Lenses and optical instruments

Laboratory course for students of the TUHH

Wladimir Banda-Barragán

Introduction

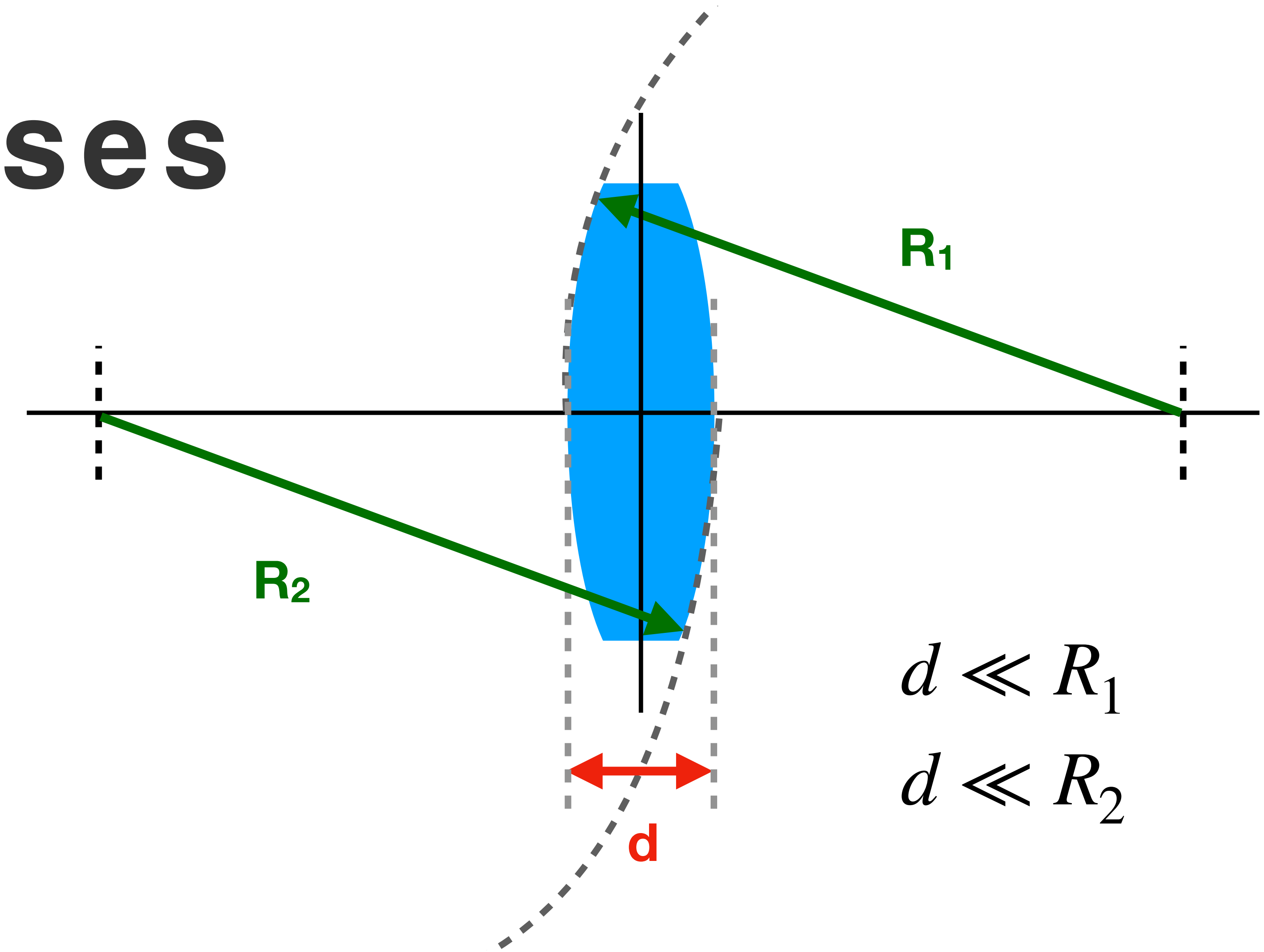
My name is Wladimir Banda-Barragán.

I will be your tutor for this laboratory session.

The class will be organised as follows:

1. Lecture with step-by-step examples (1 hour).
2. Exercises in the protocol (2.5 hours)
3. Questions and wrap-up (0.5 hours)

Thin lenses



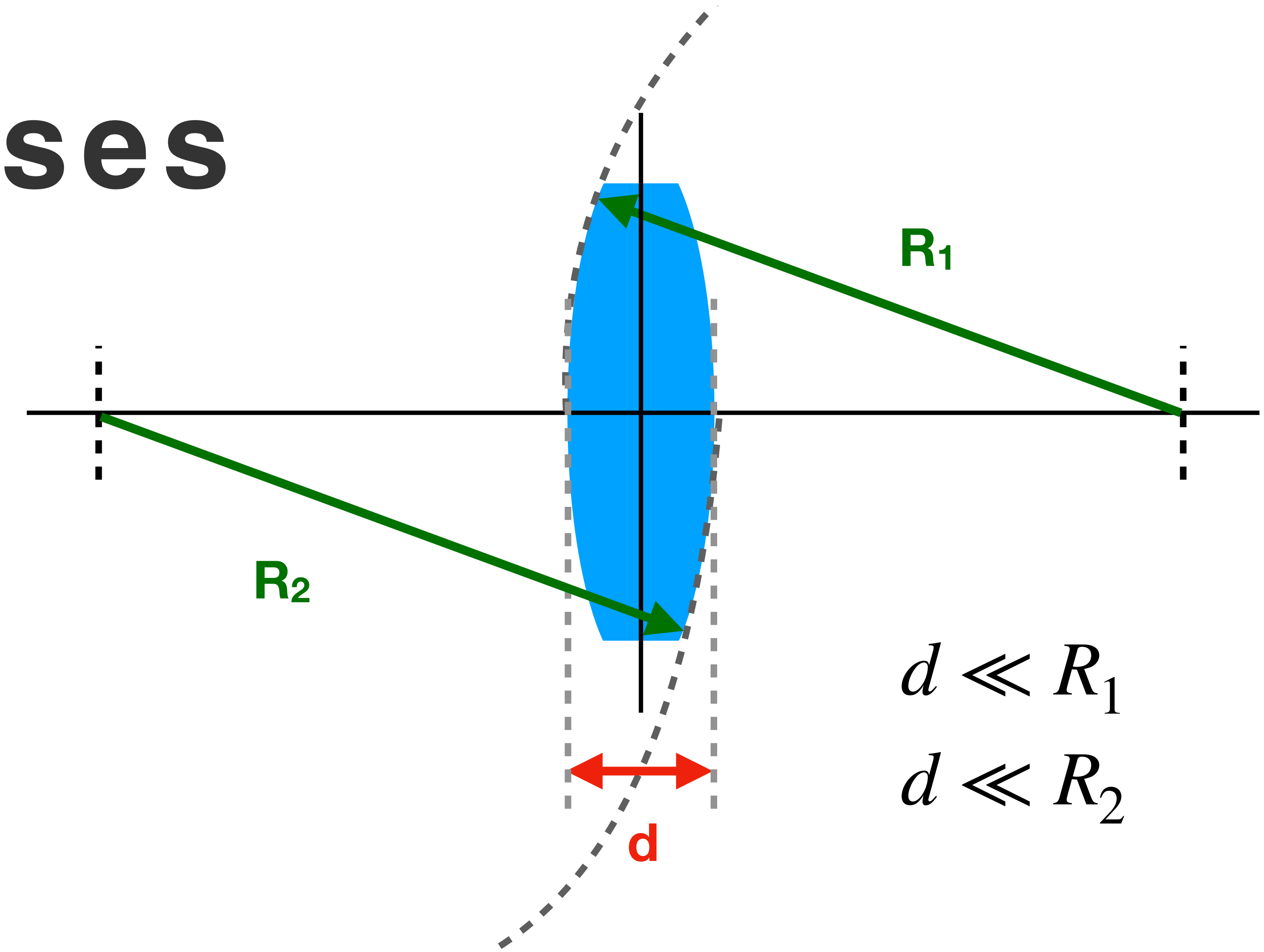
$$d \ll R_1$$
$$d \ll R_2$$

Lenses are transparent objects with aligned refracting surfaces.

Thin lenses are those with a thickness, d, that is negligible compared to the radii of curvature, $R_{1,2}$, of the lens surfaces.

Source: Wikipedia

Thin lenses



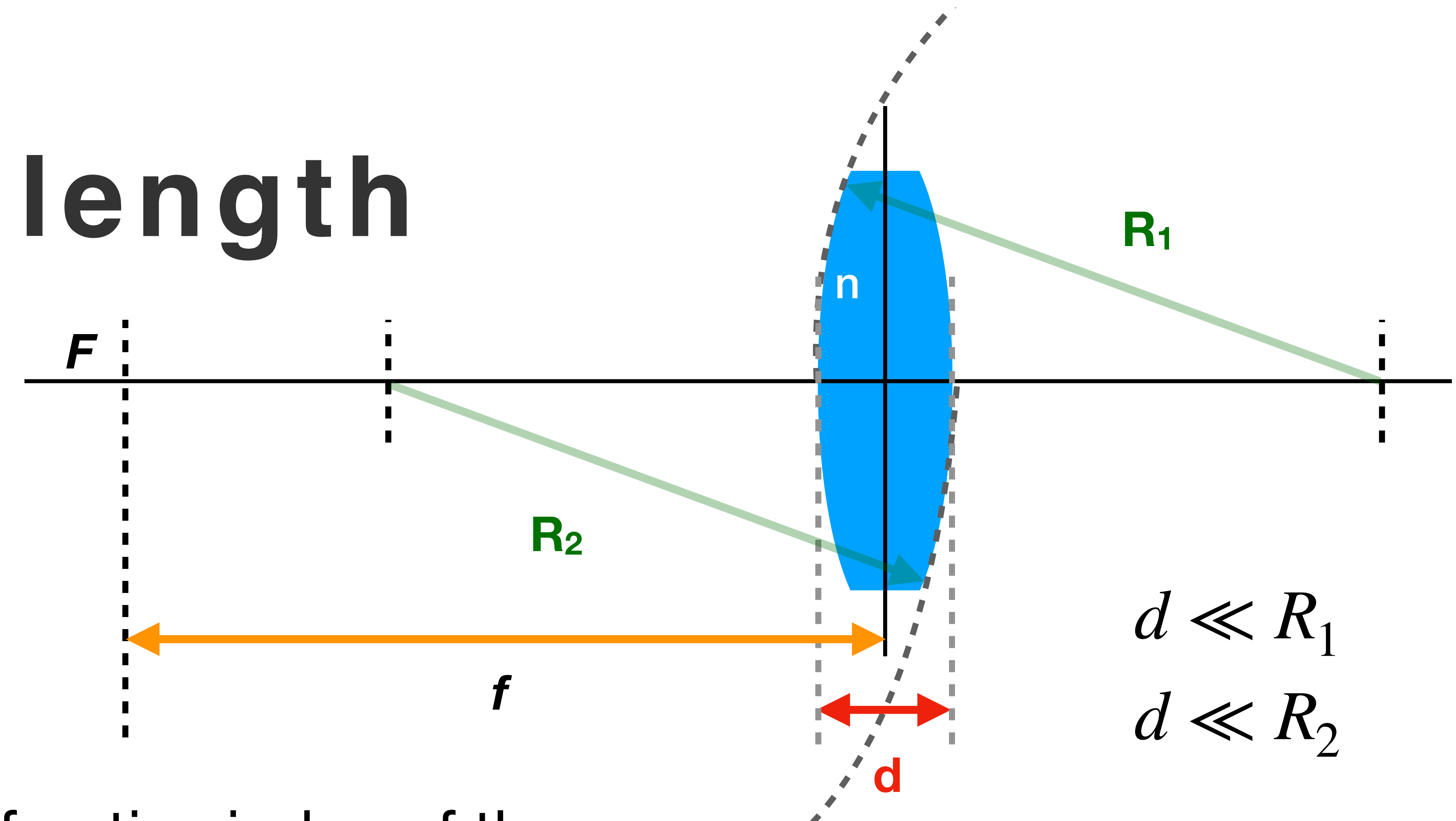
$$d \ll R_1$$
$$d \ll R_2$$

The **thin lens approximation** ignores optical effects due to the thickness of lenses and simplifies ray tracing calculations.

Lenses whose thickness is not negligible are sometimes called thick lenses.

Source: Wikipedia

Focal length



$$d \ll R_1$$
$$d \ll R_2$$

Depends on the refractive index of the lens material, n:

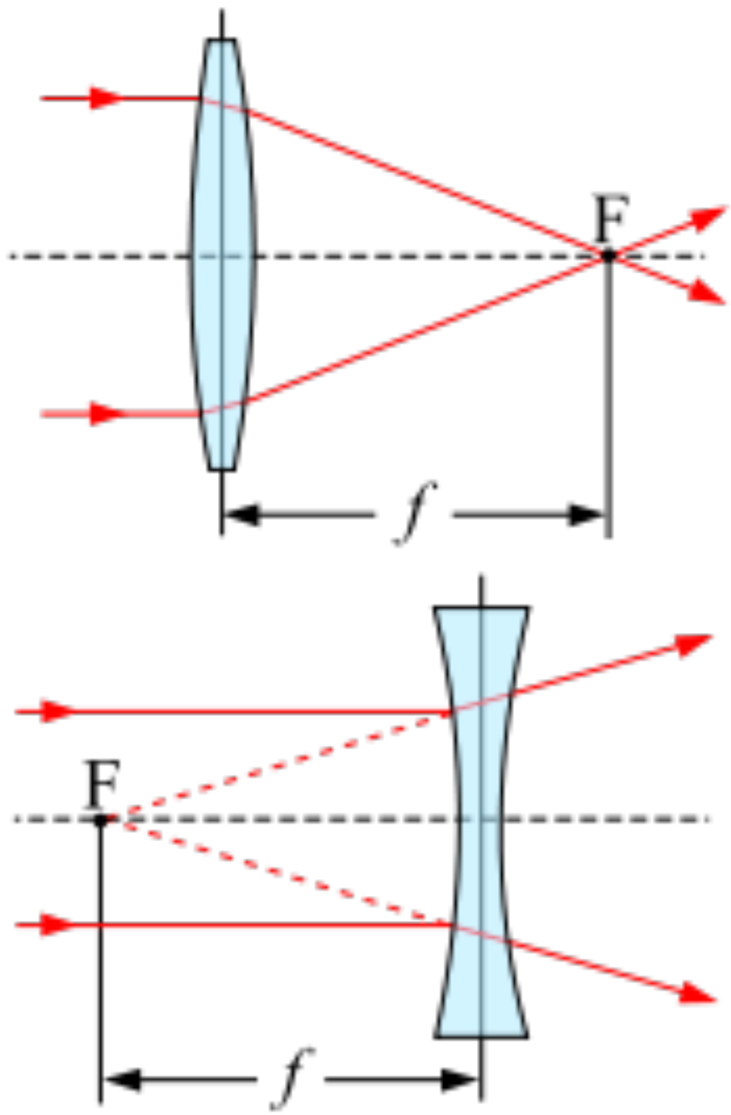
$$\frac{1}{f} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2} + \frac{(n-1)d}{nR_1R_2}\right]$$

In the thin lens approximation:

$$\frac{1}{f} \approx (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

Source: Wikipedia

Lenses types



For thin lenses, the focal length is the distance from the centre to the focal point.

Converging lenses (e.g. convex lenses) have positive focal lengths.

