Midterm Exam (part 3) - Quantum Mechanics I

NAME: _____ **SCORE:**
Date: Tuesday 14 October 2025 **Deadline:** 21h00
Credits: 10 points (2 problems) **Type of evaluation:** MT

III) Please provide solutions to the following problems:

7. **(5.0 points) The Schrödinger equation and potential energy**

A particle of mass m in a one-dimensional potential $V(x)$ has the wave function:

$$\psi(x) = Nx \exp\left(-\frac{1}{2}\alpha x^2\right), \quad \alpha > 0.$$

(a) Normalize $\psi(x)$ to determine N. What is $\langle \hat{x} \rangle$? What is $\langle \hat{x}^2 \rangle$?

(b) What is $\langle \hat{p} \rangle$? What is $\langle \hat{p}^2 \rangle$?

(c) Is $\psi(x)$ a position eigenstate? Is $\psi(x)$ a momentum eigenstate? Explain.

(d) Suppose that $V(x) = 0$. What is $\langle \hat{H} \rangle$?

(e) Suppose that nothing is known about $V(x)$, but $\psi(x)$ is an energy eigenstate. Find the potential $V(x)$ and the energy eigenvalue E, assuming $V(0) = 0$. Could $\psi(x)$ be the ground state wave function for the particle?

8. **(5.0 points) Square potential barrier**

Consider a current of particles with energies $E > V_0$ moving from $x = -\infty$ to the right, under the influence of this potential:

$$V(x) = \begin{cases} 0, & x < 0 \\ V_0, & 0 \leq x \leq L \\ 0, & x > L, \end{cases}$$

(a) Use your favourite programming tool to sketch the potential and write down the time-independent Schrödinger equation.

(b) Find the stationary state solutions for each region of interest.

(c) Express the transmitted and reflected amplitudes in terms of the incident amplitude.

(d) Use your favourite programming tool to sketch the solutions for the stationary states found in part (b), which describe particles arriving from $x = -\infty$ with energy $E > V_0$.

(e) Using the result from part (c), compute the transmission and reflection coefficients for particles with $E > V_0$. Sketch these coefficients versus the barrier width, L, and briefly discuss the results.