MISR UNIVERSITY FOR SCIENCE AND TECHNOLOGY
COLLEGE OF ENGINEERING
MECHATRONICS DEPARTMENT

MTE 506 DIGITAL CONTROL

Lab 8 – SPRING 2020

Lab 8

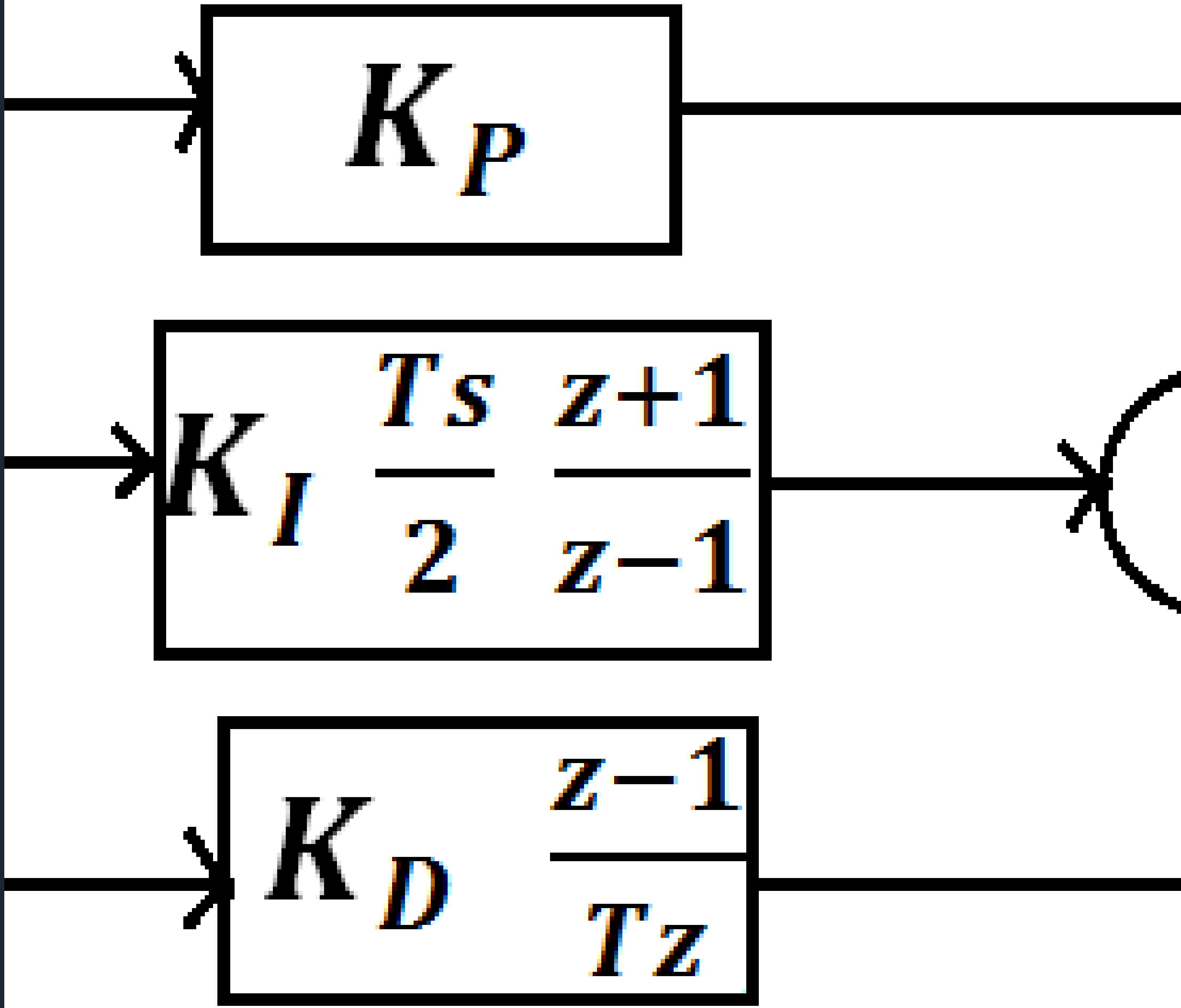
Goals of The Lab

Discrete PID

Zero Order Hold

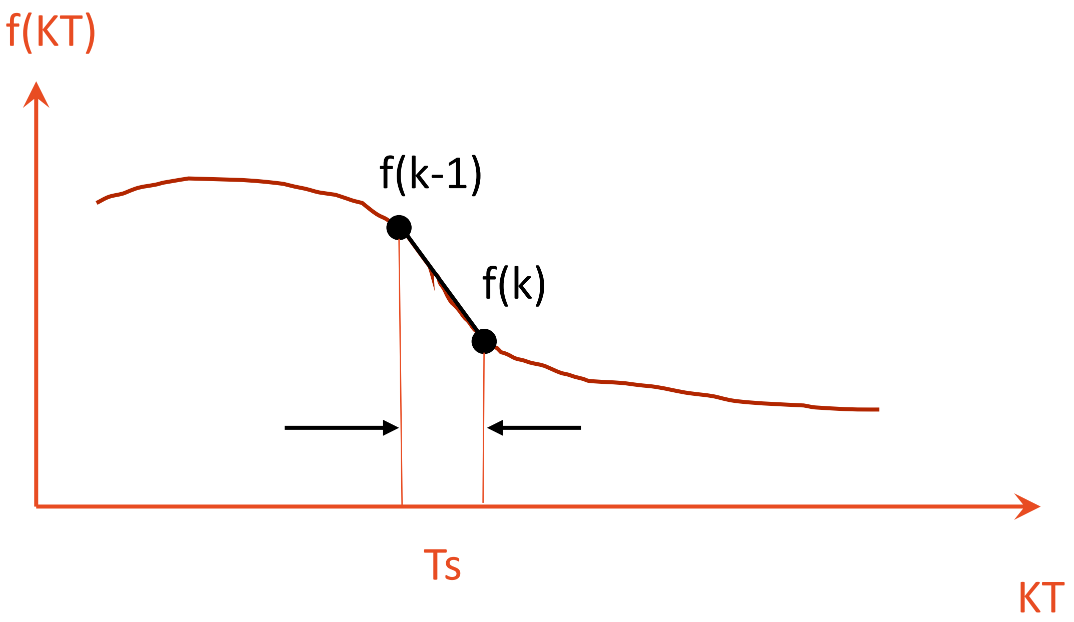
Lab 8

Discrete PID



Lab 8

Mathematical Derivation



Discrete Derivative

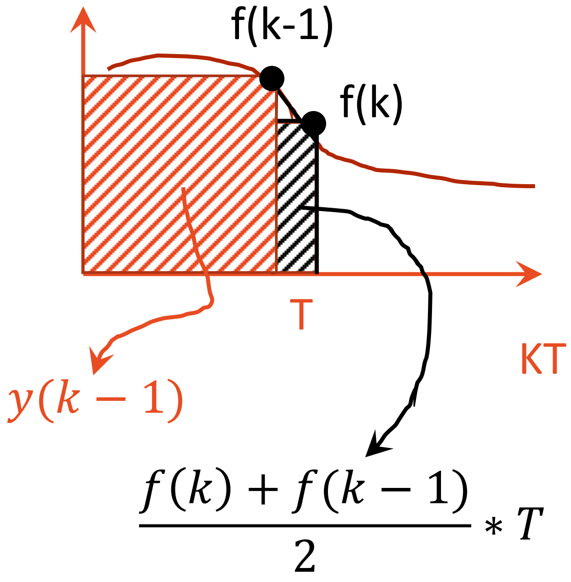
$$y(k) = \frac{df}{dt} \cong \frac{f(k) - f(k-1)}{T_s}$$

$$Y(z) = \frac{F(z) - z^{-1}F(z)}{T_s} = \frac{F(z)[1 - z^{-1}]}{T_s} = \frac{z}{z}\frac{F(z)[1 - z^{-1}]}{T_s}$$

$$Y(z) = \frac{z-1}{T_s z}F(z)$$

Mathematical Derivation

Discrete Integral (using Trapezoidal rule)



$$y(k) = \int_0^{k*T_s} f(t)dt = y(k-1) + \frac{f(k)+f(k-1)}{2}T_s$$

$$Y(z) = z^{-1}Y(z) + \frac{F(z)[1+z^{-1}]}{2}T_s \rightarrow Y(z)[1-z^{-1}] = \frac{T_s}{2}[1+z^{-1}]F(z)$$

$$Y(z) = \frac{T_s}{2}\frac{1+z^{-1}}{1-z^{-1}}F(z) = \frac{T_s}{2}\frac{z+1}{z-1}F(z)$$

Lab 8

Discrete PID controller

Analog PID

$$\mu(t) = K_p e(t) + K_i \int_{t_0}^{t_f} e(t)dt + K_d \frac{de}{dt}$$

Discrete PID

$$\mathrm{M}(z) = K_p E(z) + K_i \frac{T_s}{2}\frac{z+1}{z-1} E(z) + K_d \frac{z-1}{T_s z} E(z)$$

