Systematic Portfolio Optimization & Risk Forecasting

(Based on Robert Carver's *Smart Portfolios, Leveraged Trading, Systematic Trading*)

1. Forecast Scaling and Risk Budgeting

Forecast Normalization and Signal Strength

The expected return of an instrument is scaled by risk to get a risk-adjusted score:

$$\text{Forecast}_i = \frac{\mu_i}{\sigma_i} \tag{1}$$

This score is then multiplied by a **rule-specific scaling factor** k derived from backtesting:

$$\text{Scaled Forecast}_i = \min(\max(k \cdot \frac{\mu_i}{\sigma_i}, -20), +20) \tag{2}$$

Where:

- μ_i: expected annual return
- σ_i: annualized volatility
- k: rule-specific multiplier (e.g., breakout, carry, MAC crossover)

Example

Corn with $\mu = -12.4\%$, $\sigma = 11.9\%$: $\frac{-12.4}{11.9} = -1.042$ $\Rightarrow$ Forecast $= -1.042 \cdot 30 = -31.26 \rightarrow -20$ (capped)

Notional Position Sizing (Formula 14)

For a given portfolio risk budget R, scale the exposure proportionally:

$$\text{Position}_i = R \cdot \frac{\text{Forecast}_i}{\sum_j |\text{Forecast}_j|} \tag{3}$$

In systems where forecasts are pre-normalized, this yields a volatility-adjusted allocation directly.

2. Risk Measurement in Price Terms

Volatility in Price Units

$$\sigma_i^{\text{price}} = \sigma_i \cdot P_i \tag{4}$$

Where:

- σ_i: annualized return volatility (e.g., 0.15)
- P_i: current price

This is used in converting forecast or carry estimates into risk-adjusted forms.

Instrument Risk: the standard deviation of returns scaled by price.

$$\text{Risk-adjusted forecast} = \frac{\text{Signal (e.g., Carry)}}{\sigma^{\text{price}}} \tag{5}$$

3. Factor Exposure and Alpha Decomposition

Risk-Adjusted Alpha

Let:

$$\text{Alpha}_i^{\text{adj}} = \frac{E[R_i]}{\sigma_i}$$

- Rank instruments by $\text{Alpha}_i^{\text{adj}}$
- Size using notional risk weights from above

This is used extensively in *Smart Portfolios* to transition from pure alpha to forecasted Sharpe-based sizing.

4. Optimization Backbone

Expected Return and Risk

Let $\mathbf{w}$ be weights, $\mathbf{r}$ expected returns, Σ covariance matrix:

$$\text{Return} = \mathbf{w}^T\mathbf{r} \tag{6}$$

$$\text{Volatility} = \sqrt{\mathbf{w}^T\Sigma\mathbf{w}} \tag{7}$$

$$\text{Sharpe Ratio} = \frac{\mathbf{w}^T\mathbf{r}}{\sqrt{\mathbf{w}^T\Sigma\mathbf{w}}} \tag{8}$$

Geometric Mean Approximation

$$\mathbb{E}[G] \approx \mathbf{w}^T\mathbf{r} - \frac{1}{2}\mathbf{w}^T\Sigma\mathbf{w} \tag{9}$$

This provides a more realistic return expectation under compound growth.

5. System Design Meta-Logic

Combining Signals (From *Systematic Trading*)

Final Forecast:

