Correcting Options for Dividends

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1 Introduction

It is common practice for over-the-counter options to be "dividend-protected" such that the price of the option should not be changed based on dividend payout policy. This is typically accomplished by decreasing the strike price by the amount of the dividend on the ex-dividend date, which roughly approximates the change in value of the option, but has been shown to be incorrect [1].

Robert Geske et al. outline the shortcomings of this approach. They demonstrate that this correction, while partial, is incomplete. In a sense, the option value "shrinks," as the same percentage move above the strike price gives less return to the option. To fully correct for the payment of dividends, it is best to re-imagine the underlying as a portfolio "Σ" where dividends are reinvested at the ex-dividend price on the ex-dividend date. This theoretical option is discussed below.

2 Correcting the Theoretical Option

Purchasing more stock increases the size of the Σ portfolio that the option is based upon. To correct for this increase in total stock, the contract size of the option is increased by an equivalent amount. The total number of stocks increases by $\frac{D}{S_X}$, where D is the dividend amount and S_X is the ex-dividend price. This reflects the total number of shares repurchased with the dividend. Increasing the contract size keeps it consistent with the number of stocks in the Σ portfolio. The decrease in each price due to dividend payments is $S_X = S - D$

$$1 + \frac{D}{S_X} = \frac{S}{S_X}$$

$$S_X = \frac{S}{1 + \frac{D}{S_X}}$$

which is the same factor the contract size increases by. Decreasing the strike price E by $\frac{D}{S_X}$ completes our correction and accurately prices the option. This process is paralleled by Robert Merton [2].

3 Numerical Proof

We perform a Monte Carlo simulation to reinforce these findings. A \$100 stock with $\sigma^2 = 0.15$ and $r = 0.1$ and analyzing 10,000 iterations results in a \$100 strike call option worth 11.669 at 1 year until expiry from the Black Scholes pricing model and 11.695 from our simulation. Introducing $D = 20$ and the naive correction of $E - D$ results in a call price of 10.357. However, by adjusting the contract size and strike price as stated above for this dividend, the option price is accurately predicted at 11.677.

4 Dividend Effect on Implied Volatility

We begin our analysis by examining the effects of dividends on options' implied volatility by comparing volatility smiles for options on Duke Energy (NYSE: DUK) on the ex-dividend date and day prior for each of the quarterly dividend dates between February 2020 and August 2022. This data is obtained from IvyDB US by OptionMetrics through Wharton Research Data Services [3]. The implied volatility is calculated using Martingale Pricing, with the future value of the stock accounting for the expected dividend payments. The volatility smile at each date is constructed using the implied volatility of out of the money puts and out of the money calls on the given date.

Theoretically, the price of the stock on the ex-dividend date will be equal to the price of the stock before the dividend minus the value of the dividend. However due to the routine unpredictable fluctuations in the price of the stock, the ex-dividend stock price is often not equal to the pre-dividend stock price minus the dividend.

In these cases, illustrated in Figures 1 and 2, where the green vertical line (Pre-Dividend Price − Dividend Value) is far from the blue vertical line (Ex-Dividend Price), we can see that the volatility smiles before and after the dividend are similar, but display significant discrepancies. This result is intuitive, since large unexpected moves in the price of the stock will change the implied volatility not only because the stock is exhibiting volatile movement which may be factored into future volatility predictions, but also because as the stock price changes, the strike price for an at the money call or put changes. Implied volatility is typically lowest about at the money, meaning the smile will shift up or down with the price of the stock. This effect is illustrated in Figure 2, where ex-dividend price is higher than expected, and the volatility smile displays a minimum at a higher strike price on the ex-dividend date.

Conversely, in the cases where ex-dividend price closely approximates the pre-dividend price − dividend value, shown in Figures 3 and 4, we can see that the volatility smiles on these two dates are nearly identical. This conclusion is also intuitive, since the dividend was expected by the market and already priced into the value of the option. This fact is further illustrated in Figure 5, which shows the prices of call and put options unaffected by the dividend, as it was expected by the market.

