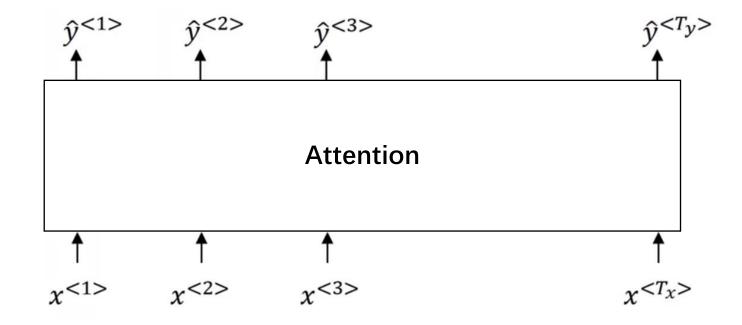
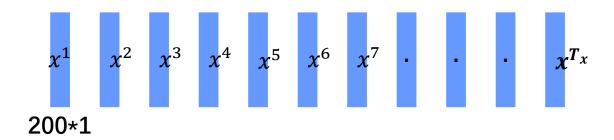
# Attention、Transformer公式推导与矩阵变换

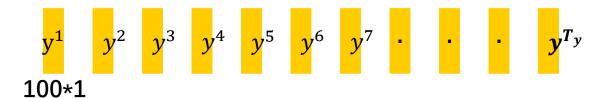
#### **RNN**

$$Q = W_q X$$
 $K = W_k X$ 
 $V = W_v X$ 
 $A = K^T Q$ 
 $A' = softmax(A)$ 
 $Y = VA'$ 

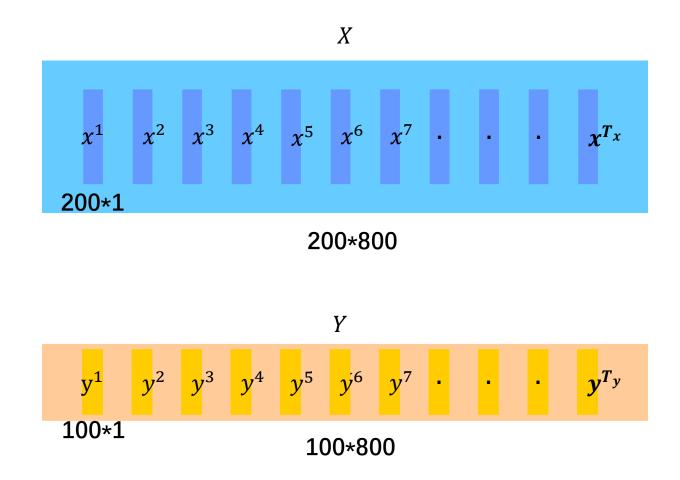


# Attention 向量矩阵化





# Attention 向量矩阵化



$$Q = W_q X$$
 $K = W_k X$ 
 $V = W_v X$ 
 $A = K^T Q$ 
 $A' = softmax(A)$ 
 $Y = VA'$ 

