EM Algorithm

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Introduction

This is the second part of Homework 3. Implement the EM algorithm. Problem (a) is explained in the writing exercise 14.2.

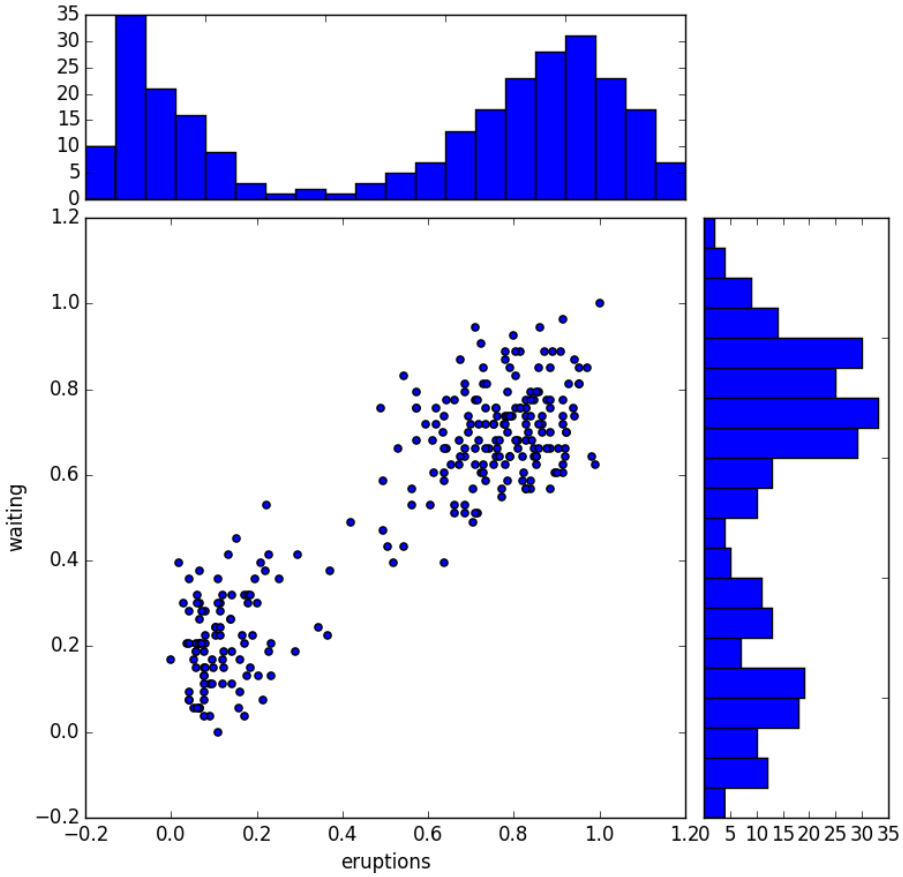
Implementation

Read Data

Read data from faithful.dat and normalize the data into 0 to 1. Plot the data points.

```
def read_data():
    data = []
    with open('faithful.dat') as filein:
        for line in filein:
            currline = line.strip('\n')
            data.append(currline)
    data = data[-272:]
    temp = []
    for e in data:
        record = e.split()[1:]
        record[0] = float(record[0])
        record[1] = float(record[1])
        temp.append(record)
    data = np.array(temp)
    min_max_scaler = preprocessing.MinMaxScaler()
    data[:,0] = min_max_scaler.fit_transform(data[:,0])
    data[:,1] = min_max_scaler.fit_transform(data[:,1])
    # print data
    return data

def plot_fig(data):
    nullfmt = NullFormatter()
    x, y = data[:,0],data[:,1]
    left, width = 0.1, 0.65
    bottom, height = 0.1, 0.65
    bottom_h = left_h = left + width + 0.02

    rect_scatter = [left, bottom, width, height]
```

```
rect_histx = [left, bottom_h, width, 0.2]
rect_histy = [left_h, bottom, 0.2, height]

plt.figure(1, figsize=(8, 8))

axScatter = plt.axes(rect_scatter)
plt.xlabel("eruptions")
plt.ylabel("waiting")
axHistx = plt.axes(rect_histx)
axHisty = plt.axes(rect_histy)
axHistx.xaxis.set_major_formatter(nullfmt)
axHisty.yaxis.set_major_formatter(nullfmt)

axScatter.scatter(x, y)

# now determine nice limits by hand:
axHistx.hist(x, bins=20)
axHisty.hist(y, bins=20, orientation='horizontal')

plt.savefig('Scatter_Plot.png')
plt.show()
```