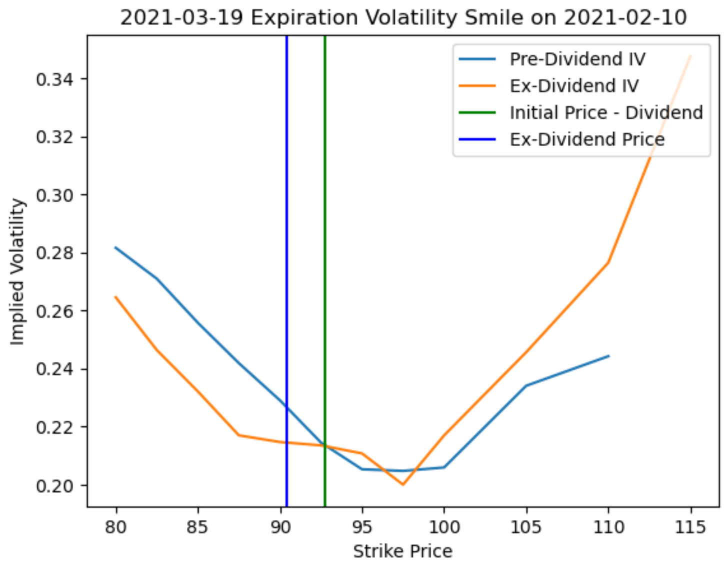

Figure 1: Implied volatility and stock price before and after February 2021 dividend