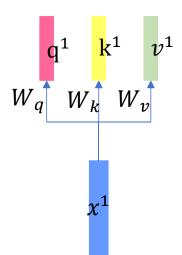


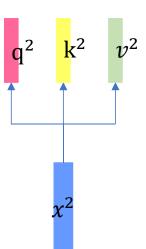
 $\chi^2$ 

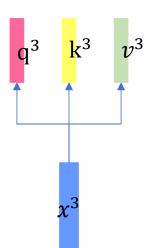
 $\chi^3$ 

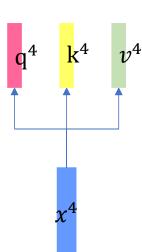
 $\chi^4$ 

$$q^i = W_q x^i$$
  
 $k^i = W_k x^i$   
 $v^i = W_v x^i$ 

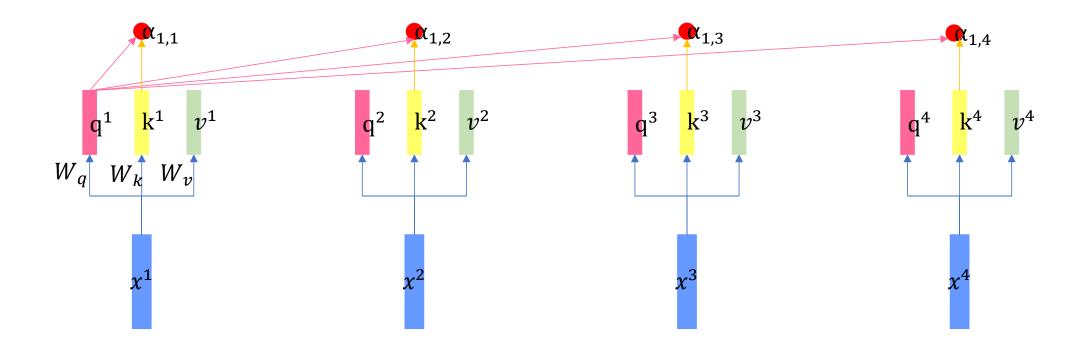


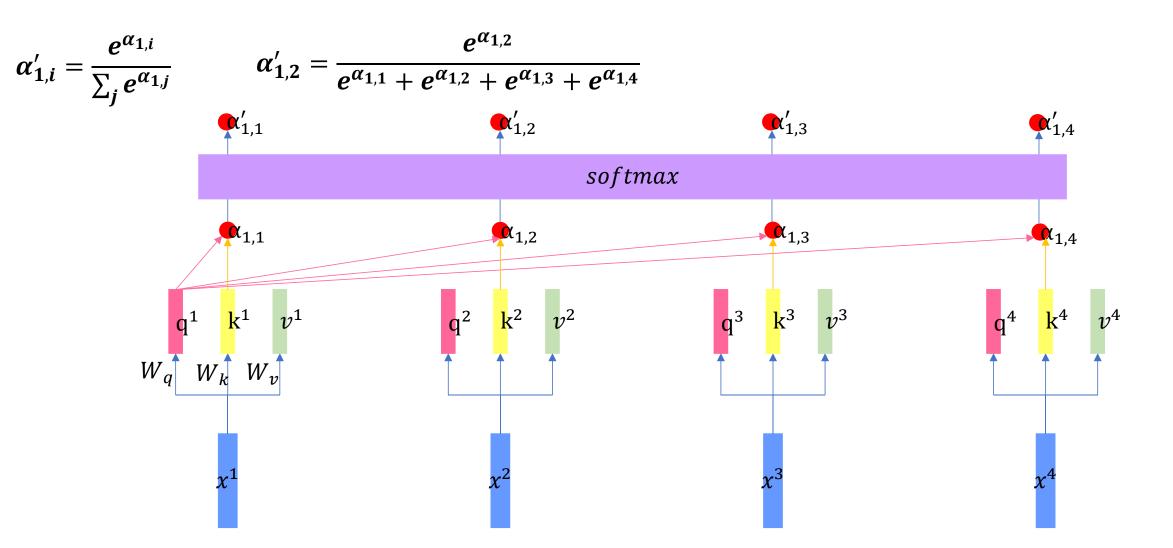


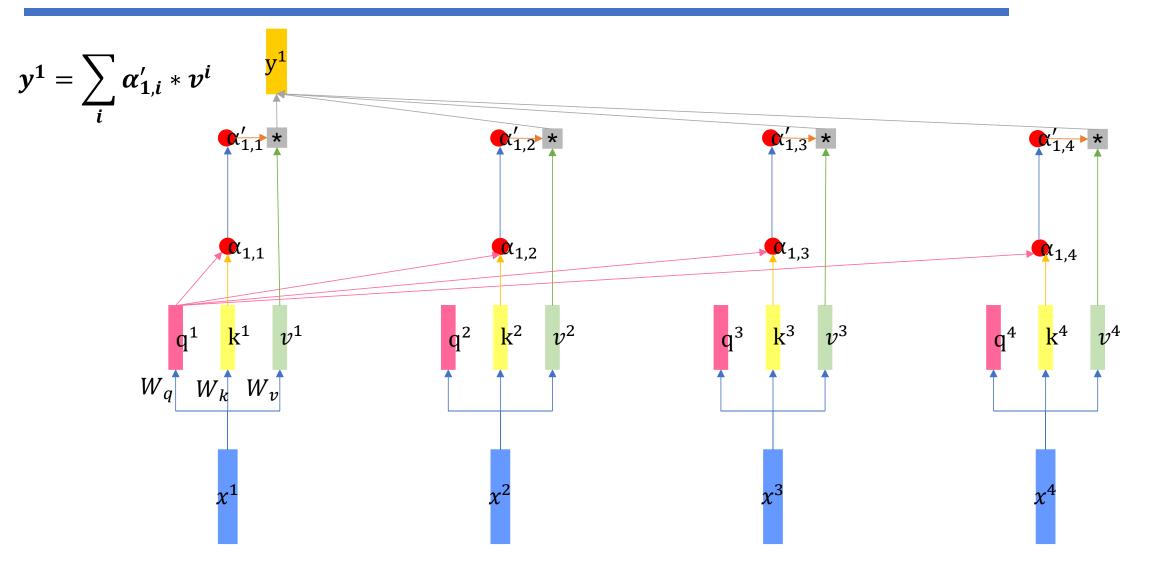




$$\alpha_{i,j}=q^i\cdot k^j$$







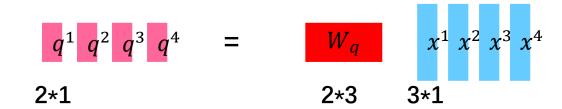


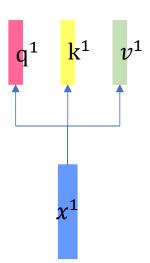


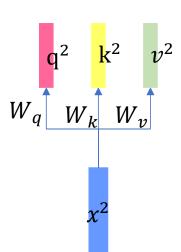


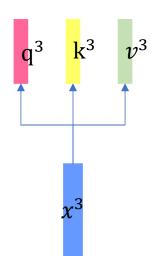


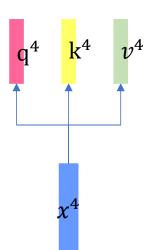
$$q^i = W_q x^i$$
  
 $k^i = W_k x^i$   
 $v^i = W_v x^i$ 

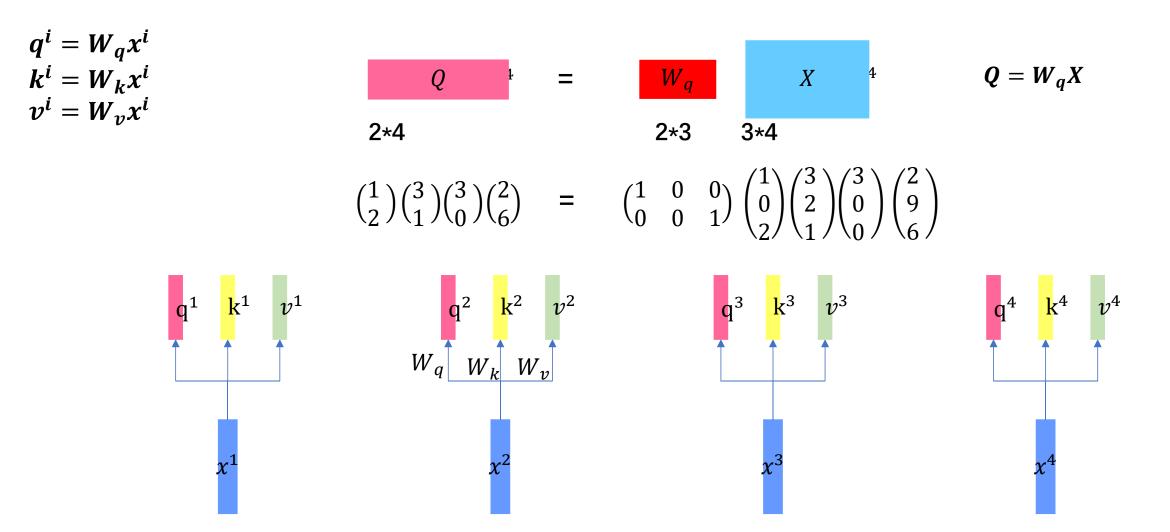


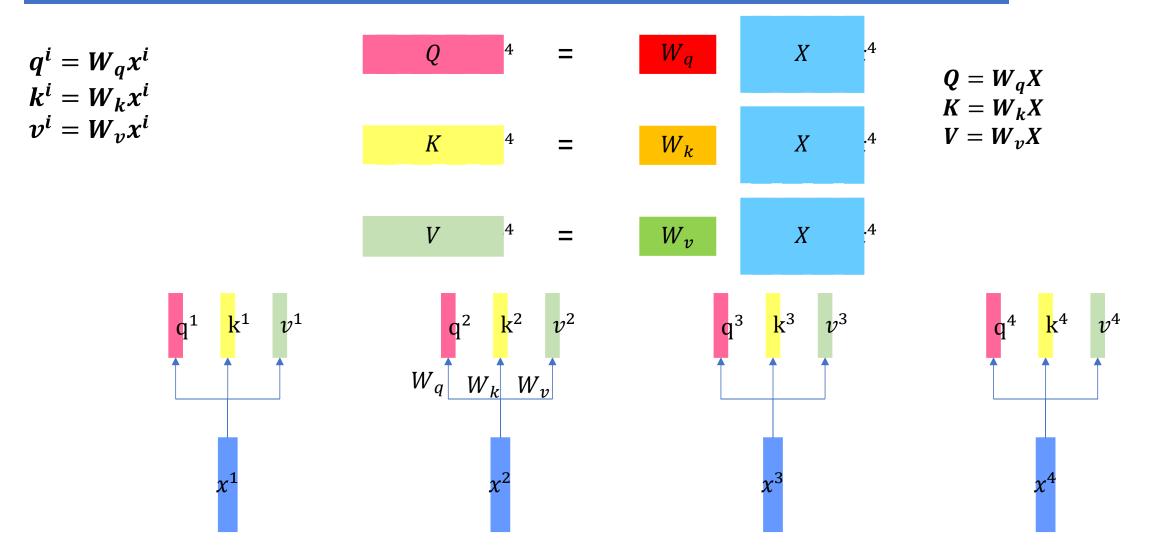


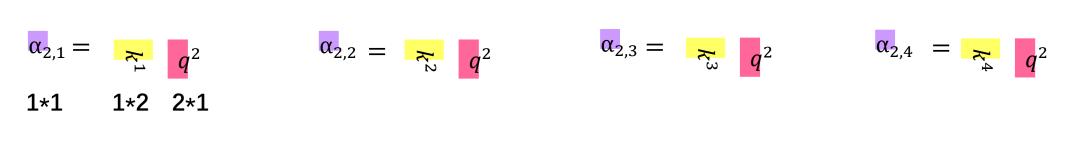


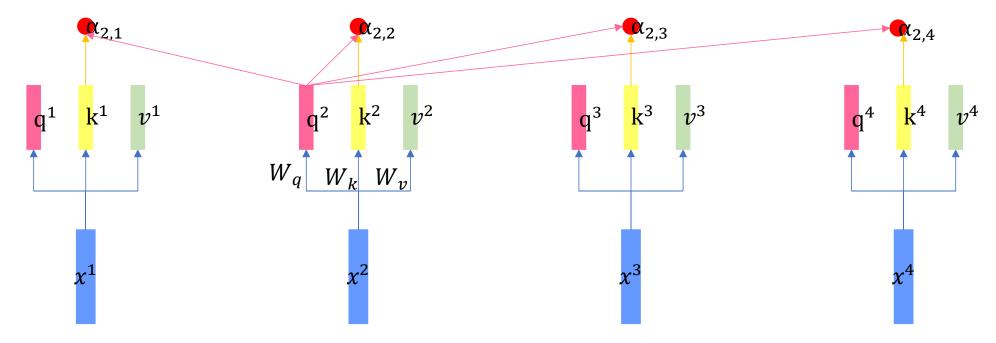


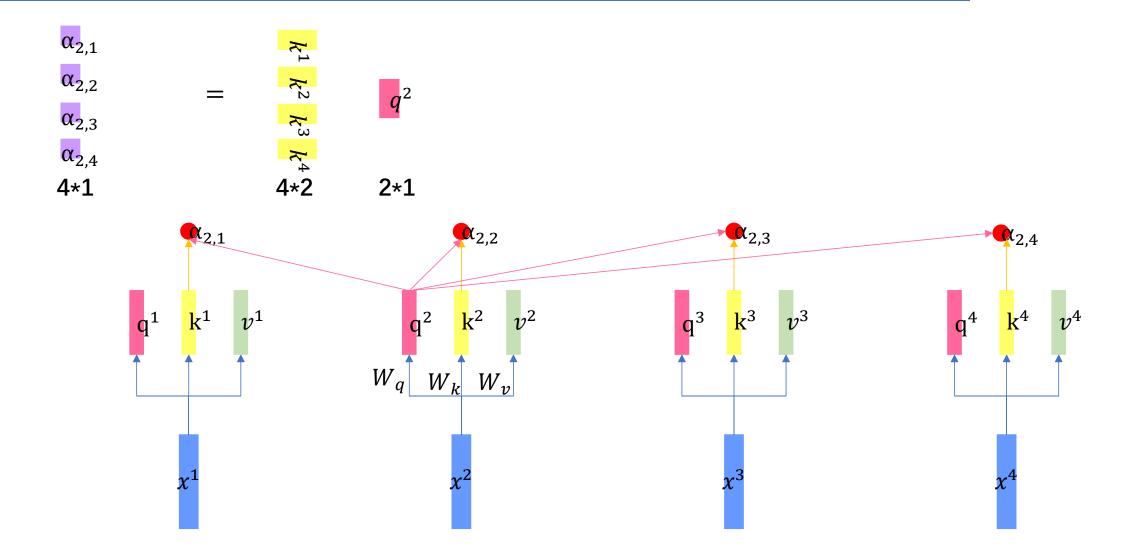


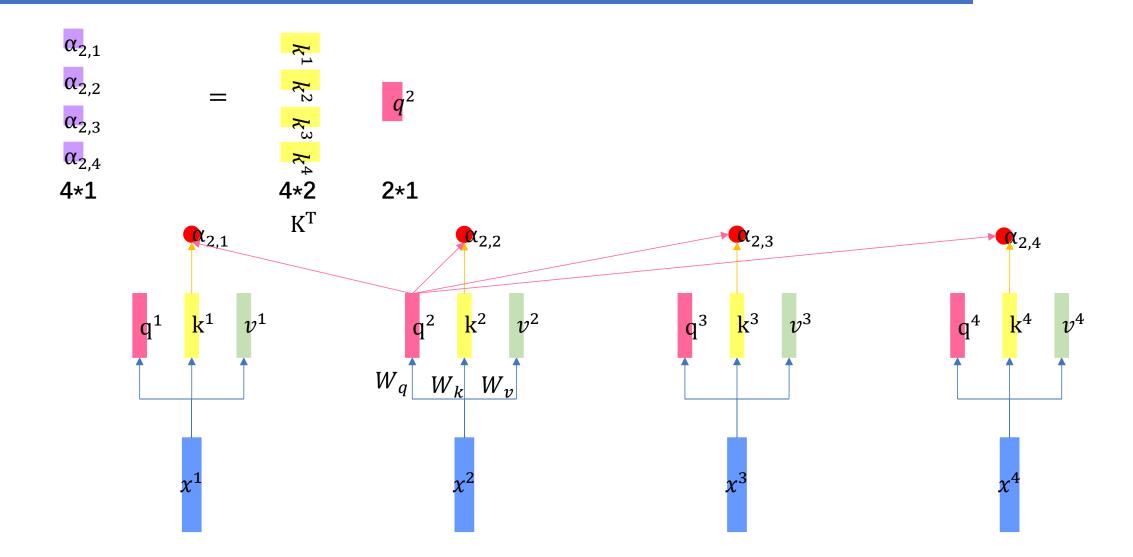


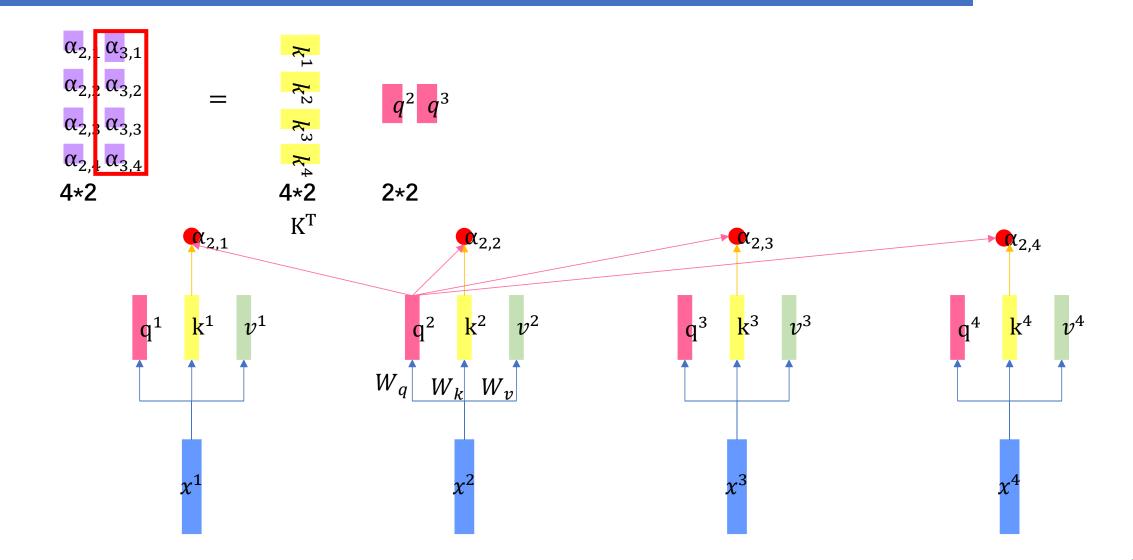


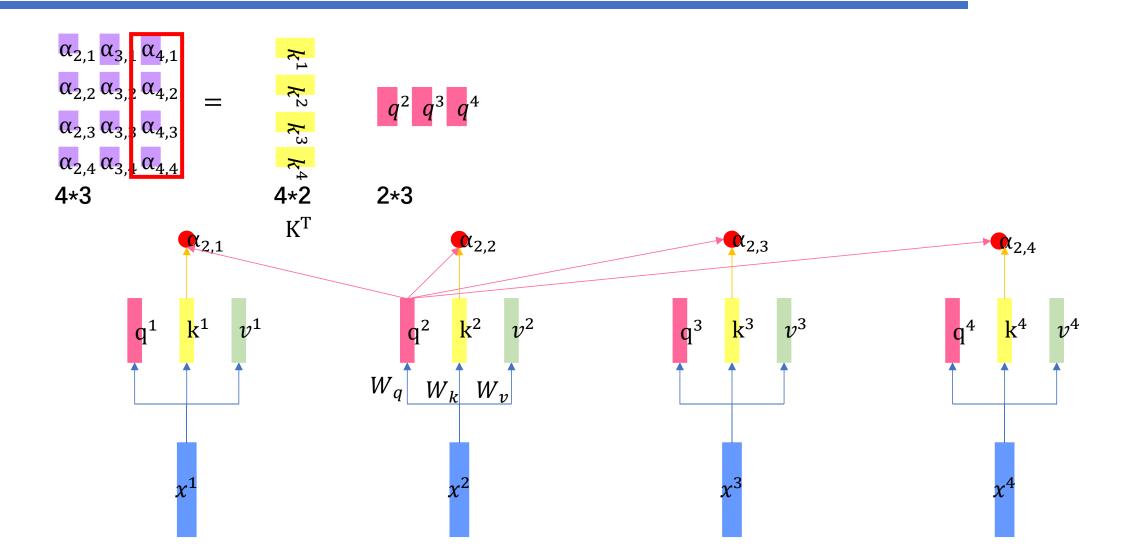


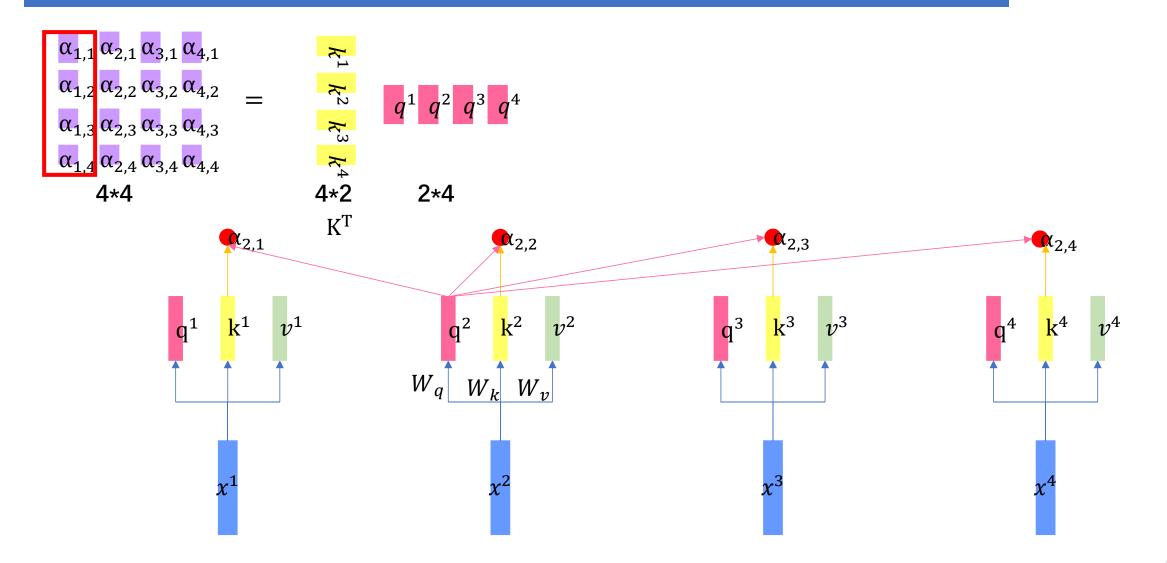


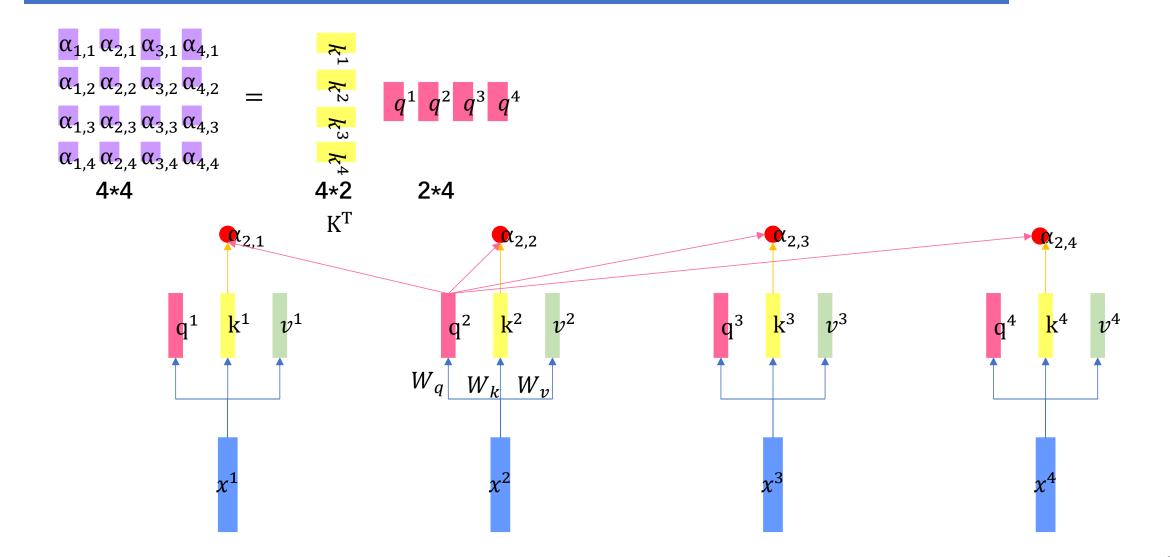


