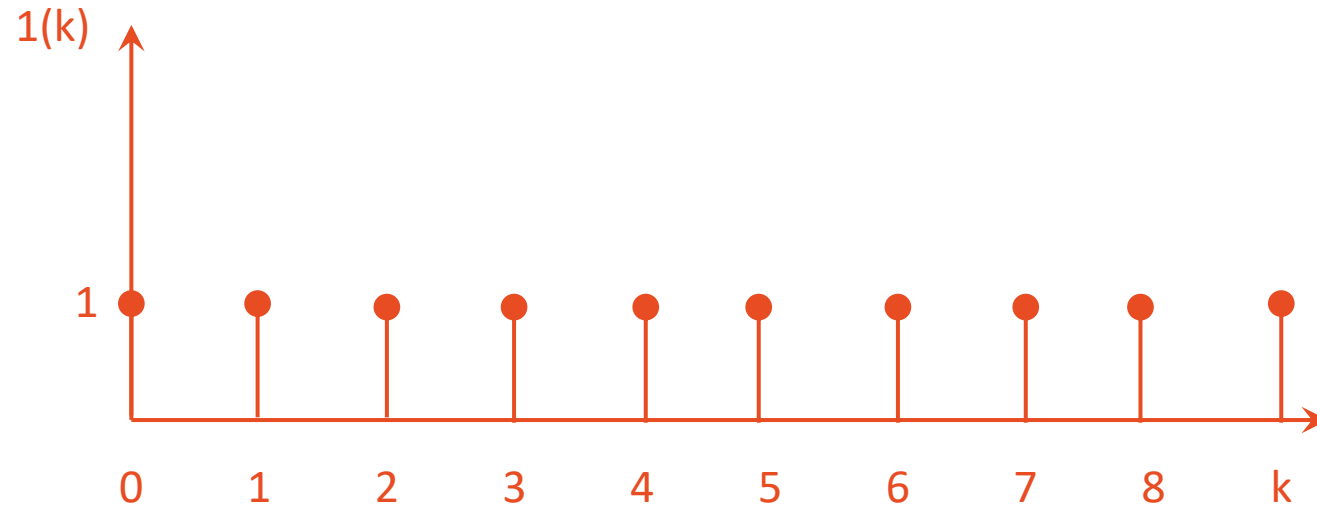# MTE 506 DIGITAL CONTROL

LAB 7 – SPRING 2020

ASSIGNMENT 3 HIGHLIGHTS

2. Find $z-transform$ of the following sequence (try to find closed form)

a) $\{0,0,0,1,1,1,\dots\}$

$$1(k) = 1 + 1\, z^{-1} + 1\, z^{-2} + \dots + 1\, z^{-k}$$
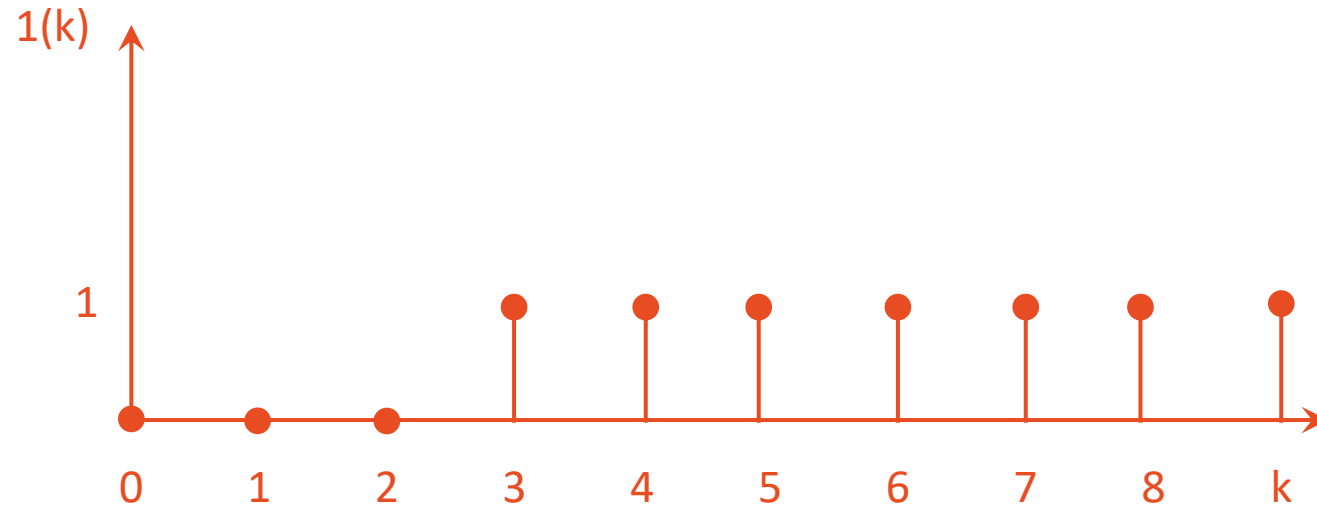
$$1(k) = 1 + z^{-1} + z^{-2} + \dots + z^{-k}$$

