Lenses and optical instruments

1. Basic Theory

The aim of this experiment is to determine the focal lengths of several different lenses. Afterwards we will use these lenses to build some simple optical instruments. First, we consider image formation by thin lenses.

The relationship between the focal length *f* of a lens, the object distance *g,* and the image distance *b* determines the ratio of the image height B to the object height G. In Fig. 1 the two right-angled triangles on the left and right sides of the lens are similar.

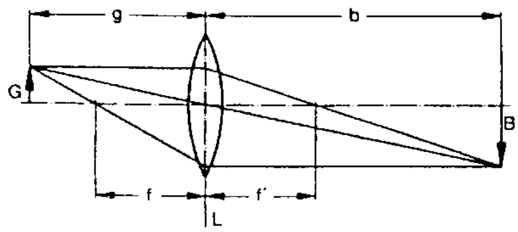

Figure 1: Construction to find the image position for a thin lens using three principal rays: i.e. the focal-, parallel-, and central rays.

$$\frac{B}{G} = \frac{b}{g} \quad and \quad \frac{G}{B} = \frac{f}{b-f} \qquad (1)$$

We can rearrange this equation to get the lens equation for thin lenses:

$$\frac{1}{f} = \frac{1}{b} + \frac{1}{g} \quad \text{or} \quad f = \frac{b.g}{b+g} \qquad (2)$$

The imaging behavior of the lens depends on whether the object is located outside the first focal point, at the focal point, or inside the focal point, as shown in this table:

Object Position *g*	**Image**			
	Type	**Position**	**Orientation**	**Relative size**
∞	real	b = f	-/-	point
$\infty > g > 2f$	real	f < b < 2f	inverted	demagnified
$g = 2f$	real	b = 2f	inverted	equal size
$f < g < 2f$	real	∞ > b > 2f	inverted	magnified
$g = f$		∞		
$g < f$	virtual	|b| > g	upright	magnified

2. Experimental Procedure

2.1 Determining the focal length of a lens

Measuring image and object distances.

In this experiment the focal lengths of two different converging lenses are to be determined by measuring the image distance *b* and object distance *g* (see Fig.1) for each lens. First a parallel beam of collimated light is produced using a lamp with the double condenser lens. Next the object (flea or slide) is positioned close to the condenser lens and a converging lens is used to produce a sharp image on the screen. The image distance and the object distance are measured from the middle of the thin lens. The measurements should be repeated for six different lens and screen positions for both of the lenses.

The focal length *f* has to be calculated using Eq. 1 for each of the measured pairs of values of *b* and *g* for each lens. Determine the average value of *f* and the standard deviation. Compare these values with the value printed on the lens (the focal length on the lens has a tolerance of ± 5%).

2.2 Bessel method

There are two possible positions in which a thin lens can produce a sharp image of an object at a distance d from the screen as illustrated in Fig. 2. For the lens in position I a sharp magnified image is produced on the screen. For the lens in position II the object distance is equal to the image distance of the lens in position I and a sharp demagnified image is produced on the screen. Hence, at the two possible positions the image and object distances are both exchanged ($g_1 = b_2$ and $g_2 = b_1$).

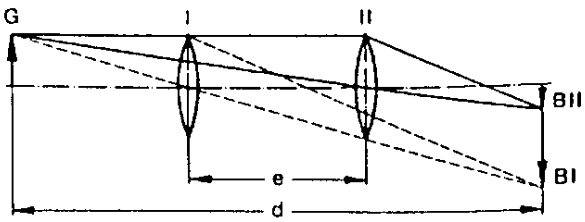

Figure 2: Focal length determination using the Bessel method.

$$g_{|} + b_{|} = d \qquad (3a)$$

The distance between lens position I and lens position II is given by e

$$b_{|} - g_{|} = e \qquad (3b)$$

Adding eqs. 3a and 3b gives

$$b_{|} = \frac{1}{2}(d + e) \qquad \text{and} \qquad g_{|} = \frac{1}{2}(d - e)$$

Substituting these values in the lens equation gives

$$f = \frac{d^2 - e^2}{4d} \qquad (4)$$

The focal length of a converging lens f_s can be determined from the vales of d and e.
This method can also be used to determine the focal length of a diverging lens f_Z by measuring the focal length of a lens system f_{ges} consisting of a diverging lens and a converging lens

$$\frac{1}{f_Z} = \frac{1}{f_{ges}} - \frac{1}{f_S} \quad \text{oder} \quad f_Z = \frac{f_{ges} \cdot f_S}{f_S - f_{ges}} \tag{5}$$

The power of the converging lens $1/f_s$ must be greater than that of the diverging lens $1/f_Z$ to produce a real image.

In this experiment you have to measure the focal length of the lens system consisting of a converging lens and a diverging lens using the Bessel method. For example, you could choose a converging lens with f_s = 100 mm. Then for a given distance d between the object and the screen you have to locate the two positions of the lens system which produce sharp images. You should choose the distance to be as large as possible and it must be at least 4 times the overall focal length of the combined lens system to produce a real image.

