

**MISR UNIVERSITY FOR SCIENCE AND TECHNOLOGY**  
**COLLEGE OF ENGINEERING**  
**MECHATRONICS DEPARTMENT**



# MTE 506 DIGITAL CONTROL

LAB 7 – SPRING 2020

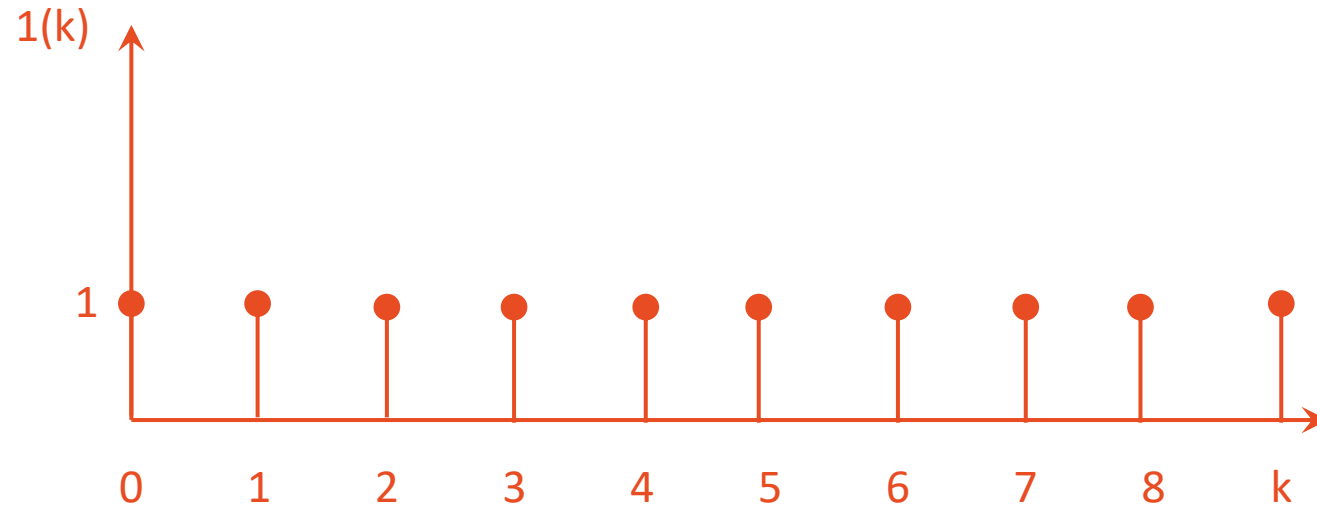
# ASSIGNMENT 3 HIGHLIGHTS

2. Find  $z$  – transform of the following sequence (try to find closed form)

a)  $\{0,0,0,1,1,1, \dots\}$

$$1(k) = 1 + 1 z^{-1} + 1 z^{-2} + \dots + 1 z^{-k}$$

$$1(k) = 1 + z^{-1} + z^{-2} + \dots + z^{-k}$$



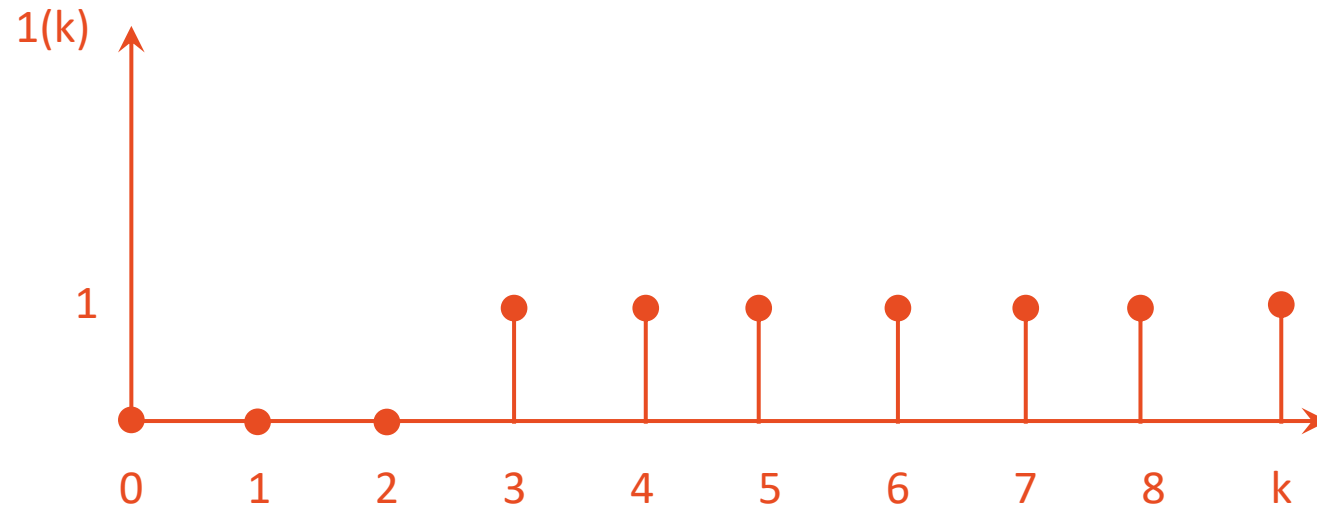