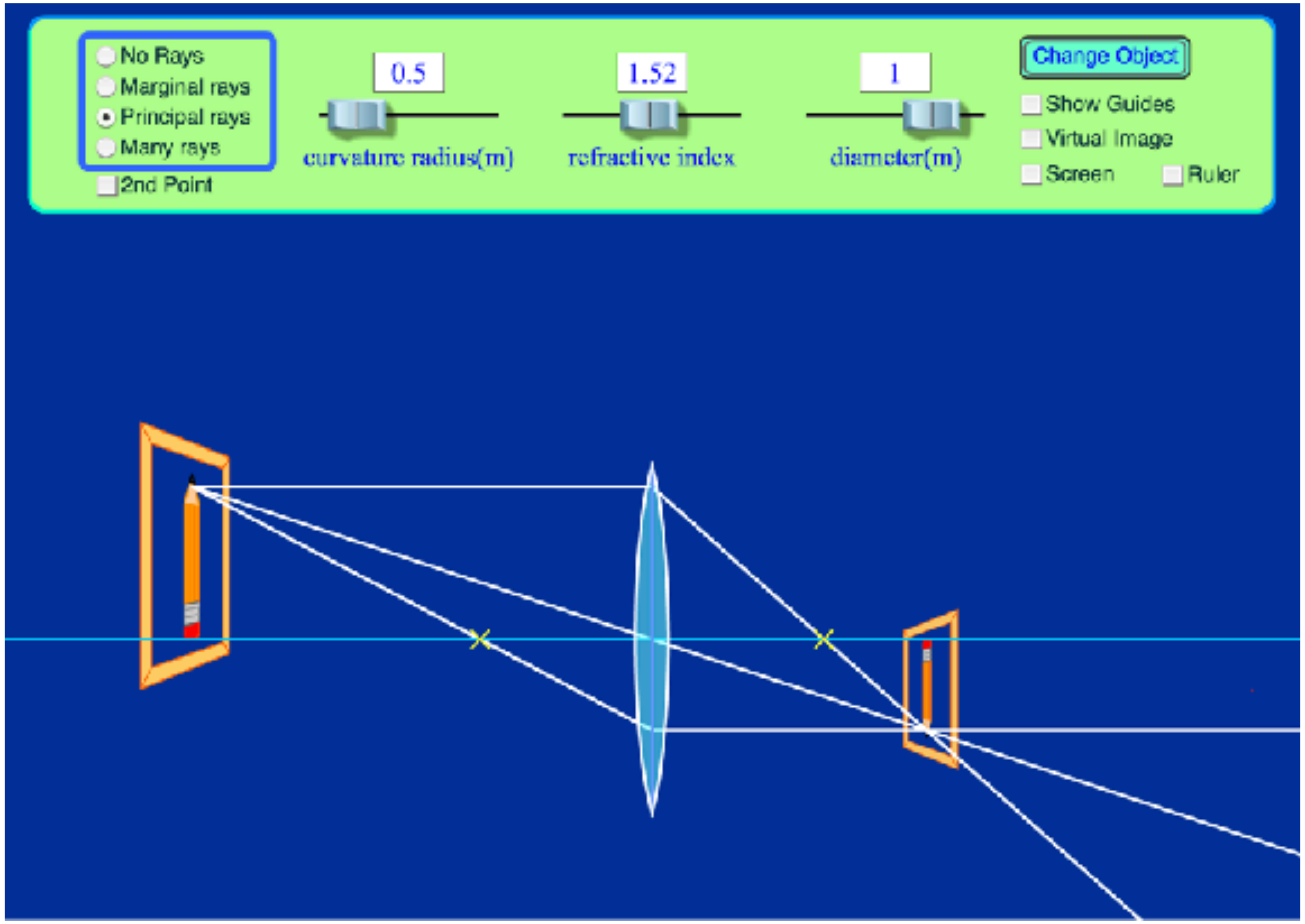
Diverging lenses (e.g. a concave lenses) have negative focal lengths.

Source: Wikipedia

Focal length: example

Let us see how "*f*" changes with "*n*"

$$\frac{1}{f} \approx (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

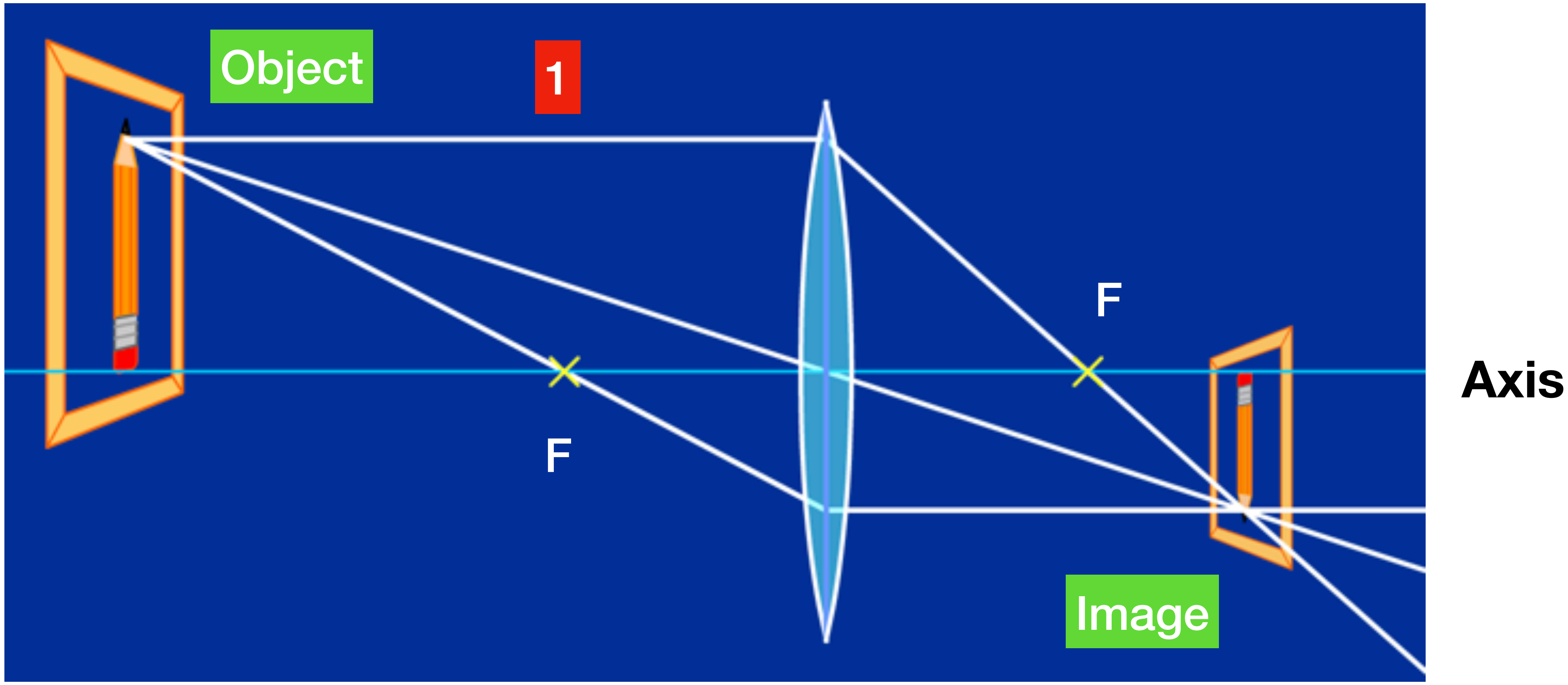


https://phet.colorado.edu/sims/geometric-optics/geometric-optics_en.html

Geometric optics

We can use the so-called principal rays:

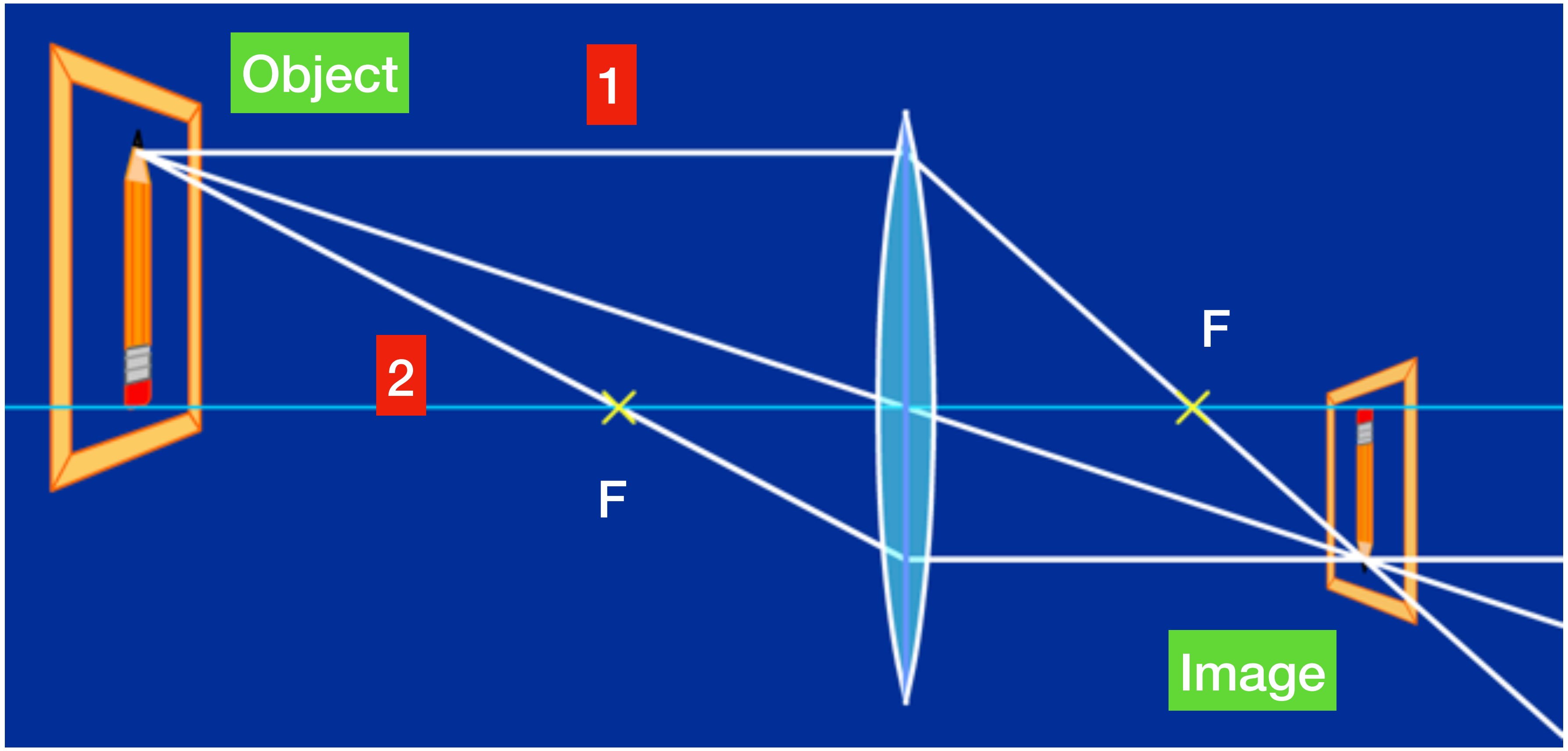
1. Any ray that enters parallel to the axis on one side of the lens proceeds towards the focal point F on the other side.



Sources: Wikipedia & https://phet.colorado.edu/sims/geometric-optics/geometric-optics_en.html

Geometric optics

2. Any ray that arrives at the lens after passing through the focal point on the front side, comes out parallel to the axis on the other side.

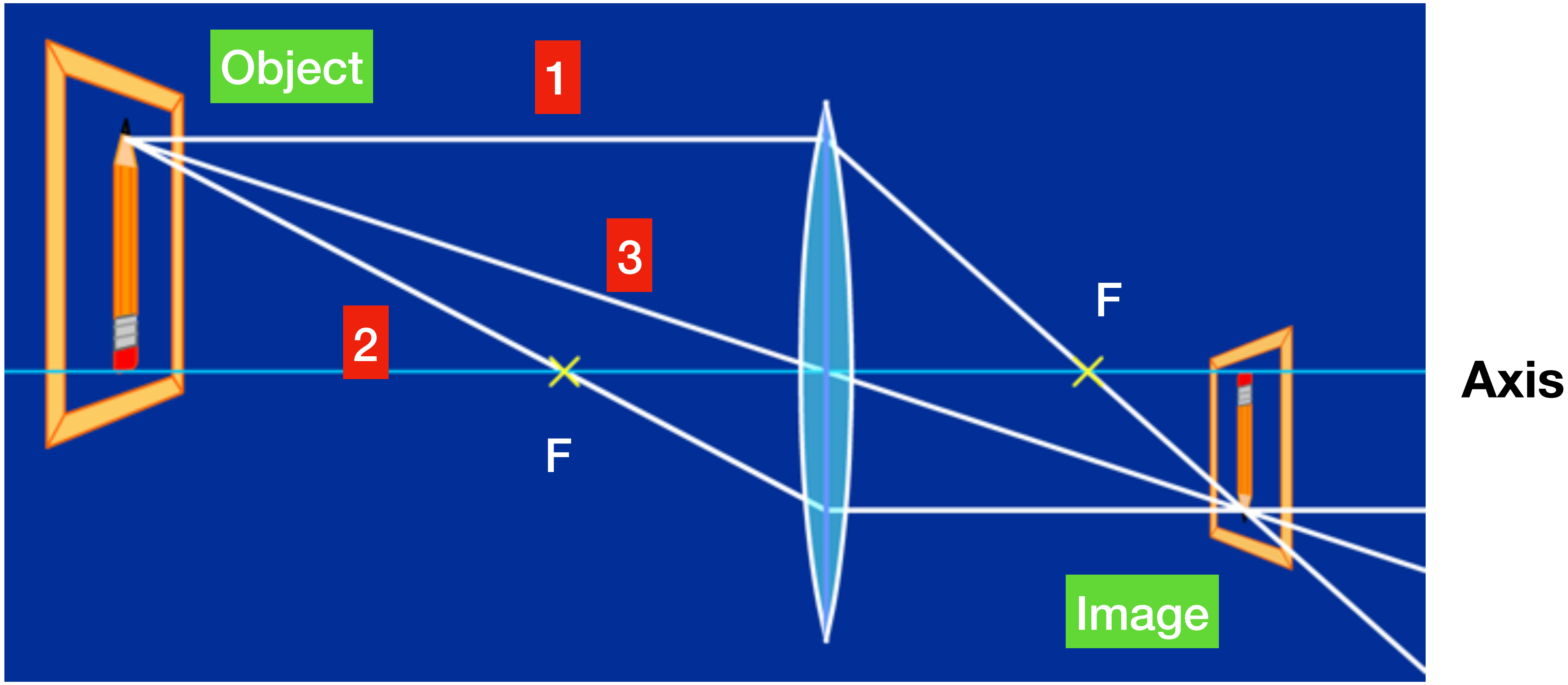


Axis

Sources: Wikipedia & https://phet.colorado.edu/sims/geometric-optics/geometric-optics_en.html

Geometric optics

3. Any ray that passes through the centre of the lens will not change its direction.

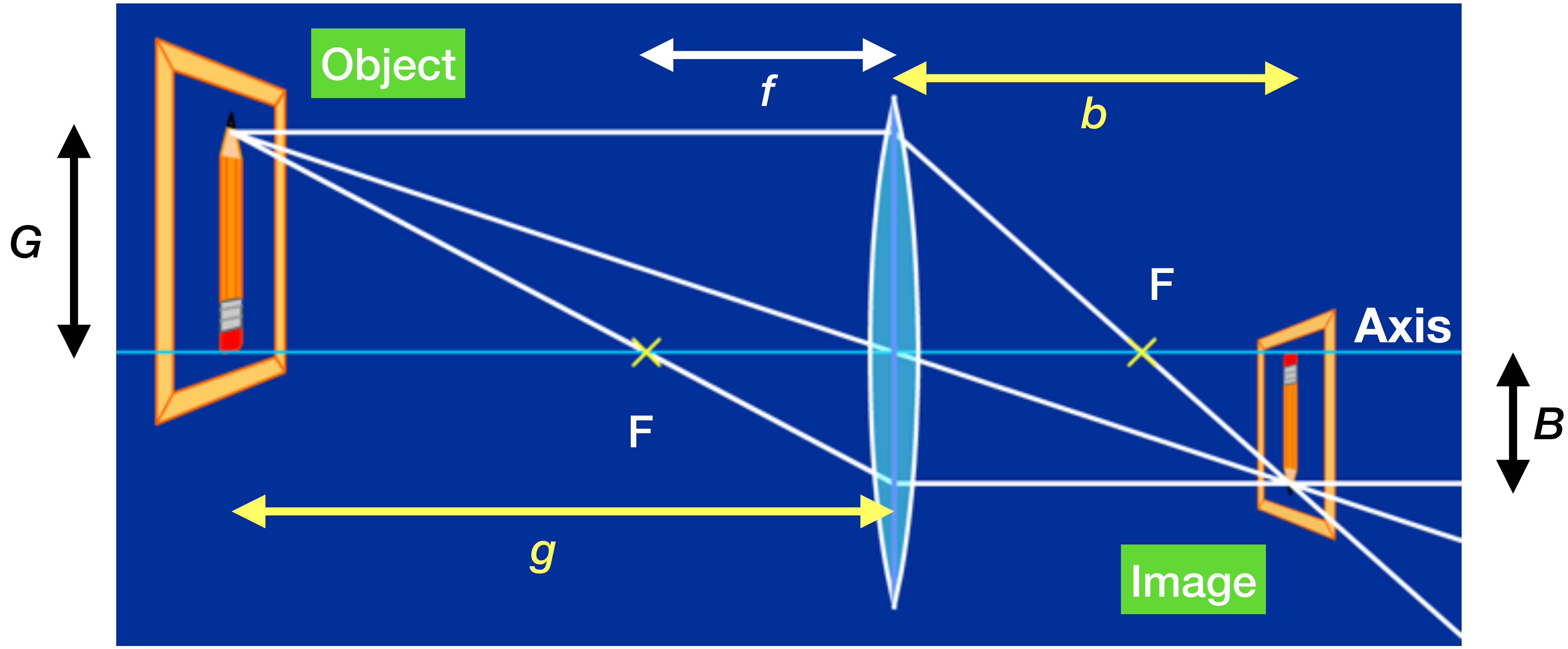


Sources: Wikipedia & https://phet.colorado.edu/sims/geometric-optics/geometric-optics_en.html

Lens laws

We can trace these rays 1, 2, and 3.