EM algorithm

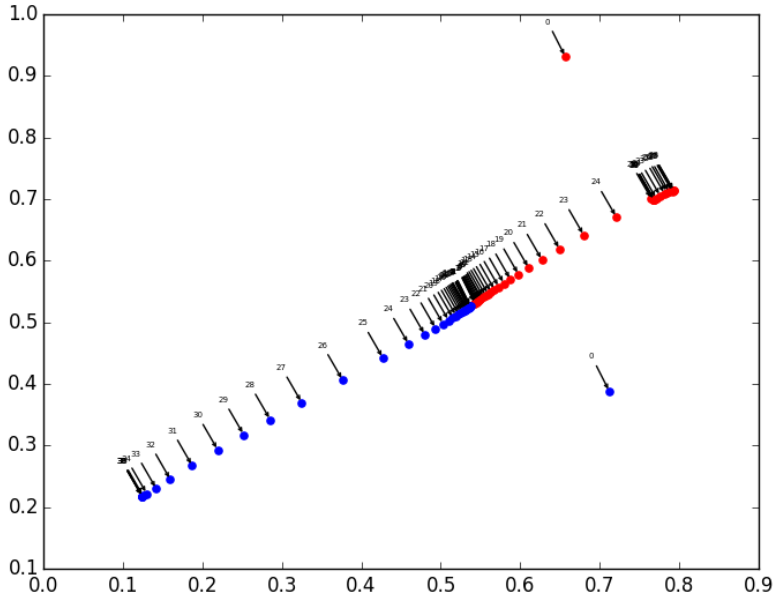
Implement EM algorithm with randomly initialized Gaussian parameters.

```
def random_init(x):
    random.seed(None)
    n,p = x.shape

    # random initialize pis
    pi = random.random()
    pis = [pi,1-pi]
    # print pis

    # random initialize mus
    mus = np.zeros((2,p))
    for i in xrange(p):
        for j in xrange(2):
            mus[j,i] = random.random()
    # print mus

    # random initialize sigmas
```

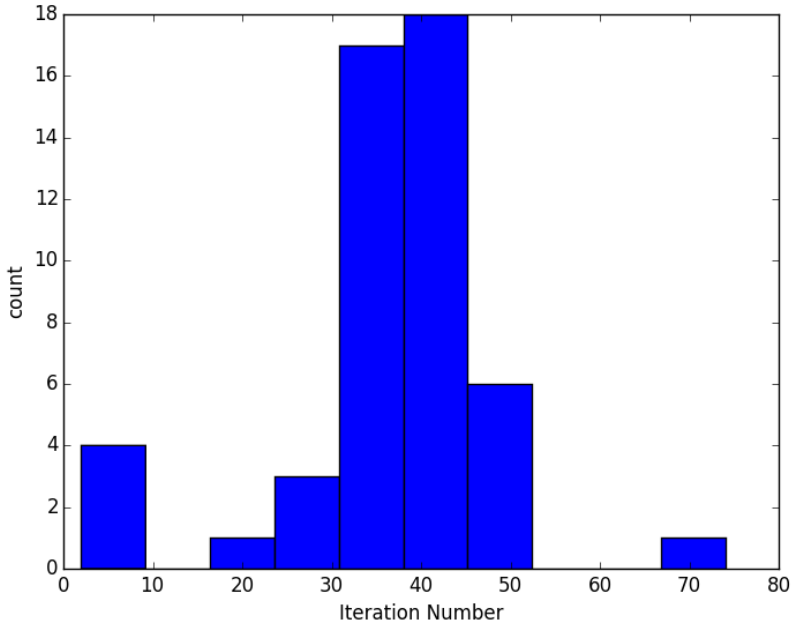
```
    sigmas = np.zeros((2,p,p))
    for j in xrange(2):
        for i in xrange(p):
            sigmas[j,i,i] = 0.5+random.random()*10
    # print sigmas
    return pis,mus,sigmas

# pis = [pi,1-pi] mus = [mu1, mu2] sigmas = [sigma1, sigma2]
# tolerance = 0.0001 default, max iteration = 100 default
def EM_algo(x,pis,mus,sigmas,tol=0.0001,maxiter=100):
    # n denotes oberservation count, p denotes oberservation dimension
    n, p = x.shape
    # k denotes normal distrubutions count
    k = len(pis)
    # ll_old denotes old log likelyhood
    ll_old = 0
    iter_count = 0
    mu_vector = []
    mu_vector.append(mus)
    for i in xrange(maxiter):
        iter_count += 1
        # E-step
        ts = np.zeros((k,n))
        for j in xrange(k):
            for i in xrange(n):
                ts[j,i] = pis[j] * mvn(mus[j],sigmas[j]).pdf(x[i])
        ts /= ts.sum(0)

        # M-step
        # update pi
        pis = np.zeros(k)
        for j in xrange(k):
            for i in xrange(n):
                pis[j] += ts[j,i]
        pis /= n

        # update mu
        mus = np.zeros((k, p))
        for j in xrange(k):
            for i in xrange(n):
                mus[j] += ts[j, i] * x[i]
            mus[j] /= ts[j, :].sum()
        mu_vector.append(mus)
        # update sigma
        sigmas = np.zeros((k,p,p))
        for j in xrange(k):
            for i in xrange(n):
                ys = np.reshape(x[i]- mus[j], (2,1))
                sigmas[j] += ts[j, i] * np.dot(ys, ys.T)
            sigmas[j] /= ts[j,:].sum()

        ll_new = 0.0
        for i in xrange(n):
            s = 0
```

```
            for j in xrange(k):
                s += pis[j] * mvn(mus[j], sigmas[j]).pdf(x[i])
            ll_new += np.log(s)

        if np.abs(ll_new - ll_old) < tol:
            break
        ll_old = ll_new
    mu_vector = np.array(mu_vector)
    # print iter_count, mus
    return iter_count,mu_vector,pis,mus,sigmas
```