# ASSIGNMENT 3 HIGHLIGHTS

2. Find  $z$  - transform of the following sequence (try to find closed form)

a)  $\{0,0,0,1,1,1, \dots\}$

$$1(k-3) = \{0 + 0 + 0\} + 1z^{-3} + 1z^{-4} + 1z^{-5} + \dots + 1z^{-k-3}$$

$$1(k-3) = z^{-3}1(k) = z^{-3} \frac{z}{z-1}$$



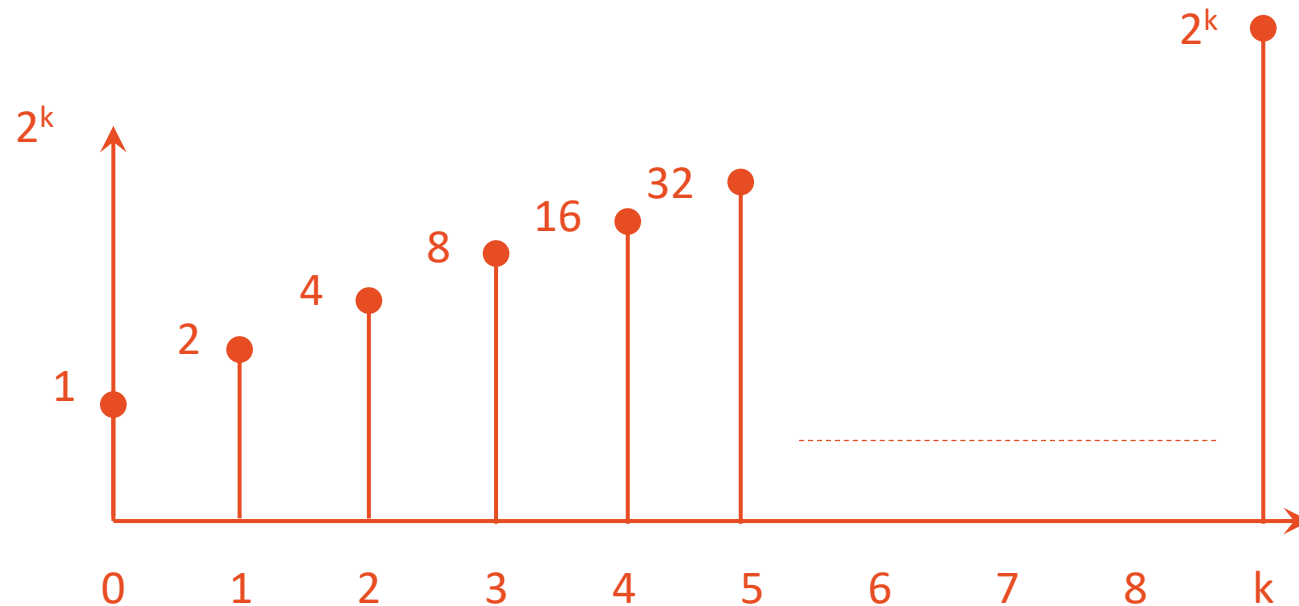
# ASSIGNMENT 3 HIGHLIGHTS

2. Find  $z$  – transform of the following sequence (try to find closed form)

b)  $\{0, 1, 2, 4, 0, 0, \dots\}$

$$2^k = 2^0 + 2^1 z^{-1} + 2^2 z^{-2} + \dots + 2^k z^{-k}$$

