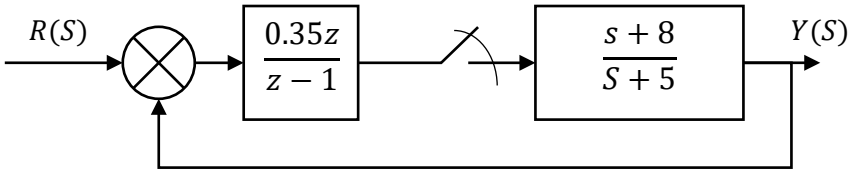
MTE 506 DIGITAL CONTROL

Solution of Assignment 5



1. Finding **the plant Zero Order Hold Transfer Function**

$$G_{ZAS}(z) = (1 - z^{-1})\mathrm{Z}\left(\mathcal{L}^{-1}\left(\frac{G(s)}{s}\right)\right) = (1 - z^{-1})\mathrm{Z}\left(\mathcal{L}^{-1}\left(\frac{s+8}{s(s+5)}\right)\right)$$

$$\because \mathcal{L}^{-1}\left(\frac{s+8}{s(s+5)}\right) = \mathcal{L}^{-1}\left(\frac{A}{s} + \frac{B}{s+5}\right)$$

$$A = \cancel{s}\frac{s+8}{\cancel{s}(s+5)}\Big]_{s=0} = \frac{s+8}{(s+5)}\Big]_{s=0} = \frac{8}{5} = 1.6$$

$$B = \cancel{(s+5)}\frac{s+8}{s\cancel{(s+5)}}\Big]_{s=-5} = \frac{s+8}{s}\Big]_{s=-5} = -\frac{3}{5} = -0.6$$

$$Sampling\ time\ is\ \mathbf{1}\ \boldsymbol{second}$$

$$\therefore \mathcal{L}^{-1}\left(\frac{s+8}{s(s+5)}\right) = \mathcal{L}^{-1}\left(\frac{8}{5}\frac{1}{s} - \frac{3}{5}\frac{1}{s+5}\right) = \mathbf{1.61}(\boldsymbol{t}) - \mathbf{0.6}\boldsymbol{e}^{-\mathbf{5}\boldsymbol{t}}\Big]_{\boldsymbol{t}=\mathbf{1}}$$

$$G_{ZAS}(z) = (1 - z^{-1})\mathrm{Z}\left(\frac{8}{5}1(k) - \frac{3}{5}e^{-5}\right)$$

$$= 1.6\left(\left(\frac{\cancel{z-1}}{\cancel{z}}\right)\frac{\cancel{z}}{\cancel{z-1}} - 0.6\left(\frac{z-1}{\cancel{z}}\right)\frac{\cancel{z}}{z - e^{-5}}\right)$$

$$G_{ZAS}(z) = 1.6 - 0.6\frac{z-1}{z-e^{-5}} = \frac{148.413\,z\ +\ 87.4479}{148.413\,z\ -\ 1}$$

2. Compute the **Closed loop transfer function**

$$G_{cl}(z) = \frac{C(z)G_{ZAS}(z)}{1 + C(z)G_{ZAS}(z)} = \frac{T(z)}{1 + T(z)}$$

$$C(z)G_{ZAS}(z) = \frac{0.35z}{z-1}\ \frac{148.413\,z\ +\ 87.4479}{148.413\,z\ -\ 1}$$

$$T(z) = \frac{(0.35\,z\ +\ 0.206227)z}{(z\ -\ 1.00674)z\ +\ 0.00673795} = \frac{0.35\,z^2\ +\ 0.206227\,z}{z^2 -\ 1.00674\,z\ +\ 0.00673795}$$

$$G_{cl}\,(z) = \frac{\dfrac{0.35\,z^2\ +\ 0.206227\,z}{z^2 -\ 1.00674\,z\ +\ 0.00673795}}{1 + \dfrac{0.35\,z^2\ +\ 0.206227\,z}{z^2 -\ 1.00674\,z\ +\ 0.00673795}}$$

$$\boldsymbol{G_{cl}\,(z) = \frac{0.259259\,z^2 +\ 0.152761\,z}{(z^2 -\ 0.592973\,z\ +\ 0.00499107)}}$$