FTT Gamma Values

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When calculating the levelised cost in any FTT module, there must be an additional component which accounts for intangible costs - costs which cannot be quantified. These are represented by a variable gamma for each shares component γ_i.

Taking, for example, the levelised cost of electricity from FTT power:

$$LCOE_i(t) = \frac{\sum_{t=0}^{\tau_i} \frac{TI_i(t)+OM_i(t)+FC_i(t)+CC_i(t)}{(1+r)^t}}{\sum_{t=0}^{\tau_i} \frac{EP_i(t)}{(1+r)^t}} + \gamma_i. \tag{1}$$

The LCOE is directly present in the shares equation, as it is used to calculate the cost difference assessed by investors, ΔC_{ij}:

$$\frac{dS_i}{dt} = \sum_j S_i S_j (A_{ij} F_i(\Delta C_{ij}) - A_{ji} F_j(-\Delta C_{ij})). \tag{2}$$

Gamma values must be found such that the rate of change of shares dS_i/dt is constant at the point where the model crosses from the historical period into the simulated period. There is only one set of gamma values for which this is true. As such, we can calculate the gamma values by solving the second derivative of S_i across the boundary between past and future. Due to the complicated nature of the equations, this may not be analytically possible; however, it is possible to do this with a root-finding algorithm such as Newton's method.

Given that the components of the shares equation are known at the points in time before and after the boundary, and that a sensible initial values for each γ_i can be chosen, one can define the second derivative of S_i at the boundary point as a function of γ_i, and then solve the following equation to find it's roots:

$$\frac{d^2 S_i(\gamma_i)}{dt^2} = 0. \tag{3}$$

From the formal definition of a derivative, the second derivative can be defined as

$$\frac{d^2 S_i}{dt^2} = \lim_{h \to 0} \frac{S_i(t+h) + S_i(t-h) - 2S_i(t)}{h^2}, \tag{4}$$

however, as we already have a definition for dS_i/dt, it is more exact to define the second derivative in terms of $\dot{S}_i$. We can also rewrite the above in a (approximate) finite-difference form, in line with the computation procedure in FTT:Power:

$$\frac{d^2 S_i(\gamma_i)}{dt^2} = \frac{d\dot{S}_i(\gamma_i)}{dt} \approx \frac{\dot{S}_i(t_{n+1}, \gamma_i) - \dot{S}_i(t_n)}{\Delta t}. \tag{5}$$

Here, t_{n+1} would be the time step after the historical period, t_n would be the last time step of the historical period (and as such no gamma value would be present), and Δt is the computational time interval used in the algorithm. Let us call the second derivative of S_i, L_i, to make the function we are focusing on clearer. Hence, we are trying to solve the problem

$$L_i(\gamma_i) \approx \frac{\dot{S}_i(t_{n+1}, \gamma_i) - \dot{S}_i(t_n)}{\Delta t} = 0. \tag{6}$$

This equation can be solved for γ_i using Newton's method: an iterative root-finding algorithm:

$$\gamma_i^{m+1} = \gamma_i^m - \frac{L_i(\gamma_i^m)}{dL_i(\gamma_i^m)/d\gamma_i^m}, \tag{7}$$

where

$$\frac{dL_i(\gamma_i)}{d\gamma_i} = \frac{1}{\Delta t} \frac{d\dot{S}_i}{d\gamma_i}. \tag{8}$$

Gamma values appear in the shares equation as follows:

$$\frac{dS_i}{dt} = \sum_j S_i S_j (A_{ij} F_i(\Delta C_{ij}, \Delta\gamma_{ij}) - A_{ji} F_j(-\Delta C_{ij}, \Delta\gamma_{ij})). \tag{9}$$

As the investor preferences function F_{ij} is approximated as a hyperbolic tangent dependent on the gamma values in the simulation, we can find its exact derivative with respect to γ_i, making our iterative routine possible to solve.

Of course, the values of gamma given by the routine may not be exact, as in reality we need to solve the problem

$$\frac{d^2\mathbf{S}}{dt^2} = 0, \tag{10}$$

where $\mathbf{S}$ is a vector of all shares, the consequence being that one gamma value will influence all the others, and vice versa. Once a root is found for a particular share, it will need to replace its initial guess when calculating the gamma value for the next share, and so on. Iteration may have to be performed multiple times to find the correct set of gamma values, i.e. until the values stop changing. Hopefully, this is possible.

This method is, in essence, similar to the method currently used to determine the gamma values, where the model is re-run, the values changed, until they converge on to the right values. Using Newton's method would allow a GUI to be avoided, and allow us to avoid rerunning the model. Something we would need is the values of parameters such as S_i, F_i and C_{ij} separate from γ_i, so we can input them into the algorithm. We would also need a strong initial guess for each γ_i, but this could be taken from previous values with ease.

There are ways to ensure that the algorithm avoids extreme values of γ_i, so this should not be a major concern. One potential problem is ensuring the values chosen by the algorithm do not interfere with other constraints on the system, such as shares adding up to one. In theory, if there is truly only one set of γ_i which ensure diffusion is constant across the boundary, the algorithm should work.

Furthermore, this is a simple root-finding algorithm, and there are a lot of potential improvements to this process that we could look in to if it is close to working.