# Formula Sheet

## Vector operations

$$\mathbf{r} + \mathbf{s} = \mathbf{s} + \mathbf{r}$$
$$2\mathbf{r} = \mathbf{r} + \mathbf{r}$$
$$\|\mathbf{r}\|^2 = \sum_i r_i^2$$

- *dot or inner product*:

$$\mathbf{r}.\mathbf{s} = \sum_i r_i s_i$$

commutative $\quad \mathbf{r}.\mathbf{s} = \mathbf{s}.\mathbf{r}$

distributive $\quad \mathbf{r}.(\mathbf{s} + \mathbf{t}) = \mathbf{r}.\mathbf{s} + \mathbf{r}.\mathbf{t}$

associative $\quad \mathbf{r}.(a\mathbf{s}) = a(\mathbf{r}.\mathbf{s})$

$$\mathbf{r}.\mathbf{r} = \|\mathbf{r}\|^2$$

$$\mathbf{r}.\mathbf{s} = \|\mathbf{r}\|\|\mathbf{s}\| \cos\theta$$

- *scalar and vector projection*:

scalar projection: $\quad \dfrac{\mathbf{r}.\mathbf{s}}{\|\mathbf{r}\|}$

vector projection: $\quad \dfrac{\mathbf{r}.\mathbf{s}}{\mathbf{r}.\mathbf{r}}\mathbf{r}$

## Basis

A basis is a set of $n$ vectors that:
(i) are not linear combinations of each other
(ii) span the space
The space is then n-dimensional.

## Matrices

$$A\mathbf{r} = \mathbf{r}'$$
$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} e \\ f \end{pmatrix} = \begin{pmatrix} ae + bf \\ ce + df \end{pmatrix}$$
$$A(n\mathbf{r}) = n(A\mathbf{r}) = n\mathbf{r}'$$
$$A(\mathbf{r} + \mathbf{s}) = A\mathbf{r} + A\mathbf{s}$$

Identity: $\quad I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

clockwise rotation by $\theta$: $\quad \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

determinant of 2x2 matrix: $\quad \det\begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$

inverse of 2x2 matrix: $\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \dfrac{1}{ad - bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

- *summation convention for multiplying matrices a and b*:

$$ab_{ik} = \sum_j a_{ij} b_{jk}$$

## Change of basis

Change from an original basis to a new, primed basis. The columns of the transformation matrix $B$ are the new basis vectors in the original coordinate system. So

$$B\mathbf{r}' = \mathbf{r}$$

where $r'$ is the vector in the $B$-basis, and $r$ is the vector in the original basis. Or;

$$\mathbf{r}' = B^{-1}\mathbf{r}$$

If a matrix $A$ is *orthonormal* (all the columns are of unit size and orthogonal to eachother) then:

$$A^T = A^{-1}$$

## Gram-Schmidt process for constructing an orthonormal basis

Start with $n$ linearly independent basis vectors $\mathbf{v} = \{\mathbf{v}_1, \mathbf{v}_2, ..., \mathbf{v}_n\}$. Then

$$\mathbf{e}_1 = \frac{\mathbf{v}_1}{\|\mathbf{v}_1\|}$$

$$\mathbf{u}_2 = \mathbf{v}_2 - (\mathbf{v}_2.\mathbf{e}_1)\mathbf{e}_1 \quad \text{so} \quad \mathbf{e}_2 = \frac{\mathbf{u}_2}{\|\mathbf{u}_2\|}$$

... and so on for $\mathbf{u_3}$ being the remnant part of $\mathbf{v}_3$ not composed of the preceding $\mathbf{e}$-vectors, etc. ...

## Transformation in a Plane or other object

First transform into the basis referred to the reflection plane, or whichever; $E^{-1}$.
Then do the reflection or other transformation, in the plane of the object $T_E$.
Then transform back into the original basis $E$.
So our transformed vector $r' = ET_E E^{-1}r$.

## Eigenstuff

To investigate the characteristics of the $n$ by $n$ matrix $\mathbf{A}$, you are looking for solutions the the equation,

$$\mathbf{A}\mathbf{x} = \lambda\mathbf{x}$$

where $\lambda$ is a scalar eigenvalue. Eigenvalues will satisfy the following condition

$$(\mathbf{A} - \lambda\mathbf{I})\mathbf{x} = 0$$

where $\mathbf{I}$ is an $n$ by $n$ dimensional identity matrix

- *PageRank*
To find the dominant eigenvector of link matrix $\mathbf{L}$, the Power Method can be iteratively applied, starting from a uniform initial vector $\mathbf{r}$.

$$\mathbf{r}^{i+1} = \mathbf{L}\mathbf{r}^i$$

A damping factor, $d$, can be implement to stabilize this method as follows.

$$\mathbf{r}^{i+1} = d\mathbf{L}\mathbf{r}^i + \frac{1 - d}{n}$$