ASSIGNMENT 3 HIGHLIGHTS

2. $\text{Find } z - transform \text{ of the following sequence } (try \text{ to } find \text{ closed } form)$

$a) \{0,0,0,1,1,1, \dots\}$

$$1(k-3) = \{0 + 0 + 0\} + 1z^{-3} + 1\,z^{-4} + 1\,z^{-5} + \dots + 1\,z^{-k-3}$$

$$1(k-3) = z^{-3}1(k) = z^{-3}\frac{z}{z-1}$$

ASSIGNMENT 3 HIGHLIGHTS
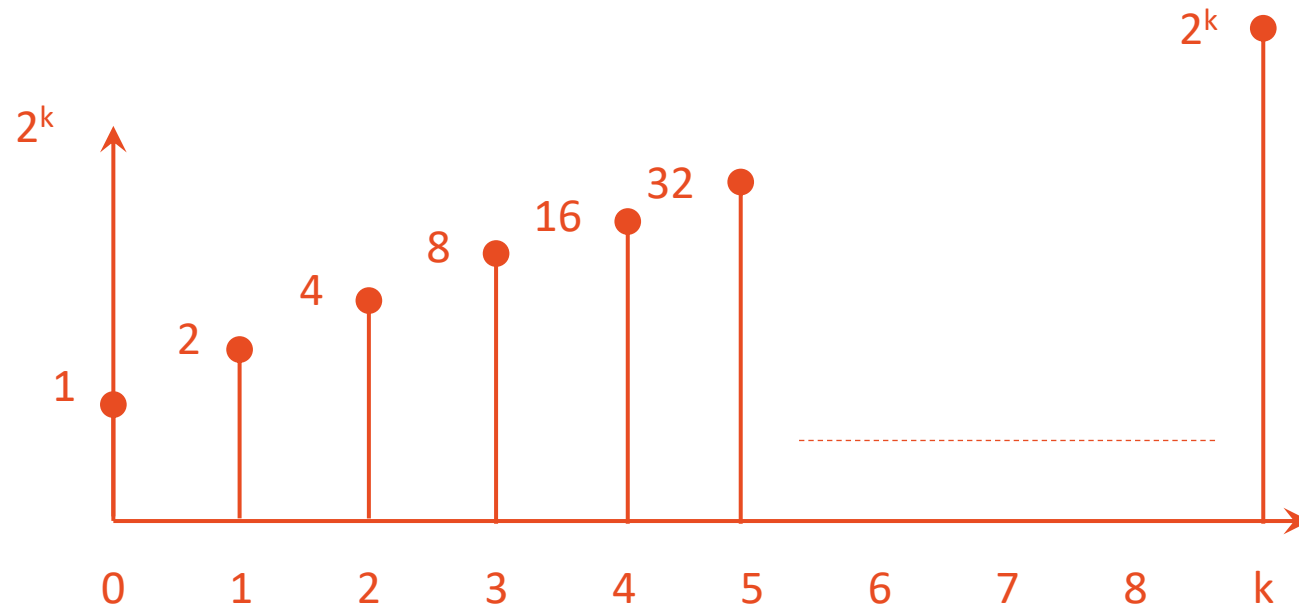
2. Find $z-transform$ of the following sequence ($try\ to\ find\ closed\ form$)