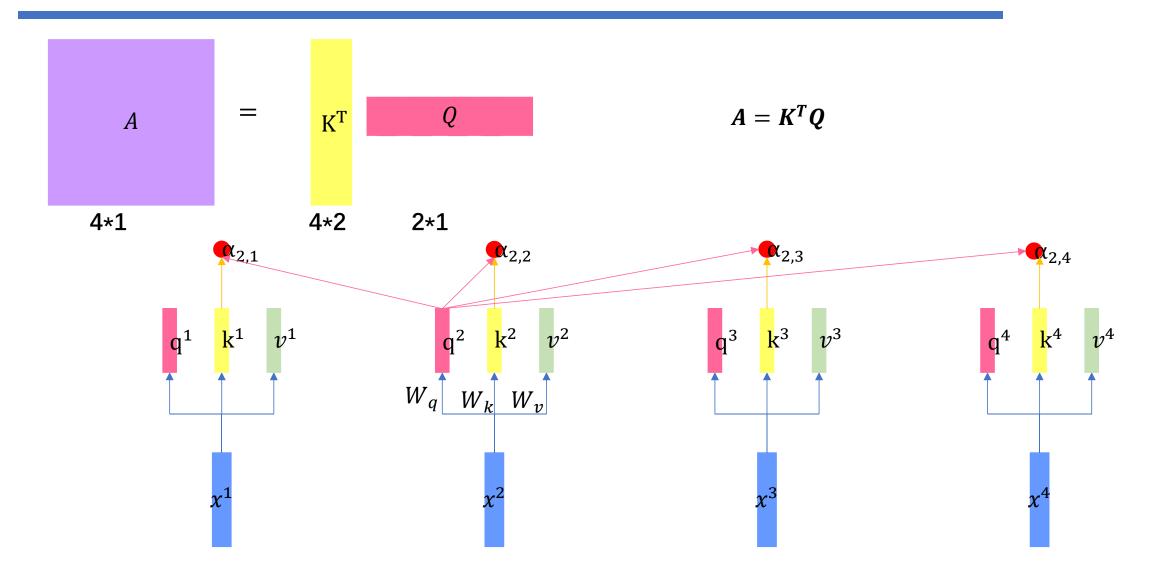


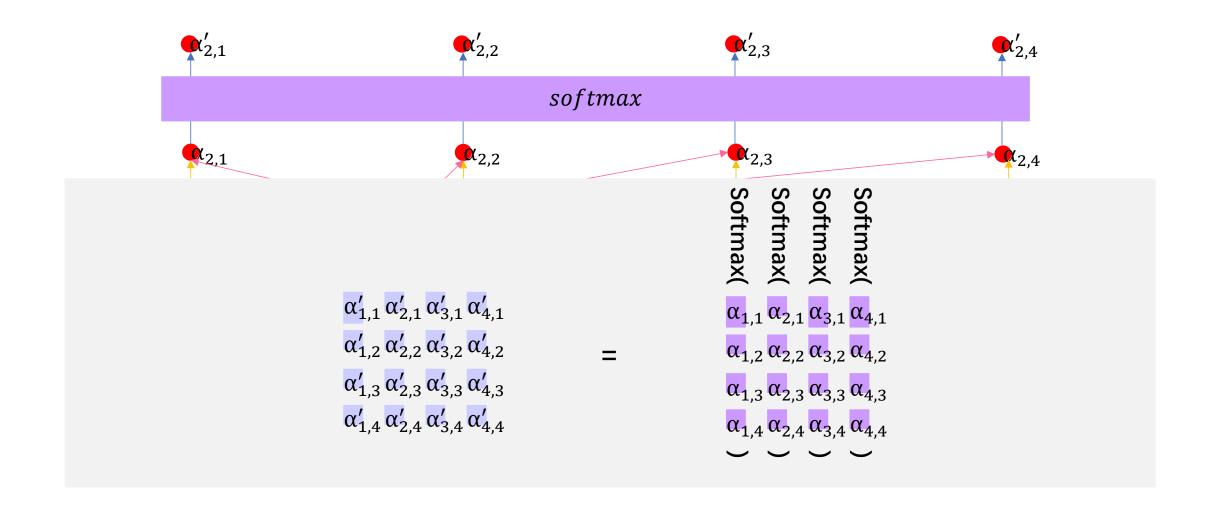


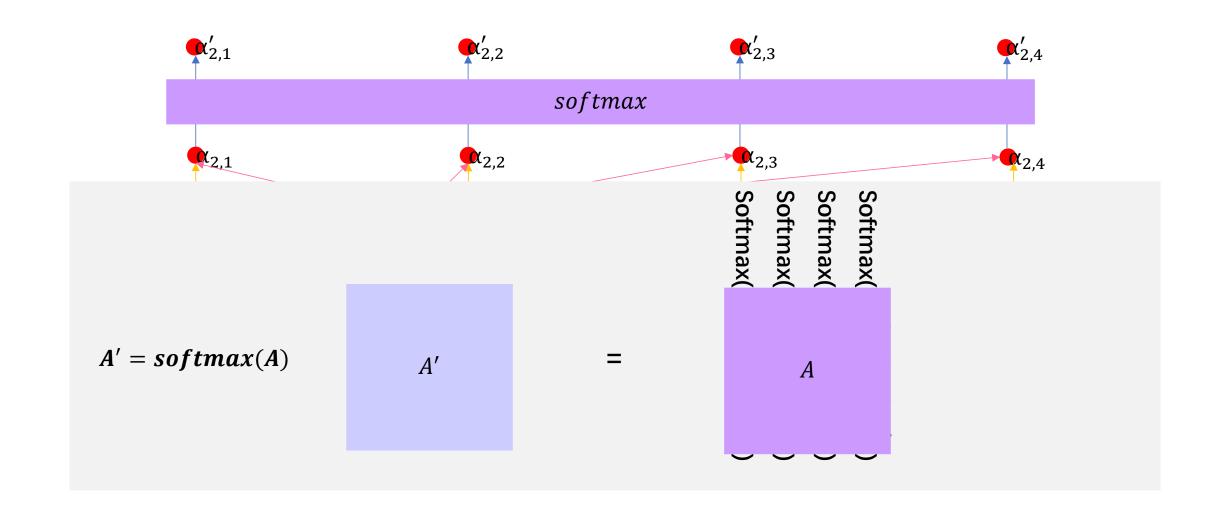


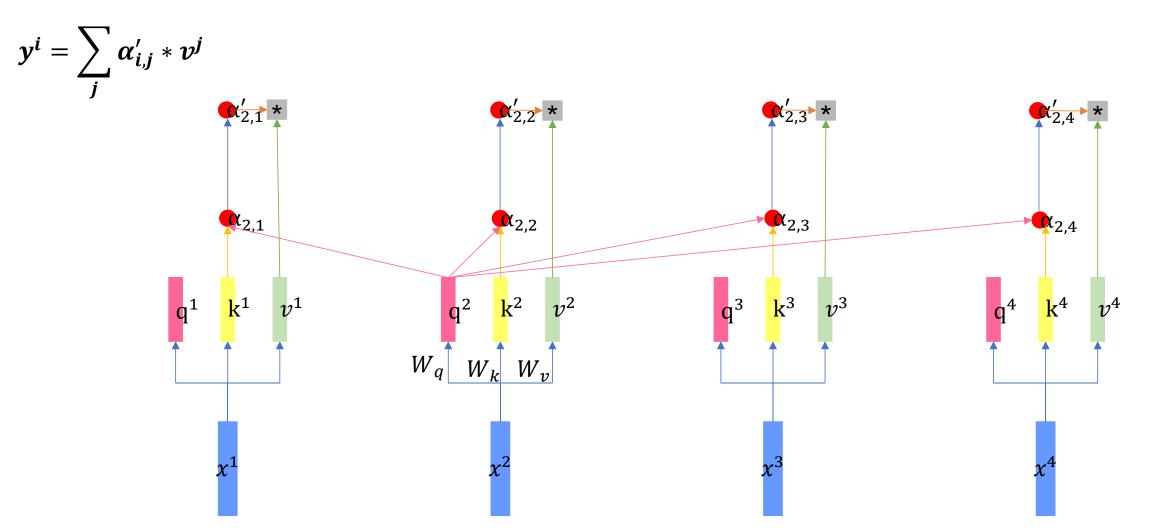


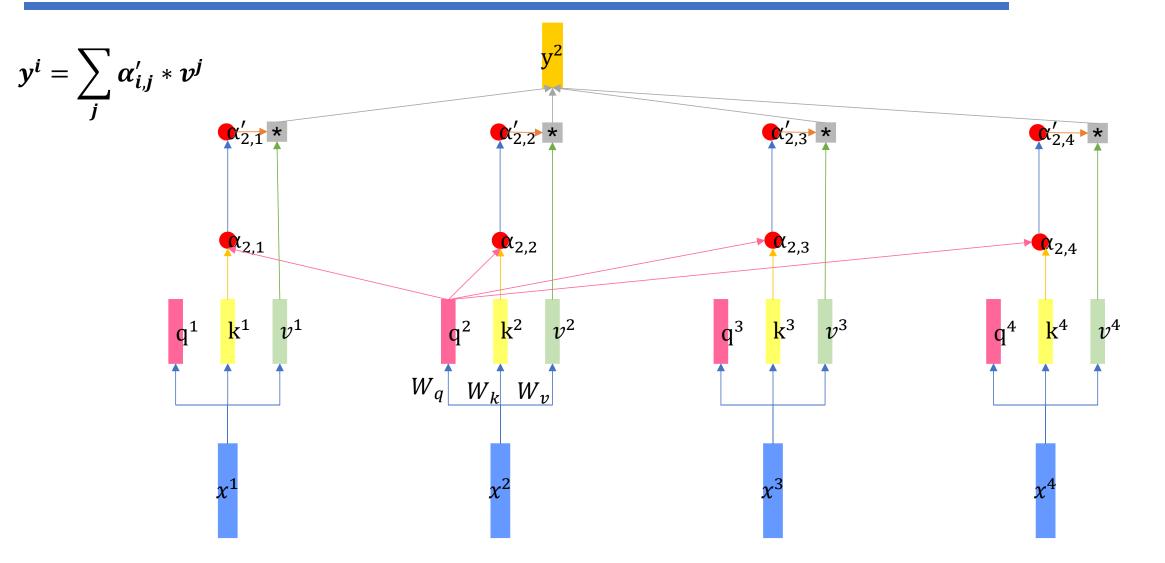


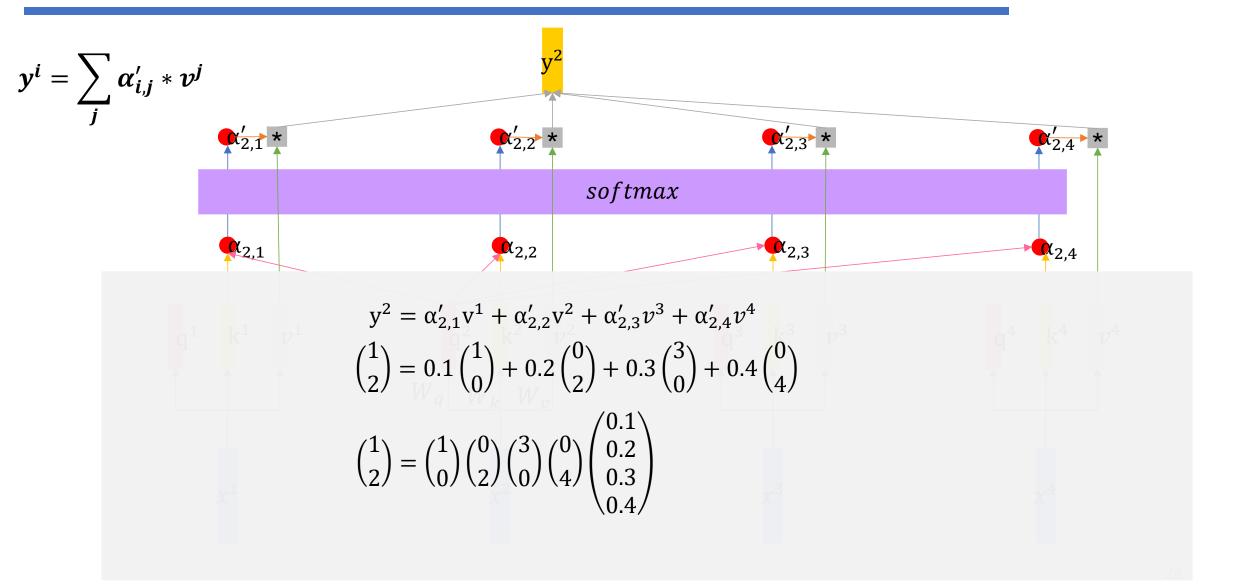


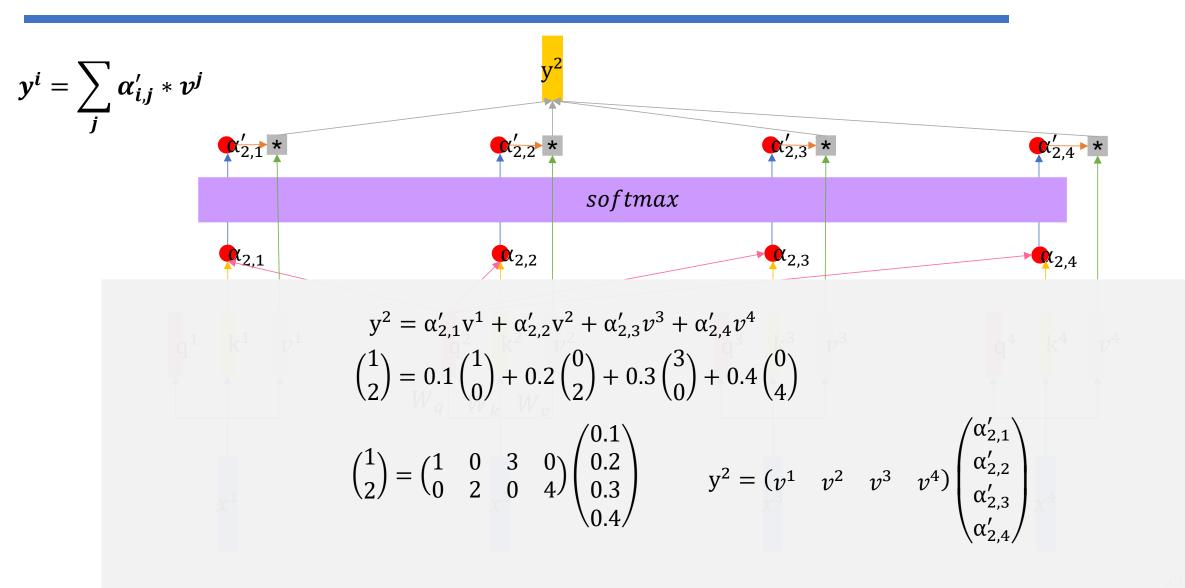












$$y^i = \sum_j$$

$$\binom{1}{2} = \binom{1}{0} \quad \binom{0}{2} \quad \binom{0}{0} \quad \binom{0}$$

$$y^{2} = (v^{1} \quad v^{2} \quad v^{3} \quad v^{4}) \begin{pmatrix} \alpha_{2,1} \\ \alpha'_{2,2} \\ \alpha'_{2,3} \\ \alpha'_{2,4} \end{pmatrix}$$

$$v^1$$
  $v^2$   $v^3$   $v$ 

$$\alpha'_{2,2}$$

$$\alpha'_{2,3}$$

$$\alpha'_{2,4}$$

$$y^i = \sum_i$$

$$\binom{1}{2}\binom{0.9}{0} = \binom{1}{0} \quad \binom{0}{2} \quad \binom{0}{0} \quad \binom{0}{$$

$$y^{2}y^{3} = (v^{1} \quad v^{2} \quad v^{3} \quad v^{4}) \begin{pmatrix} \alpha'_{2,1} \\ \alpha'_{2,2} \\ \alpha'_{2,3} \\ \alpha'_{2,4} \end{pmatrix} \begin{pmatrix} \alpha'_{3,1} \\ \alpha'_{3,2} \\ \alpha'_{3,3} \\ \alpha'_{3,4} \end{pmatrix}$$

$$y^{2} y^{3} = v^{1} v^{2} v^{3} v^{4} \frac{\alpha'_{2,2} \alpha'_{3,2}}{\alpha'_{2,3} \alpha'_{3,3}} \frac{\alpha'_{2,4} \alpha'_{3,4}}{\alpha'_{2,4} \alpha'_{3,4}}$$