Discrete PID controller

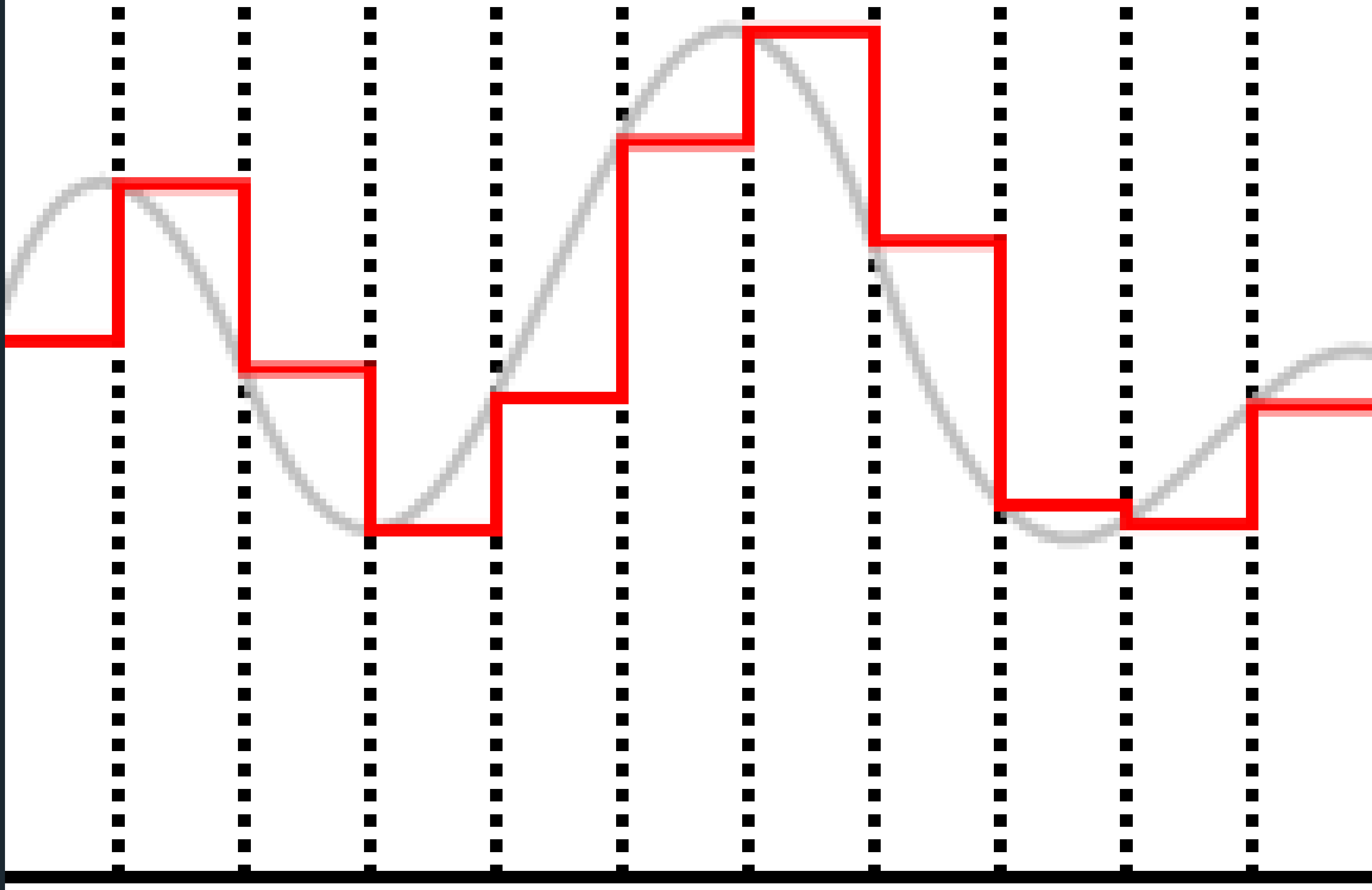
Implemented PID

$$\underbrace{U(t)}_{} = \underbrace{U(t - T_s)}_{\text{Previous Controller Action}} + \underbrace{K_p * e(t)}_{\text{Proportional Controller}} + \underbrace{K_i * \left[\frac{e(t) + e(t - T_s)}{2} * T_s\right]}_{\text{Integral Controller}} + \underbrace{K_d * \frac{e(t) - e(t - T_s)}{T_s}}_{\text{Differential Controller}}$$