The focal length *f*, the object distance *g,* and the image distance *b* determine the ratio of the image height B to the object height G.



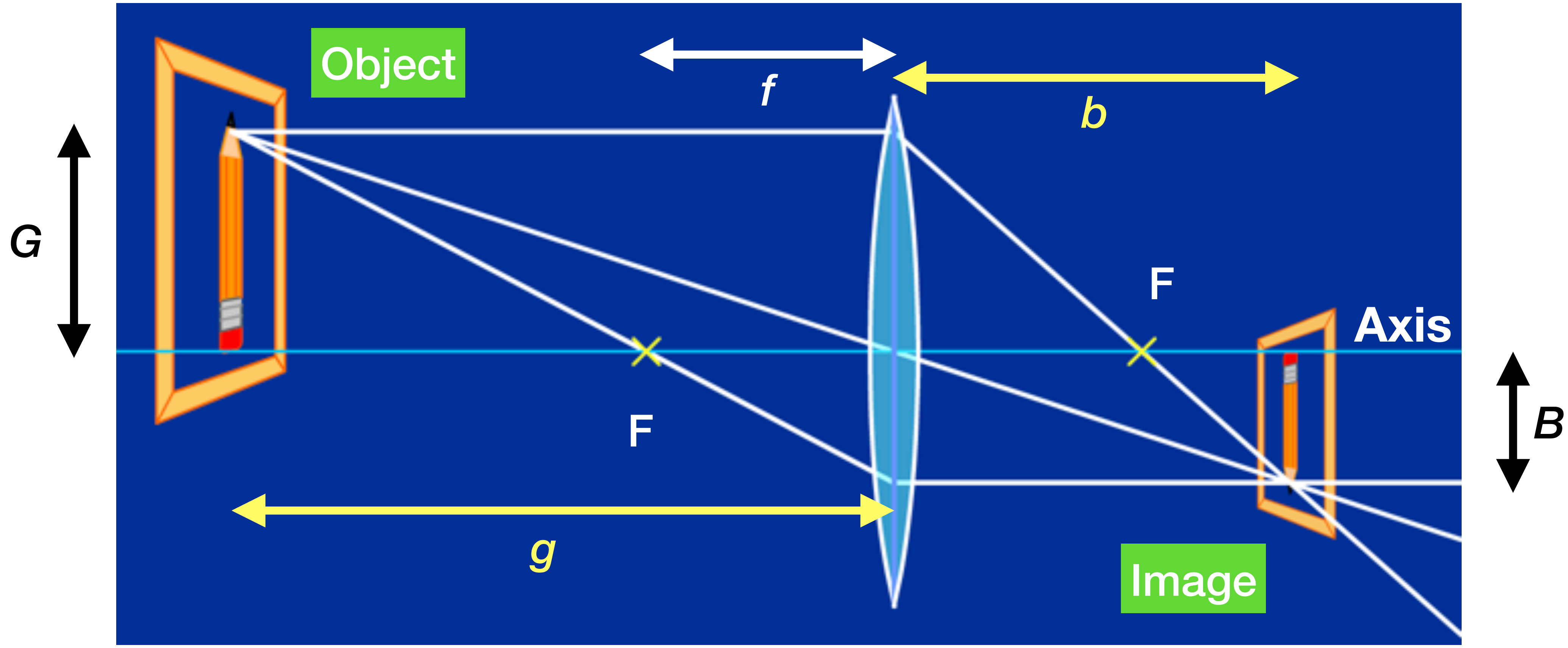
Sources: Wikipedia & https://phet.colorado.edu/sims/geometric-optics/geometric-optics_en.html

Lens laws

Using geometry, we obtain the lens equation for thin lenses

$$\frac{B}{G} = \frac{b}{g} \quad \textit{and} \quad \frac{G}{B} = \frac{f}{b-f} \qquad (1)$$

$$\frac{1}{f} = \frac{1}{b} + \frac{1}{g} \quad \text{or} \quad f = \frac{b.g}{b+g} \qquad (2)$$



Sources: Wikipedia & https://phet.colorado.edu/sims/geometric-optics/geometric-optics_en.html

Images

The imaging behaviour of the lens depends on whether the object is located outside the focal point, at the focal point, or inside the focal point, as shown in this table:

Object Position g	**Image**			
	Type	**Position**	**Orientation**	**Relative size**
∞	real	$b = f$	-/-	point
$\infty > g > 2f$	real	$f < b < 2f$	inverted	demagnified
$g = 2f$	real	$b = 2f$	inverted	equal size
$f < g < 2f$	real	$\infty > b > 2f$	inverted	magnified
$g = f$		∞		
$g < f$	virtual	$\lvert b \rvert > g$	upright	magnified

Images

Object	Image			
Position g	Type	Position	Orientation	Relative size
∞	real	$b = f$	-/-	point
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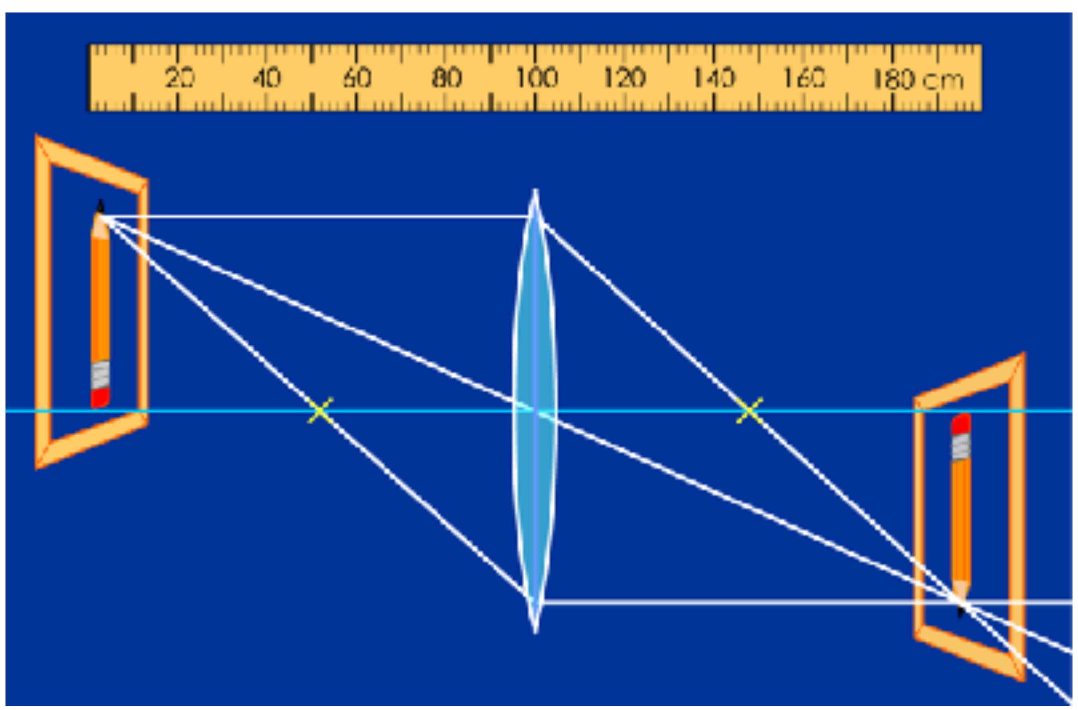
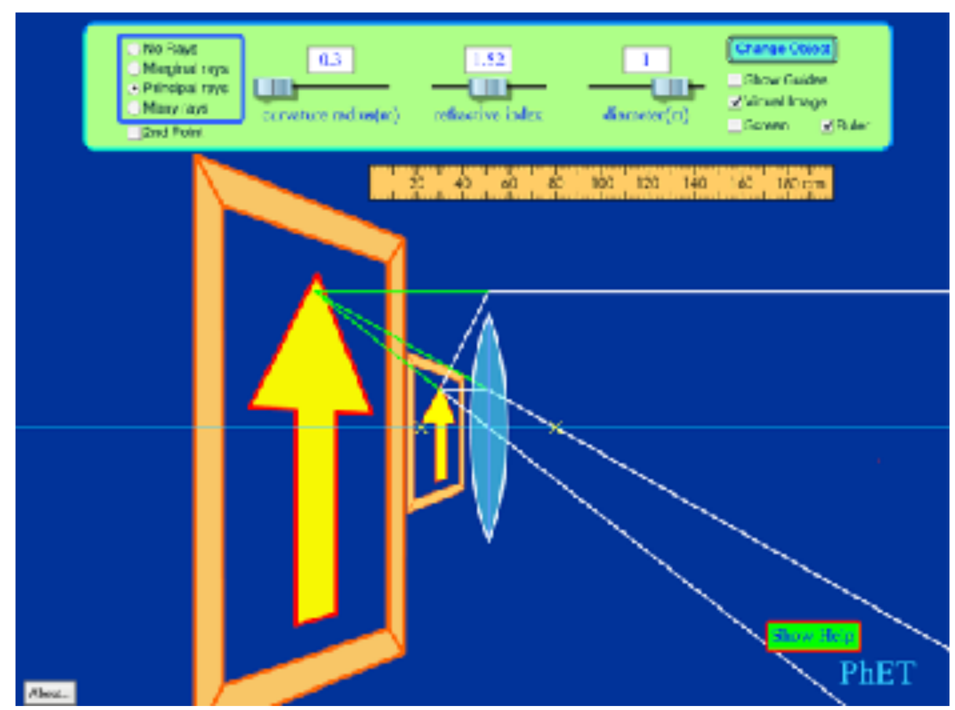
$g = f$		∞		
$g < f$	virtual	$\|b\| > g$	upright	magnified

$g = 2f$

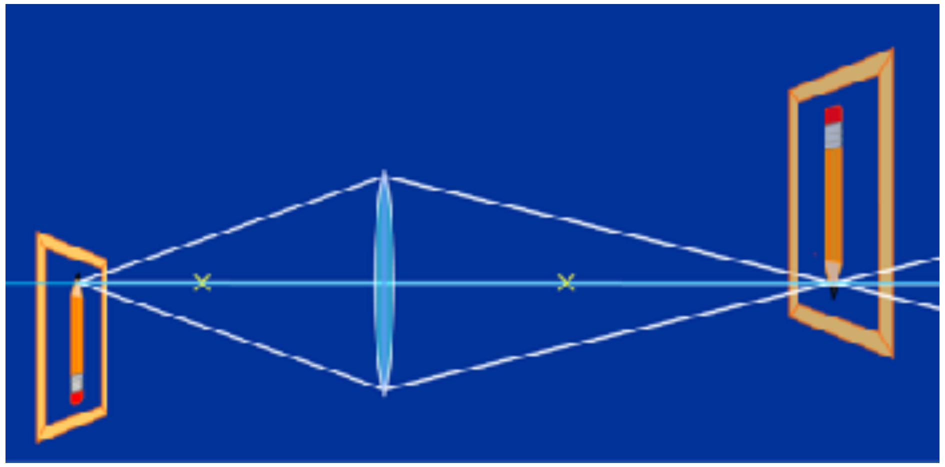


$g < f$

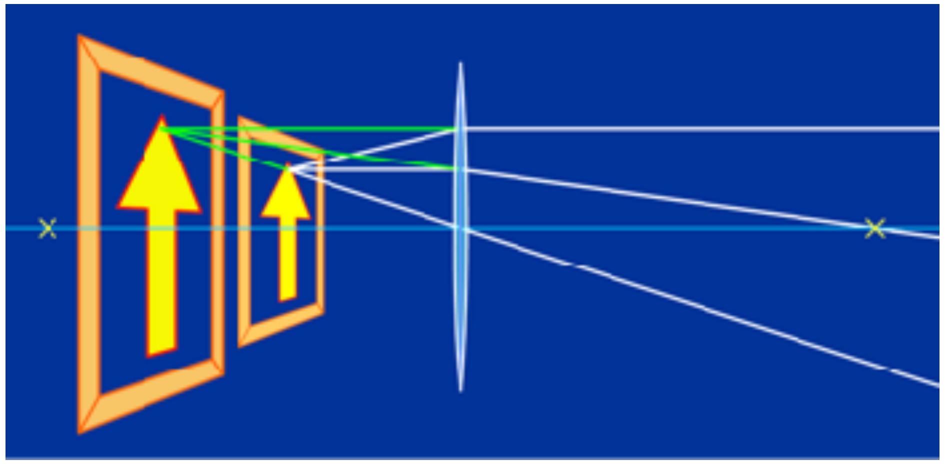


Real and virtual images

Real images are those where light actually converges



Virtual images are locations from where light appears to have converged.



Source: https://web.pa.msu.edu/courses/2000fall/PHY232/lectures/lenses/images.html