You should repeat the measurements for 6 different values of *d.*

Calculate the individual values and then the average value of f_{ges}. Then using the averaged value for f_s obtained in 2.1 you can calculate the averaged value of f_Z for the diverging lens. You should also calculate the standard deviation of f_Z.

2.3 Slide projector

Put the slide directly in front of the condenser lens and position the objective lens L_2 (f_2 = + 100 mm) so that the image covers the entire area of the screen. To achieve the best image illumination, the condenser lens should be positioned so that the image of the lamp is located in the plane of objective lens L_2 (see Fig. 3).

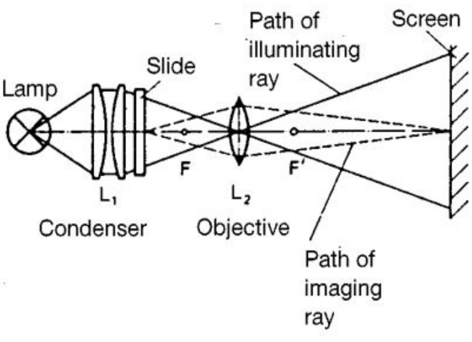

Figure 3: Rays of light in a slide projector.

In this experiment the magnification of the slide projector should be determined by directly measuring B and G.

$$V = \frac{B}{G} = \frac{b}{g} = \frac{b-f}{f} \tag{6}$$

The magnification should also be determined by measuring g, b and f. Finally you should compare the results from all three calculations.

2.4 Microscope

A light microscope is an optical instrument which produces highly magnified images of very small objects. The objective lens L_1 has a short focal length (e.g. f_1 = +20 mm), so the overall length of microscope is small. As shown in Fig. 4, a real intermediate image is formed which is viewed through the eyepiece lens L_2 (f_2 = +50 mm), which acts as a magnifying glass.

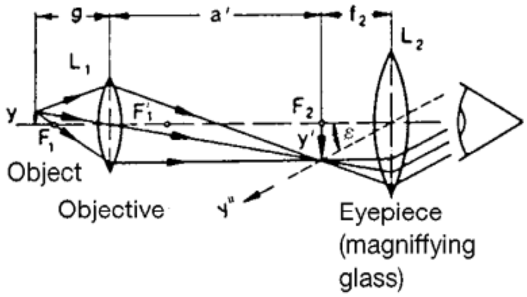

Figure 4: An optical microscope.

In this experiment, L_1 should be mounted on the optical bench close to the object, but the object distance must be slightly larger than f_1 to produce a real magnified intermediate image.

The intermediate image *y*' should be viewed first on the screen to be able to measure the intermediate image distance *a'* and the object distance *g*, so that the magnification can be calculated later. After this the intermediate image can be viewed through the eyepiece L_2. The screen is removed and eyepiece should be positioned about f_2 = 50 mm further away from the screen position (see Fig. 4). The illumination of the object by the lamp and condenser lens should be reduced with a matt glass screen.

The size of the magnified image *y''* can be determined by simultaneously observing a ruler (length scale) at the near-point distance (25 cm) using the left eye, while viewing the image through the microscope with the right eye. With a bit of practice, it is possible to see both superimposed on top of each other at the same time.

$$V = \frac{y''}{y} \tag{7}$$

The overall magnification can be calculated from the image height *y''* *and* the object height (*y*).

The overall magnification V_{mikr} of the microscope is found by multiplying the magnification from the microscope objective

$$\beta_{Obj} = \frac{Y'}{Y''} = \frac{a'}{g} = \frac{a'}{f_1} - 1 \tag{8}$$

with the angular magnification of the eyepiece Γ_L = 250 mm / f_2:

$$V_{mikr} = \beta_{Obj}.\Gamma_L = \left(\frac{a'}{f_1} - 1\right).\frac{250\,mm}{f_2} \tag{9}$$

With the available lenses magnifications up to *V* = 60 can be achieved. You should calculate the magnification *V* from your experimental data and compare it to the value given by equation (9).

2.5 Astronomical refracting telescope

The principle of the astronomical refracting telescope is shown in Fig. 5. It consists of two converging lenses the first of which, the objective, forms a real inverted image which is examined using the second lens, the eyepiece.

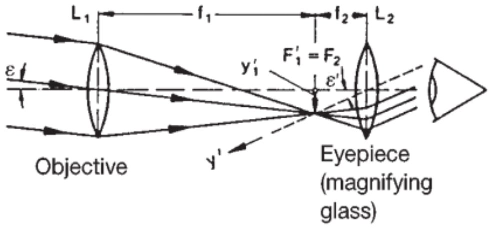

Figure 5: The astronomical refracting telescope

In this experiment a converging lens with a rather long focal length f_1 (e.g. +300 mm), and another one with a shorter focal length f_2 (e.g. +50 mm) are fixed on the optical bench separated by $f_1 + f_2$. Looking through the lens with shorter focal length, you can see an inverted, magnified image of a distant object. A sharp image can be obtained by changing the lens separation. The objective lens L_1 projects an inverted real image with height Y_1' of a distant object. The intermediate image is viewed through the eyepiece L_2, which acts like a magnifying glass.