$$y^i = \sum_i$$

$$\binom{1}{2}\binom{0.9}{0}\binom{0}{1.6} = \binom{1}{0} \quad \binom{0}{0} \quad \binom{0}{0$$

$$y^{2}y^{3}y^{4} = (v^{1} \quad v^{2} \quad v^{3} \quad v^{4}) \begin{pmatrix} \alpha'_{2,1} \\ \alpha'_{2,2} \\ \alpha'_{2,3} \\ \alpha'_{2,4} \end{pmatrix} \begin{pmatrix} \alpha'_{3,1} \\ \alpha'_{3,2} \\ \alpha'_{3,3} \\ \alpha'_{3,4} \end{pmatrix} \begin{pmatrix} \alpha'_{4,1} \\ \alpha'_{4,2} \\ \alpha'_{4,3} \\ \alpha'_{4,4} \end{pmatrix}$$

$$y^{2} y^{3} y^{4} = v^{1} v^{2} v^{3} v^{4}$$

$$= v^{1} v^{2} v^{3} v^{4}$$

$$= \alpha'_{2,1} \alpha'_{3,1} \alpha'_{4,1}$$

$$\alpha'_{2,2} \alpha'_{3,2} \alpha'_{4,2}$$

$$\alpha'_{2,3} \alpha'_{3,3} \alpha'_{4,3}$$

$$\alpha'_{2,4} \alpha'_{3,4} \alpha'_{4,4}$$

$$y^i = \sum_i$$

$$\binom{0.1}{0}\binom{1}{2}\binom{0.9}{0}\binom{0}{1.6} = \binom{1}{0} \quad 0 \quad 3 \quad 0 \\ 0 \quad 2 \quad 0 \quad 4$$

$$\binom{0.1}{0}\binom{0.1}{0}\binom{0.1}{0.2}\binom{0}{0.3}\binom{0}{0}\binom{0}{0}$$