$$2^k = 1 + 2 z^{-1} + 4 z^{-2} + \dots + 2^k z^{-k}$$

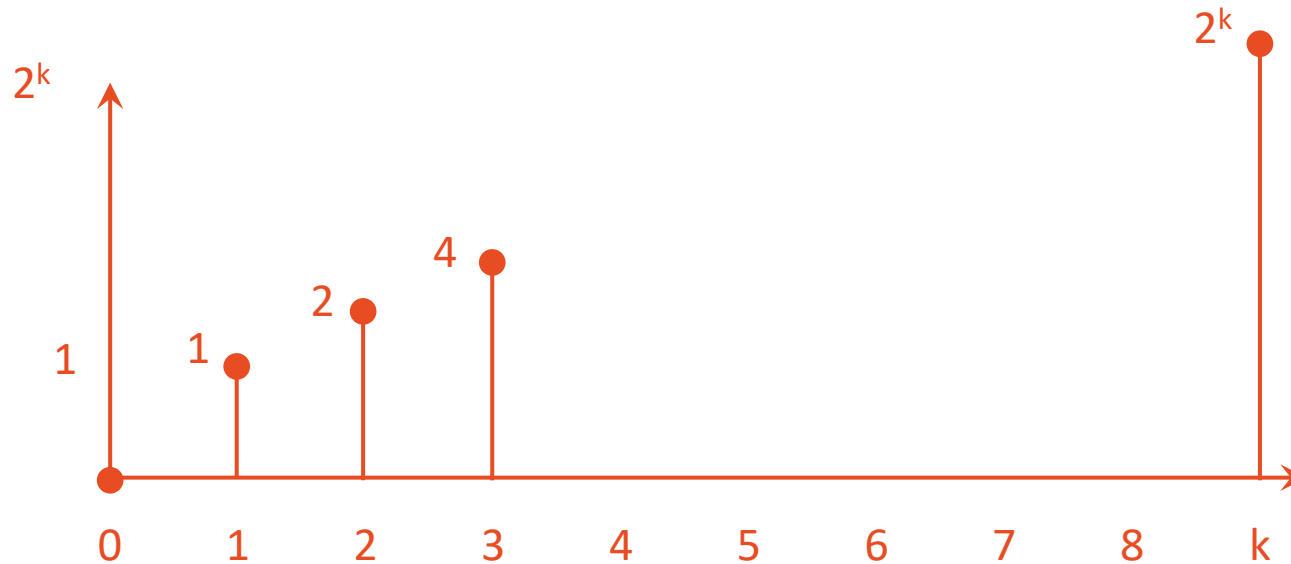


# ASSIGNMENT 3 HIGHLIGHTS

2. Find  $z$  – transform of the following sequence (try to find closed form)

b)  $\{0, 1, 2, 4, 0, 0, \dots\}$

$$2^{k-1} = \{0\} + 2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} \dots + 2^k z^{-k-1}$$



# ASSIGNMENT 3 HIGHLIGHTS

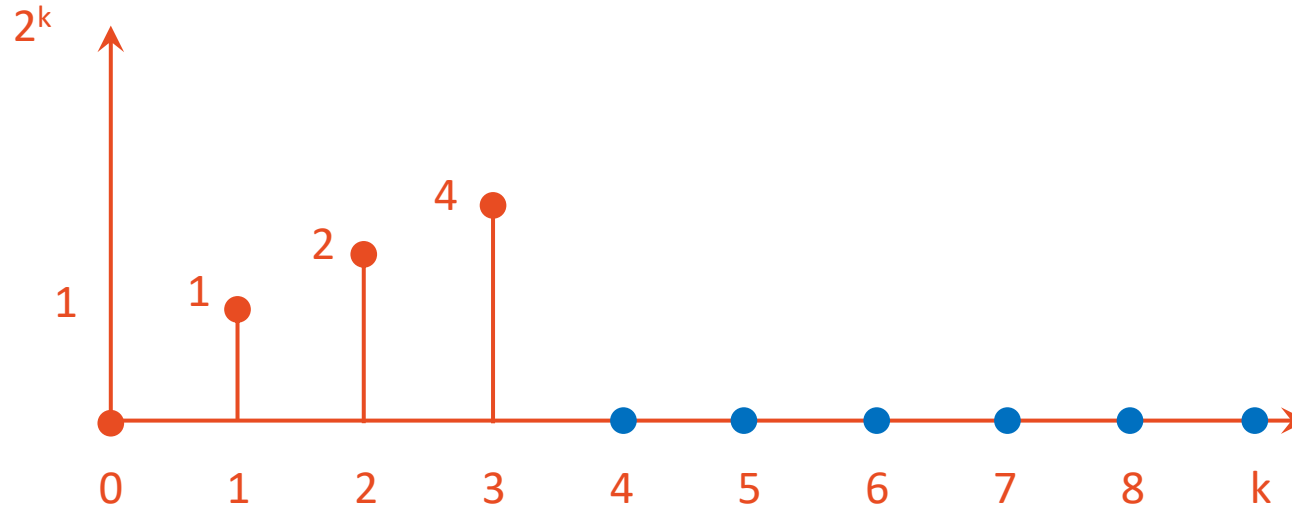
2. Find  $z$  - transform of the following sequence (try to find closed form)