The angular magnification (for small angles) is given by

$$\Gamma = \frac{\varepsilon'}{\varepsilon} = \frac{Y_1'/f_2}{Y_1'/f_1} = \frac{f_1}{f_2} \qquad (10)$$

Make a note of the objects viewed through your telescope (e.g. a window or a tree), their approximate distance and the approximate size of the image and compare the magnification with the one calculated theoretically one using equation (10).

<u>Alternative determination of the angular magnification:</u>

View the scale on the door with 25 horizontal equally spaced parallel lines through the telescope and focus on the line pattern by slightly changing the lens separation. Use one eye to look through the telescope and with the other eye look along the telescope directly at the line pattern on the door. You must keep your eye close to the lens. You should simultaneously see both the lines on the door and the magnified line pattern through the telescope. By a slightly moving your head, you can align the top edge of one of the magnified lines through the telescope with the top edge of the line pattern on the door. Next you have to count how many magnified lines correspond to the height of the entire line pattern on the door. Divide 25 by this number and you have the angular magnification Γ.

2.6 Galilean telescope (opera glass)

The earliest telescope was constructed in 1609. The objective is a converging lens, but the eyepiece is a diverging lens placed so that the focal points of the two lenses coincide beyond the eyepiece, as shown in Fig. 6. An erect image is obtained at infinity and there is no intermediate image. The small overall length and the erect image make this design suitable for opera glasses with a magnification of two to three times.

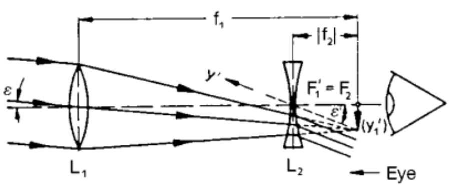

Figure 6: The Galilean Telescope.

In this experiment a converging lens with a rather long focal length f_1 (e.g. +300 mm) and a diverging lens with a short focal length f_2 (e.g. -50 mm) are set up with a separation given by $f_1 - |f_2|$ (see Fig. 6). After focusing you can see an erect magnified image of distant objects through the eyepiece. The magnification of the telescope is again given by:

$$\Gamma = \frac{f_1}{|f_2|} \qquad (11)$$

View the same object as you used for the astronomical telescope and compare the image size with the calculated magnification Γ.

3. Preparation

Task	Result	
2.1	f_{bg}	for two lenses + comparison with value on lens
2.2	f_Z	using the Bessel method
2.3	V	from the three different calculations
2.4	V	+ comparison with V_{mikr}
2.5	Γ	+ comparison with the image size
2.6	Γ	+ comparison with the image size

For 2.1 and 2.2 the average values and the standard deviations must be calculated.

4. Appendix: Average values and standard deviations

For the correct calculation of the average values and the standard deviations please read the documentation on data analysis.

5. Self-test questions

These questions are primarily intended so that you can check whether you understand the basic principles and really know what you are doing. Similar questions might be asked by the experiment supervisor

(1) What does a lens do from the point of view of geometrical optics?

(2) What are the basic types of lenses and what are their shapes?

(3) How do you specify the typical parameter of a lens and what does it mean?

(4) Why do we assign a typical parameter to a diverging lens and why is this negative? What is the difference between a diverging lens and a converging lens?

(5) Which physical process takes place when a ray of light passes through a lens?

(6) What is a thin lens and what is the lens equation for a thin lens?

(7) From which point in a thin lens are object distance g and image distance b determined?

(8) What is the minimum value of g that gives a real image? When do you obtain magnified images, demagnified images, and images with the same size as the object?

(9) Why is the Bessel method for determining the focal length of a combination of two lenses better than the simple method of measuring the object and image distances? (Tip: Is it necessary to measure g and b in the Bessel method?)

(10) Explain how the focal length determination is performed using the Bessel method.

(11) Explain the optics of a microscope. What should the distance be between the object and the first lens? Is it possible view objects in a microscope with normal room lighting?

(12) Explain the optics of an astronomical telescope. Why does it have this name? (Tip: property of image)?

(13) Explain the optics of a Galilean telescope. Why is this telescope also called a terrestrial telescope or an opera glass?

6. Preparation for the Experiments

Here are suggestions for things that must be done well before you have to actually perform the measurements.

- Read this instruction manual carefully several times.
- Plan your measurements and how you will tabulate the data.
- For the experiment using the Bessel method to determine the focal length of a lens system consisting of a converging lens and a diverging lens you should calculate the overall focal length of the lens system using the given focal lengths of the two lenses. Then you should calculate the minimum distance d between the object and the screen.
- Read the information on data analysis.

Compliance with these points will save unnecessary extra work. The main reason for having to repeat this experiment is incomplete data analysis.

7. Performing the Experiments

This experiment requires all the time available because the measurements have to be performed carefully.

- For the different experiments you should use the lenses suggested in this document.
- You can save time by assigning tasks. For example, one student can make the measurements, the second writes down the results, and the third student calculates the values using the equations given in this document. In this way errors can be spotted quickly, so you don't waste time.
- Prepare the tables for recording the data before the day of the experiment.
- Follow the instructions given in Sect. 6.