$$\mathbf{y^{1}}\mathbf{y^{2}}\mathbf{y^{3}}\mathbf{y^{4}} = (v^{1} \quad v^{2} \quad v^{3} \quad v^{4}) \begin{pmatrix} \alpha'_{1,1} \\ \alpha'_{1,2} \\ \alpha'_{1,3} \\ \alpha'_{1,4} \end{pmatrix} \begin{pmatrix} \alpha'_{2,1} \\ \alpha'_{2,2} \\ \alpha'_{2,3} \\ \alpha'_{2,4} \end{pmatrix} \begin{pmatrix} \alpha'_{3,1} \\ \alpha'_{3,2} \\ \alpha'_{3,3} \\ \alpha'_{3,4} \end{pmatrix} \begin{pmatrix} \alpha'_{4,1} \\ \alpha'_{4,2} \\ \alpha'_{4,3} \\ \alpha'_{4,4} \end{pmatrix}$$

$$y^{1} y^{2} y^{3} y^{4} = v^{1} v^{2} v^{3} v^{4} \frac{\alpha'_{1,1} \alpha'_{2,1} \alpha'_{3,1} \alpha'_{4,1}}{\alpha'_{1,2} \alpha'_{2,2} \alpha'_{3,2} \alpha'_{4,2}} \frac{\alpha'_{1,1} \alpha'_{2,1} \alpha'_{2,1} \alpha'_{3,1} \alpha'_{4,1}}{\alpha'_{1,3} \alpha'_{2,3} \alpha'_{3,3} \alpha'_{4,3}} \frac{\alpha'_{1,4} \alpha'_{2,4} \alpha'_{3,4} \alpha'_{4,4}}{\alpha'_{1,4} \alpha'_{2,4} \alpha'_{3,4} \alpha'_{4,4}}$$

$$y^i = \sum_i$$

$$\begin{pmatrix} 0.1 & 1 & 0.9 & 0 \\ 0 & 2 & 0 & 1.6 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 3 & 0 \\ 0 & 2 & 0 & 4 \end{pmatrix} \begin{pmatrix} 0.1 & 0.1 & 0 & 0 \\ 0 & 0.2 & 0 & 0 \\ 0 & 0.3 & 0.3 & 0 \\ 0 & 0.4 & 0 & 0.4 \end{pmatrix}$$

