Homework 1 - Quantum Mechanics I

NAME: ____ **SCORE:** ____________

Deadline: Friday 12 September 2025 by 13:00
Credits: 32 -> 20 points **Number of problems:** 4 **Type:** Formative Evaluation

- This homework includes problems on unit 1 of the QM course programme.
- This assignment should be submitted individually by the deadline.
- If you work with classmates, please state their names and write your own reports.
- Please submit a professional homework report and clearly highlight all answers.
- All calculation steps should be justified to get full credits.
- Any code used should be attached to get full credits.

1. **(8 points) Black body radiation**
This problem consists of determining the temperature of our Sun (which is an almost perfect black body) based on a regression performed on its spectrum. The Solar spectrum that you will analyse was obtained from satellite instruments, so it does not contain atmospheric absorption. Please download the data file from: https://github.com/wbandabarragan/physics-teaching-data/blob/main/1D-data/sun_spectra.csv

This data file contains 6 columns, but we will use only the first three:

1. Wavelength (λ in nm)
2. Extraterrestrial ($J_\lambda \equiv$ spectral radiosity in $\mathrm{W\,m^{-2}\,nm^{-1}}$)
3. Global-tilt ($J_\lambda \equiv$ spectral radiosity in $\mathrm{W\,m^{-2}\,nm^{-1}}$)

As you know from quantum theory, the spectral radiance of a black body is described by Planck's law, so this would be a natural fitting model to be used for the regression. Planck's law reads:

$$B_\lambda = \frac{2hc^2}{\lambda^5}\frac{1}{\exp\left(\frac{hc}{\lambda k_{\mathrm{B}} T}\right) - 1}$$

where B_λ is in units of $\mathrm{W\,m^{-2}\,nm^{-1}\,sr^{-1}}$ in the SI system. To obtain the spectral radiosity, you need to integrate the spectral radiance over the solid angle (Ω, in units of steradian, sr) subtended by the emitting object (which in this case is the Sun), so:

$$J_\lambda = \int_\Omega B_\lambda \, d\Omega$$

We are far away from the Sun, so you should consider the Sun size on the sky to obtain the correct solid angle. For a celestial body, the solid angle (in units of sr) can be computed from of its radius, $R = 696340\,\mathrm{km}$ and the average Earth-Sun distance (i.e. the distance from the observer to the object), $d = 151.35 \times 10^6\,\mathrm{km}$, using:

$$\Omega = 2\pi\left(1 - \frac{\sqrt{d^2 - R^2}}{d}\right)$$

(a) Calculate the solid angle Ω subtended by the Sun in units of sr from the Sun radius, R, and the Earth-Sun distance, d.

(b) Use your favourite programming tool to open the above CSV data file and take the first three columns from it, i.e. the wavelength (λ in nm) and extraterrestrial and global-tilt spectral radiosities (J_λ in $\mathrm{kW\,m^{-2}\,nm^{-1}}$). Then, make a high-quality labeled plot of the extraterrestrial spectral radiosity (in the Y-axis) versus wavelength (in the X-axis). Does the relation between the two look linear? Hints: The conversion to kW will help later on.

(c) Define a physically-motivated fitting function (i.e. J_λ in units of $\mathrm{kW\,m^{-2}\,nm^{-1}}$ as obtained from the Planck law). Which variable should be the free parameter for the regression? Hints: Be very careful in handling the units and do not forget to use the Sun's solid angle computed in (a) to obtain the spectral radiosity from the spectral radiance.

(d) Carry out the regression using your favourite programming tool. Report the best-fit function, what is the temperature of the Sun? Hint: since the fitting function is not a simple polynomial function, providing an initial guess for the regression may help some regression algorithms in Mathematica or Python.

(e) Make a high-quality labeled plot of spectral radiosity (in the Y-axis) versus wavelength (in the X-axis) showing both the experimental data and the best-fit model (obtained from Planck's law for the fitted temperature).

(f) Finally, you will compare the results to predictions based on classical theory. As you know, the black body radiation in classical theory is described by the Rayleigh-Jeans law. Calculate the spectral radiosity based on temperature, but now according to the Rayleigh-Jeans function given below. Hint: As before, be careful with the units and do not forget to use the Sun's solid angle computed in (a) to obtain the spectral radiosity, J_λ, from the spectral radiance:

$$B_\lambda = \frac{2ck_BT}{\lambda^4}$$

(g) Make a high-quality labeled plot of spectral radiance (in the Y-axis) versus wavelength (in the X-axis) showing the experimental data, the best-fit model (i.e. Planck's law for the fitted temperature), and the classical model obtained in (f). Does classical theory correctly describe the black body spectrum of the Sun? Hint: You may wish to limit the Y-axis of the plot to be able to compare the lines.

(h) To finish, you will add the global-tilt spectral radiosity to a new plot with the same lines as those in (e). Comment on the results and investigate the origin of possible discrepancies between the extraterrestrial radiosity, Planck's prediction and the global-tilt radiosity.