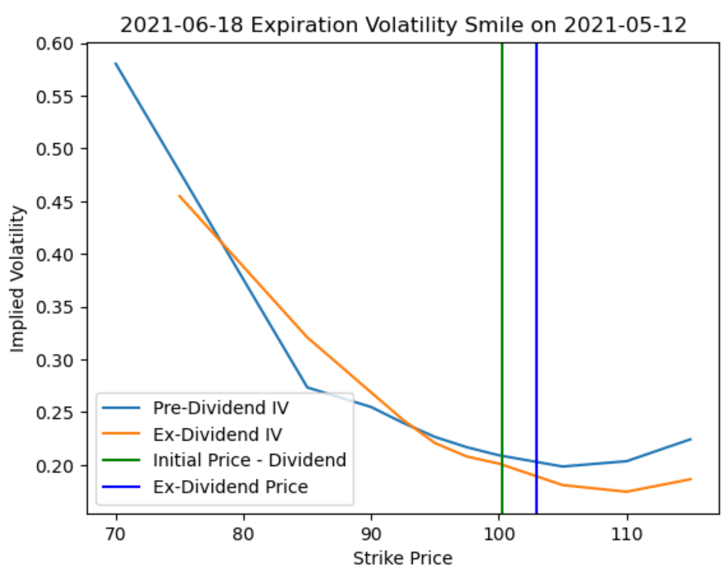

Figure 2: Implied Volatility and stock price before and after May 2021 dividend

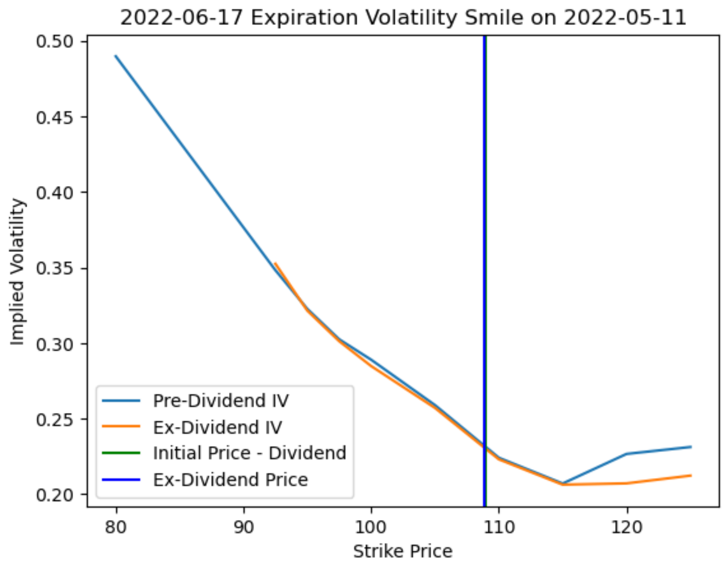

Figure 3: Implied Volatility and stock price before and after May 2022 dividend

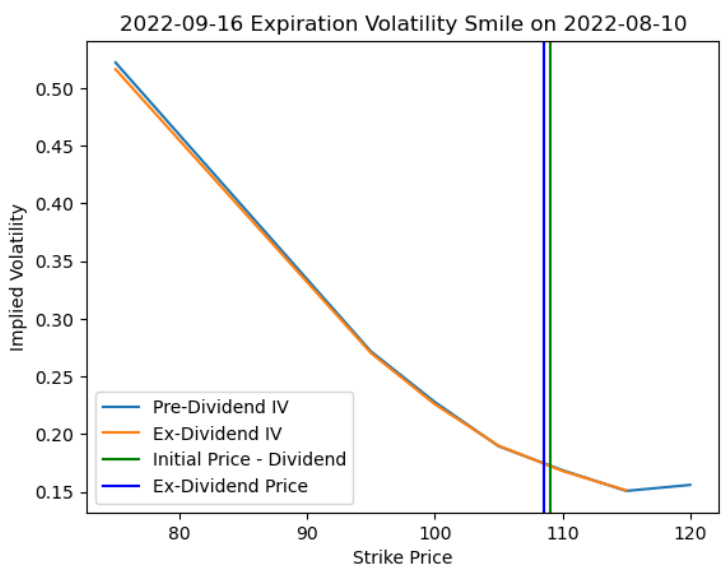

Figure 4: Implied Volatility and stock price before and after August 2022 dividend

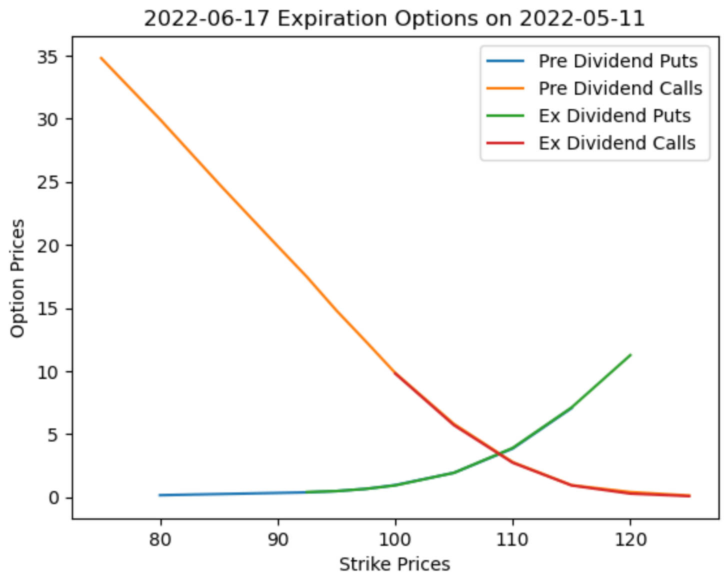

Figure 5: Options prices before and after August 2022 dividend

5 Pricing Options on Σ

From the analysis in the section above, we can determine that the volatilities used in pricing options before and after dividends remain the same.

Let $C(F, E, R, T, \sigma)$ be the price of a call option, where:

- F is the future price of the stock
- E is the strike price of the option
- R is the risk-free rate
- T is the time until the option expires
- σ is the volatility of the stock

Using the option proposed above with the reinvested-dividend Σ portfolio as the underlying asset, we obtain the following equality between options on the stock itself and on the portfolio of reinvested dividend:

$$C(Se^{RT}, E, R, T, \sigma) = \lambda * C(F, \frac{E}{\lambda}, R, T, \sigma) \tag{1}$$

Where S is the spot price of the stock and $\lambda = 1 + \frac{D}{S_X}$ with D as the value of the dividend and S_X as the ex-dividend price. We assume there is only one dividend before expiry, as with multiple dividends, we introduce multiple λs.