b)  $\{0, 1, 2, 4, 0, 0, \dots\}$

$$2^{k-1} = \{0\} + 2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} \dots + 2^k z^{-k-1}$$

$$f(k) = 2^{k-1}, 0 \leq k \leq 3 \rightarrow f(z) = z^{-1} f(2^k) = z^{-1} \frac{z}{z-2}$$

$$f(k) = 0, k > 3$$



Lab 7

# Goals of The Lab



Inverse Z-Transform



Solved examples and  
assignment

## Inverse Z- Transform

$$\begin{array}{r}
 1 + 0.5z^{-1} + 0.25z^{-2} + \dots \\
 -0.5 \overline{) z} \\
 \underline{z - 0.5} \\
 0.5 \\
 0.5 - 0.25z^{-1} \\
 \underline{0.25z^{-1}} \\
 0.25z^{-1} - 0.125z^{-2} \\
 \underline{\phantom{0.25z^{-1} - 0.125z^{-2}}} \\
 \vdots
 \end{array}$$



# Problem Approach

$$y(k+n) + \dots = u(k+m) + \dots$$

Difference Equation

Z-Transform

$$\frac{Y(z)}{z} = \frac{k}{z-a} + \dots$$

$$z^m + \dots \Big) \frac{az^{-1} + bz^{-2} + \dots}{z^n + \dots}$$

$$y(k) = u(k) + e^{-3k} + \dots$$

Inverse Z-Transform

## Finding Inverse Z-Transform using *partial fraction expansion*

*Disctinct poles*  $\rightarrow Y(z) = \frac{z}{z - 0.4} + \frac{z}{z - e^{-2T}}$

*Repeated poles*  $\rightarrow Y(z) = \frac{z}{(z - 1)^2} + \frac{z}{z - 0.3}$

# Finding Inverse Z-Transform using *partial fraction expansion*

$$F(z) = \frac{z + 1}{z^2 + 0.3z + 0.02}$$

*Solutin pattern*  $\rightarrow F(z) \rightarrow \frac{F(z)}{z}$

$$F(z) = \frac{z}{z} \frac{z + 1}{z^2 + 0.3z + 0.02} \rightarrow \frac{F(z)}{z} = \frac{A}{z} + \frac{B}{z + 0.1} + \frac{C}{z + 0.2}$$

## Finding Inverse Z-Transform using *partial fraction expansion*

$$F(z) = \frac{z}{z} \frac{z+1}{z^2 + 0.3z + 0.02} \rightarrow \frac{F(z)}{z} = \frac{A}{z} + \frac{B}{z+0.1} + \frac{C}{z+0.2}$$

*Finding partial fraction coefficients*

$$A = z \left. \frac{F(z)}{z} \right|_{z=0} = F(0) = \frac{0+1}{0^2 + 0.3(0) + 0.02} = \frac{1}{0.02} = 50$$

$$A = 50$$

## Finding Inverse Z-Transform using *partial fraction expansion*

$$F(z) = \frac{z}{z} \frac{z+1}{z^2 + 0.3z + 0.02} \rightarrow \frac{F(z)}{z} = \frac{A}{z} + \frac{B}{z+0.1} + \frac{C}{z+0.2}$$