b) $\{0,1,2,4,0,0,\dots\}$

$$2^k = 2^0 + 2^1 z^{-1} + 2^2 z^{-2} + \ \dots + 2^k\ z^{-k}$$

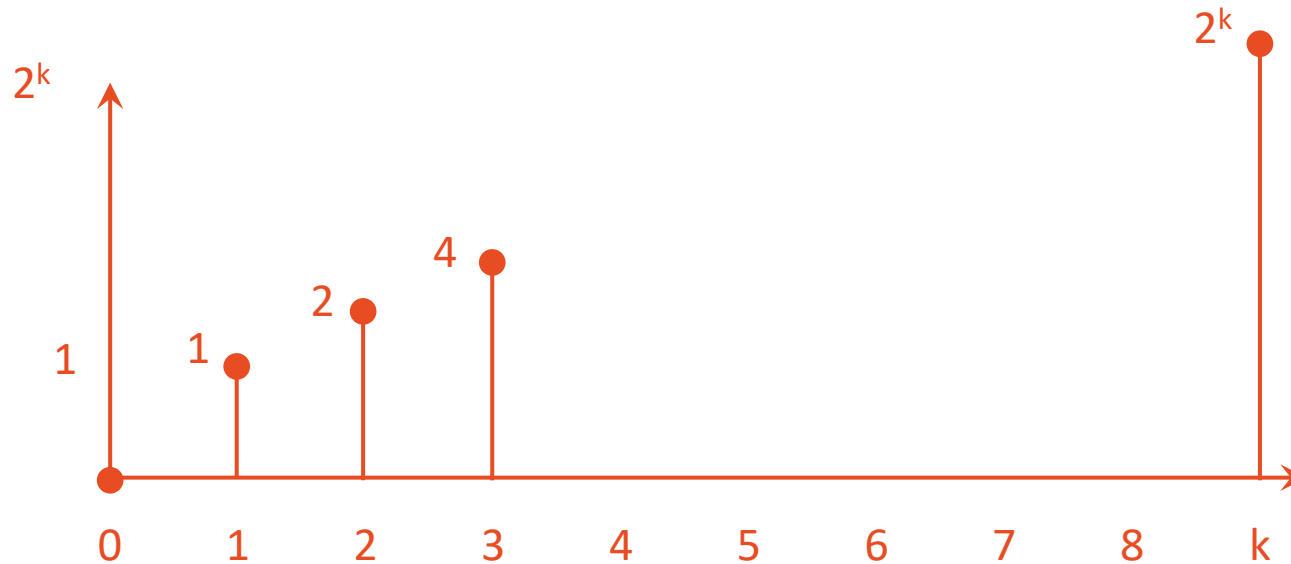$$2^k = 1 + 2\ z^{-1} + 4\ z^{-2} + \ \dots + 2^k\ z^{-k}$$

ASSIGNMENT 3 HIGHLIGHTS

2. Find $z - transform$ of the following sequence (try to find closed form)

b) $\{0,1,2,4,0,0,...\}$

$$2^{k-1} = \{0\} + 2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} \ ... + 2^k \ z^{-k-1}$$