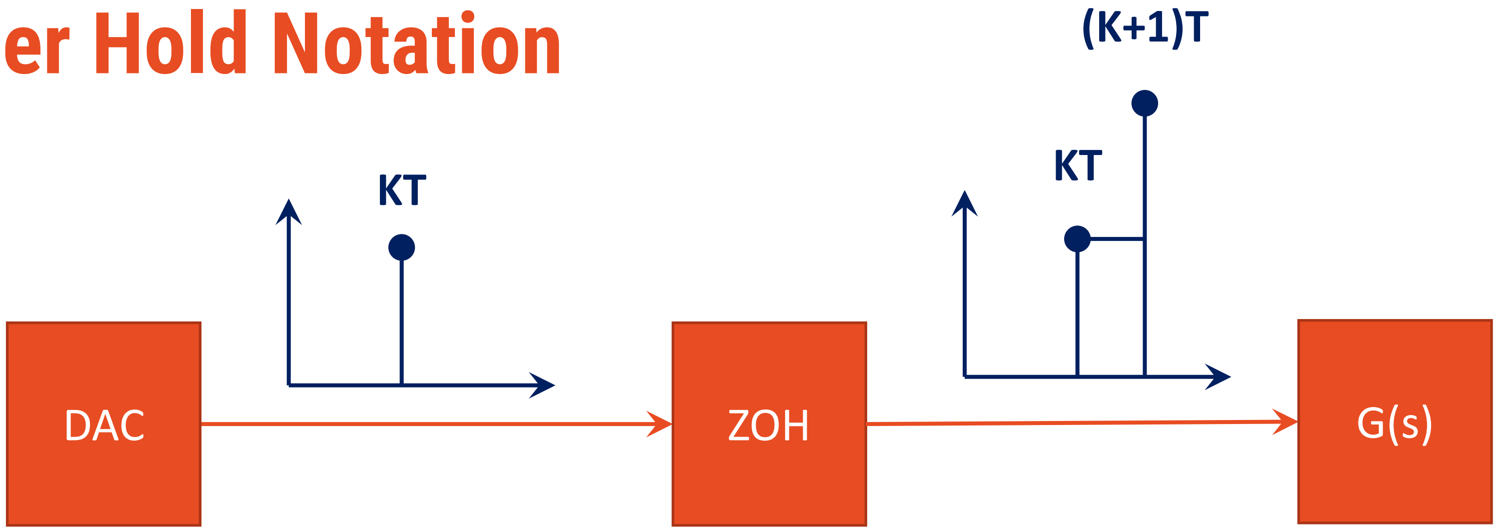
A Low Pass Filter Must Be Added If Differential Controller Is Used

Lab 8

Zero Order Hold

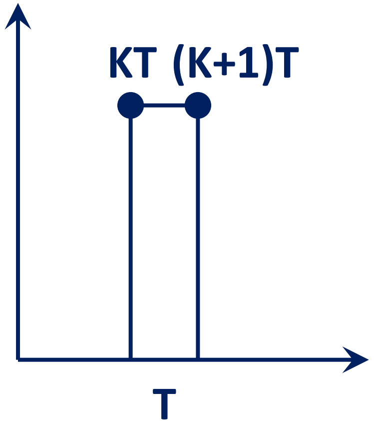


Zero Order Hold Notation



Modeing the pulse

$$G_{ZOH}(s) = \mathcal{L}\big(1(t) - 1(t-T)\big) = \frac{1}{s} - \frac{1}{s}e^{-sT} = \frac{1-e^{-sT}}{s}$$



Zero Order Hold Notation

Modeing the pulse

$$G_{ZOH}(s) = \mathcal{L}\big(1(t) - 1(t-T)\big) = \frac{1}{s} - \frac{1}{s}e^{-sT} = \frac{1 - e^{-sT}}{s}$$

