model, individuals do not need to recognize whether they have interacted with an individual before and do not remember what strategy this individual applied. In fact, individuals do not seek friends, and have little control regarding which individuals they stay close to and, therefore, which individuals they interact with. Their neighborhood can, in fact, change significantly. Moreover, in our model individuals are always exposed to their neighbors, i.e., they cannot avoid further interactions with neighbors who defected in the past, if they do not migrate (but then, they will usually lose contact to their friends as well). If an individual decides to migrate, the new neighbors will be interaction partners, no matter whether they are happy with this or not. Therefore, success-driven migration makes weaker and less favorable assumptions for cooperation than partner selection. Although the latter can be very selective and allows the creation of an individualized interaction network, migration can only choose among available neighborhoods, which also means that one may have to accept the presence of defectors, if there are no better choices.

## Migration and Property Games in Densely Populated Worlds ( $d \geq 0.9$ )

In the success-driven migration game, a fully populated world ( $d = 1$ ) is only feasible when property violation exists ( $s > 0$ ), since an individual willing to move must expel another individual. In absence of property violation ( $s = 0$ ), the game only involves updating strategies with no migration. When  $d = 1$  and  $s > 0$ , we find that cooperators cannot strive and disappear after less than 15 iterations.

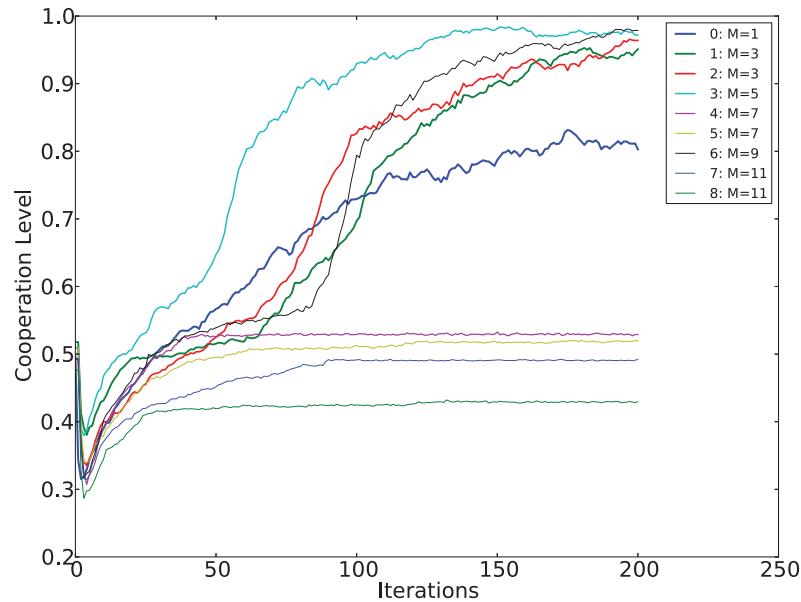
In the limit of population density  $d \rightarrow 1$ , success driven migration with property violation  $s > 0$  leads to a systematic collapse of cooperation. However, for highly densely populated worlds (e.g.,  $d = 0.9$ ), even without property violation ( $s = 0$ ), while cooperators can resist to defectors, they may however **not** be able to invade completely the grid (actually, they may be stuck at an intermediate level, usually  $0.4 \leq c \leq 0.6$ ). Cooperators successfully strive and invade almost completely the grid with some probability, which deserves further scrutiny (see Figure 4 for a comparison of games with  $d = 1$  and with  $d = 0.9$  with same initial conditions but different outcomes).

Here, we question whether the ability by cooperators to invade the grid, is determined by the initial conditions, or on the contrary, if this capability is the result of a chaotic process, when individuals randomly take their turn for migration and strategy update as the simulation proceed. We performed the simulations ten times for  $M = 3$  and  $M = 5$  with random initial conditions, and we repeated 10 times the simulations with each of the 10 initial conditions generated. We then counted the number of simulations that ended with  $c > 0.9$ , conditioned on random conditions on the one hand, and on fixed initial conditions on the other hand.