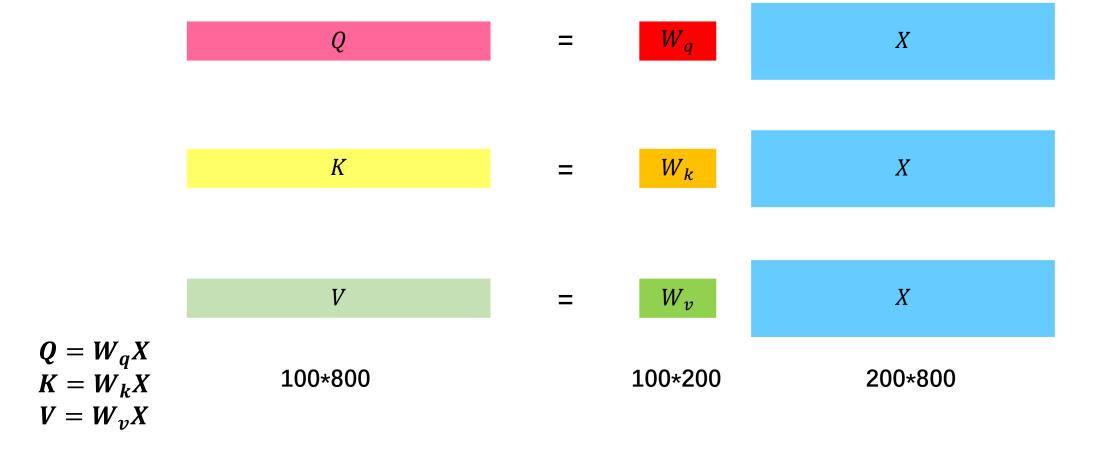
$$y^{1}y^{2}y^{3}y^{4} = (v^{1} \quad v^{2} \quad v^{3} \quad v^{4}) \begin{pmatrix} \alpha'_{1,1} & \alpha'_{2,1} & \alpha'_{3,1} & \alpha'_{4,1} \\ \alpha'_{1,2} & \alpha'_{2,2} & \alpha'_{3,2} & \alpha'_{4,2} \\ \alpha'_{1,3} & \alpha'_{2,3} & \alpha'_{3,3} & \alpha'_{4,3} \\ \alpha'_{1,4} & \alpha'_{2,4} & \alpha'_{3,4} & \alpha'_{4,4} \end{pmatrix}$$

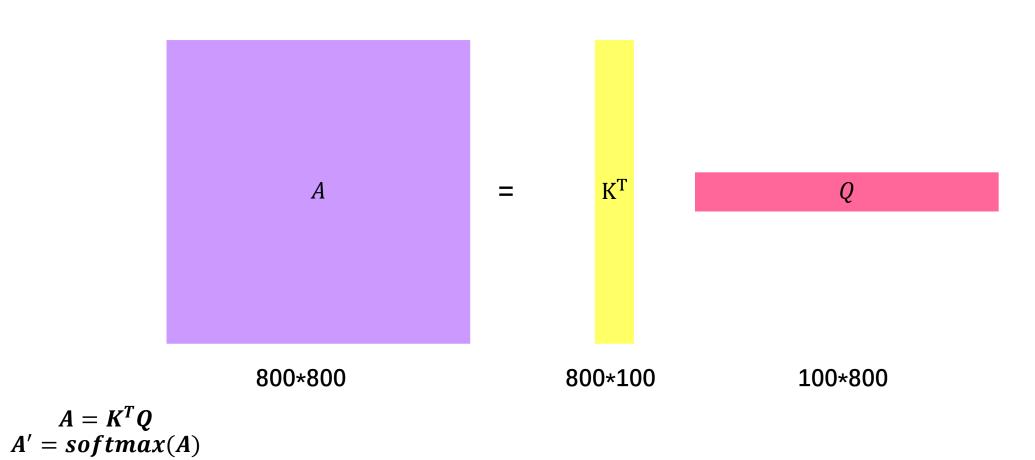
$$Y = VA'$$

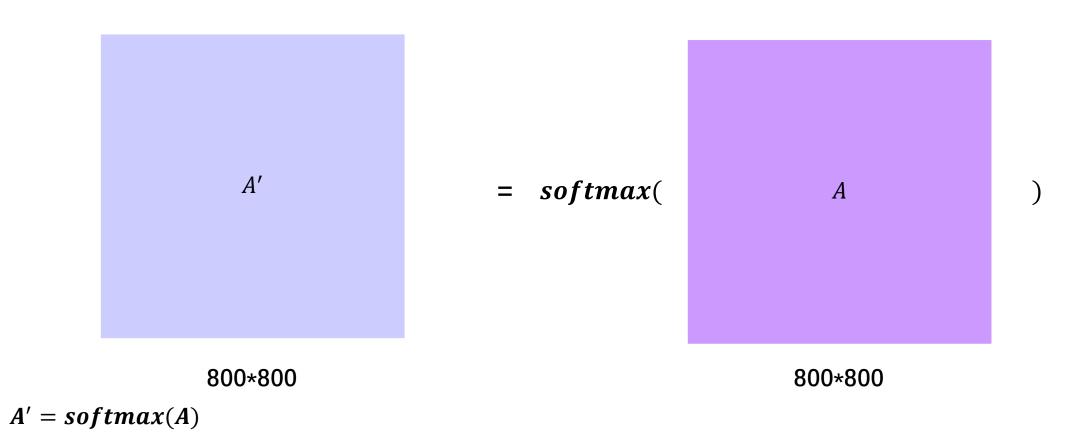
4

7

A'

