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Abstract

In many biological and human systems, organisms and individuals find ways to enjoy exclusive *private property* rights on the consumption of their own resources. These resources are however subjected to theft. In most human societies, examples of property violations include pickpockets and real-estate thefts, violation of intellectual property, online identity frauds and privacy violation. These private property violations undermine trust and cooperation for the production of collective value. Therefore, besides individual protection against private property violations, most human societies set a level of legal, executive and judicial enforcement, which is considered as sufficient to maintain cooperation. But what should be the “right” level of private property right enforcement to maintain cooperation among individuals? Or conversely, how much private property violation a cooperative society can afford? And how is this level of enforcement is influenced by the migration range of agents, as well as their density ?

We address this question with a *mobility public goods game* in which agents can steal a neighboring occupied site with some probability s. We find that cooperation levels exhibits a sharp phase transition, and disappears for some value $s > s*$. The critical value $s*$ varies as a function of the migration distance (positively) and the density of occupied sites (negatively). When we introduce noise (of types I and II), which has been found to help the emergence of cooperation [], we find that $s*$ drops by one order of magnitude. In other words, societies that promote the emergence of cooperation are more sensitive to private property violations.

to do:

- explain qualitatively the origins of the phase transition, including why games with noise are more sensitive to private property violations.
- Since cooperation is very sensitive to even small values of s in case of noise, can we determine a maximum value of s beyond which cooperation does no longer emerge? $\rightarrow$ redo the emergence of cooperation experiment with various values of s. (I don't think it makes sense to try this because the result is already somehow there: since there is this drop, we know at about which level of cooperators the phase transition occurs. It's very unlikely that cooperation can grow if it is below this critical level.)
- Given that $s*$ varies as a function of density ρ, can we introduce a growth function, which involves progressive replication of agents (AB models), and we check the evolution of the right level of enforcement? (next paper).

Related Work

- Entropy / Shannon Entropy
- Synchronies [**?**]

- Social Neuroscience / Hyperscanning / Game Theory [**?**], [**?**]
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Introduction

Private Property in Nature and Society

Private property is a fundamental principle in most human societies. Similarly, in nature, biological systems have developed strategies to enjoy exclusive rights on resources.

Social norms and laws entitle people to protect their rights to enjoy exclusive possession of goods, freedom to move, to keep their actions and thoughts for themselves, or on the contrary, express themselves, without interference of anyone else. Because the right to private property is so precious, it has to be defended against potential violators. The enforcement of private property can occur at the individual level or through social norms and institutions. And the ability to enforce efficiently private property is often seen as factor for stability in (Western) societies, and as such as a factor of trust in societies. [].

The ability to enjoy exclusive consumption of resources is a fundamental principle of nature and society. [**drop examples from nature here**]

principle in nature and in most human societies. Biological organisms, people and organizations enjoy exclusive rights on resources, such as energy, land, intellectual property, and the right to privacy. Because it confers some exclusive advantage, private property is often challenged, in a number of ways, such as robberies, data breaches, and more generally, privacy violations. Social norms and laws entitle individuals to enforce their exclusive property rights, by themselves, or through institutions.

Enforcement of Private Property

To ensure trust and cooperation between entities, enforcement occurs

- Immune system → make a foreign entity cannot invade a living organism

- laws & law enforcement

Private Property vs. Cooperation

Cooperation doesn't involve sharing, apart sharing "trust".

There is a fundamental tension between cooperation and the violation of private property. Cooperation can only be sustained if an agent can trust other agents, if she knows that other agents cannot take profit from her exclusive right on a private property.

How much private property violation can a society society?

We formulate the hypothesis that the violation of private property has a negative influence (undermines) on cooperation in nature and society.

Implementation

For that, we propose a public good game, in which, agents can steal the "site" of a another agent, with some probability s.

- importance of saturated versus resourceful environments (maybe to be put further down in the paper) → tragedy of the commons !

Below, we model cooperation in a game-theoretical game way, and we integrate the influence of the violation of exclusive rights, as well as the level of resources available (to some extent, maybe rephrase as "opportunities").

This is motivated by the observation that individuals "best-placed" (with higher pay-off) trigger envy and jealousy by others. $\rightarrow$ actually, there are clusters of cooperators that defectors aim to destroy **intentionally** (versus by chance in previous models).

To improve their situation, individuals are often willing to migrate to a more favorable place. What if this place is already occupied and/or has become favorable precisely because it is occupied (think of a field well labored for years, compared to one, which has been abandoned for a long time) ?

So far, the role of private property has received little attention in game theory.

[**Insert a paragraph on cooperation/tragedy of the commons / private property and how they connect to the model**]

how much "property enforcement" (by whatever means) is needed ?