Plot the mean vectors trajectories.



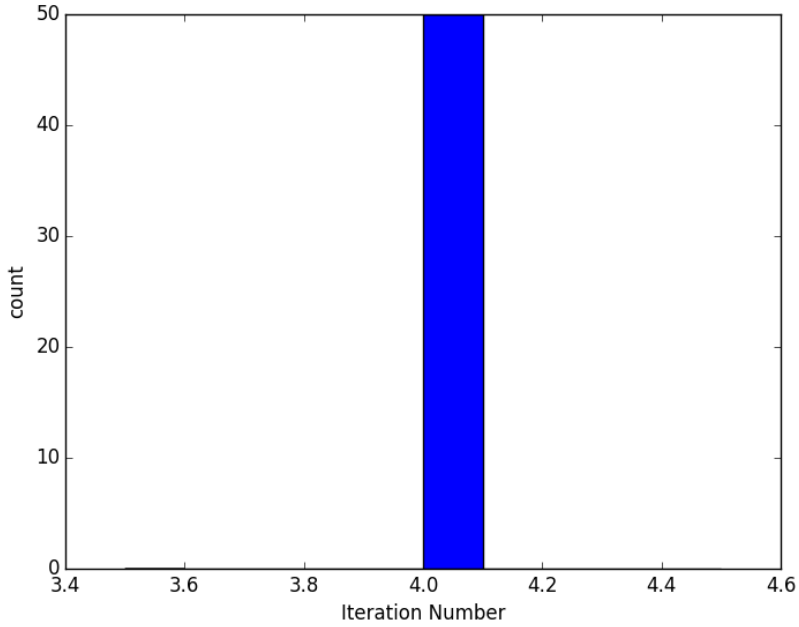
Plot the iterations histogram.



KMeans Initial Guess

```
def kmeans_estimate(x):
    labels = k_means(x,2)[1]
    # print labels
    cluster = [[],[]]
    for i in xrange(len(labels)):
        cluster[labels[i]].append(x[i])
    cluster[0] = np.array(cluster[0])
    cluster[1] = np.array(cluster[1])
    pi = len(cluster[0])*1.0/len(labels)
    pis = [pi,1-pi]
    means = []
    sigmas = []
    for i in xrange(2):
        curr = cluster[i]
        mean = np.average(curr,axis=0)
        means.append(mean)

        sigma = np.zeros((2,2))
        for i in xrange(len(curr)):
            y = np.reshape(curr[i]-mean, (2,1))
            sigma += np.dot(y, y.T)
        sigma /= len(curr)
        sigmas.append(sigma)

    pis = np.array(pis)
    means = np.array(means)
    sigmas = np.array(sigmas)
```

```
print means
return pis,means,sigmas
```



Compare with the random initialized guess, using KMeans can take very few steps for EM algorithm to converge. From the 2 histograms, random guess sometimes take 50 more iterations to converge but KMeans guess only take 4 iterations to converge.