*Finding partial fraction coefficients*

$$B = (z+0.1) \left. \frac{F(z)}{z} \right|_{z=-0.1} = (z+0.1) \left. \frac{z+1}{z(z+0.1)(z+0.2)} \right|_{z=-0.1}$$

$$B = \frac{-0.1+1}{(-0.1)(-0.1+0.2)} = \frac{0.9}{-0.01} = -90$$

$$B = -90$$

$$A = 50$$

## Finding Inverse Z-Transform using *partial fraction expansion*

$$F(z) = \frac{z}{z} \frac{z+1}{z^2 + 0.3z + 0.02} \rightarrow \frac{F(z)}{z} = \frac{A}{z} + \frac{B}{z+0.1} + \frac{C}{z+0.2}$$

*Finding partial fraction coefficients*

$$C = (z+0.2) \frac{F(z)}{z} \Bigg|_{z=-0.2} = (z+0.2) \frac{z+1}{z(z+0.1)(z+0.2)} \Bigg|_{z=-0.2} \quad C = 40$$

$$C = \frac{-0.2+1}{(-0.2)(-0.2+0.1)} = \frac{0.8}{0.02} = 40 \quad B = -90$$

$$A = 50$$

## Finding Inverse Z-Transform using *partial fraction expansion*

$$F(z) = \frac{z}{z} \frac{z+1}{z^2 + 0.3z + 0.02} \rightarrow \frac{F(z)}{z} = \frac{A}{z} + \frac{B}{z+0.1} + \frac{C}{z+0.2}$$

$$\frac{F(z)}{z} = 50 \frac{1}{z} - 90 \frac{1}{z+0.1} + 40 \frac{1}{z+0.2}$$

$$F(z) = 50 \frac{z}{z} - 90 \frac{z}{z+0.1} + 40 \frac{z}{z+0.2}$$

$$C = 40$$

$$B = -90$$

$$A = 50$$

$$f(t) = 50\delta(k) - 90(-0.1)^k + 40(-0.2)^k$$

# Finding Inverse Z-Transform using *partial fraction expansion*

$a^k$	$\frac{1}{1 - az^{-1}}$
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Kronecker delta $\delta_0(k)$	1
1 $k = 0$	
0 $k \neq 0$	

From Z-Transform Table

$$F(z) = 50 \frac{z}{z} - 90 \frac{z}{z + 0.1} + 40 \frac{z}{z + 0.2}$$

$$C = 40$$

$$B = -90$$

$$A = 50$$

$$f(t) = 50\delta(k) - 90(-0.1)^k + 40(-0.2)^k$$



## Finding Inverse Z-Transform using *partial fraction expansion*

*For repeated roots*

$$A_{1j} = \frac{1}{(j-1)!} \frac{d^{j-1}}{dz^{j-1}} (z - z_l)^r F(z) \Big|_{z \rightarrow z_l}, j = 1, 2, \dots, r$$

$A_{1j} \dots j^{th}$  coefficient of repeated root

# Finding Inverse Z-Transform using *partial fraction expansion*

$$F(z) = \frac{1}{z^2(z - 0.5)} \quad A_{1j} = \frac{1}{(j-1)!} \frac{d^{j-1}}{dz^{j-1}} (z - z_l)^r F(z) \Big|_{z \rightarrow z_l}, j = 1, 2, \dots, r$$

$$\frac{F(z)}{z} = \frac{1}{z^3(z - 0.5)} \rightarrow \frac{A_{11}}{z^3} + \frac{A_{12}}{z^2} + \frac{A_{13}}{z} + \frac{B}{z - 0.5}$$

$$A_{11} = \frac{1}{0!} z^3 \frac{F(z)}{z} \Big|_{z=0} = z^3 \frac{1}{zz^2(z - 0.5)} \Big|_{z=0} = \frac{1}{0 - 0.5} = -2$$

$$A_{11} = -2$$

# Finding Inverse Z-Transform using *partial fraction expansion*

$$F(z) = \frac{1}{z^2(z - 0.5)} \quad A_{1j} = \frac{1}{(j-1)!} \frac{d^{j-1}}{dz^{j-1}} (z - z_l)^r F(z) \Big|_{z \rightarrow z_l}, j = 1, 2, \dots, r$$