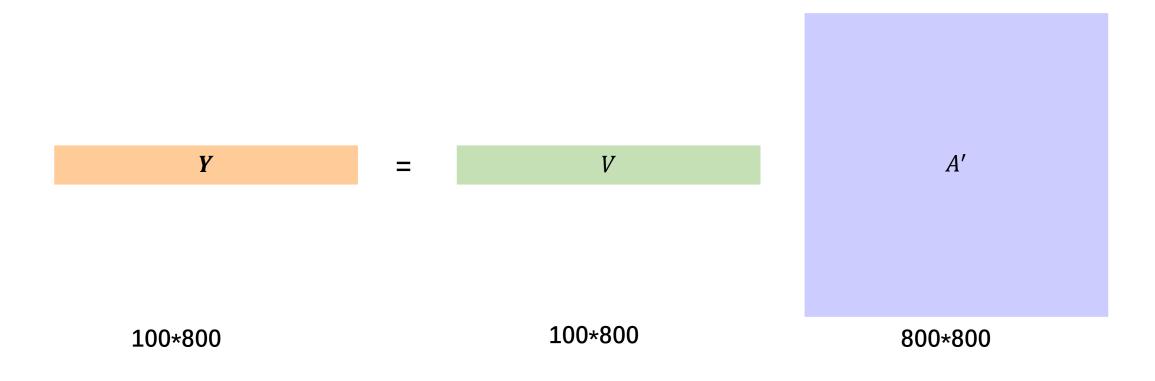




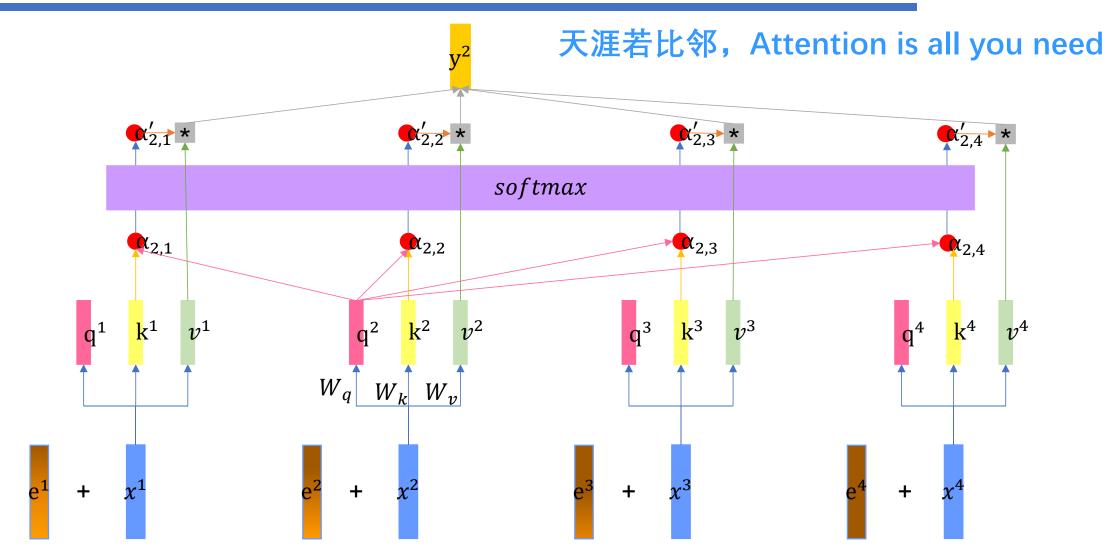


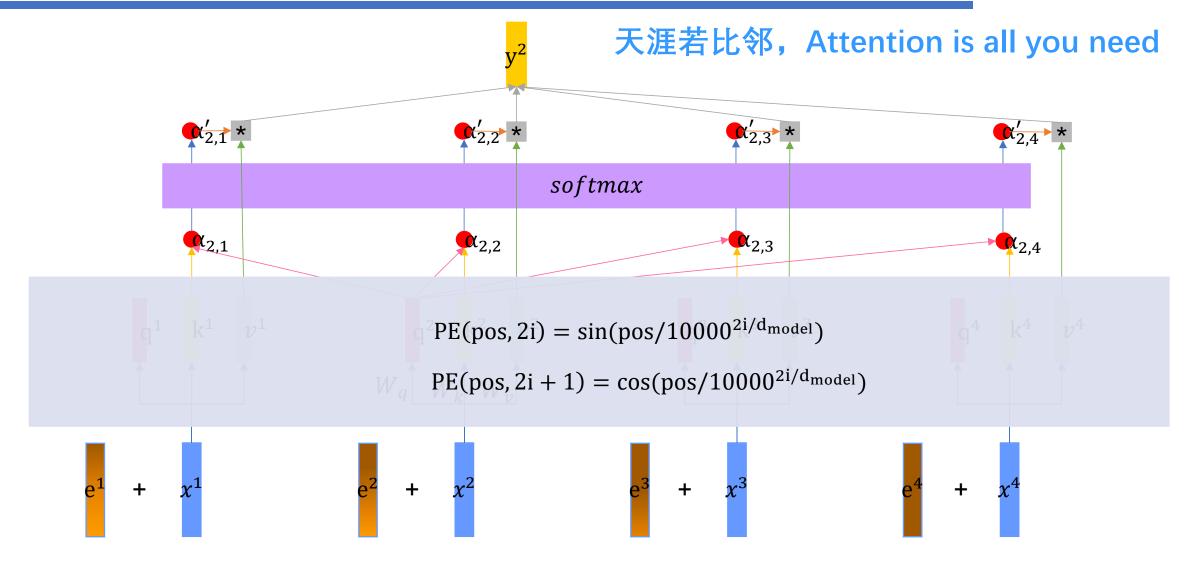
### **Self-Attention**

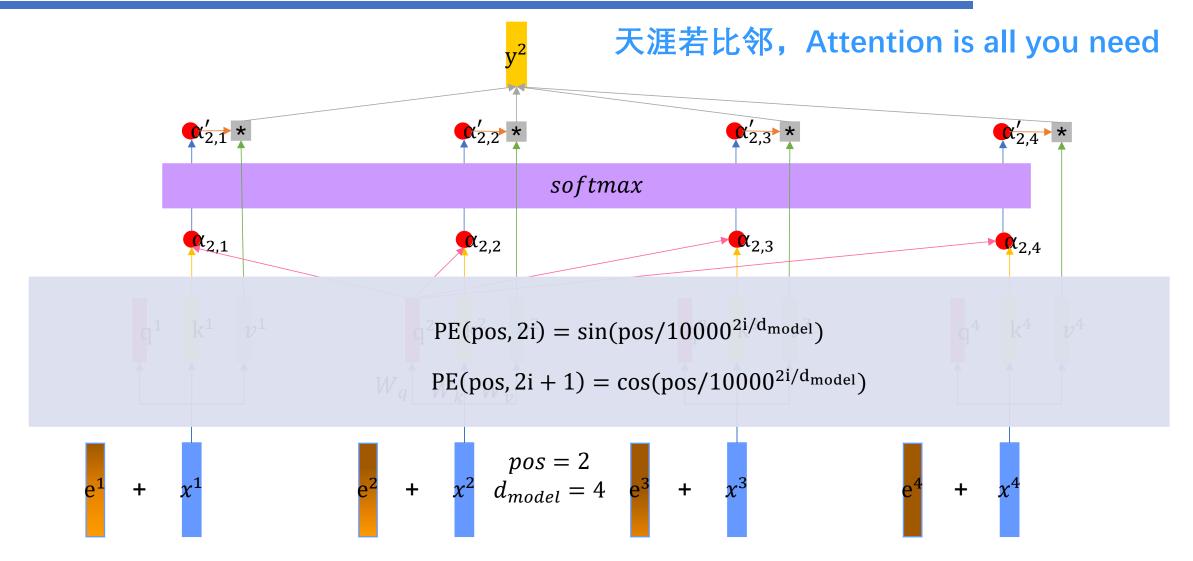
Y = VA'

