

Formula Sheet

Vector operations

$$\mathbf{r} + \mathbf{s} = \mathbf{s} + \mathbf{r}$$

$$2\mathbf{r} = \mathbf{r} + \mathbf{r}$$

$$\|\mathbf{r}\|^2 = \sum_i r_i^2$$

- dot or inner product:

$$\mathbf{r} \cdot \mathbf{s} = \sum_i r_i s_i$$

$$\text{commutative } \mathbf{r} \cdot \mathbf{s} = \mathbf{s} \cdot \mathbf{r}$$

$$\text{distributive } \mathbf{r} \cdot (\mathbf{s} + \mathbf{t}) = \mathbf{r} \cdot \mathbf{s} + \mathbf{r} \cdot \mathbf{t}$$

$$\text{associative } \mathbf{r} \cdot (a\mathbf{s}) = a(\mathbf{r} \cdot \mathbf{s})$$

$$\mathbf{r} \cdot \mathbf{r} = \|\mathbf{r}\|^2$$

$$\mathbf{r} \cdot \mathbf{s} = \|\mathbf{r}\| \|\mathbf{s}\| \cos \theta$$

- scalar and vector projection:

$$\begin{aligned} \text{scalar projection: } & \frac{\mathbf{r} \cdot \mathbf{s}}{\|\mathbf{r}\|} \\ \text{vector projection: } & \frac{\mathbf{r} \cdot \mathbf{s}}{\mathbf{r} \cdot \mathbf{r}} \mathbf{r} \end{aligned}$$

Basis

A basis is a set of n vectors that:

(i) are not linear combinations of each other

(ii) span the space

The space is then n -dimensional.

Matrices

$$\mathbf{A}\mathbf{r} = \mathbf{r}'$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} e \\ f \end{pmatrix} = \begin{pmatrix} ae + bf \\ ce + df \end{pmatrix}$$

$$\mathbf{A}(n\mathbf{r}) = n(\mathbf{A}\mathbf{r}) = n\mathbf{r}'$$

$$\mathbf{A}(\mathbf{r} + \mathbf{s}) = \mathbf{A}\mathbf{r} + \mathbf{A}\mathbf{s}$$

$$\text{Identity: } \mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\text{clockwise rotation by } \theta: \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$$

$$\text{determinant of 2x2 matrix: } \det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$$

$$\text{inverse of 2x2 matrix: } \begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

- summation convention for multiplying matrices a and b :

$$ab_{ik} = \sum_j a_{ij} b_{jk}$$

Change of basis

Change from an original basis to a new, primed basis. The columns of the transformation matrix B are the new basis vectors in the original coordinate system. So

$$B\mathbf{r}' = \mathbf{r}$$

where \mathbf{r}' is the vector in the B -basis, and \mathbf{r} is the vector in the original basis. Or;

$$\mathbf{r}' = B^{-1}\mathbf{r}$$

If a matrix A is *orthonormal* (all the columns are of unit size and orthogonal to each other) then:

$$A^T = A^{-1}$$

Gram-Schmidt process for constructing an orthonormal basis

Start with n linearly independent basis vectors $\mathbf{v} = \{\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_n\}$. Then

$$\mathbf{e}_1 = \frac{\mathbf{v}_1}{\|\mathbf{v}_1\|}$$

$$\mathbf{u}_2 = \mathbf{v}_2 - (\mathbf{v}_2 \cdot \mathbf{e}_1)\mathbf{e}_1 \quad \text{so} \quad \mathbf{e}_2 = \frac{\mathbf{u}_2}{\|\mathbf{u}_2\|}$$

... and so on for \mathbf{u}_3 being the remnant part of \mathbf{v}_3 not composed of the preceding \mathbf{e} -vectors, etc. ...

Transformation in a Plane or other object

First transform into the basis referred to the reflection plane, or whichever; E^{-1} .

Then do the reflection or other transformation, in the plane of the object T_E .

Then transform back into the original basis E .

So our transformed vector $\mathbf{r}' = ET_E E^{-1}\mathbf{r}$.

Eigenstuff

To investigate the characteristics of the n by n matrix \mathbf{A} , you are looking for solutions the the equation,

$$\mathbf{A}\mathbf{x} = \lambda\mathbf{x}$$

where λ is a scalar eigenvalue. Eigenvalues will satisfy the following condition

$$(\mathbf{A} - \lambda\mathbf{I})\mathbf{x} = 0$$

where \mathbf{I} is an n by n dimensional identity matrix

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To find the dominant eigenvector of link matrix \mathbf{L} , the Power Method can be iteratively applied, starting from a uniform initial vector \mathbf{r} .

$$\mathbf{r}^{i+1} = \mathbf{L}\mathbf{r}^i$$

A damping factor, d , can be implement to stabilize this method as follows.

$$\mathbf{r}^{i+1} = d\mathbf{L}\mathbf{r}^i + \frac{1-d}{n}$$