$$\frac{F(z)}{z} = \frac{1}{z^3(z - 0.5)} \rightarrow \frac{A_{11}}{z^3} + \frac{A_{12}}{z^2} + \frac{A_{13}}{z} + \frac{B}{z - 0.5}$$

$$A_{12} = \frac{1}{1!} \frac{d}{dz} z^3 \frac{F(z)}{z} \Big|_{z=0} = \frac{d}{dz} z^3 \frac{1}{z^3(z - 0.5)} \Big|_{z=0} =$$

$$\frac{(0 - 0.5)(0) - (1)(1)}{(0 - 0.5)^2} = \frac{1}{0.25} = 4$$

$$A_{12} = 4$$

$$A_{11} = -2$$

# Finding Inverse Z-Transform using *partial fraction expansion*

$$F(z) = \frac{1}{z^2(z - 0.5)} \quad A_{1j} = \frac{1}{(j-1)!} \frac{d^{j-1}}{dz^{j-1}} (z - z_l)^r F(z) \Big|_{z \rightarrow z_l}, j = 1, 2, \dots, r$$

$$\frac{F(z)}{z} = \frac{1}{z^3(z - 0.5)} \rightarrow \frac{A_{11}}{z^3} + \frac{A_{12}}{z^2} + \frac{A_{13}}{z} + \frac{B}{z - 0.5}$$

$$A_{13} = \frac{1}{2!} \frac{d^2}{dz^2} z^3 \frac{F(z)}{z} \Big|_{z=0} = \frac{1}{2!} \frac{d^2}{dz^2} z^3 \frac{1}{z^3(z - 0.5)} \Big|_{z=0} = \frac{1}{2} \frac{d}{dz} \frac{1}{(z - 0.5)^2} \Big|_{z=0} \quad A_{13} = 8$$

$$= \frac{1}{2} \frac{(-1)(-2)}{(0 - 0.5)^3} = \frac{2}{0.125} = 8 \quad A_{12} = 4$$

$$A_{11} = -2$$

# Finding Inverse Z-Transform using *partial fraction expansion*

$$F(z) = \frac{1}{z^2(z - 0.5)} \quad A_{1j} = \frac{1}{(j-1)!} \frac{d^{j-1}}{dz^{j-1}} (z - z_l)^r F(z) \Big|_{z \rightarrow z_l}, j = 1, 2, \dots, r$$

$$\frac{F(z)}{z} = \frac{1}{z^3(z - 0.5)} \rightarrow \frac{A_{11}}{z^3} + \frac{A_{12}}{z^2} + \frac{A_{13}}{z} + \frac{B}{z - 0.5}$$

$$\begin{aligned} B &= (z - 0.5) \frac{F(z)}{z} \Big|_{z=0.5} = (z - 0.5) \frac{1}{z^3(z - 0.5)} \Big|_{z=0.5} \\ &= \frac{1}{0.5^3} = 8 \end{aligned}$$

$$B = 8$$

$$A_{13} = 8$$

$$A_{12} = 4$$

$$A_{11} = -2$$

# Finding Inverse Z-Transform using *partial fraction expansion*

$$F(z) = \frac{1}{z^2(z - 0.5)} \quad A_{1j} = \frac{1}{(j-1)!} \frac{d^{j-1}}{dz^{j-1}} (z - z_l)^r F(z) \Big|_{z \rightarrow z_l}, j = 1, 2, \dots, r$$

$$\frac{F(z)}{z} = -2 \frac{1}{z^3} + 4 \frac{1}{z^2} + 8 \frac{1}{z} + 8 \frac{1}{z - 0.5}$$

$$F(z) = -2 \frac{1}{z^2} + 4 \frac{1}{z} + 8 + 8 \frac{z}{z - 0.5}$$

$$F(z) = -2(1)z^{-2} + 4(1)z^{-1} + 8(1) + 8 \frac{z}{z - 0.5}$$

$$f(k) = -2\delta(k - 2) + 4\delta(k - 1) + 8\delta(k) + 8(0.5)^k$$

$$B = 8$$

$$A_{13} = 8$$

$$A_{12} = 4$$

$$A_{11} = -2$$

# Finding Inverse Z-Transform using *long division*

$$F(z) = \frac{z + 1}{z^2 + 0.2z + 0.1}$$

$$\begin{array}{r}
 z^{-1} + 0.8 z^{-2} - 0.26 z^{-3} \\
 \hline
 z^2 + 0.2z + 0.1 \overline{) \quad z + 1} \\
 \underline{z + 0.2 + 0.1z^{-1}} \\
 0.8 - 0.1 z^{-1} \\
 \underline{0.8 + 0.16 z^{-1} + 0.08 z^{-2}} \\
 -0.26 z^{-1} + \dots
 \end{array}$$