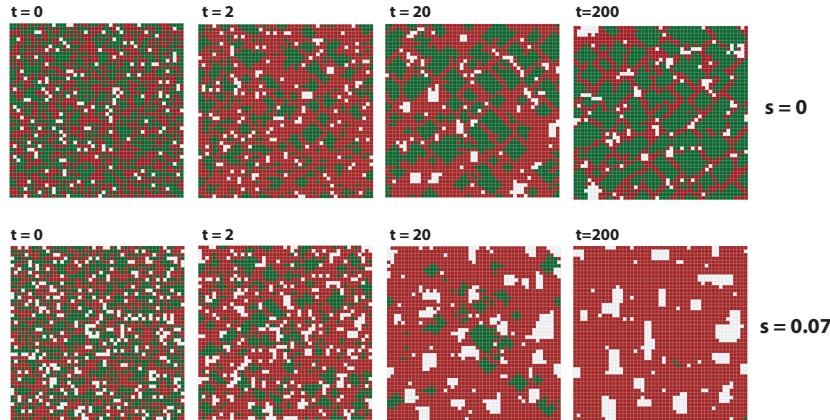
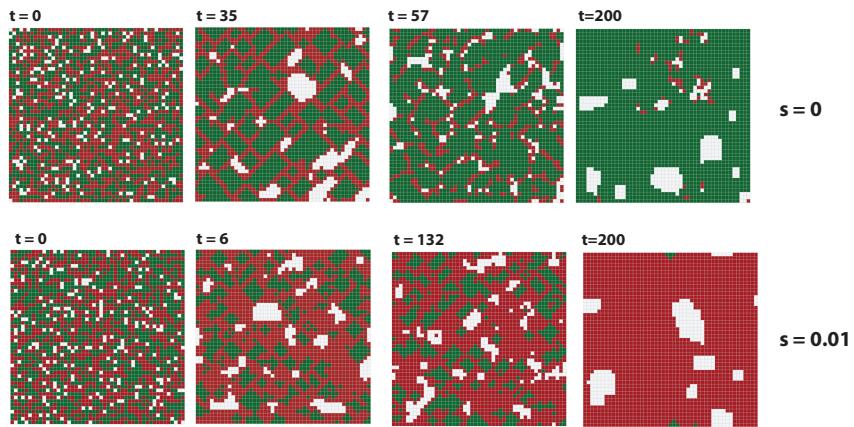
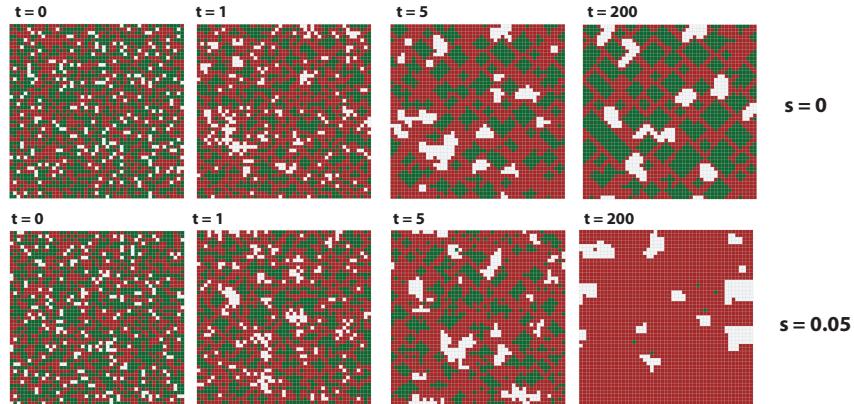
For  $M = 3$  (resp.  $M = 5$ ), we find that high cooperation achievement  $c > 0.9$  results at XX% (resp. XX%) from the initial conditions, and XX% (resp. XX %) from the random strategy and migration updates by individuals. The latter contribution may be seen as an instance of a chaotic process.

We shall study in more details the origins of this chaotic behavior, leading either to  $0.4 \leq c \leq 0.6$  after  $t = 200$  iterations or to  $c > 0.9$ . We inspect how migration patterns in the latter case, influence departure from the former dynamics.

**Figure 4.** Evolution of cooperation for  $M = \{1, 5, 9\}$  for  $d = 1$  and for  $d = 0.9$ , starting from the same initial conditions of a focal simulation (bold line) for which cooperation could invade more than 90% of the grid.



