Midterm Exam (part 2) - Quantum Mechanics I

NAME: _____ **SCORE:**
Date: Tuesday 14 October 2025 **Deadline:** 21h00
Credits: 10 points (2 problems) **Type of evaluation:** MT

II) Please provide solutions to the following problems:

5. **(5.0 points) Momentum expectation value $\langle \hat{p} \rangle$**

(a) The spatial wave function of a particle is square-integrable and real up to an arbitrary multiplicative phase:

$$\psi(x) = e^{i\alpha}\phi(x),$$

with α real and constant and $\phi(x)$ real. What is the expectation value of the momentum, $\langle \hat{p} \rangle$?

(b) Consider instead the wave function:

$$\psi(x) = \phi_1(x) + e^{i\alpha}\phi_2(x),$$

where $\phi_1(x)$ and $\phi_2(x)$ are each real and square-integrable. What is $\langle \hat{p} \rangle$? Hint: the answer can be expressed as an integral.

(c) In the above case, for what values of α can we be sure that $\langle \hat{p} \rangle$ is zero without having further information about ϕ_1 and ϕ_2?

(d) Consider this time the wave function:

$$\psi(x) = e^{ikx}\phi(x),$$

with k real and constant wavenumber and $\phi(x)$ real. Calculate $\langle \hat{p} \rangle$.

6. **(5.0 points) Schrödinger equation and stationary states**

Consider a particle of mass m in the potential:

$$V(x) = \begin{cases} \infty, & x < 0 \\ -\frac{50\,\hbar^2}{m\,a^2}, & 0 \le x \le a \\ 0, & x > a, \end{cases}$$

(a) Sketch $V(x)$ versus x.

(b) Explain the types of stationary state solutions allowed by this potential.

(c) Write down the time-independent Schrödinger equation and find the bound state solutions.

(d) How many bound states are there?