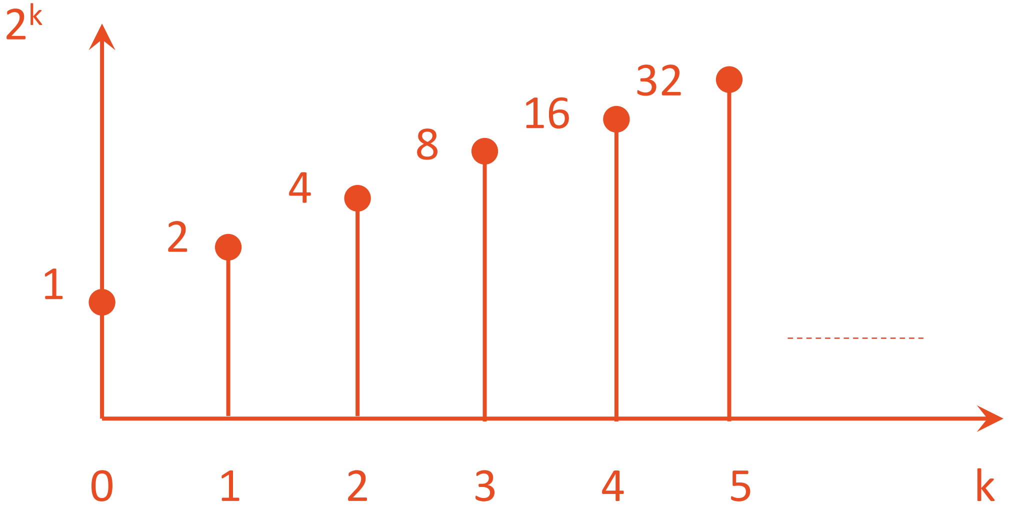
$$G_{ZA}(s) = G(s)G_{ZOH}(s) = (1 - e^{-sT})\frac{G(s)}{s}$$

$$G_{ZA}(z) = \ (1 - z^{-T})z\left[\mathcal{L}^{-1}\left(\frac{G(s)}{s}\right)\right]$$

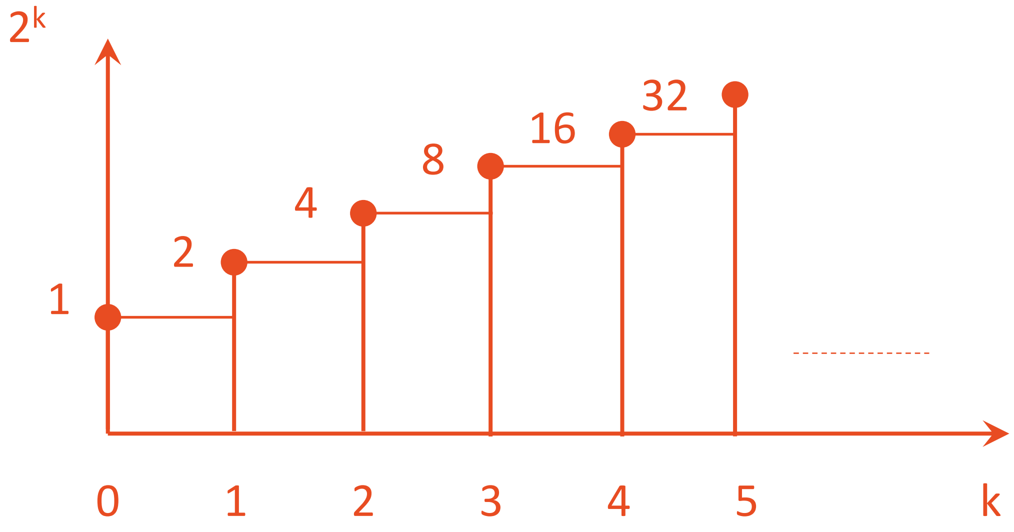
$G_{ZA}(s) = G^{*}(z)$ … *Zero Order Hold of Analog Signal*