As we will show, cooperation can only be sustained for when high levels of property enforcement exist. We study the influence of property violation in the migration game.

Model

[**pretty much copied from Helbing 2009**] Our study is carried out for the prisoner's dilemma game (PD), which has often been used to model selfish behavior of individuals in situations where it is risky to cooperate and tempting to defect, but where the outcome of mutual defection is inferior to cooperation on both sides. Formally, the so-called "reward" R represents the payoff for mutual cooperation, while the payoff for defection of both sides is the "punishment" P. T represents the "temptation" to unilaterally defect, which results in the "sucker's payoff" S for the cooperating individual. The inequalities $T > R > P > S$ and $2R > T + S$ define the classical prisoner's dilemma, in which it is more profitable to defect, no matter what strategy the other individual selects.

[**pretty much copied from Helbing 2009**] Rational individuals are expected to defect when they meet once. however, defection by everyone is implied as well by the game-dynamical replicator equation [**what's that?**], which takes in to account imitation of superior strategies, or payoff-driven birth-and-death processes.

[**pretty much copied from Helbing 2009**] In contrast, a coexistence of cooperators and defectors is predicted for the snowdrift game (SD). Although it is also used to study social cooperation, its payoffs are characterized by $T > R > S > T$ [**and so what ?**].

[**pretty much copied from Helbing 2009**] Cooperation can be supported by repeated interactions [], by intergroup competition with or without altruistic punishment [], and by network reciprocity based on the clustering of cooperators []. In the latter case, the cooperation in 2-dimensional spatial games is further enhanced by "disordered environments" ($\approx 10\%$ inaccessible empty locations) [], and by diffusive mobility, provided that the mobility parameters is in a suitable range [].

[**pretty much copied from Helbing 2009**] However, strategy mutations, random relocations, and other sources of stochasticity ("noise") can significantly challenge the formation and survival of cooperative clusters. When no mobility or undirected, random mobility are considered , the level of cooperation in the spatial games is sensitive to noise [**?**], as favorable correlations between cooperative neighbors are destroyed.

[**pretty much copied from Helbing 2009**] *success-driven* migration, in contrast, is a robust mechanism. By leaving unfavorable neighborhoods, seeking more favorable ones, and remaining in cooperative neighborhoods, it supports cooperative clusters very efficiently against the destructive effects of noise, thus preventing defector invasion in a large area of payoff parameters. **Here it should reconnect wigh the property game**

[**pretty much copied from Helbing 2009**] We assume N individuals on a square with periodic boundary conditions and LxL sites, which are either empty or occupied by one individual. Individuals are updated asynchronously, in a random sequential order, and statistically, each individual gets updated i times (i.e., the number of Monte Carlo steps $MCS = i * L^2$). At each step, the randomly selected individual performs simultaneous interactions with the $m = 4$ direct neighbors and compares the overall payoff with that of the m neighbors. The strategy of the best performing neighbor is copied with probability $1 - r$ (i.e., imitation), if the own payoff was lower. With probability r, the strategy is randomly "reset" (**noise 1**): the individual spontaneously cooperate (with probability q) or to defect (with probability $1 - q$). The resulting strategy mutations reflect deficient imitation or trial-and-error behavior. As a side effect, such noise leads to an independence of the final cooperation level from the initial one (at $t = 0$), and a *qualitatively different* pattern formation dynamics for the same payoff values, update rules, and initial conditions (c.f. SI Fig.1). Using the alternative Fermi update rule [] would have been possible as well. However, resetting strategies rather than inverting them, combined with values q much smaller than 0.5, creates particularly adverse conditions for cooperation.

[**summarize the results of [?] here**]

[**pretty much copied from Helbing 2009**] *"success-driven migration"* has been implemented as follows []. Before the imitation step, an individual explores the expected payoffs for *all* sites in the Moore neighborhood $(2M+1)x(2M+1)$ of range M. If the fictitious payoff is higher than in the current location, the individual is assumed to move to the site with the highest payoff with probability m. If the site is already occupied by another individual, this individual will be expelled with probability q to the best empty site in her own Moore neighborhood (to study the specific effects of q on cooperation, with have set $m = 1$).

In some sense, the property game is a "generalization" of Figure 2 E-F in [?]