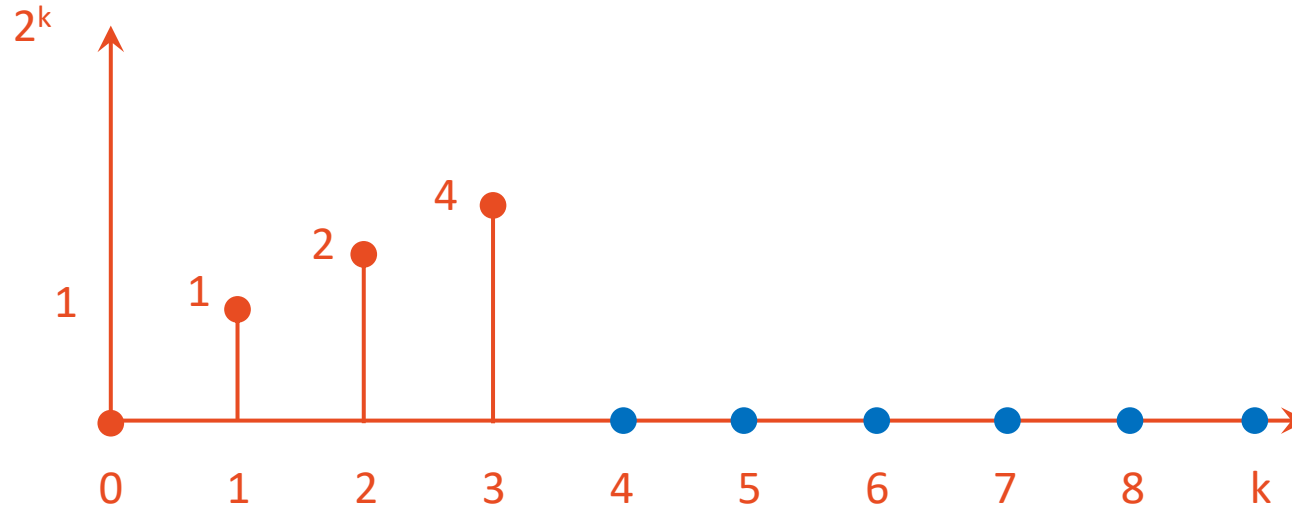# ASSIGNMENT 3 HIGHLIGHTS

2. Find $z-transform$ of the following sequence ($try\ to\ find\ closed\ form$)

b) $\{0,1,2,4,0,0,\dots\}$

$$2^{k-1} = \{0\} + 2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} \dots + 2^k z^{-k-1}$$

$$f(k) = 2^{k-1}, 0 \leq k \leq 3 \rightarrow f(z) = z^{-1}f(2^k) = z^{-1}\frac{z}{z-2}$$

$$f(k) = 0, k > 3$$

Goals of The Lab

Inverse Z-Transform

Solved examples and assignment

Inverse Z- Transform

$$-0.5 \overline{)\begin{array}{l} 1 + 0.5z^{-1} + 0.25z^{-2} + \cdots \\ z \end{array}}$$

$$z - 0.5$$

$$\overline{}$$

$$0.5$$

$$0.5 - 0.25z^{-1}$$

$$\overline{0.25z^{-1}}$$

$$0.25z^{-1} - 0.125z^{-1}$$

$$\overline{}$$

$$\vdots$$

Problem Approach

Difference Equation

$$y(k + n) + \cdots = u(k + m) + \cdots$$

Z-Transform

$$\frac{Y(z)}{z} = \frac{k}{z - a} + \cdots$$

$$z^m + \cdots \big) \, \frac{az^{-1} + bz^{-2} + \cdots}{z^n + \cdots}$$

$$y(k) = u(k) + e^{-3k} + \cdots$$

Inverse Z-Transform

Finding Inverse Z-Transform using *partial fraction expansion*

$$Disctinct\ poles \rightarrow Y(z) = \frac{z}{z - 0.4} \quad + \quad \frac{z}{z - e^{-2T}}$$

$$Repeated\ poles \rightarrow Y(z) = \frac{z}{(z - 1)^2} \quad + \quad \frac{z}{z - 0.3}$$

Finding Inverse Z-Transform using *partial fraction expansion*

$$F(z) = \frac{z+1}{z^2 + 0.3z + 0.02}$$

$$\textit{Solutin pattern} \rightarrow F(z) \rightarrow \frac{F(z)}{z}$$

$$F(z) = \frac{z}{z} \frac{z+1}{z^2 + 0.3z + 0.02} \rightarrow \frac{F(z)}{z} = \frac{A}{z} + \frac{B}{z+0.1} + \frac{C}{z+0.2}$$

Finding Inverse Z-Transform using *partial fraction expansion*

$$F(z) = \frac{z}{z} \frac{z+1}{z^2 + 0.3z + 0.02} \rightarrow \frac{F(z)}{z} = \frac{A}{z} + \frac{B}{z+0.1} + \frac{C}{z+0.2}$$

Finding partial fraction coefficients

$$A = z \frac{F(z)}{z} \Bigg]_{z=0} = F(0) = \frac{0+1}{0^2 + 0.3(0) + 0.02} = \frac{1}{0.02} = 50$$

$$A = 50$$

Finding Inverse Z-Transform using *partial fraction expansion*

$$F(z) = \frac{z}{z}\frac{z+1}{z^2 + 0.3z + 0.02} \rightarrow \frac{F(z)}{z} = \frac{A}{z} + \frac{B}{z+0.1} + \frac{C}{z+0.2}$$