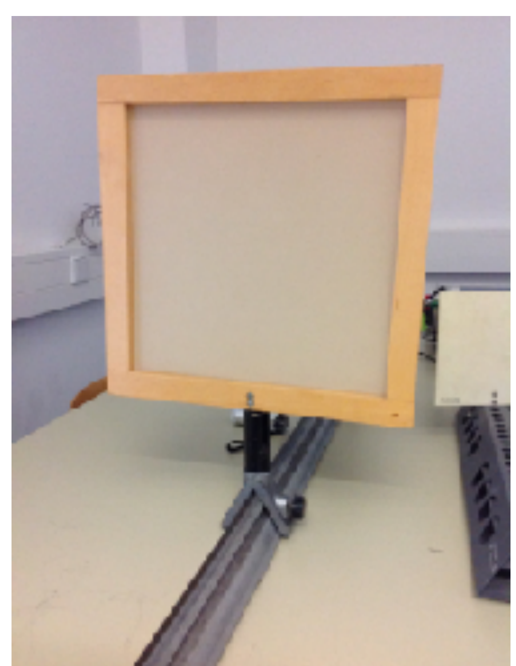
Experimental procedure

Lens holder

Lenses

Lamp

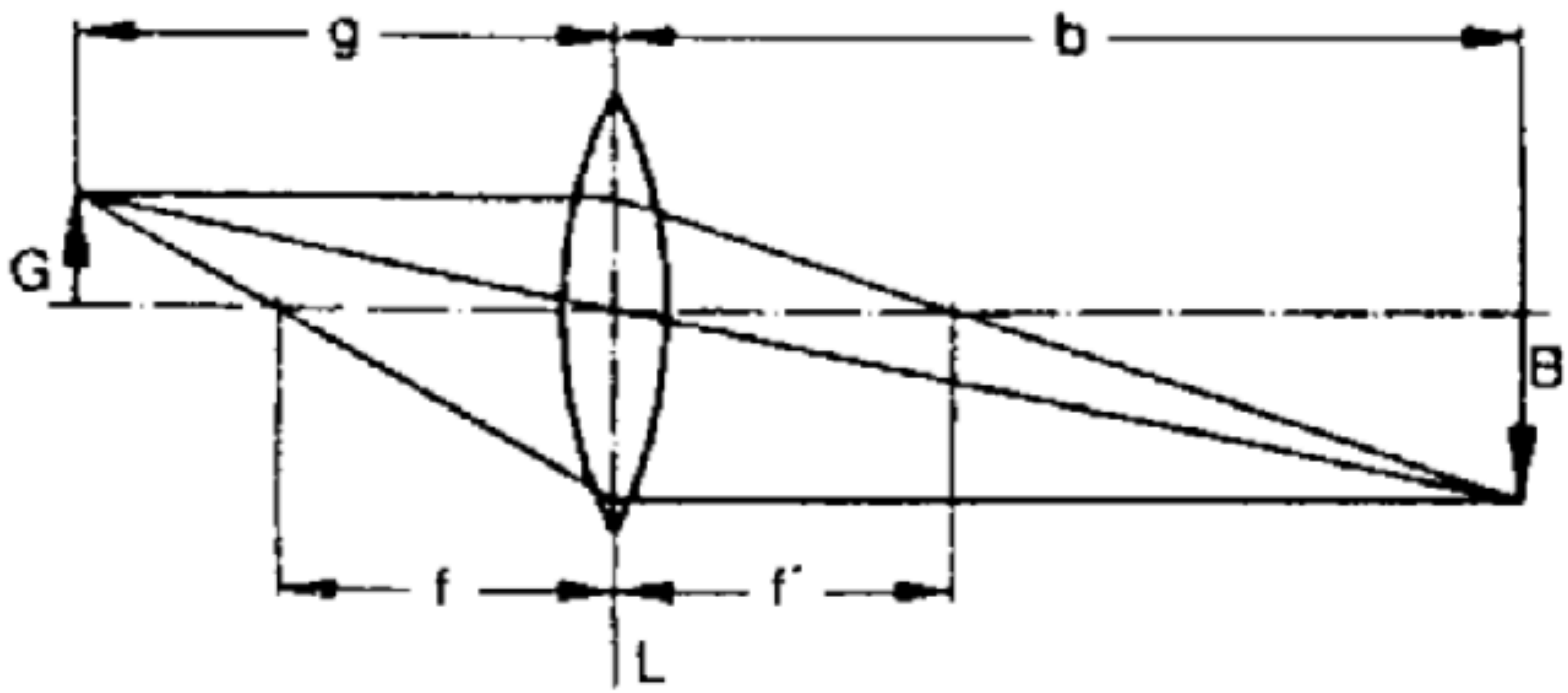
Screen



Objects

Experimental procedure

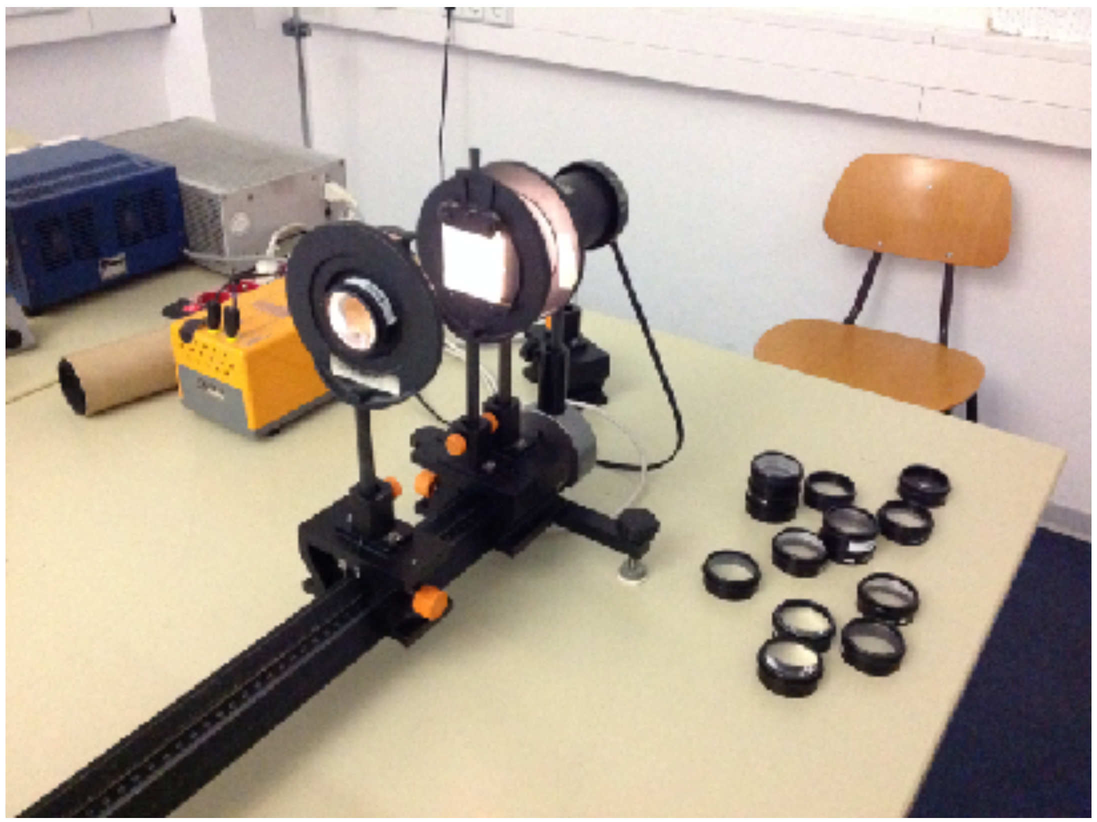
1. Determining the focal length of a lens:



The focal lengths of two different converging lenses are to be determined by measuring the image distance ***b*** and object distance ***g*** for each lens.

Experimental procedure

1. Determining the focal length of a lens:

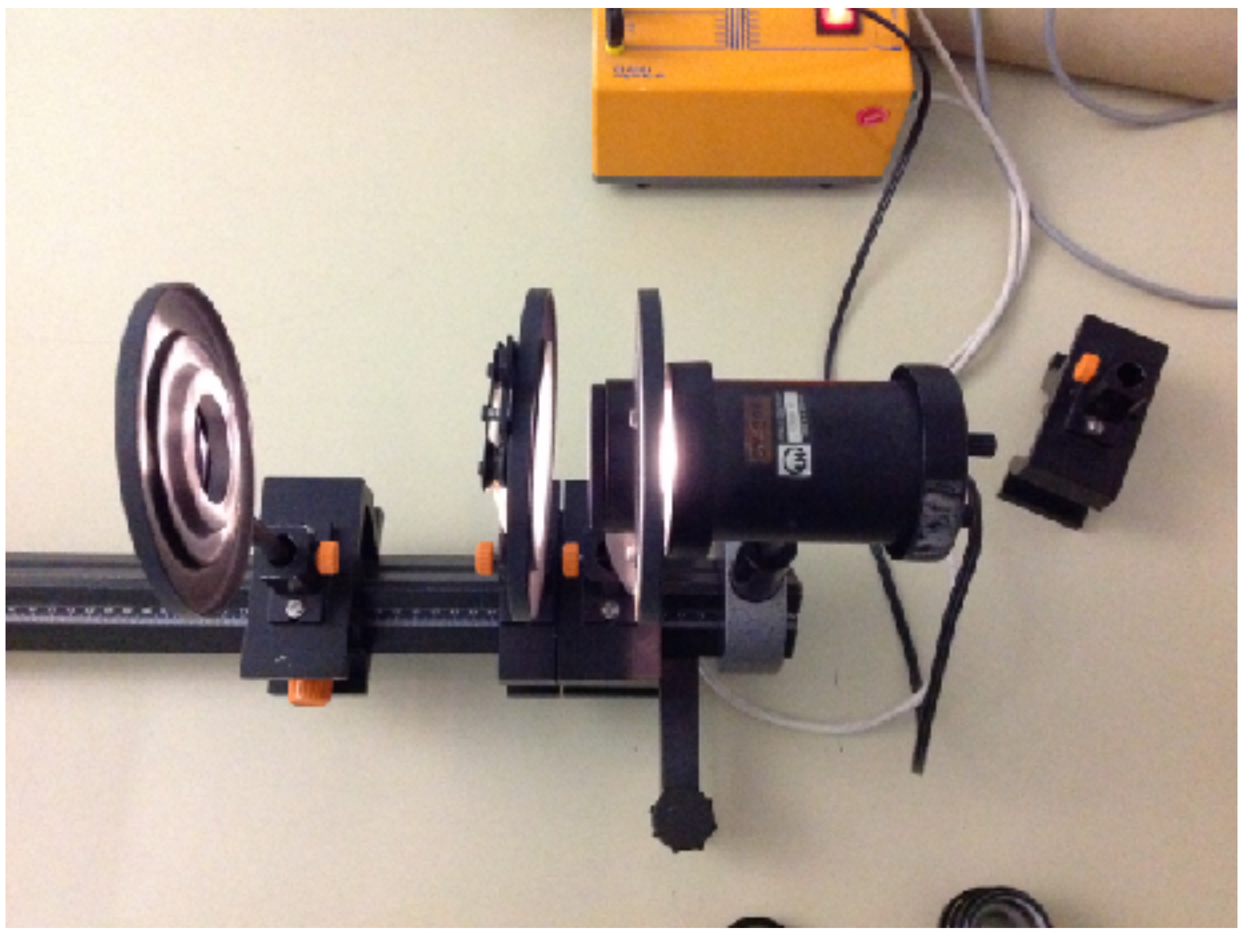


First a parallel beam of collimated light is produced using a lamp with the double condenser lens.

Next the object is positioned close to the condenser lens and a converging lens is used to produce a sharp image on the screen.

Experimental procedure

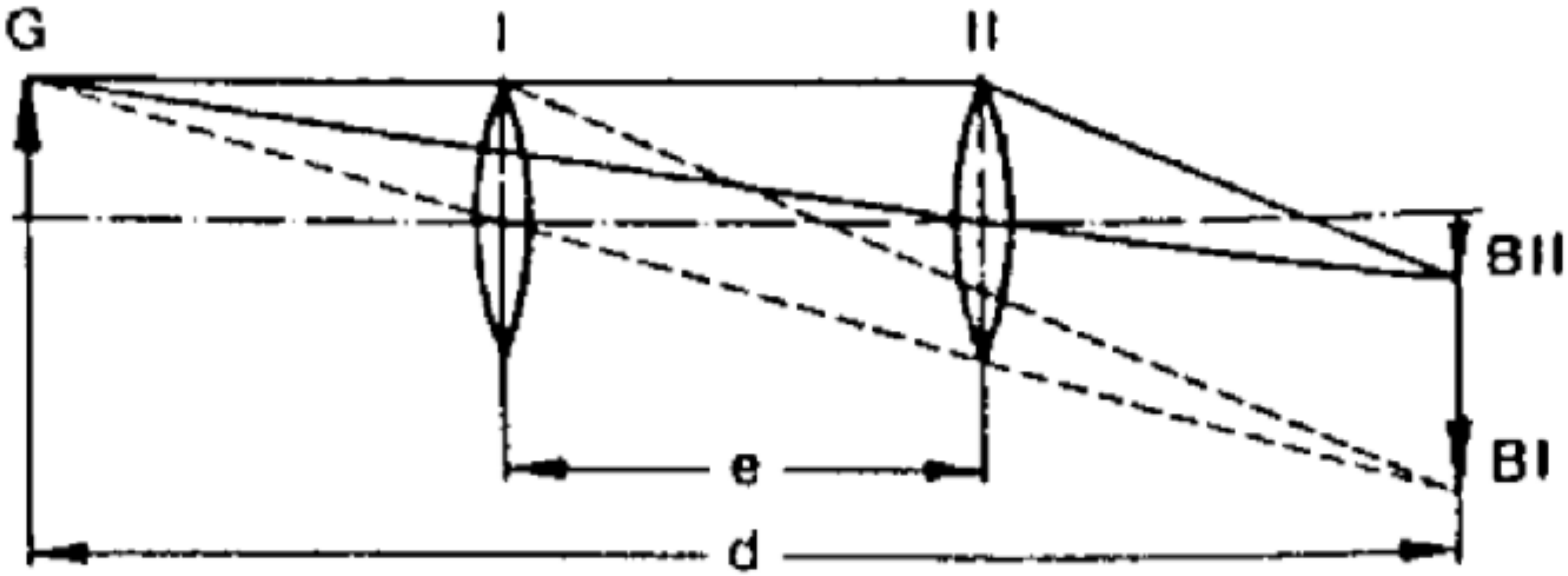
1. Determining the focal length of a lens:



The image distance and the object distance are measured from the middle of the thin lens.

Experimental procedure

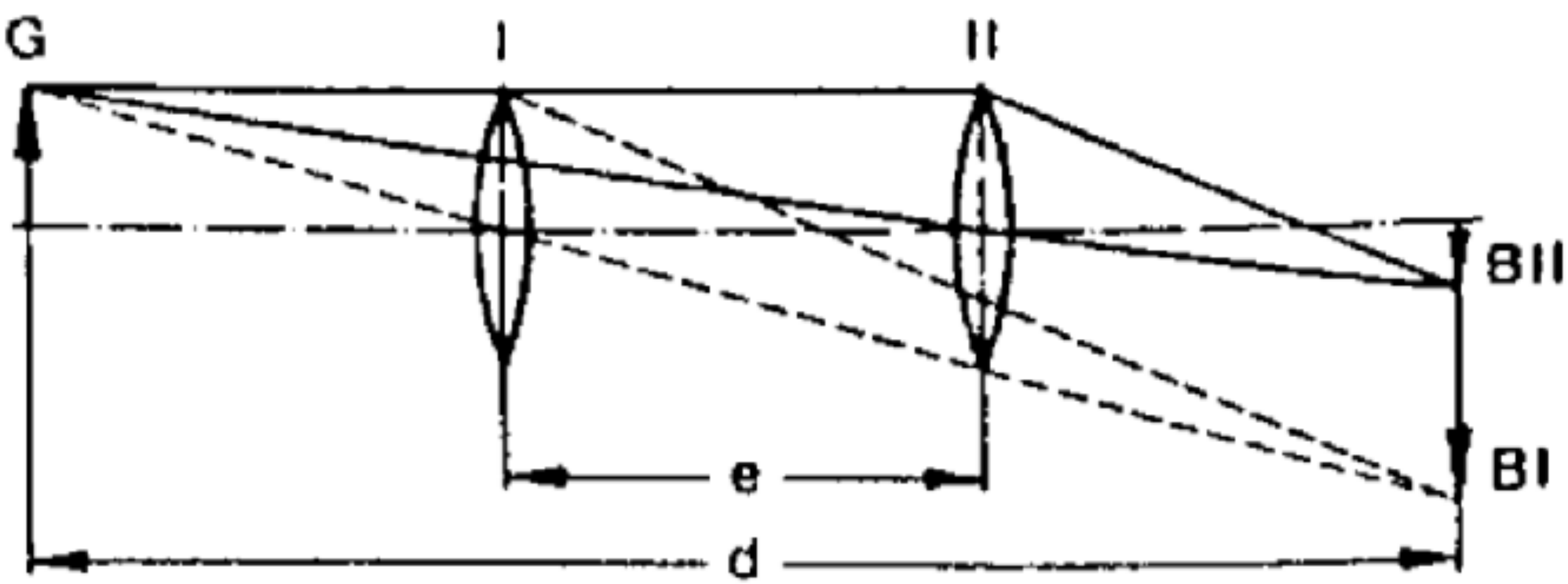
2. Bessel method:



There are two different possible positions I and II in which a thin lens can produce a sharp image of an object at a distance ***d*** from the screen.

Experimental procedure

2. Bessel method:



For the lens in position I a sharp magnified image BI is produced on the screen.

For the lens in position II the object distance is equal to the image distance of the lens in position I and a sharp demagnified image BII is produced on the screen.