Zero Order Hold Notation

$\boldsymbol{Zero\ Order\ Hold\ vs.Impulse\ Response}$



$$\boldsymbol{Y(z)\ \ldots impulse}$$

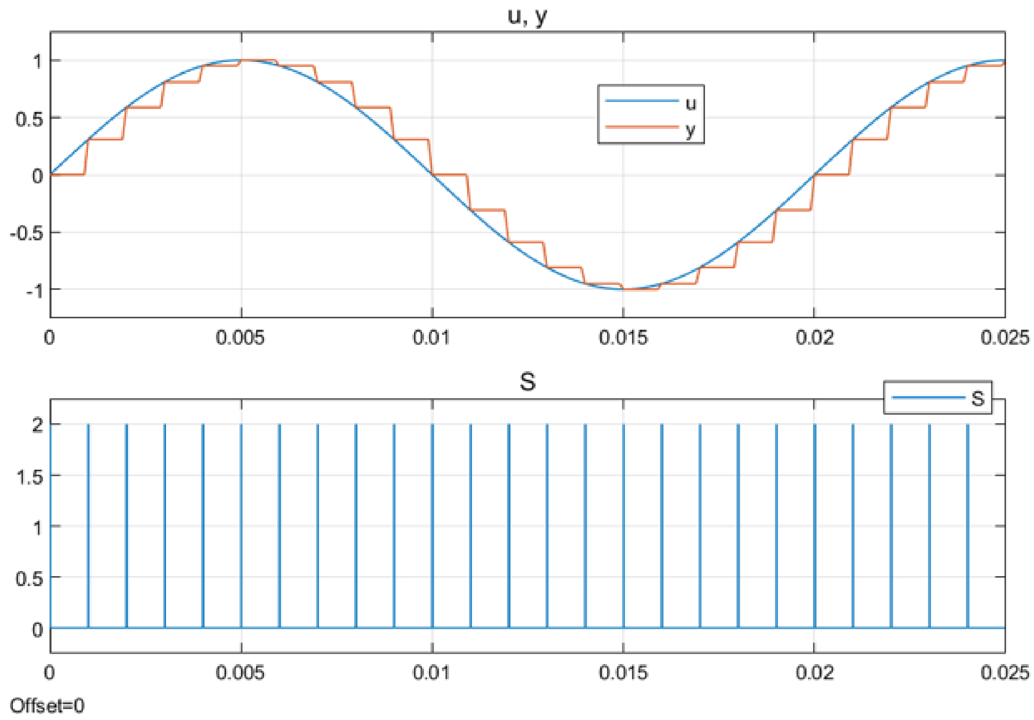


$$\boldsymbol{Y(z)\ \ldots Zero\ Order\ Hold}$$

Lab 8

Zero Order Hold Notation

What is the real life response to follow



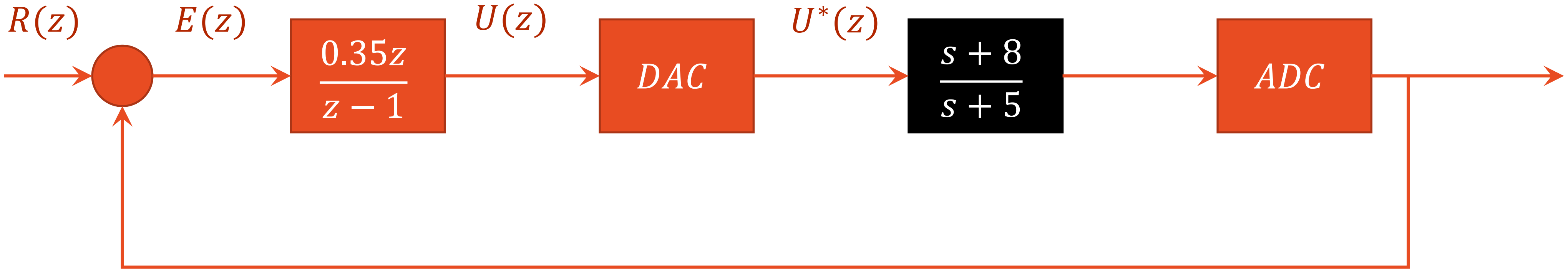
The acquisition system is based on ADC (latching between samples)

The analog output is also holding values between samples (DAC or PWM)

Latching → Zero Order Hold

Lab 8

Zero Order Hold for Closed Loop System



$Find\ the\ z-transform\ function\ for\ the\ analog\ system\ with\ DAC\ and\ ADC$

$$\boldsymbol{G_{ZA}(z) = \left(1 - z^{-T}\right) z\left[\mathcal{L}^{-1}\left(\frac{G(s)}{s}\right)\right] \rightarrow \frac{G(s)}{s} = \frac{1}{s}\left(\frac{s+8}{s+5}\right)}$$

Zero Order Hold for Closed Loop System

$$Find\ the\ z-transform\ function\ for\ the\ analog\ system\ with\ DAC\ and\ ADC$$

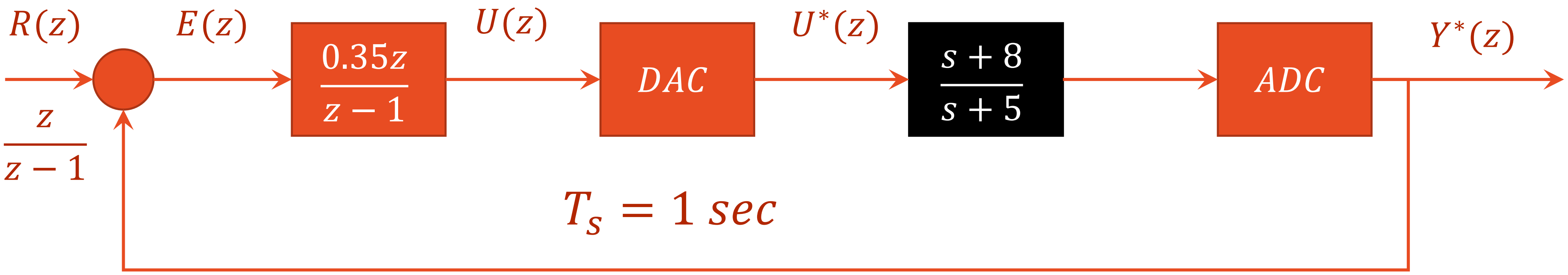
$$\frac{G(s)}{s}=\frac{s+8}{s(s+5)}=\frac{\boldsymbol{A}}{s}+\frac{\boldsymbol{B}}{s+5}=\mathbf{1.6}\frac{1}{s}-\mathbf{0.6}\frac{s}{s+5}$$