Initial configuration

1. **Grid and iterations:** Grid Size = 49x49, Moore's distance = 5, iterations = 200
2. **Prisoners' dilemma:** $T > R > P > S$ and $2R > T + S$, actually $T = 1.3$, $R = 1$, $P = 0.1$, $S = 0$
3. **empty sites:** variable

execution steps

1. Select a random agent on grid
2. Play with 4 nearest neighbors and find best site
3. with probability m, explore neighborhood within Moore's distance: with probability $1 - s$ find best empty site, and with probability s find best site (incl. both empty and occupied sites)
4. if site with higher pay-off is found, move to this new site. If the site is occupied, expel agent to empty site with highest pay-off in expelled agent's Moore's distance.
5. with probability $1 - r$ copy best strategy form 4 nearest neighbors, and with probability q, spontaneously cooperate.

We test two scenarios :

1. no noise 1, no noise 2
2. noise 1, noise 2
3. **empty sites:** variable
4. Is there a case where no property violation would be less beneficial for the emergence of cooperation than a little property violation ?

and we study the evolution of cooperation as a function of s, the probability to expell another agent. Because the probability to expel is conditioned by the number of filled sites around (the more sites are filled, the higher the probability to expel (to steal an occupied site).

Results

Computer simulations of the "property game" model show that even low probability q migration to non-empty sites, considerably undermines cooperation: in the *empty-cell migration only* configuration ($q = 0$), cluster of cooperators form with defectors at the boundaries (Fig. 1A). However when $q > 0$, defectors can move inside clusters of cooperators (Fig. 1B). For sufficiently large q, clusters of cooperators are invaded and get destroyed.

Should we study q versus the size of clusters? $\rightarrow$ what level of "robustness" is needed to cope with a given level of q? $\rightarrow$ maybe understanding the effects of property violations boils down to uncovering the tradeoff between configurations that generate large clusters, and the intensity of property violation $q \rightarrow$ it naturally extends to a dynamics formulation of the problem: how fast clusters can re-generate (or migrate) as they get invaded by defectors (do clusters actually migrate?).

show results with no imitation and no noise1 (migration only + property violation

show results with migration, imitation + no noise1

Show results with migration, imitation and noise1

Measure distribution of cluster sizes? Migration of clusters vs. evolution of their size (gravity center displacement)? $\rightarrow$ more migration implies more threat therefore more move?

[**Supplementary Materials?**] Figure 3 shows the abrupt transition from highly sustained cooperation levels to the successful invasion of defectors as q increases. [**Show also a distribution of the probability of time before cooperators disappear**]

[**Supplementary Materials?**] **Effects of grid sparsity? $\rightarrow$ more sparse grids prevent larger clusters? or maybe not? surely, more sparse grids reduce the probability of *expelling*.**

It is interesting that, while PD games with imitation and noise together promote cooperation (and the outbreak of cooperation) [**?**], they are also much more sensitive to property violations (Fig. 3).

Show difference between property violations and noise2 (probability to move to a randomly chosen site (free or occupied) without considering the expected success).

property violations break (clusters of cooperators) $\rightarrow$ is there a chance that breaking clusters (of cooperators) can in fact "help" the outbreak of cooperation? $\rightarrow$ I guess, it's unlikely. Still, it should be maybe studied

We want to analyze how cooperation can still emerge in the property game.

1. Emergence of cooperation under constraint of property violation ?

2. properties of phase transitions $\rightarrow$ probability distribution of survival time ?

Figure 1. Representative simulation results for the spatial prisoners's dilemma with payoffs $T = 1.3$, $R = 1$, $P = 0.1$, and $S = 0$ after $t = 200$ iterations. The simulations are for 49x49 grids with 50% empty sites (see Figure **??** for other percentages of grid sparsity). At time $t = 0$, we assumed that 50% of the individuals were cooperators and 50% were defectors. **For reasons of comparison, all simulations shown were performed with identical initial conditions and random numbers (red, defector, blue, cooperator; white, empty site; green defector who became a cooperator in the last iteration; yellow, cooperator who turned into a defector $\rightarrow$ So far, the initial conditions are randomly set. Shall I always keep the same initial conditions, at least, for presentation purpose?**.

Figure 2. **(a)** Phase transitions at different levels of $s*$ for various level of grid sparsity $(perc_filled = \{0.1, 0.3, 0.5, 0.7, .0.9\})$ for $M = 5$. **(b) Phase transitions for various levels of Moore neighborhood** $M = \{3, 5, 7, 9\}$.

Discussion

Materials and Methods

Acknowledgments

Figures

Tables

Figure 3. **(a)** Typical time series for $s << s*$,$s < s*$, $s \approx s*$, and $s > s*$ (for $perc_filled = 0.5$, $M = 5$, $q = 0$, and $r = 0$). **(b)**

Supplementary Material

Spatial Influence

Influence of Grid Density

Influence of Moore's Distance M

Combined Influence of Grid Density and M

Volatility / Resilience versus Private Property