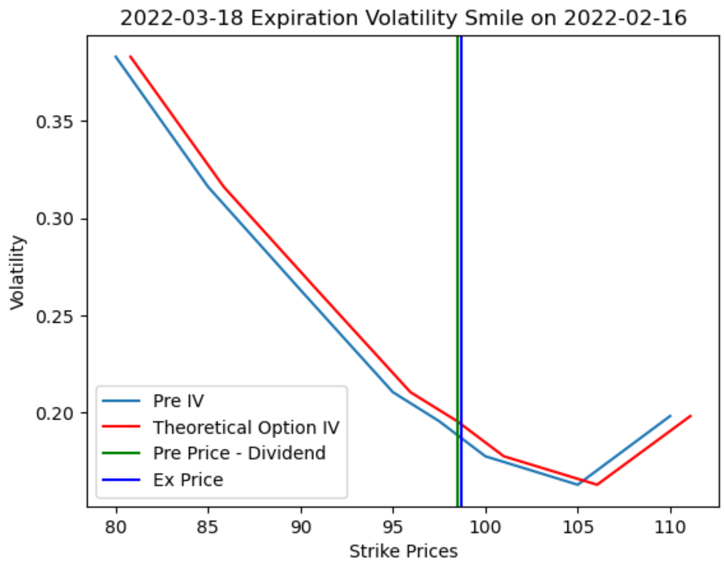

Figure 6: Volatility smiles for market option and option on sigma portfolio

The left hand side of this equation represents a call on the Σ portfolio, with $F = Se^{RT}$ since we assume growth of the stock price at the risk free rate, and ignore dividends as they do not affect the value of the portfolio. The right hand side of the equation represents a call option using the future price of the stock accounting for dividends, with strike $\frac{E}{\lambda}$ and contract size λ. As demonstrated above, this equality must hold, because after the dividend is paid the call on the left will be exchanged for the call on the right to protect against the dividend.

6 Implied Volatility of Options on Σ

A key assumption in this model is constant volatility, and while the assumption of constant volatility is violated in the real world, equation (1) can still hold for any given E. Effectively, the implied volatility of an option on Σ with strike price E must equal the implied volatility of an option on the stock with strike price $\frac{E}{\lambda}$, since we know the implied volatility of the options on the stock will not change after the dividend as shown in Section 4. This results in the smiles shown in Figure 6, where the volatility smile for options on Σ mirrors the volatility smile for market-traded options, with strike prices shifted by a factor of λ.

This implies that for two otherwise identical stocks, where one pays dividends and the other does not, options on the dividend-paying stock will have a volatility smile shifted by a factor of λ. This is an intuitive conclusion, as introducing a dividend reduces the future price used to calculate the price of the option.

7 Conclusion

While existing market traded options can be correctly for the predictable dividends paid by firms, there still exists the possibility of rational early exercise of American calls given a large enough dividend. Further, because dividend payout policy affects the implied volatility and price of options, there is a potential agency problem. For example, a manager could buy puts and artificially increase their value by announcing a large dividend before expiration. While in practice these problems are minimized since dividends typically represent a small portion of a stock's value, neither of these problems exists for an option on the Σ portfolio.

One issue which we largely ignore in this analysis is the empirical phenomenon that stock values typically decline by less than the full dividend value due to taxes and other market frictions. Thus, an adjustment using λ does present an opportunity for imperfect pricing.

While the current contract structure of market-traded options is perhaps more simple and accessible for investors, options on the Σ portfolio would remove agency issues as well as incentives for early exercise. Thus, options on the Σ portfolio of a stock could present a superior alternative to current market-traded options.

References

[1] Robert Geske. "Over-the-Counter Option Market Dividend Protection and 'Biases' in the Black-Scholes Model: A Note." In: *The Journal of Finance* 38.4 (1983), pp. 1271–1277. DOI: `https://doi.org/10.2307/2328024`.

[2] Robert C. Merton. "Theory of Rational Option Pricing". In: *The Bell Journal of Economics and Management Science* 4.1 (1973), pp. 141–183. DOI: `https://doi.org/10.2307/3003143`.

[3] OptionMetrics. "IvyDB US". In: *Wharton Research Data Services* (2024).