$$f(k) = \{1, 0.8, -0.26, \dots\}$$

## Assignment 4

*Solve the following difference equations:*

$$y(k + 1) - 0.8 y(k) = 0, y(0) = 1$$

$$y(k + 1) - 0.8 y(k) = 1(k), y(0) = 0$$

$$y(k + 1) - 0.8 y(k) = 1(k), y(0) = 1$$

$$y(k + 2) + 0.7 y(k + 1) + 0.06 y(k) = \delta(k), y(0) = 0, y(1) = 2$$

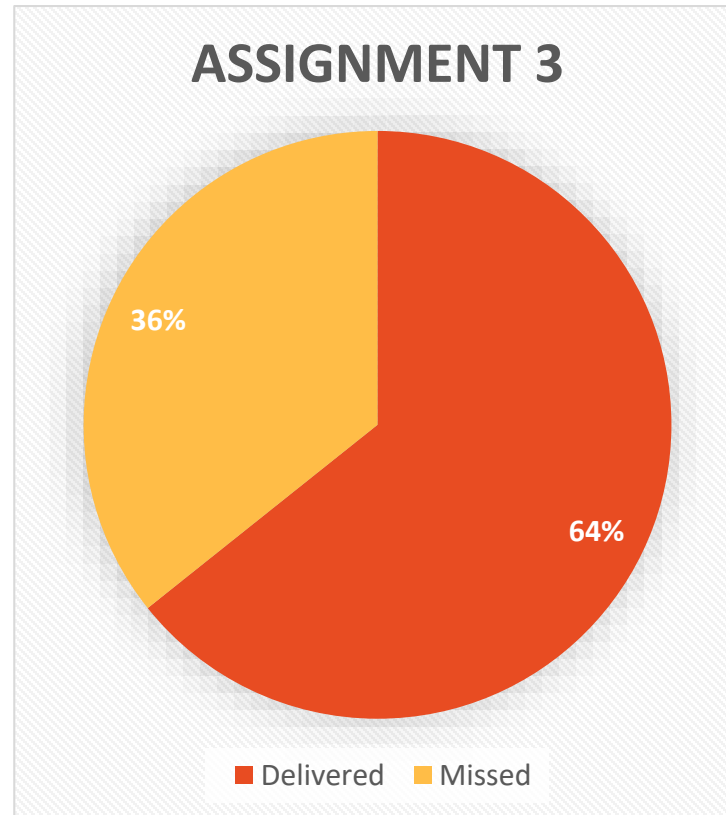
*Due before : 07 – 04 – 2020*

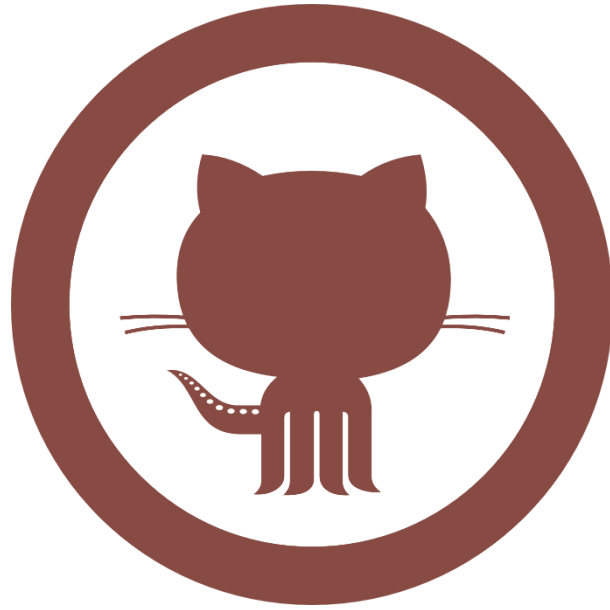


# Past Assignments



## Quick Feedback about past assignments





Don't forget to pull the lab update from.

<http://github.com/wbadry/mte506>

# END OF Lab 7