Final Exam - Quantum Mechanics for Nanosciences

NAME: ____ SCORE:

Deadline: Tuesday 30 January 2023 (by the end of the day)
Credits: 56 points, Number of questions: 13

Part A. Choose the correct answer to each question or statement given below, and briefly justify your choice in the space assigned to each of them. Unjustified answers do not count.

1. **(2 points) Compton scattering**
   How much energy is transferred to an electron (e-, initially at rest) by a photon in a Compton scattering experiment? The wavelength of the photon before the collision is $\lambda = 0.01\,\mathrm{nm}$, and the scattering angle is $\theta = 60^\circ$ ($h = 6.626 \times 10^{-34}\,\mathrm{J\,s}$, $c = 3 \times 10^8\,\mathrm{m\,s^{-1}}$).

   A. $0\,\mathrm{J}$, no energy is transferred to the e-.
   B. $2.15 \times 10^{-15}\,\mathrm{J}$
   C. $1.77 \times 10^{-14}\,\mathrm{J}$
   D. $1.99 \times 10^{-14}\,\mathrm{J}$

2. **(2 points) Photoelectric effect**
   What is the maximum kinetic energy of an electron (e-) removed from a cooper (Cu) metal surface illuminated by light of $\lambda = 600\mathrm{nm}$? The work function, Φ, of Cu is $4.7\,\mathrm{eV}$ ($1\mathrm{eV} = 1.6 \times 10^{-19}\,\mathrm{J}$).

   A. The e- cannot be ejected by light of this λ
   B. $-4.22 \times 10^{-19}\,\mathrm{J}$
   C. $7.53 \times 10^{-19}\,\mathrm{J}$
   D. $3.32 \times 10^{-19}\,\mathrm{J}$

3. **(2 points) de Broglie wavelength**
   A proton has four times the momentum of an electron. If the electron has a de Broglie wavelength λ_e, what is the de Broglie wavelength of the proton?

   A. λ_e
   B. $\frac{\lambda_e}{16}$
   C. $4\lambda_e$
   D. $\frac{\lambda_e}{4}$

4. **(2 points) Infinite square well potential**
   A particle is in an infinite square well potential with walls at $x = 0$ and $x = L$. If the particle is in the state $\psi(x) = A \sin\left(\frac{3\pi x}{L}\right)$, where A is a constant, what is the probability that the particle is between $x = \frac{1}{3}L$ and $x = \frac{2}{3}L$?

   A. $\frac{1}{3}$
   B. 1
   C. $\frac{1}{\sqrt{3}}$
   D. $\frac{2}{3}$

5. **(2 points) The hydrogen atom**
The energy required to knock out the electron in the third orbit of a hydrogen atom is equal to:

A. $+13.6\,\mathrm{eV}$
B. $+\frac{13.6}{9}\,\mathrm{eV}$
C. $-13.6\,\mathrm{eV}$
D. $+\frac{13.6}{3}\,\mathrm{eV}$

6. **(2 points) Spectrum of hydrogen**
Which of the following statements is true?

A. The Lyman series is a continuous spectrum.
B. The Paschen series is a discrete spectrum in the infrared.
C. The Balmer series is a discrete spectrum in the ultraviolet.
D. The Lyman series is a discrete spectrum in the optical.

7. **(2 points) Spectrum of hydrogen**
An electron jumps from the 4th orbit to the 2nd orbit of the hydrogen atom. The frequency in Hz of the emitted radiation will be: (Recall that: $\mathcal{R} = 10^7\,\mathrm{m}^{-1}$, $c = 3 \times 10^8\,\mathrm{m\,s^{-1}}$)

A. $\frac{3}{16} \times 10^5$
B. $\frac{3}{16} \times 10^{15}$
C. $\frac{9}{16} \times 10^{15}$
D. $\frac{3}{4} \times 10^{15}$

8. **(2 points) Normalisation and Spherical Harmonics**
A quantum system is known to be in the (unnormalised) state described by the wave function $\psi(\theta,\phi) = 5Y_4^3 + Y_6^3 - 2Y_6^0$, where the $Y_\ell^m(\theta,\phi)$ are the spherical harmonics. The probability of finding the system in a state with azimuthal orbital quantum number $m = 3$ is:

A. $\frac{5}{\sqrt{30}}$
B. $\frac{5}{6}$
C. $\frac{6}{\sqrt{30}}$
D. $\frac{13}{15}$

Part B. Provide <u>concise answers</u> to the following items:

9. **(5 points) Bound states and scattering states**
   (a) Write down 2 differences between bound states and scattering states.

   (b) Provide one example of a potential that allows: i) only bound states, ii) only scattering states, and iii) both bound and scattering states.

10. **(6 points) Solutions for the hydrogen atom**
    Briefly describe (a) the terms of the potential of the hydrogen atom,

    (b) how we solved the Schrödinger equation for the hydrogen atom, and

    (c) what solutions we found.

Part C. Solve the following problems:

11. **(10 points) Square potential barrier: transmission and reflection coefficients**
Let us consider a time-independent square potential barrier, $V(x)$, given by the following piecewise function:

$$V(x) = \begin{cases} 0, & x < 0 \\ V_0, & 0 \leq x \leq L \\ 0, & x > L, \end{cases}$$

(a) Use python to plot $V(x)$, labelling the three regions of interest as I, II, and III.

(b) Find the stationary states that describe particles arriving from $x = -\infty$ with energy $E > V_0$, and sketch the solutions using python

(c) Analyse the boundary conditions to compute the transmission and reflection coefficients. Sketch these coefficients versus the barrier width, L, and briefly discuss the results.

(d) Find the stationary states that describe particles arriving from $x = -\infty$ with energy $E < V_0$, and sketch the solutions using python.

(e) Compute the transmission coefficient, and discuss how this quantum result differs with respect to classical expectations.

12. **(9 points) Spherical harmonics**

(a) Construct all the possible spherical harmonics, $Y_\ell^m(\theta, \phi)$, for $\ell = 3$.

(b) Choose two of them and check that they are normalised and orthogonal.

(c) Using python, make plots of all of them.

13. **(10 points) Hydrogen atom**

(a) Construct all the possible spatial wave functions, $\psi_{nlm}(r, \theta, \phi) = R_{n\ell}(r)Y_\ell^m(\theta, \phi)$, of the hydrogen atom for $(n, \ell, m) = (2, 1, m)$ and $(n, \ell, m) = (4, 2, m)$.

(b) Using python, make density plots of all of these states.

(c) Calculate the energy levels of these states in units of eV and plot the energy ladder.

(d) In terms of the Bohr radius, find $\langle r \rangle$, $\langle x \rangle$, $\langle r^2 \rangle$ and $\langle x^2 \rangle$ for an electron in the ground state of hydrogen.

(e) Find $\langle x^2 \rangle$ in the states $(n, \ell, m) = (2, 1, m)$ and $(n, \ell, m) = (4, 2, m)$ with the highest possible value of m allowed in each of them.