Experimental procedure

2. Bessel method:

Hence, at the two possible positions the image and object distances are both exchanged (i.e., $g_1 = b_2$ and $g_2 = b_1$).

$$g_I + b_I = d \qquad (3a)$$

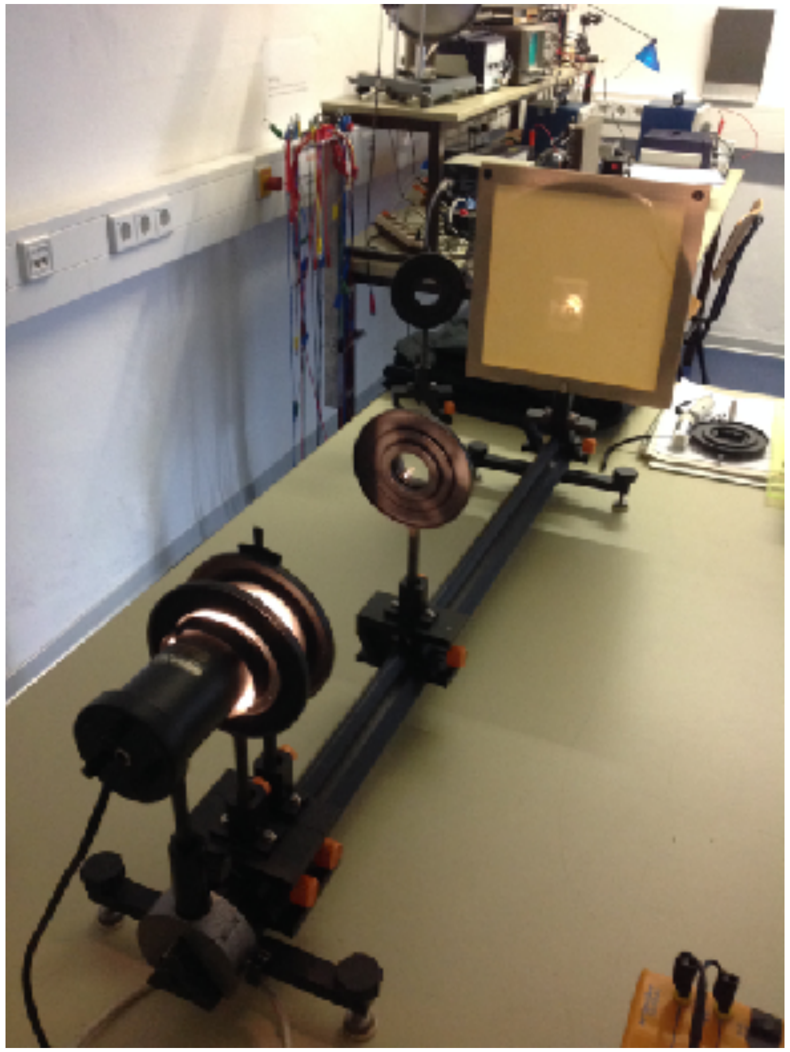
$$b_I - g_I = e \qquad (3b)$$

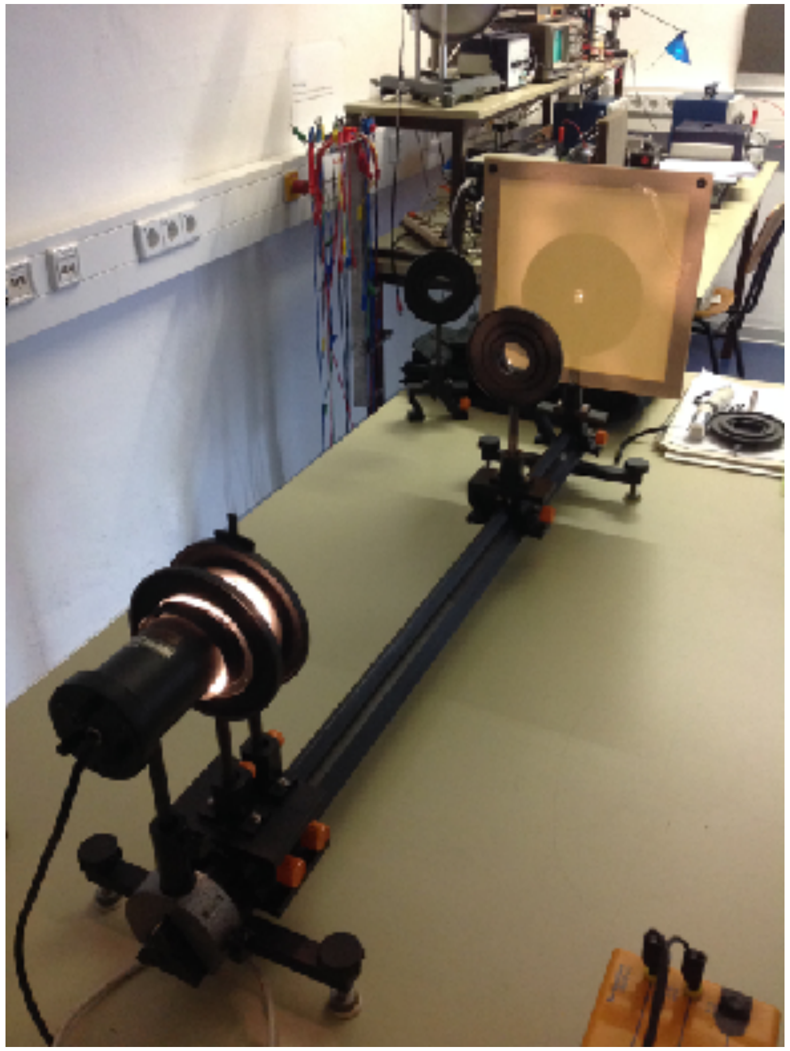
$$b_I = \frac{1}{2}(d + e) \quad \text{and} \quad g_I = \frac{1}{2}(d - e)$$

$$f = \frac{d^2 - e^2}{4d} \qquad (4)$$

Experimental procedure

2. Bessel method:





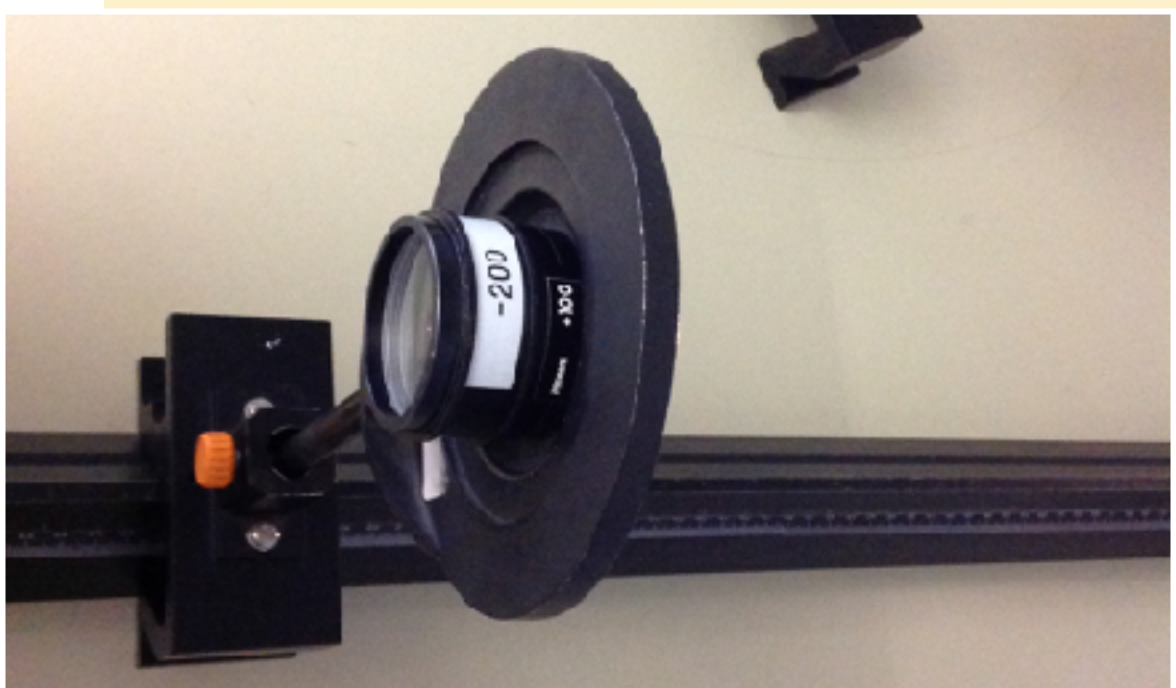
Thus, the focal length of a converging lens $\boldsymbol{f}_s$ can be determined from the vales of ***d*** and ***e***.

Experimental procedure

2. Bessel method:

This method can also be used to determine the focal length of a diverging lens f_Z by measuring the focal length of a lens system f_{ges} consisting of a diverging lens and a converging lens:

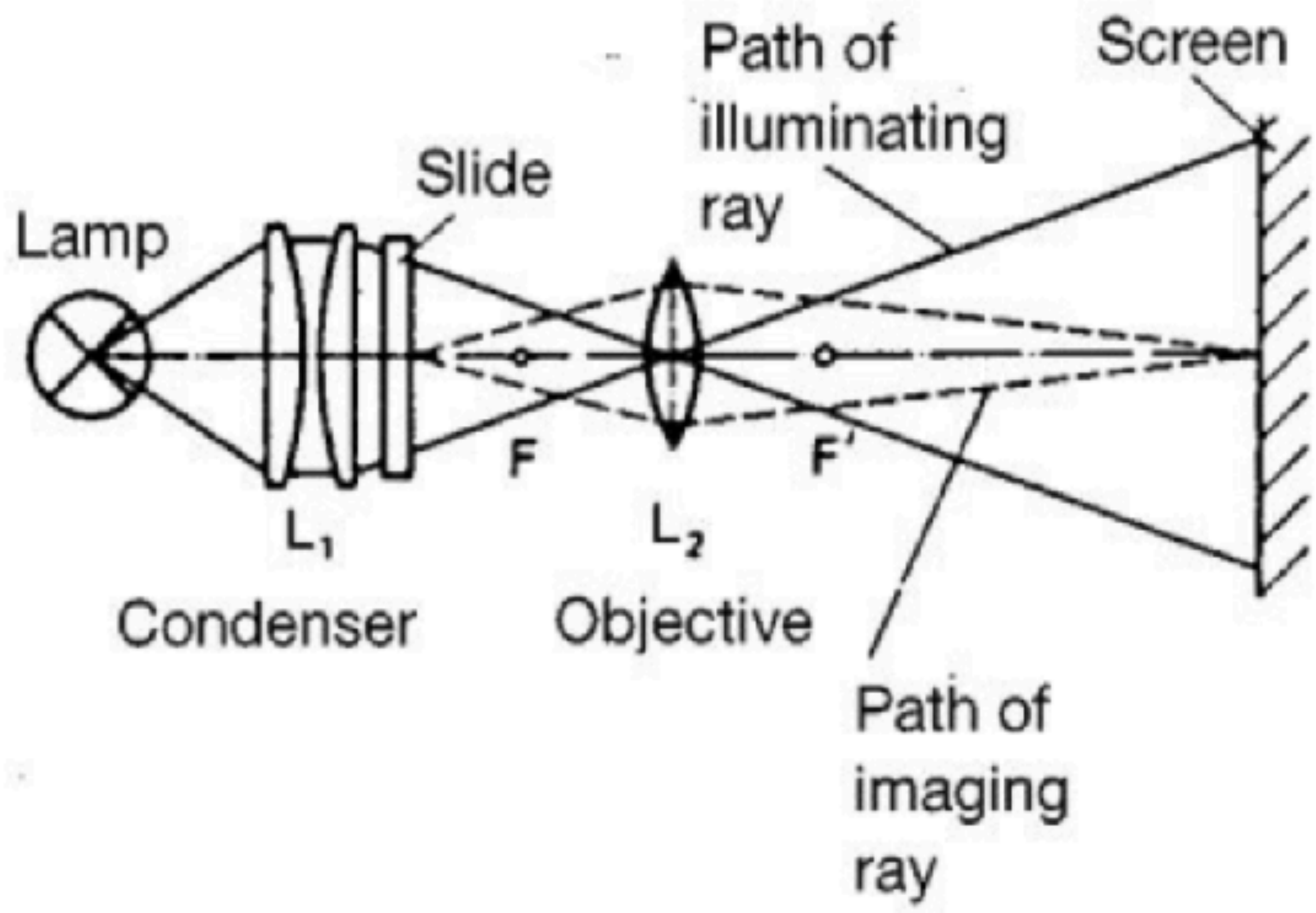
$$\frac{1}{f_Z} = \frac{1}{f_{ges}} - \frac{1}{f_S} \quad \text{oder} \quad f_Z = \frac{f_{ges} \cdot f_S}{f_S - f_{ges}} \tag{5}$$



The power of the converging lens $1/f_s$ must be greater than that of the diverging lens $1/f_Z$ to produce a real image.

Experimental procedure

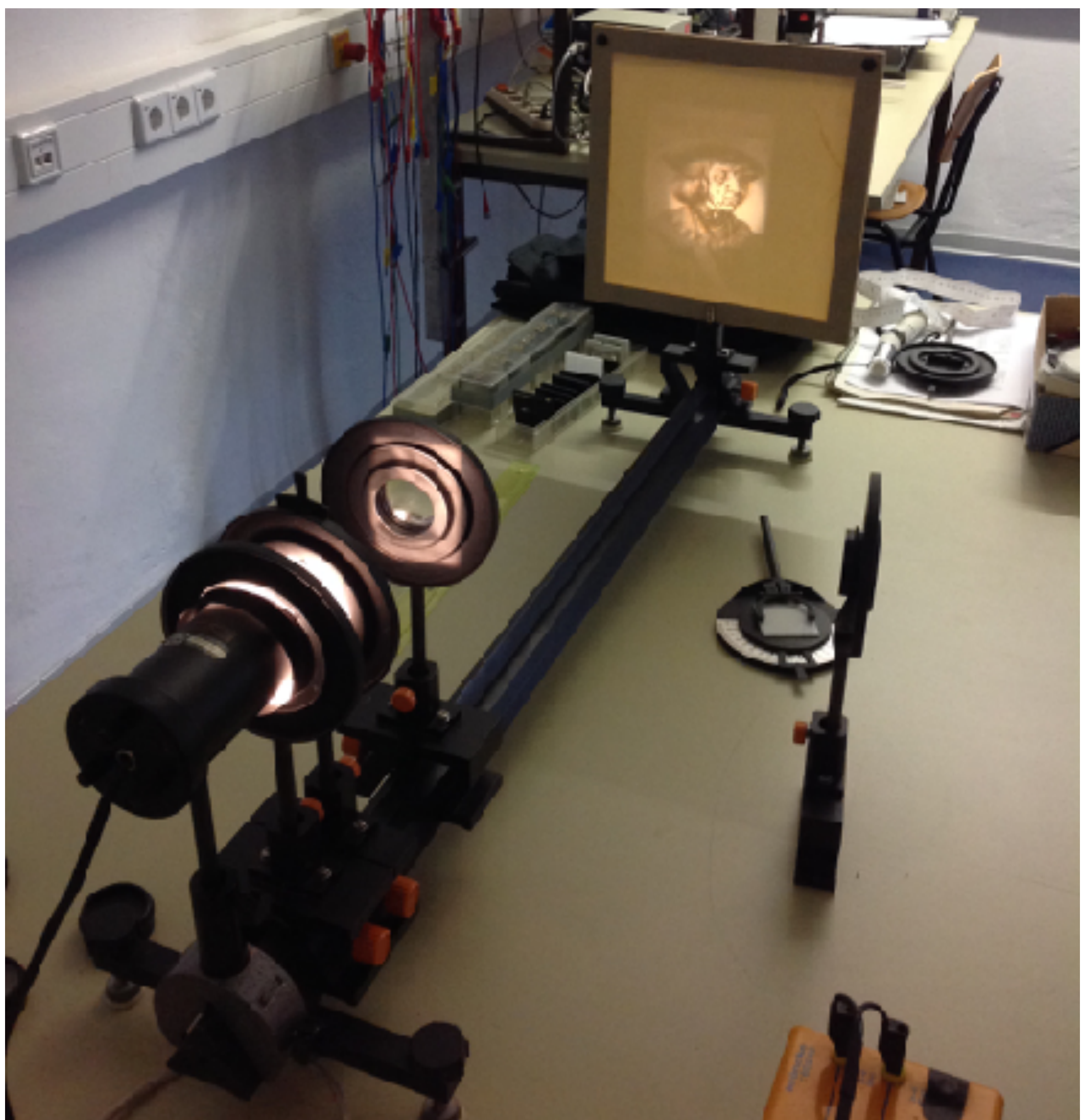
3. Slide projector:



The magnification ***V*** of the slide projector can be determined by directly measuring ***B*** and ***G.***