Finding partial fraction coefficients

$$B = (z+0.1)\frac{F(z)}{z}\Bigg|_{z=-0.1} = (z+0.1)\frac{z+1}{z(z+0.1)(z+0.2)}\Bigg|_{z=-0.1}$$

$$B = \frac{-0.1+1}{(-0.1)(-0.1+0.2)} = \frac{0.9}{-0.01} = -90$$

$$B = -90$$

$$A = 50$$

Finding Inverse Z-Transform using *partial fraction expansion*

$$F(z) = \frac{z}{z} \frac{z+1}{z^2 + 0.3z + 0.02} \rightarrow \frac{F(z)}{z} = \frac{A}{z} + \frac{B}{z+0.1} + \frac{C}{z+0.2}$$

Finding partial fraction coefficients

$$C = (z+0.2)\frac{F(z)}{z}\Bigg]_{z=-0.2} = (z+0.2)\frac{z+1}{z(z+0.1)(z+0.2)}\Bigg]_{z=-0.2}$$

$$C = \frac{-0.2+1}{(-0.2)(-0.2+0.1)} = \frac{0.8}{0.02} = 40$$

$$C = 40$$

$$B = -90$$

$$A = 50$$

Finding Inverse Z-Transform using *partial fraction expansion*

$$F(z) = \frac{z}{z}\frac{z+1}{z^2 + 0.3z + 0.02} \rightarrow \frac{F(z)}{z} = \frac{A}{z} + \frac{B}{z + 0.1} + \frac{C}{z + 0.2}$$

$$\frac{F(z)}{z} = 50\frac{1}{z} - 90\frac{1}{z + 0.1} + 40\frac{1}{z + 0.2}$$

$$F(z) = 50\frac{z}{z} - 90\frac{z}{z + 0.1} + 40\frac{z}{z + 0.2}$$

$$C = 40$$

$$B = -90$$

$$f(k) = 50\delta(k) - 90(-0.1)^k + 40(-0.2)^k$$

$$A = 50$$

Finding Inverse Z-Transform using *partial fraction expansion*

a^k	$\dfrac{1}{1-az^{-1}}$

From Z-Transform Table

Kronecker delta $\delta_0(k)$ 1 $k = 0$ 0 $k \neq 0$	1

$$F(z) = 50\,\frac{z}{z} - 90\,\frac{z}{z+0.1} + 40\,\frac{z}{z+0.2}$$

$$f(k) = 50\delta(k) - 90(-0.1)^k + 40(-0.2)^k$$

$C = 40$

$B = -90$

$A = 50$

Finding Inverse Z-Transform using *partial fraction expansion*

For repeated roots

$$A_{1j} = \frac{1}{(j-1)!}\frac{d^{j-1}}{dz^{j-1}}(z-z_l)^r F(z)\Big|_{z \to z_l} , j = 1, 2, \dots, r$$

$A_{1j} \dots j^{th}\ coefficient\ of\ repeated\ root$

Finding Inverse Z-Transform using *partial fraction expansion*

$$F(z) = \frac{1}{z^2(z-0.5)}$$

$$A_{1j} = \frac{1}{(j-1)!}\frac{d^{j-1}}{dz^{j-1}}(z-z_l)^r F(z)\bigg|_{z \to z_l}, j = 1, 2, \ldots, r$$

$$\frac{F(z)}{z} = \frac{1}{z^3(z-0.5)} \rightarrow \frac{A_{11}}{z^3} + \frac{A_{12}}{z^2} + \frac{A_{13}}{z} + \frac{B}{z-0.5}$$

$$A_{11} = \frac{1}{0!}z^3\frac{F(z)}{z}\bigg|_{z=0} = z^3\frac{1}{zz^2(z-0.5)}\bigg|_{z=0} = \frac{1}{0-0.5} = -2$$

$$A_{11} = -2$$

Finding Inverse Z-Transform using *partial fraction expansion*

$$F(z) = \frac{1}{z^2(z - 0.5)}$$

$$A_{1j} = \frac{1}{(j-1)!}\frac{d^{j-1}}{dz^{j-1}}(z - z_l)^r F(z)\bigg|_{z \to z_l}, j = 1, 2, \ldots, r$$

$$\frac{F(z)}{z} = \frac{1}{z^3(z - 0.5)} \rightarrow \frac{A_{11}}{z^3} + \frac{A_{12}}{z^2} + \frac{A_{13}}{z} + \frac{B}{z - 0.5}$$