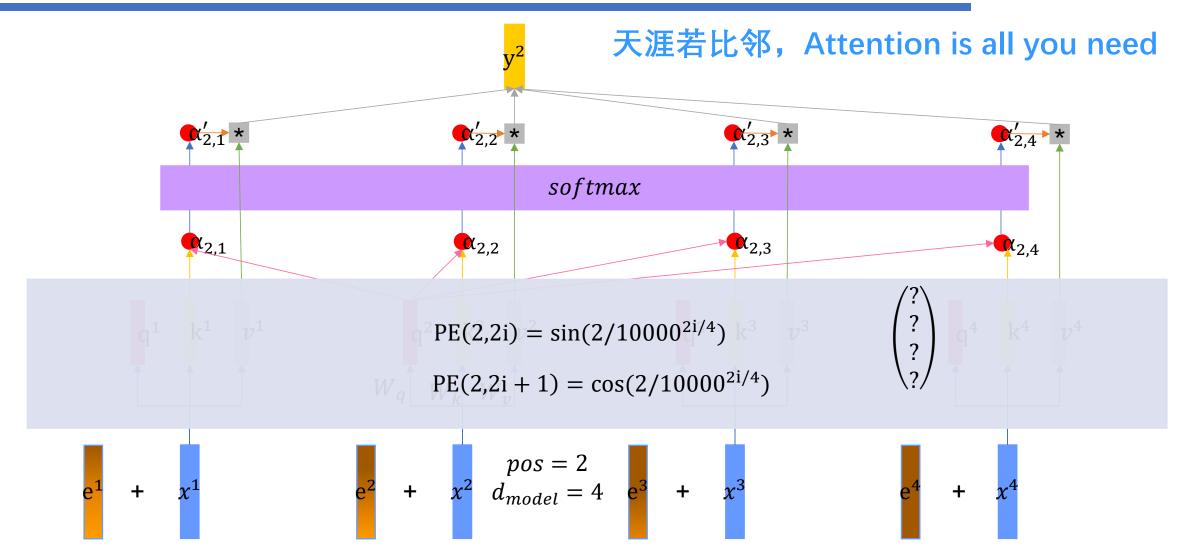


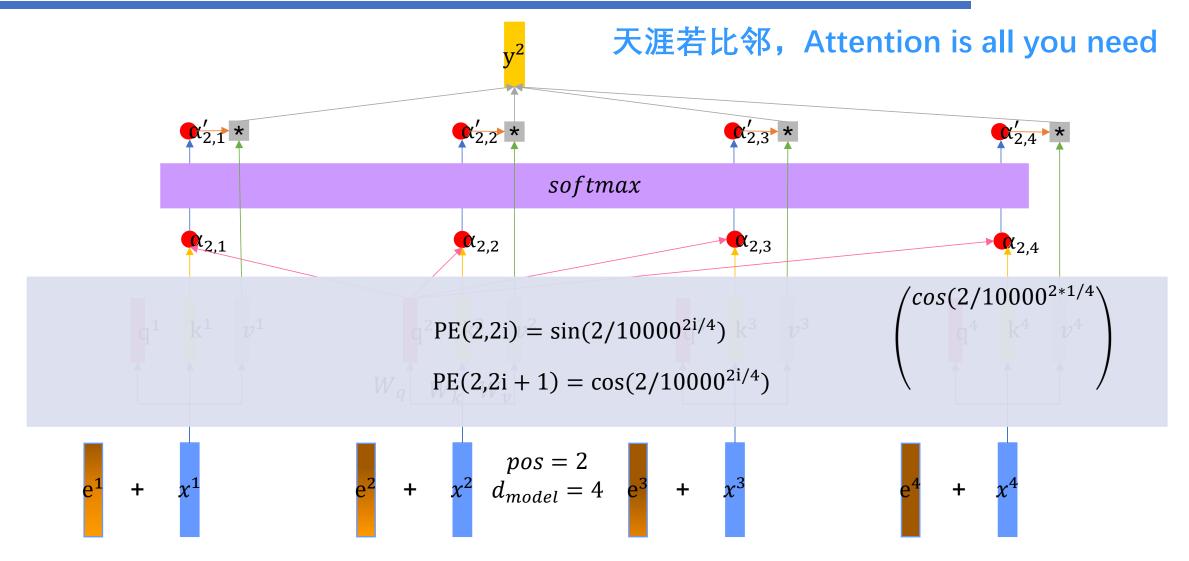


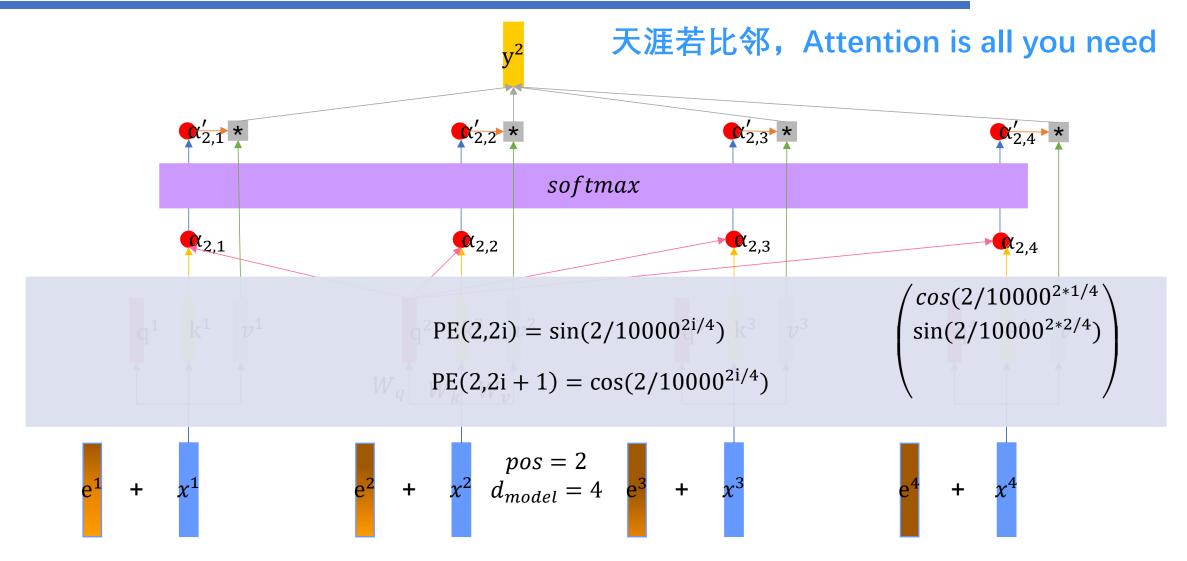


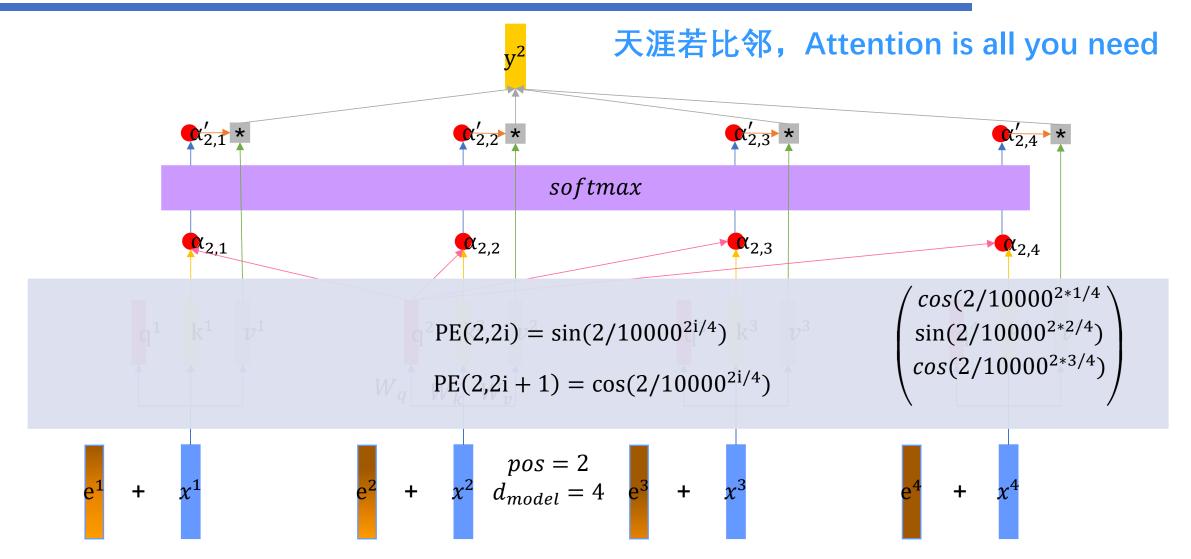


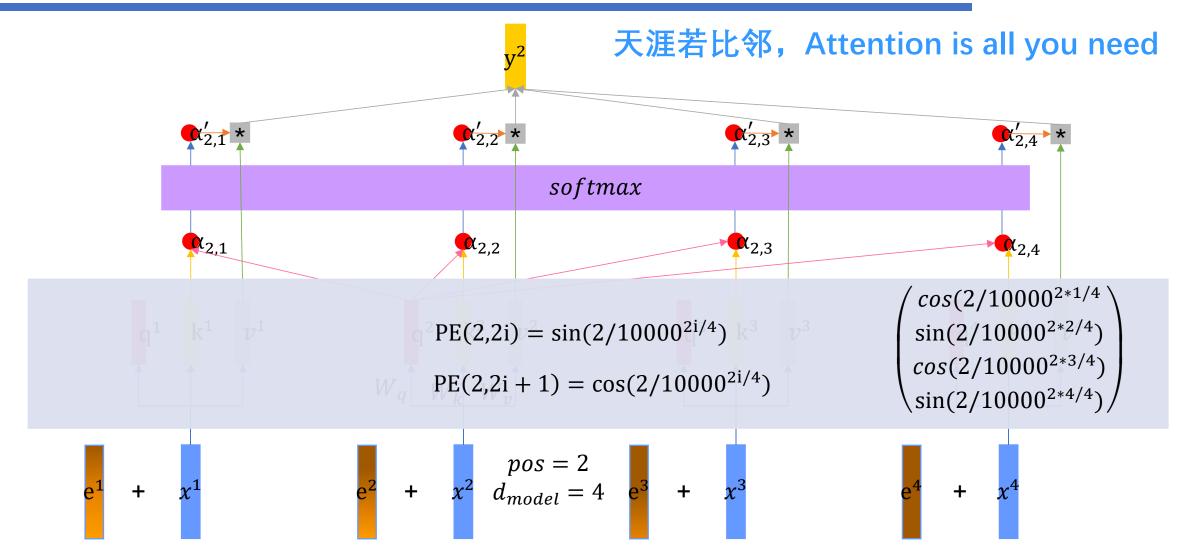


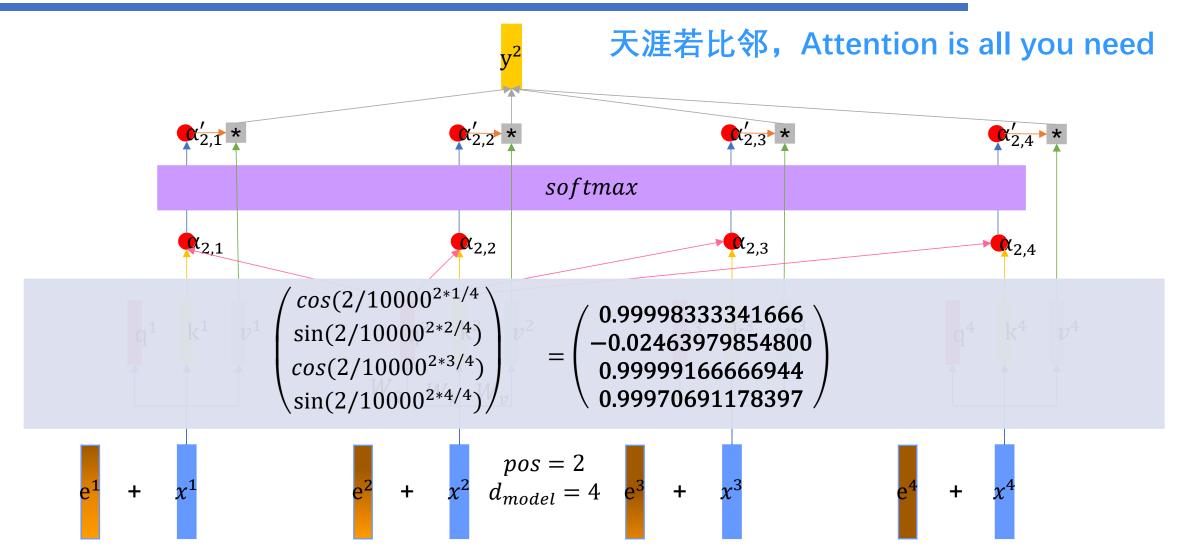


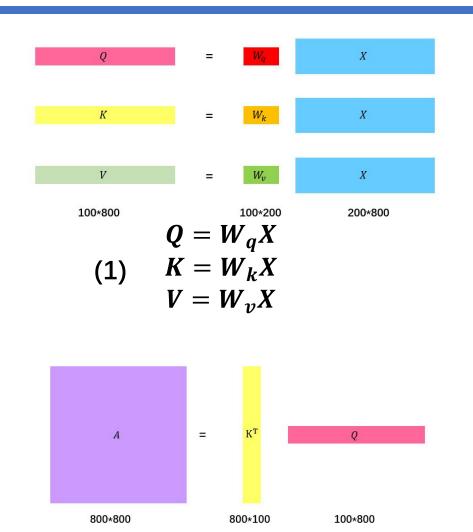






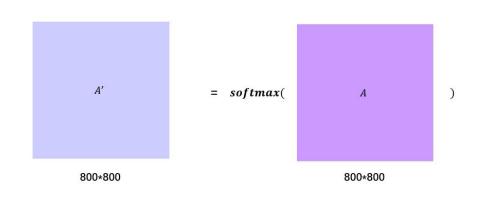




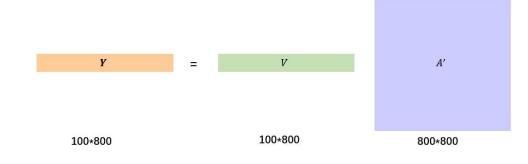


 $A = K^T Q$ 

(2)



(3) 
$$A' = softmax(A)$$



$$(4) Y = VA'$$