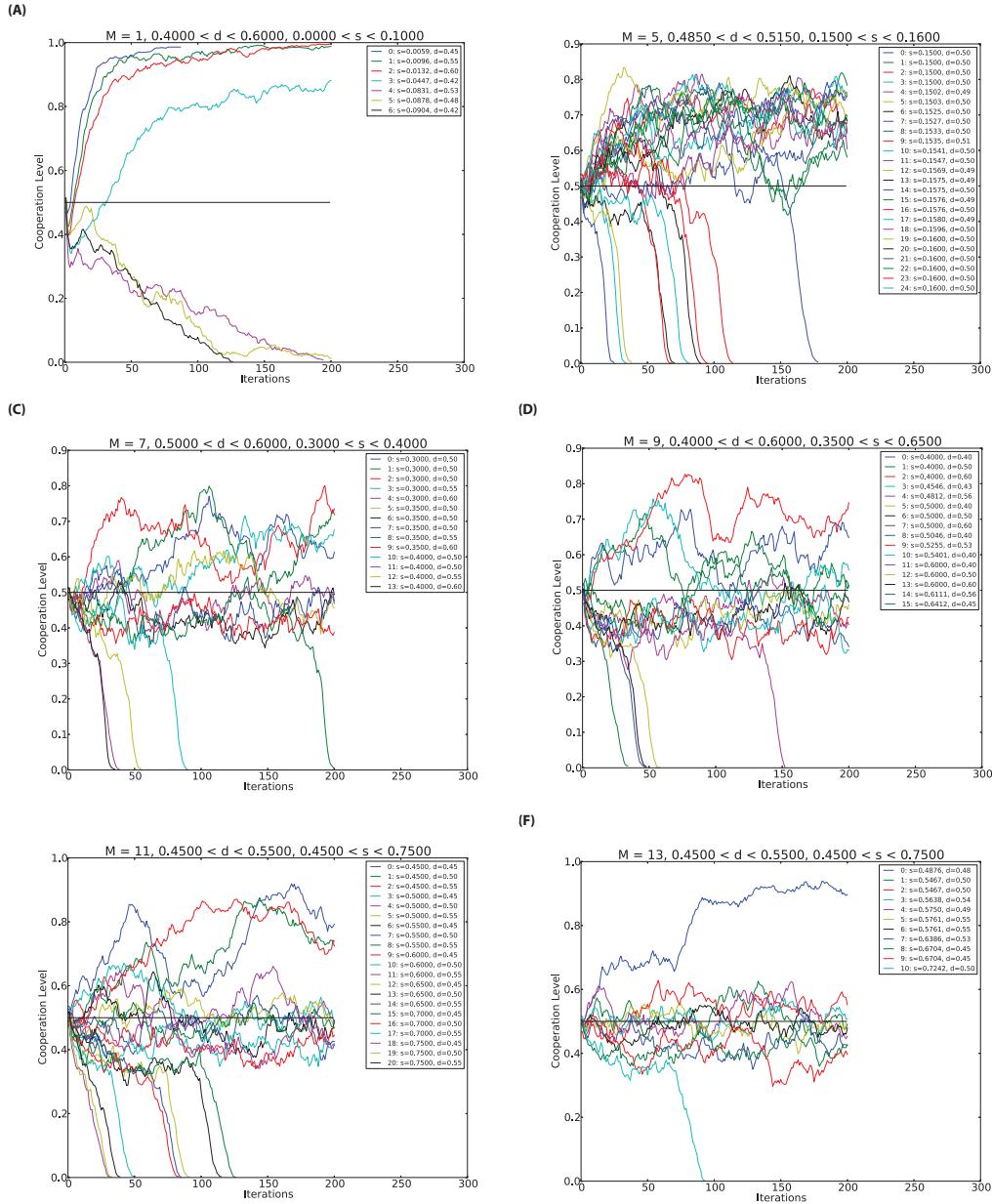
**Figure 5.** Evolution of cooperation for  $d = 0.9$ , with  $M = \{1, 3, 5, 7, 9, 11\}$ . Cooperation does best for medium migration ranges  $M = \{3, 5\}$ . For  $M \geq 7$ , cooperation is stuck between 40% and 55%, at the notable exception of  $M = 9$ , which performs as well as  $M = \{3, 5\}$ . It looks like that, for all  $M$ , cooperators counter defector invasion by creating small clusters, which grow up to a saturation where these clusters can survive surrounded by defectors. In some cases  $M = \{3, 5, 9\}$ , cooperators manage to unlock the situation and further break the belt of defectors. [The conditions for this to happen remain unclear to me so far, especially that  $M = 9$  performs like  $M = \{3, 5\}$  rather than like  $M = \{7, 11\}$ ].