2. **(8 points) Double-slit interference**

(a) Electrons of momentum p pass through a pair of slits separated by a distance d. What is the distance, Δy, between adjacent maxima of the interference fringe pattern formed on a screen at a distance D beyond the slits? Assume that the width of the slits is much less than the de Broglie wavelength of the electron.

(b) In an experiment performed by Jönsson in 1961, electrons were accelerated through a 50 kV potential towards two slits separated by a distance $d = 2 \times 10^{-4}$ cm, then detected on a screen $D = 35$ cm beyond the slits. Calculate the electron's de Broglie wavelength, λ, and the fringe spacing, Δy.

(c) What values would d, D, and Δy take if Jönsson's apparatus were simply scaled up for use with visible light rather than electrons?

(d) What values would d, D, and Δy take if Jönsson's apparatus were simply scaled down for use with "buckyballs" (C_{60} molecules) rather than electrons? Please have a look at Arndt et al. 1999 paper and compare your results to the values reported in the paper (https://atomfizika.elte.hu/akos/orak/archive/atfsz/dualitas/fulleren.pdf)

3. **(8 points) Compton and de Broglie wavelengths**

(a) Calculate the Compton wavelengths of a proton, a neutron, and an alpha particle. How much energy (in both J and eV) would photons with those wavelengths have?

(b) Suppose we have an experiment in which monochromatic light is scattered by a proton at an angle of 120°. What is the fractional increase in the wavelength, $\frac{\Delta\lambda}{\lambda_0}$, if the incident light has: 1) a $\lambda_0 = 100\,\mathrm{pm}$, and 2) a $\lambda_0 = 1.0\,\mathrm{fm}$? Why are these wavelengths so different, and for which wavelength is the Compton scattering less apparent?

(c) Calculate the de Broglie wavelength of each of the following particles: 1) an automobile of 1.5 metric tons traveling at $50\,\mathrm{km\,h^{-1}}$, 2) a sphere of mass 12 g moving at $9\,\mathrm{cm\,s^{-1}}$, 3) a smoke particle of diameter $2 \times 10^{-5}\,\mathrm{cm}$ and a density of $0.3\,\mathrm{g\,cm^{-3}}$ moving in air at temperature $T = 290\,\mathrm{K}$, where the particle has the same translational kinetic energy as the thermal average of the air molecules, $K_E = \frac{3}{2}k_B T$, and 4) an ^{87}Rb atom that has been laser cooled to a temperature of $T = 90\,\mu\mathrm{K}$, for which we can also assume that $K_E = \frac{3}{2}k_B T$.

(d) Use your favourite programming language to make a 2D phase plot of the Compton and de Broglie wavelengths of all the above particles mentioned in items (a-c), plus electrons. The particle mass should be in the X axis and the wavelengths on the Y axis. For simplicity, you may assume that all particles move at a non-relativistic speed of $10^4\,\mathrm{m\,s^{-1}}$. Calculate and add the Planck length and mass to the plot, and briefly comment on the results.

4. **(8 points) The photoelectric effect**

As reviewed in class, the photoelectric effect experiment consists of setting up an electric circuit embedding 2 metal plates. We illuminate the first plate with a beam of photons of certain wavelength, λ, and as the photons interact with the metal plate, electrons are ejected from it. The electrons then reach the second plate and an electric current emerges. To prevent these electrons from reaching the second plate, we can increase the voltage until the electric current becomes zero in the circuit. The voltage value at which this happens is called "stopping potential", V_0, which is defined as the potential needed to stop the photoelectrons with the largest kinetic energy (so $K_{\max} = e\,V_0$) at a specific wavelength λ.

(a) Carry out photoelectric effect experiments for 2 different metals: Cesium (Cs) and Niobium (Nb), and collect 8 data points for each, using:
https://applets.kcvs.ca/photoelectricEffect/PhotoElectric.html
To collect the data, choose a metal, fix a wavelength, and vary the voltage until the current becomes zero. When this happens, push "record data points". Then, vary the wavelength and repeat the process. When you have 8 data points for the first metal, choose another metal and repeat the experiment. At the end, you should have a data table with 16 data points, 8 for each metal.

(b) Attach your CSV data file to your homework submission.

Using your favourite programming language:

(c) Open and read the data file containing the experimental results, and make two high-quality labeled scattered plots, one for each metal, with the maximum kinetic energy ($K_{\max}$) in the Y-axis and frequency on the X-axis.

(d) For each metal, define a good model to describe the data. Carry out a regression and find the function that best fits the data.

(e) Report the fitting functions for each metal and make two labeled plots, one for each metal, containing the experimental data and their fits. Make a new figure combining the data and fitting functions for both metals.

(f) Which metal has a higher cutoff frequency? What does the slope of the curves represent?

Using the fitting functions, carry out the following calculations:

(g) Calculate the work function, ϕ, and the cutoff wavelength, λ_{cutoff}, for each metal, and the relative errors with respect to the known values (research what these values are).