Experimental procedure

3. Slide projector:



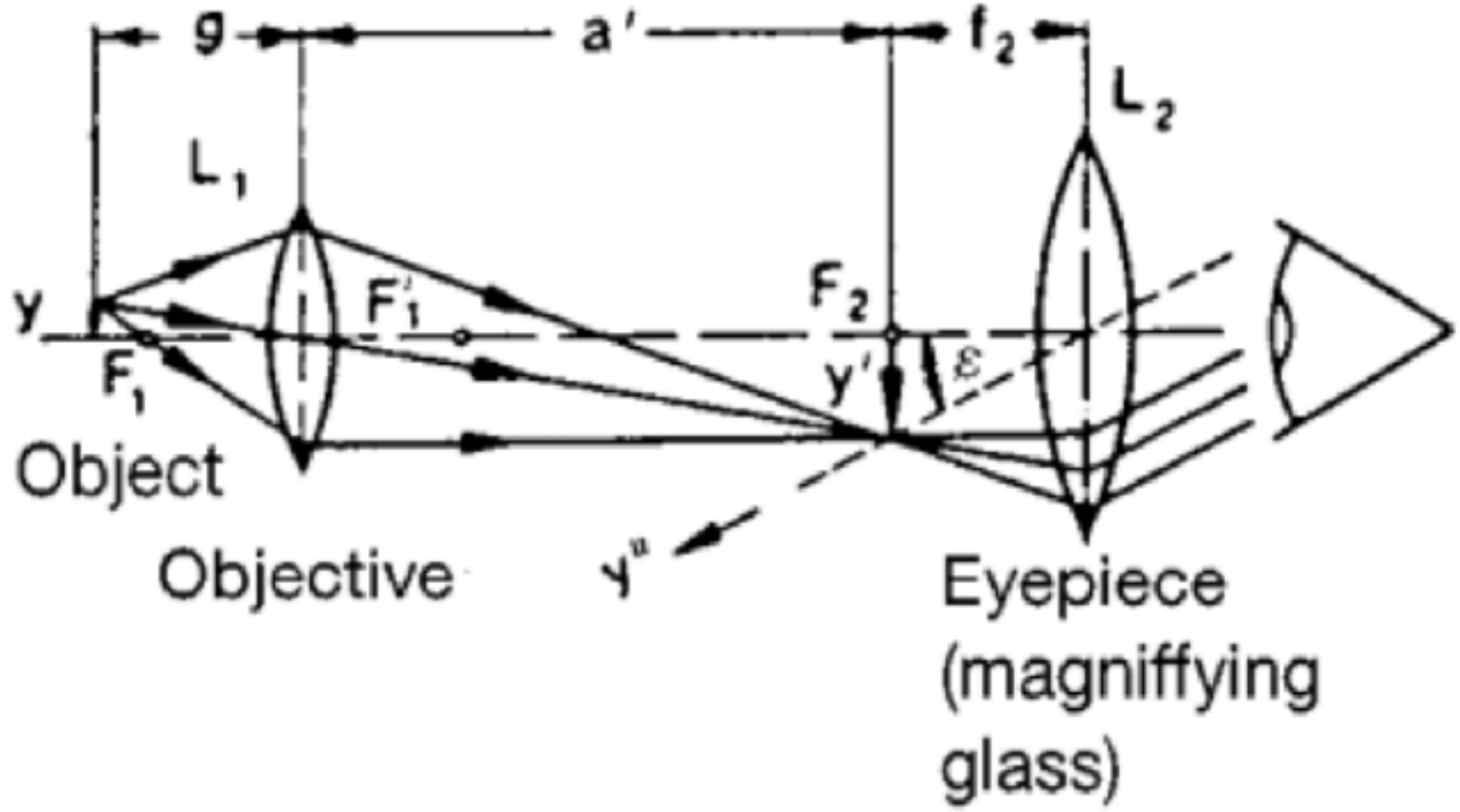
The magnification **V** can also be determined by measuring ***g***, ***b*** and ***f***.

$$V = \frac{B}{G} = \frac{b}{g} = \frac{b-f}{f} \qquad (6)$$

Eq. 6.1 6.2 6.3

Experimental procedure

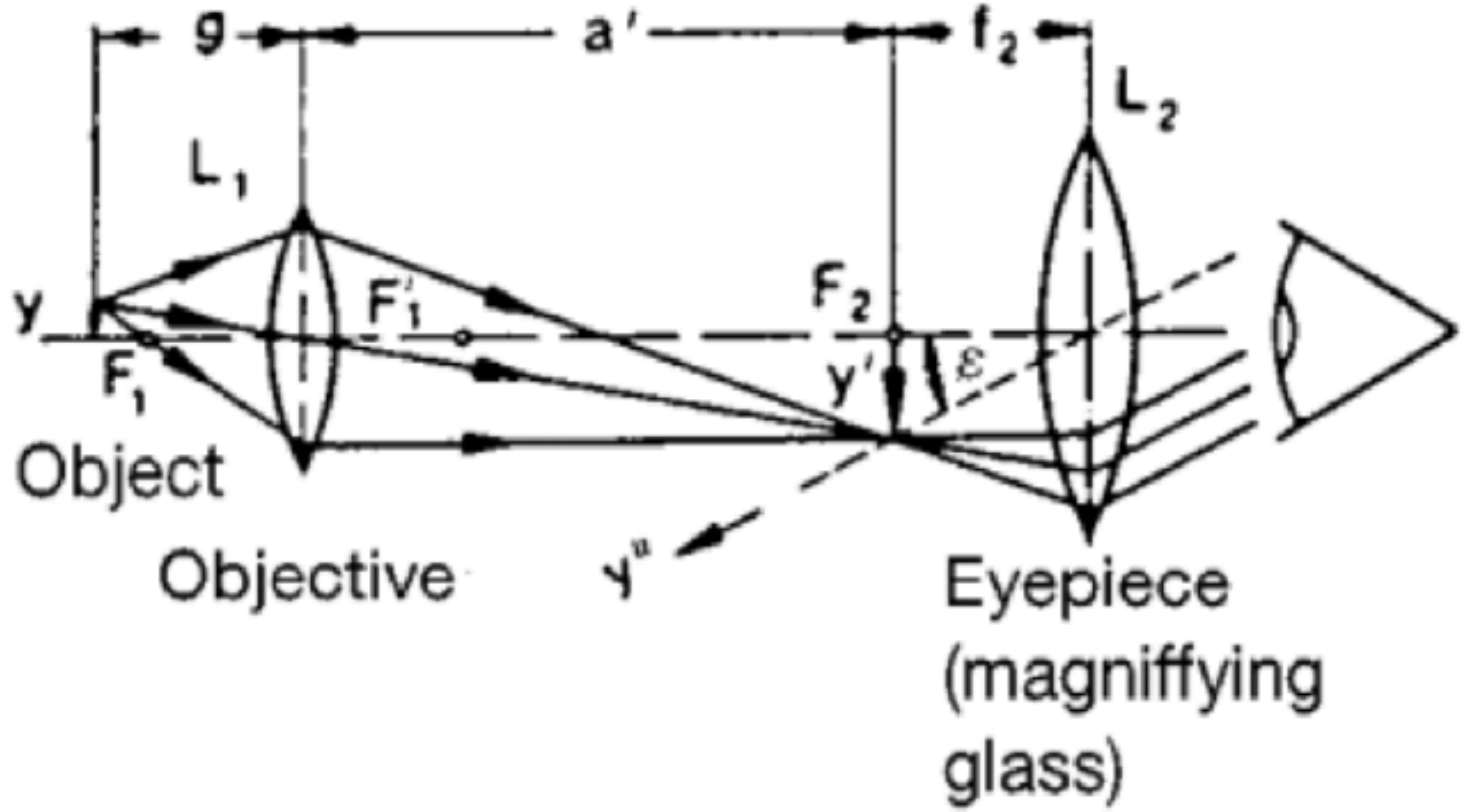
4. Microscope



A light microscope is an optical instrument which produces highly magnified images of very small objects.

Experimental procedure

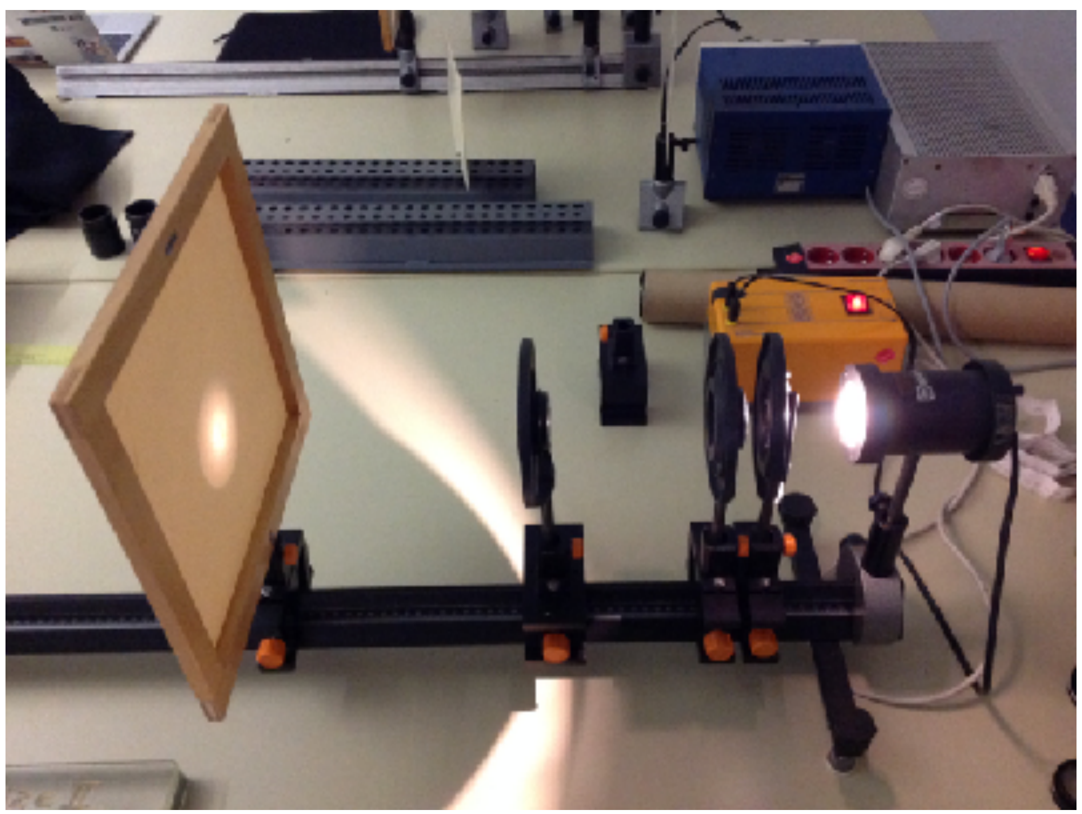
4. Microscope



The objective lens L_1 has a short focal length (e.g. f_1=+20 mm). A real intermediate image is viewed through the eyepiece lens L_2(f_2= +50 mm), which acts as a magnifying glass.

Experimental procedure

4. Microscope



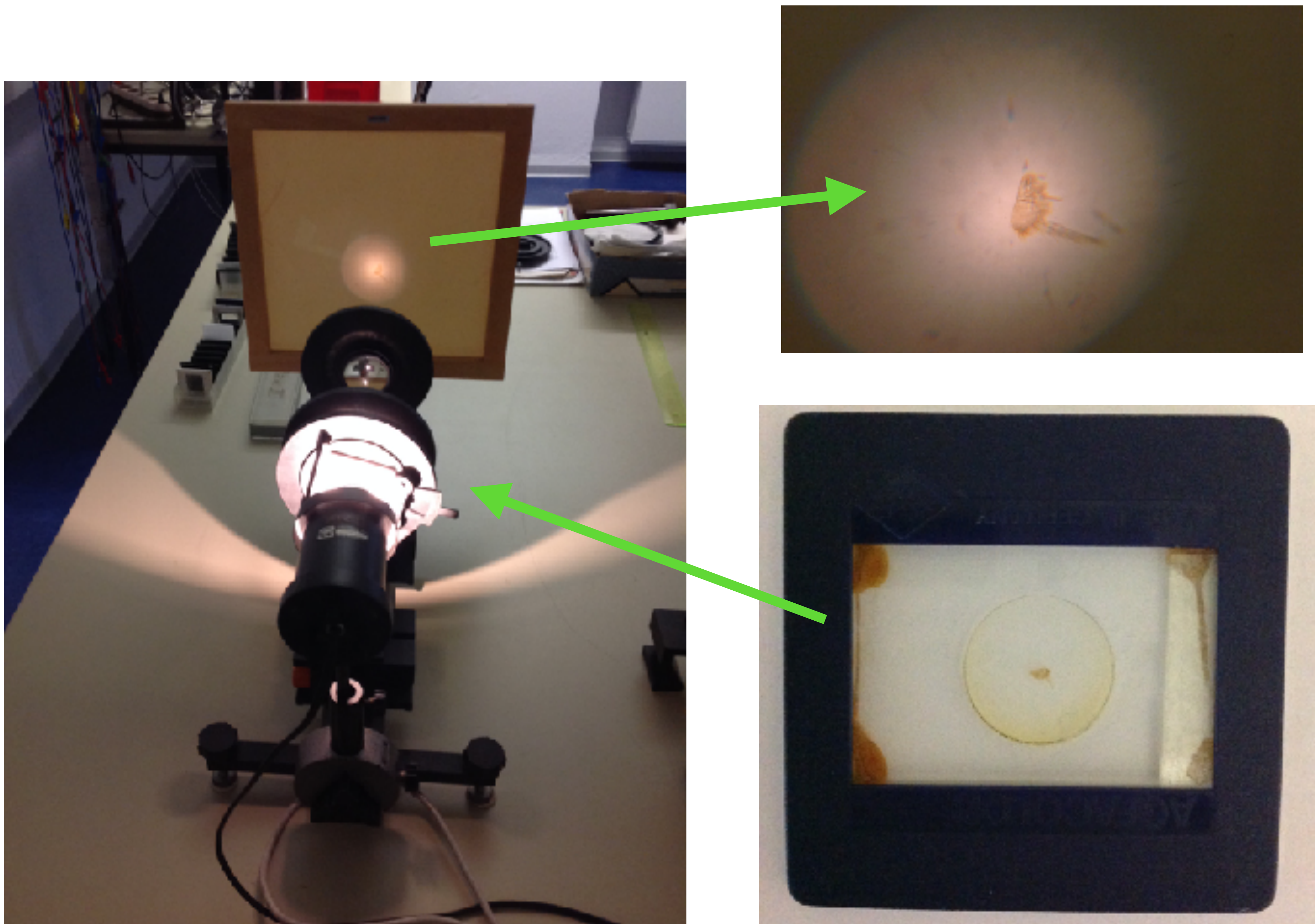
The overall magnification can be calculated from the image height *y" and* the object height (*y*).