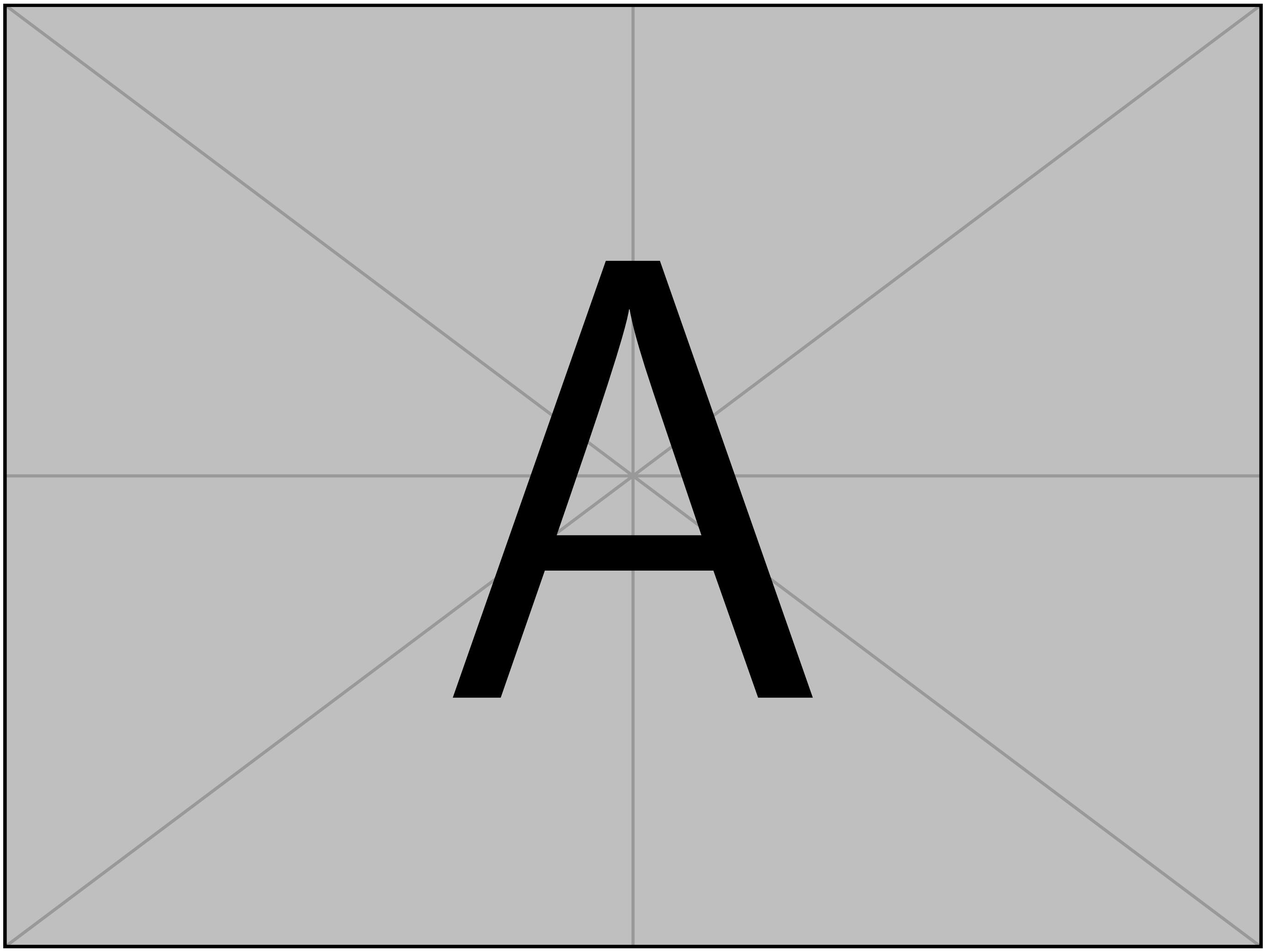
$$F_i = \sum_{k=1}^{K} \text{weight}_k \cdot \text{Signal}_{ik}$$

Use correlation-aware weighting or average if signals are orthogonal.

Bootstrapped Weights (*Smart Portfolios*)

Simulate returns → Optimize weights over each path → Average across simulations.

6. Cheat Flowchart



- Forecast μ_i, estimate volatility σ_i
- Compute forecast $\frac{\mu_i}{\sigma_i}$
- Scale: Forecast $\cdot$ Factor (capped ± 20)
- Normalize: `weights = scaled forecast / sum(abs(forecast))`
- Multiply by portfolio volatility target

"Only bet as much as your volatility can absorb." — Carver