$$A_{12} = \frac{1}{1!}\frac{d}{dz}z^3\frac{F(z)}{z}\bigg|_{z=0} = \frac{d}{dz}z^3\frac{1}{z^3(z - 0.5)}\bigg|_{z=0} =$$

$$\frac{(0 - 0.5)(0) - (1)(1)}{(0 - 0.5)^2} = \frac{1}{0.25} = 4$$

$$A_{12} = 4$$

$$A_{11} = -2$$

Finding Inverse Z-Transform using *partial fraction expansion*

$$F(z) = \frac{1}{z^2(z - 0.5)}$$

$$A_{1j} = \frac{1}{(j-1)!}\frac{d^{j-1}}{dz^{j-1}}(z - z_l)^r F(z)\Big|_{z \to z_l}, j = 1, 2, \dots, r$$

$$\frac{F(z)}{z} = \frac{1}{z^3(z - 0.5)} \rightarrow \frac{A_{11}}{z^3} + \frac{A_{12}}{z^2} + \frac{A_{13}}{z} + \frac{B}{z - 0.5}$$

$$A_{13} = \frac{1}{2!}\frac{d^2}{dz^2}z^3\frac{F(z)}{z}\Big|_{z=0} = \frac{1}{2!}\frac{d^2}{dz^2}z^3\frac{1}{z^3(z - 0.5)}\Big|_{z=0} = \frac{1}{2}\frac{d}{dz}\frac{1}{(z - 0.5)^2}\Big|_{z=0} \quad A_{13} = 8$$

$$= \frac{1}{2}\frac{(-1)(-2)}{(0 - 0.5)^3} = \frac{2}{0.125} = 8 \qquad A_{12} = 4$$

$$A_{11} = -2$$

Finding Inverse Z-Transform using *partial fraction expansion*

$$F(z) = \frac{1}{z^2(z-0.5)}$$

$$A_{1j} = \frac{1}{(j-1)!}\frac{d^{j-1}}{dz^{j-1}}(z-z_l)^r F(z)\Big|_{z \to z_l}, j = 1, 2, \ldots, r$$

$$\frac{F(z)}{z} = \frac{1}{z^3(z-0.5)} \rightarrow \frac{A_{11}}{z^3} + \frac{A_{12}}{z^2} + \frac{A_{13}}{z} + \frac{B}{z-0.5}$$

$$B = (z-0.5)\frac{F(z)}{z}\Big|_{z=0.5} = (z-0.5)\frac{1}{z^3(z-0.5)}\Big|_{z=0.5}$$

$$= \frac{1}{0.5^3} = 8$$

$$B = 8$$
$$A_{13} = 8$$
$$A_{12} = 4$$
$$A_{11} = -2$$

Finding Inverse Z-Transform using *partial fraction expansion*

$$F(z) = \frac{1}{z^2(z - 0.5)}$$

$$A_{1j} = \frac{1}{(j-1)!}\frac{d^{j-1}}{dz^{j-1}}(z - z_l)^r F(z)\bigg|_{z \to z_l}, j = 1, 2, \dots, r$$

$$\frac{F(z)}{z} = -2\frac{1}{z^3} + 4\frac{1}{z^2} + 8\frac{1}{z} + 8\frac{1}{z - 0.5}$$

$$F(z) = -2\frac{1}{z^2} + 4\frac{1}{z} + 8 + 8\frac{z}{z - 0.5}$$

$$B = 8$$

$$F(z) = -2(1)z^{-2} + 4(1)z^{-1} + 8(1) + 8\frac{z}{z - 0.5}$$

$$A_{13} = 8$$

$$A_{12} = 4$$

$$f(k) = -2\delta(k - 2) + 4\delta(k - 1) + 8\delta(k) + 8(0.5)^k$$

$$A_{11} = -2$$

Finding Inverse Z-Transform using *long division*

$$F(z) = \frac{z + 1}{z^2 + 0.2z + 0.1}$$

$$z^2 + 0.2z + 0.1 \enclose{longdiv}{}$$

$$\begin{array}{l} z^{-1} + 0.8\,z^{-2} - 0.26\,z^{-3} \\ \hline z \quad + \quad 1 \\ z \quad + \quad 0.2 \quad + \quad 0.1z^{-1} \\ \hline \qquad\qquad 0.8 \quad - \quad 0.1 \quad z^{-1} \\ \qquad\qquad 0.8 \quad + \quad 0.16\,z^{-1} \quad + \quad 0.08\,z^{-2} \\ \hline \qquad\qquad\qquad\qquad -0.26\,z^{-1} \quad + \quad \ldots \end{array}$$

$$f(k) = \{0, 1, 0.8, -0.26, \ldots\}$$

Assignment 4

Solve the following difference equations:

$y(k + 1) - 0.8\,y(k) = 0, y(0) = 1$

$y(k + 1) - 0.8\,y(k) = 1(k), y(0) = 0$
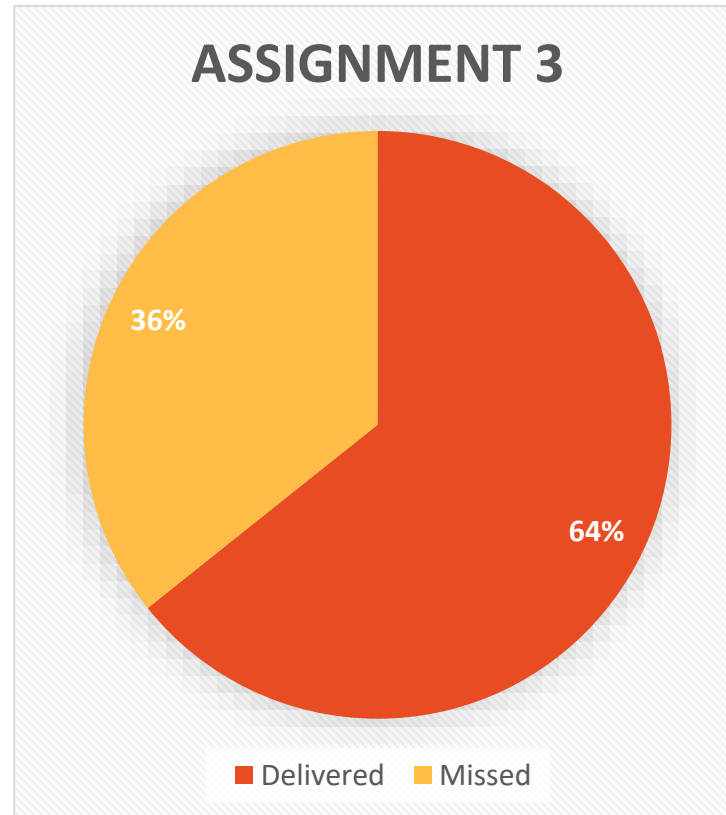
$y(k + 1) - 0.8\,y(k) = 1(k), y(0) = 1$

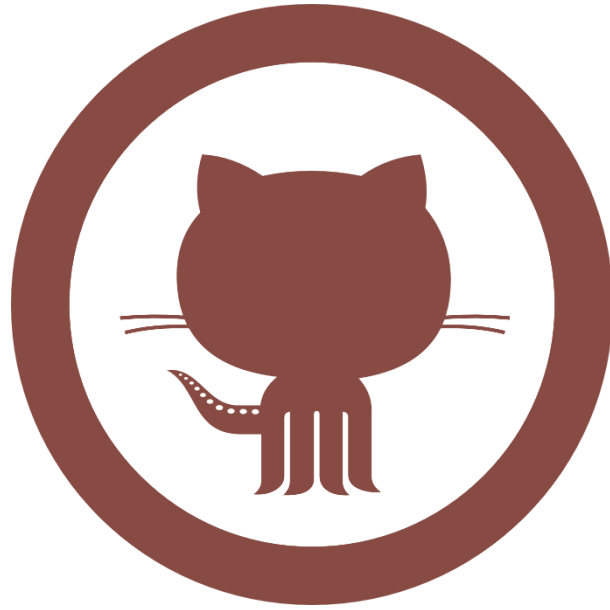$y(k + 2) + 0.7\,y(k + 1) + 0.06\,y(k) = \delta(k), y(0) = 0, y(1) = 2$

Due before : $15 - 04 - 2020$

Lab 7

Past Assignments

Quick Feedback about past assignments

ASSIGNMENT 3

36%

64%

■ Delivered   ■ Missed

Don't forget to pull the lab update from.

http://github.com/wbadry/mte506

Waleed El-Badry

END OF Lab 7