$$G_{ZAS}(z)=(1-z^{-1})\mathcal{Z}\left(\frac{G(s)}{s}\right)=\frac{z-1}{z}\mathcal{Z}(1.6-0.6e^{-5})=\frac{z-1}{z}\left(1.6\frac{z}{z-1}-0.6\frac{z}{z-e^{-5}}\right)$$

$$G_{ZAS}(z)=1.6-0.6\frac{z-1}{z}\frac{z}{z-0.0067}=1.6-\frac{z-1}{z-0.0067}=\frac{0.6z+0.989}{z-0.0067}$$

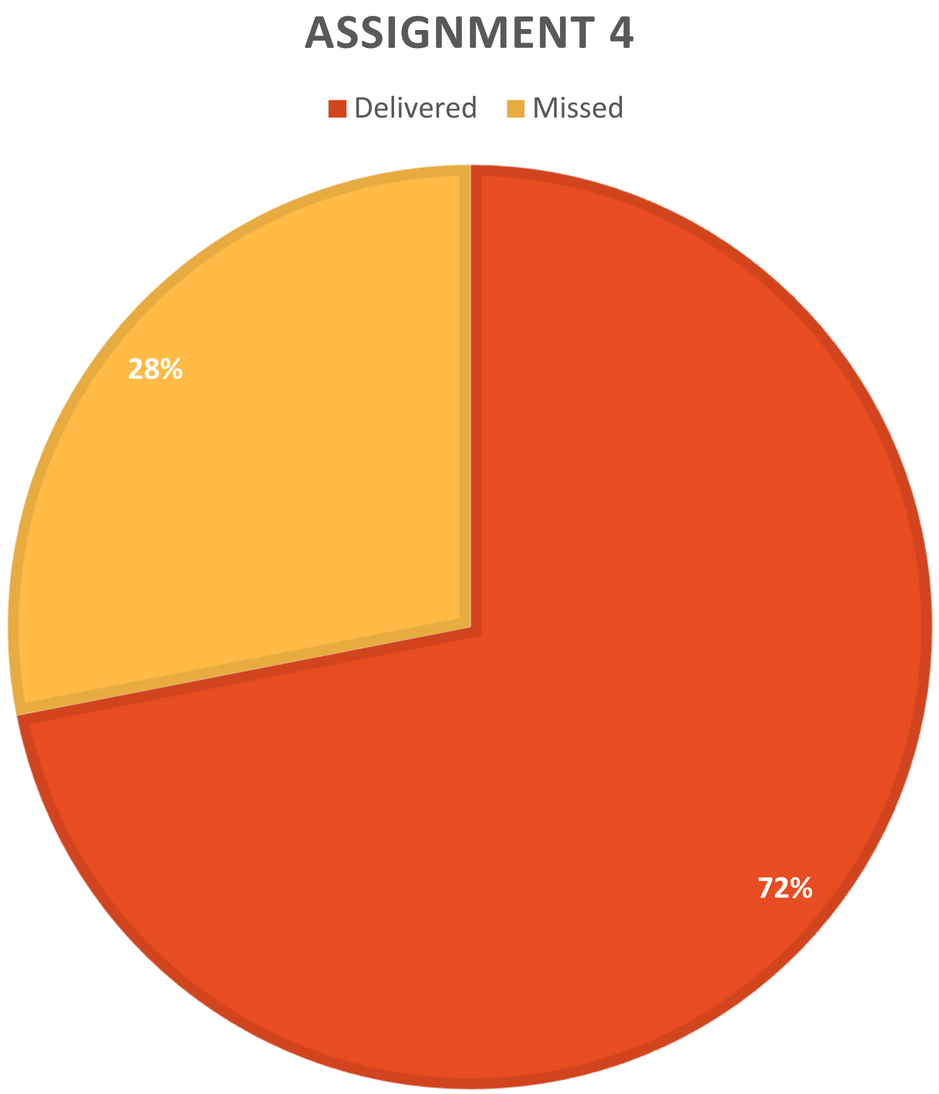
Lab 8

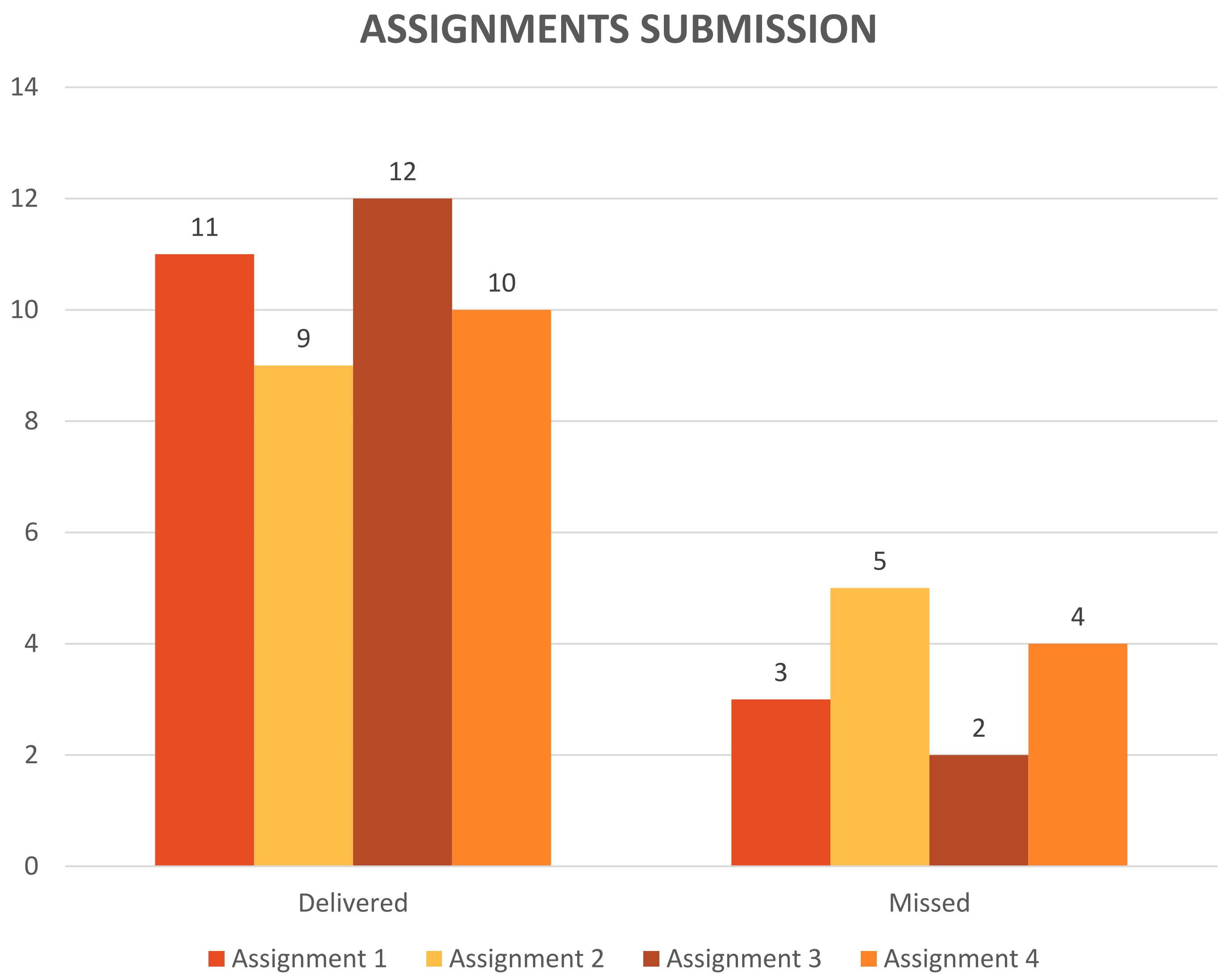
Assignment



*Find the closed loop transfer function (**paper work** and using **Simulink**)*

*Due **Before** 24 – 04 – 2020*


ASSIGNMENT 4
Delivered
Missed
28%
72%


ASSIGNMENTS SUBMISSION
14
12
10
8
6
4
2
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Delivered
Missed
Assignment 1
Assignment 2
Assignment 3
Assignment 4

Don’t forget to pull the lab update from.

http://github.com/wbadry/mte506

END OF Lab 8