$$V = \frac{y''}{y} \quad (7)$$

$$V_{mikr} = \left(\frac{a'}{f_1} - 1\right) . \frac{250\ mm}{f_2} \quad (9)$$

Experimental procedure

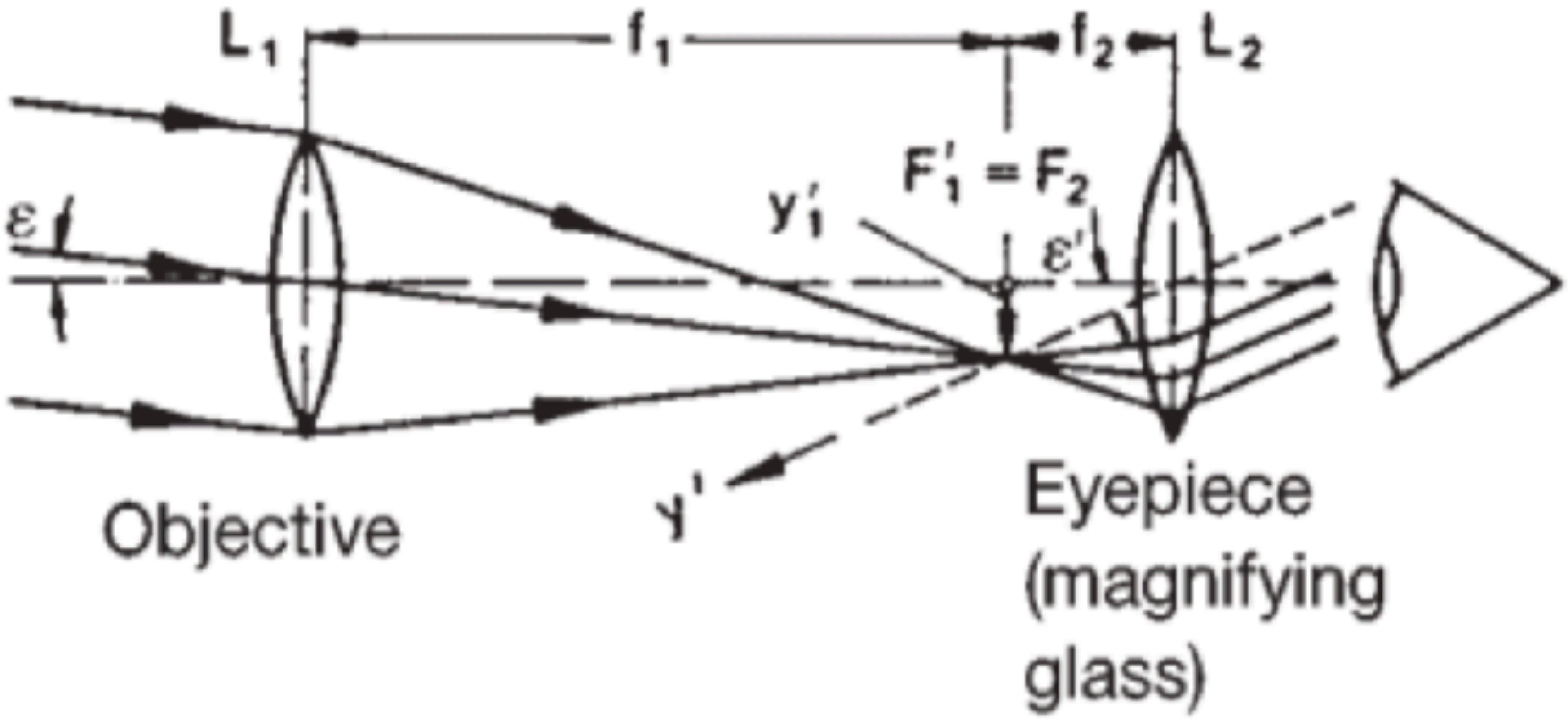
4. Microscope



Magnified image

Experimental procedure

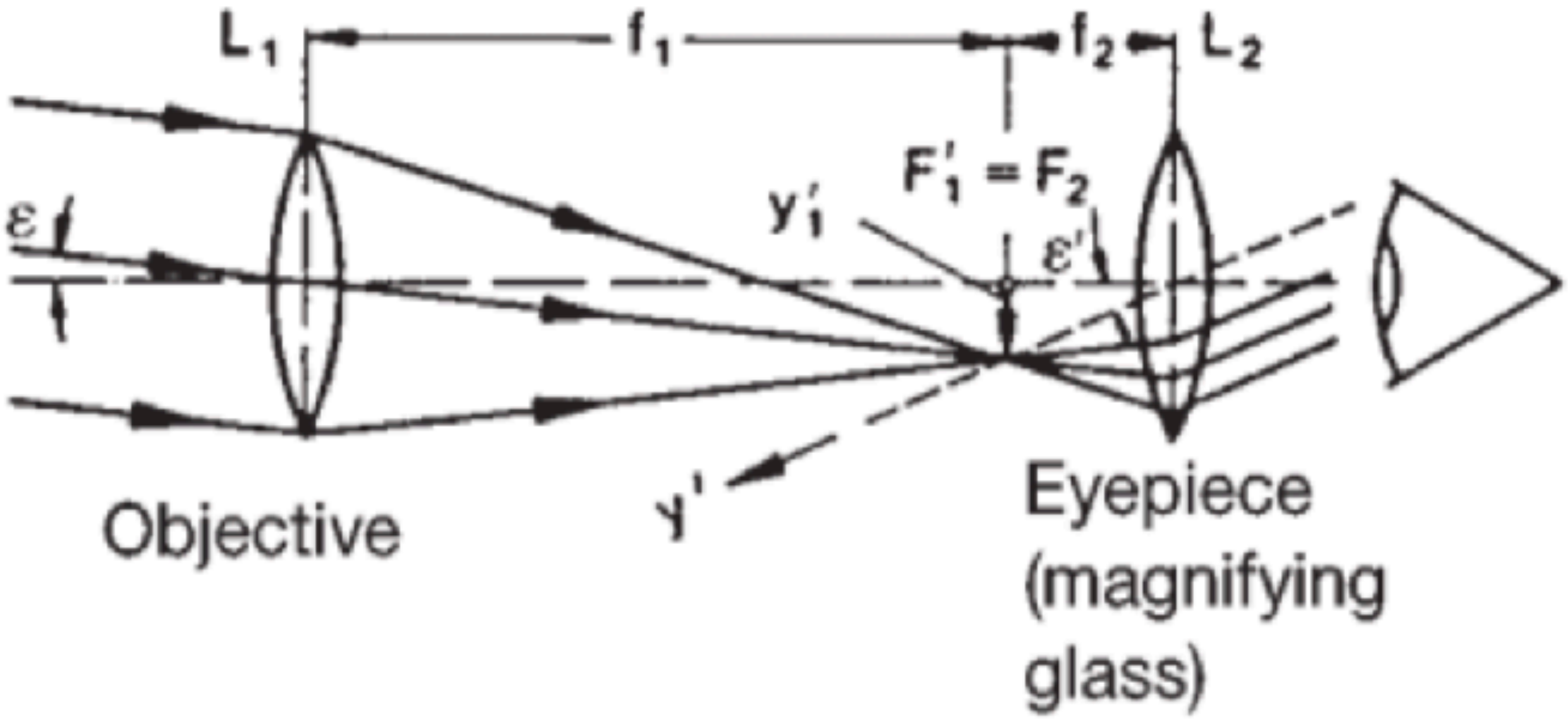
5. Astronomical refracting telescope:



An astronomical refracting telescope consists of two converging lenses the first of which, the **objective**, forms a real inverted image which is examined using the second lens, the **eyepiece**.

Experimental procedure

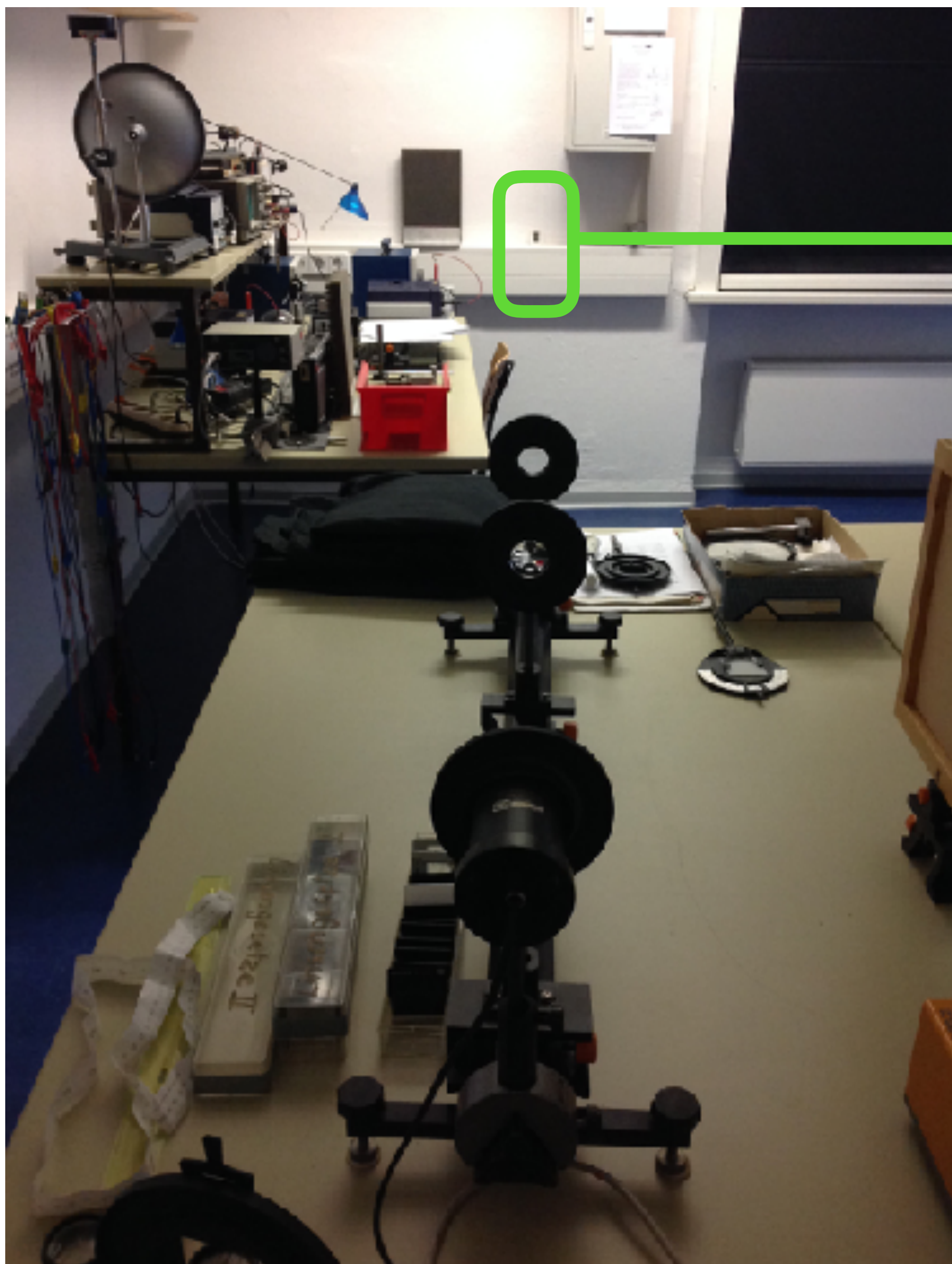
5. Astronomical refracting telescope:



A converging lens with a rather long focal length f_1 (e.g. +300 mm), and another one with a shorter focal length f_2 (e.g. +50 mm) are separated by $f_1 + f_2$.

Experimental procedure

5. Astronomical refracting telescope:



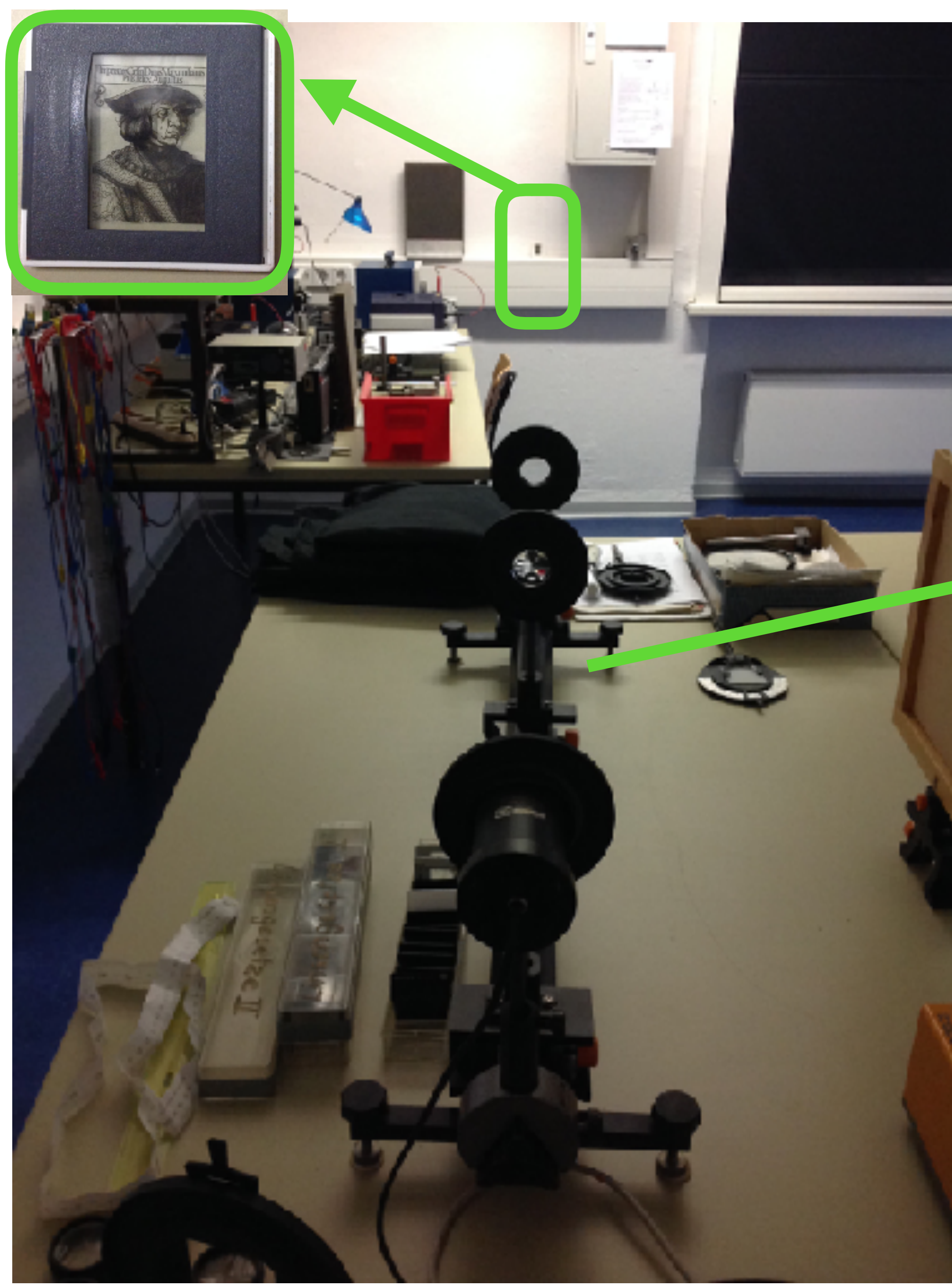
The angular magnification (for small angles) is given by:

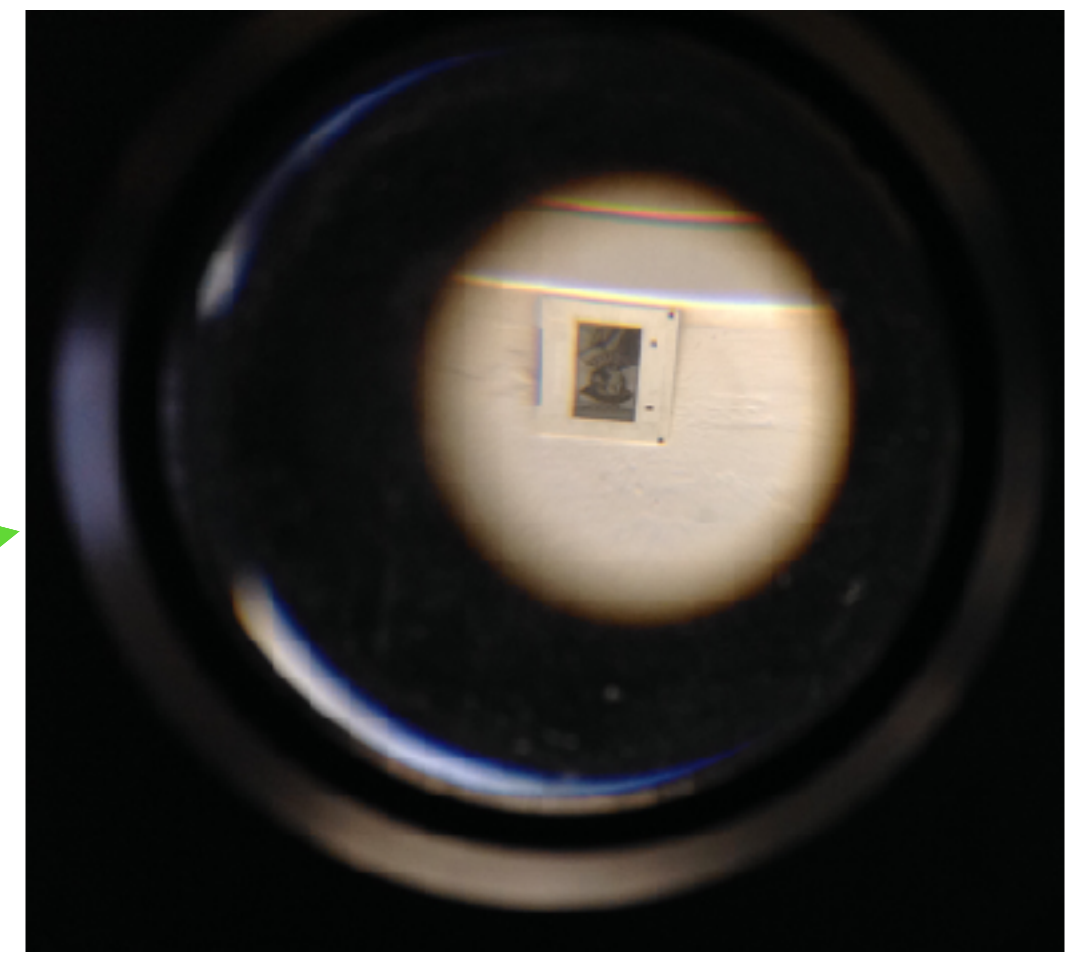
$$\Gamma = \frac{\varepsilon'}{\varepsilon} = \frac{Y_1'/f_2}{Y_1'/f_1} = \frac{f_1}{f_2} \qquad (10)$$