(A)  $d = 0.90, M = 1$ (B)  $d = 0.90, M = 5$ (C)  $d = 0.90, M = 11$ 

**Figure 6.** Effects of migration in densely populated grids ( $d = 0.9$ ). With small migration range  $M = 1$  (see B.), clusters of cooperators form quickly, along with defectors around these clusters. The small migration range does not allow cooperators to jump over defectors. When the migration range is larger  $M = 5$  (see C.), similar clusters of cooperators form at first ( $t = 35$ ), but the larger migration range allows creating connected clusters ( $t = 56$ ), which in turn help overcome most defectors ( $t = 200$ ).

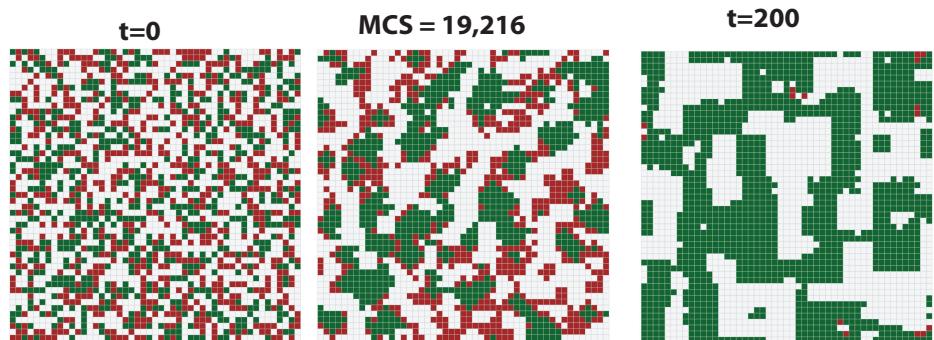
**Migration and Property Games in Medium Populated Worlds ( $0.4 \leq d \leq 6$ )**

[Some text here]



**Figure 7.** Evolution of cooperation for average grid density  $0.40 \leq d < 0.60$  over  $N = 200$  iterations for migration ranges  $M = \{1, 3, 5, 7, 9, 11, 13\}$  and for property violation values  $s$  close to  $s^*(M)$  (migration probability  $m = 1$ ). The presented time series illustrate how the property violation phase transition occurs as a function of the migration range. Populations with a small migration range  $M = 1$  can enhance cooperation after an initial drop. Yet little migration capabilities, make populations very sensitive to property violation: for  $s^* > 0.05$ , defectors win quickly (see A.). For  $M = 5$ , we observe two scenarios regarding property violation: Either cooperative populations win  $s < s^* = 0.158$  or on the contrary, they disappear for  $s > s^*$ . It appears that populations, which cannot sustain a cooperation level higher than 0.5, almost surely defect (see B.). As migration range gets larger ( $M = \{7, 9\}$ ), an intermediary state appears, in which populations can sustain clusters of cooperation for some time, while defectors are in majority (see C. and D.). For even larger migration ranges ( $M = \{11, 13\}$ ), this intermediary state appears even more clearly (see E. and F.), with cooperative populations in minority ( $c \approx 0.45$  for  $M = 11$  and  $c \approx 0.40$  for  $M = 13$ ), yet clustered in a world of defectors (see Figure ?? C and G).

**d = 0.50, M=1, s=0**



**d = 0.50, M=5, s=0**

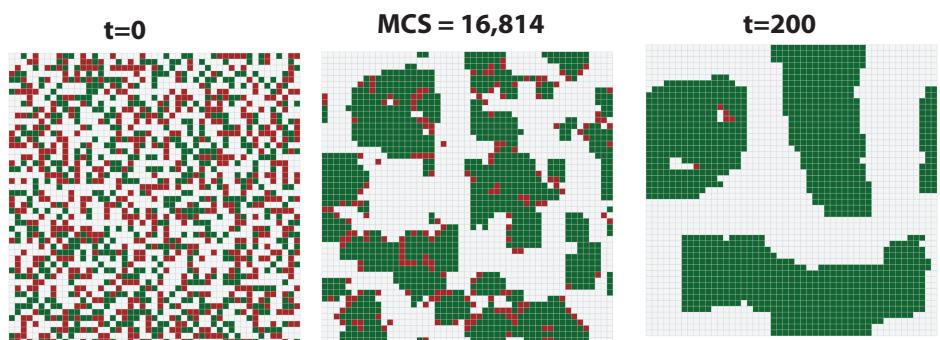


Figure 8