Eq. 10.1 10.2 10.3

Experimental procedure

5. Astronomical refracting telescope:

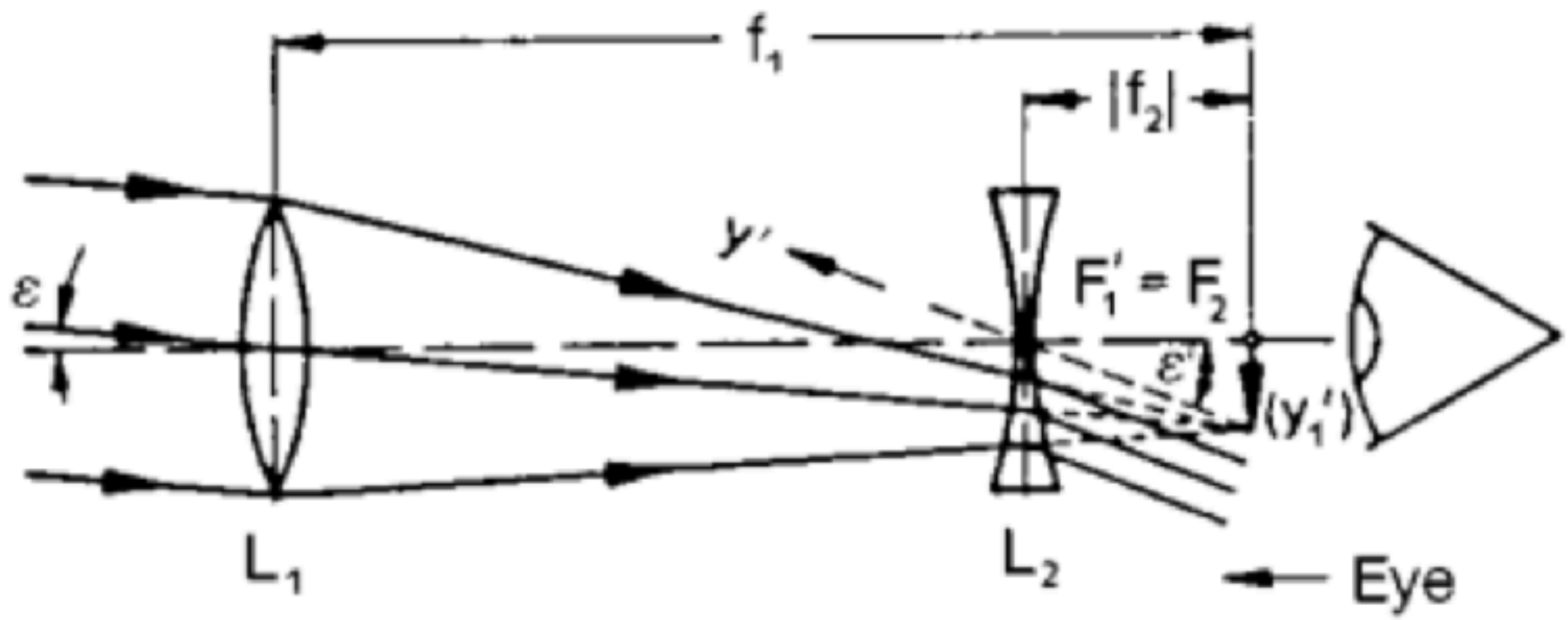




Looking through the lens with shorter focal length, you can see an inverted, magnified image of a distant object.

Experimental procedure

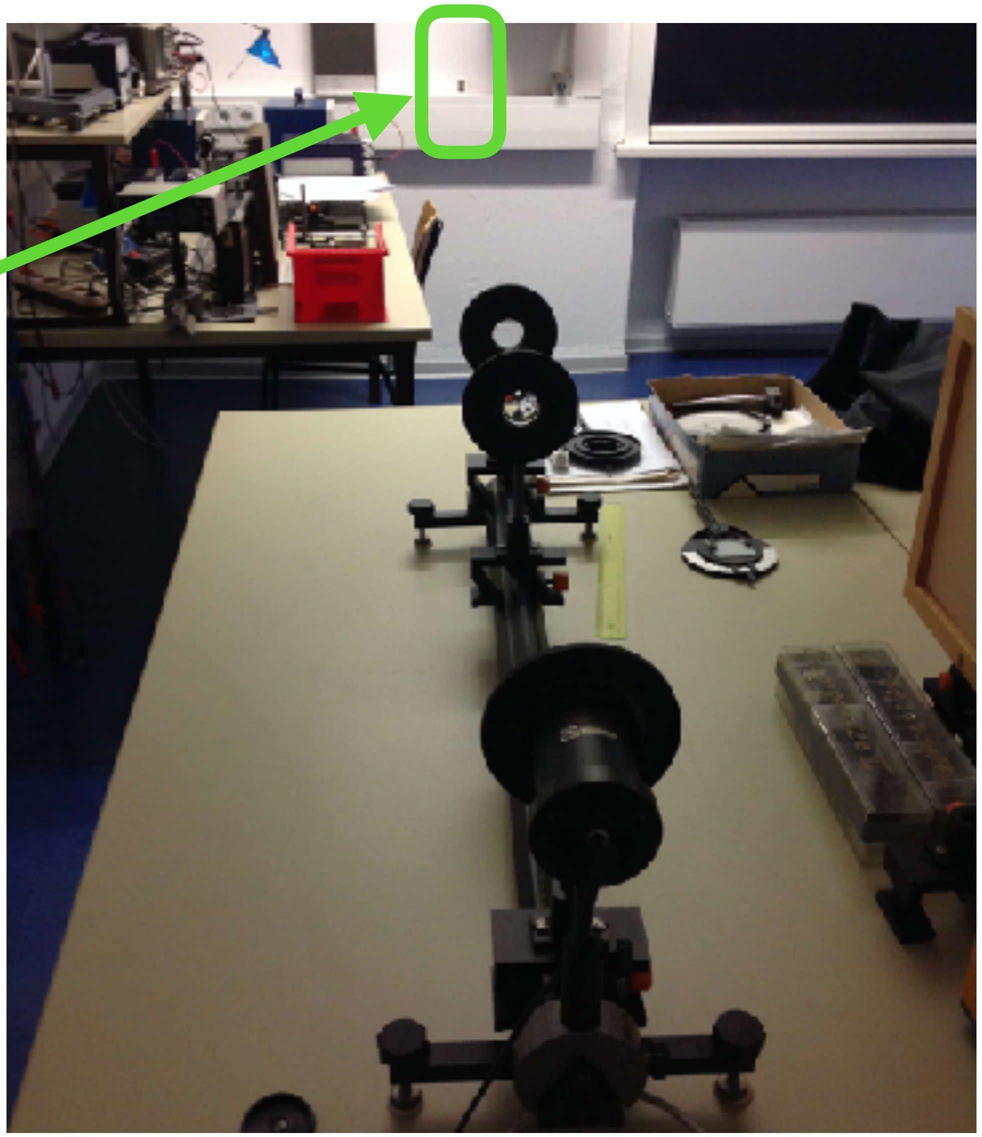
6. Galilean telescope:



Constructed in 1609. The objective is a converging lens, and the eyepiece is a diverging lens placed so that the focal points of the two lenses coincide beyond the eyepiece.

Experimental procedure

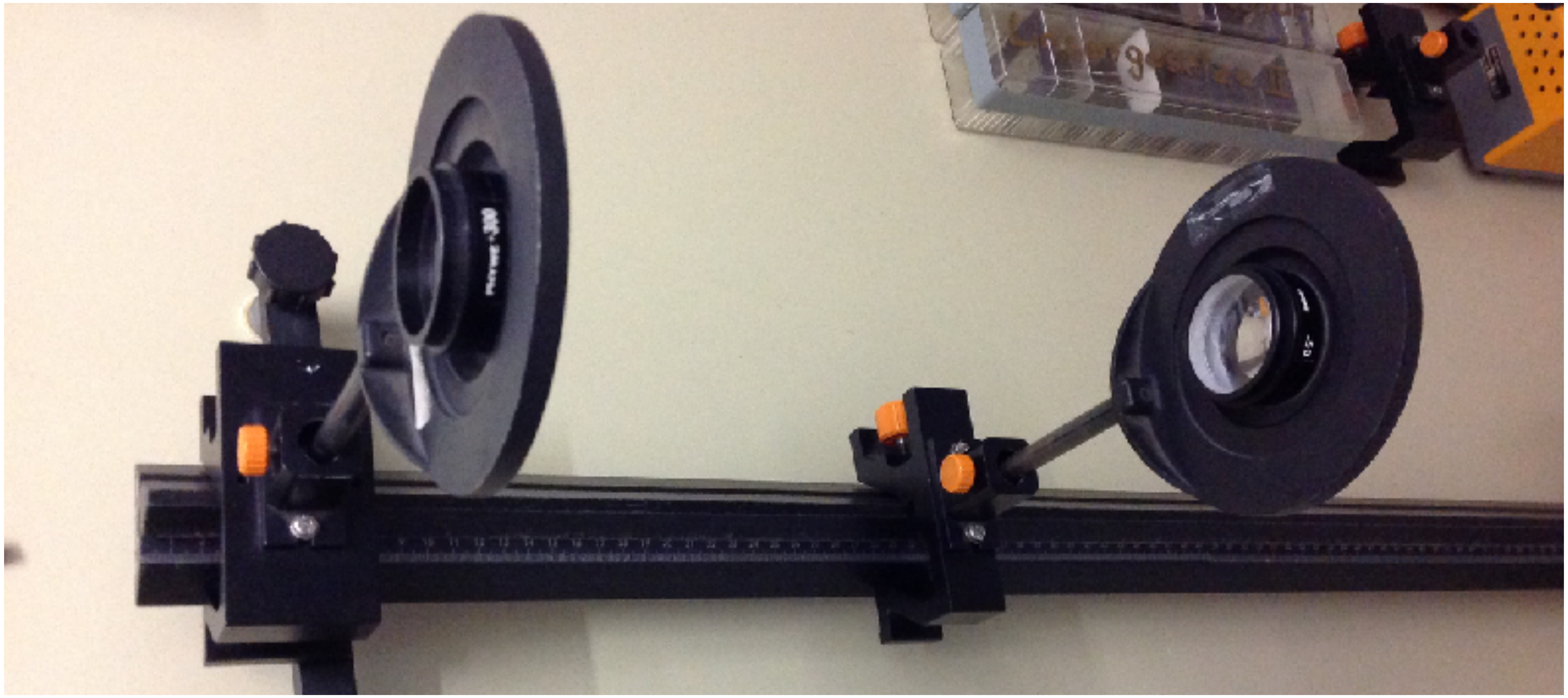
6. Galilean telescope:



An upright image is obtained at infinity and there is no intermediate image. The small overall length and the upright image make this design suitable for opera glasses with a magnification of two to three times.

Experimental procedure

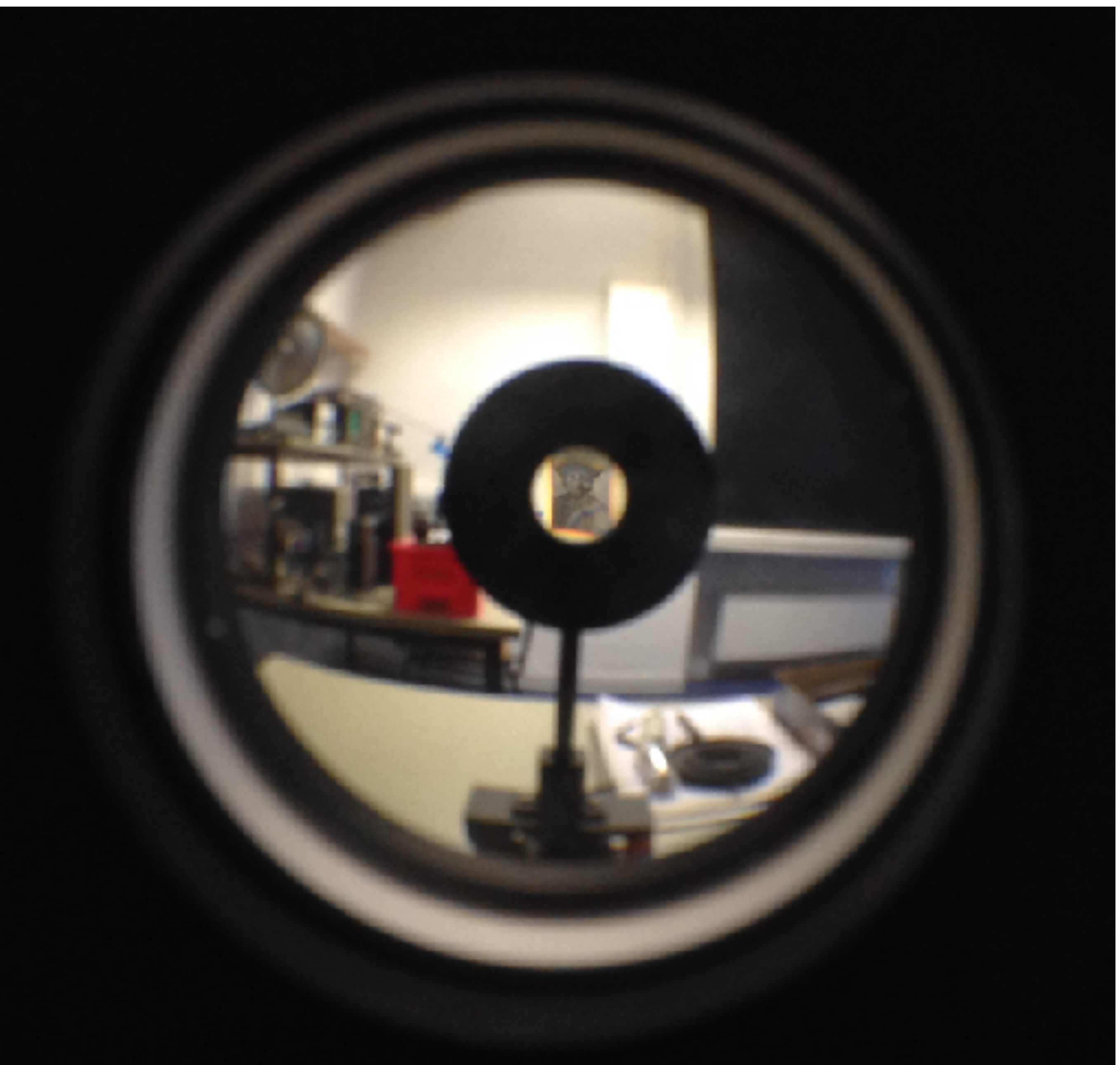
6. Galilean telescope:



A converging lens with a rather long focal length f_1 (e.g. +300 mm) and a diverging lens with a short focal length f_2 (e.g. -50 mm) are set up with a separation given by $f_1 - |f_2|$.

Experimental procedure

6. Galilean telescope:



After focusing you can see an upright magnified image of distant objects through the eyepiece. The magnification of the telescope is again given by:

$$\Gamma = \frac{f_1}{|f_2|} \qquad (11)$$

Protocol 1

Please use this applet:

https://ricktu288.github.io/ray-optics/

1. Follow experiment 1 to determine the focal length of two converging lenses (L_1: +100mm, L_2: +300mm).

2. Repeat the measurements for six different lens and screen positions for both of the lenses.

3. Calculate the focal length ***f*** using equation 1 for each of the measured pairs of values of ***b*** and ***g*** for each lens.

4. Determine the average value of ***f*** and its standard deviation.

5. Compare these values with the theoretical values on the lenses (assume they have a tolerance of ± 5%).

Protocol 2

Please use this applet:

https://ricktu288.github.io/ray-optics/

1. Measure the focal length of a converging lens (L_1: +100mm) using the Bessel method
2. Repeat the measurements for six different values of ***d***.
3. Compare the average value of the focal length $\boldsymbol{f_s}$ with the one obtained for the same lens in the previous experiment.

Protocol 3

Please use this applet:

https://ricktu288.github.io/ray-optics/

1. Measure the focal length of the **lens system** consisting of a converging lens (+100 mm) and a diverging lens (-300 mm) using the Bessel method.

2. For a given distance ***d*** between the object and the screen, locate the two positions of the lens system which produce sharp images. Repeat the measurements for 6 different values of ***d***.

3. Calculate the individual values and the average value of $\boldsymbol{f_{ges}}$. Using the averaged value for $\boldsymbol{f_s}$ obtained in protocol 2 you can calculate the averaged value of $\boldsymbol{f_z}$ for the diverging lens.

4. Calculate the standard deviation of $\boldsymbol{f_z}$ (use error propagation).

Protocol 4

Please use this applet:

https://ricktu288.github.io/ray-optics/

1. Build a slide projector and compare the results from all three calculations (i.e. with equations 6.1, 6.2, and 6.3)

2. Build a microscope using the lenses mentioned in experiment 4, and compare the magnification values obtained with equations 7 and 9.

3. Build an astronomical refracting telescope using the lenses mentioned in experiment 5, and compare the angular magnification calculated with equations 10.1, 10.2, and 10.3.

4. Build a Galilean telescope using the lenses mentioned in experiment 6, and compare the theoretical and experimental magnification values (equation 11).