Fast Generation of Big Random Binary Trees

UCL Computer Science Research Note RN/20/01

13 January 2020 arXiv:2001.04505

random_tree() is a linear time and space C++ implementation able to create trees of up to a billion nodes for genetic programming and genetic improvement experiments. A 3.60GHz CPU can generate more than 18 million random nodes for GP program trees per second.

W. B. Langdon

Slides for Software Systems Engineering SSE Reading Group, 15 Jan 2020

Why do we care?

- Tree universal data structure
- Space of trees is huge but sampling it is not simple
- Lessons
  - Linear O(n) 1999 code fine for trees of 100 nodes not usable with trees of a million nodes
  - Solving problem with real pseudo random number generators (PRNGs)
    - although we are in a state of sin (Von Neumann), call it random from now on.

Back Ground 1996-2000

- Genetic Programming needs random trees
  - Random start to the population
  - Subtree replacement mutation
- John Koza propose “ramped half and half”
- Walter Bohm and Andreas Geyer-Schulz [FOGA 4] and Hitoshi Iba [PPSN 96] suggest uniform random trees.
- 1997 I implement Iba’s in Andy Singleton’s GPquick
- Sean Luke [2000] says not fast
  - Who cares trees tiny trees, over head small.

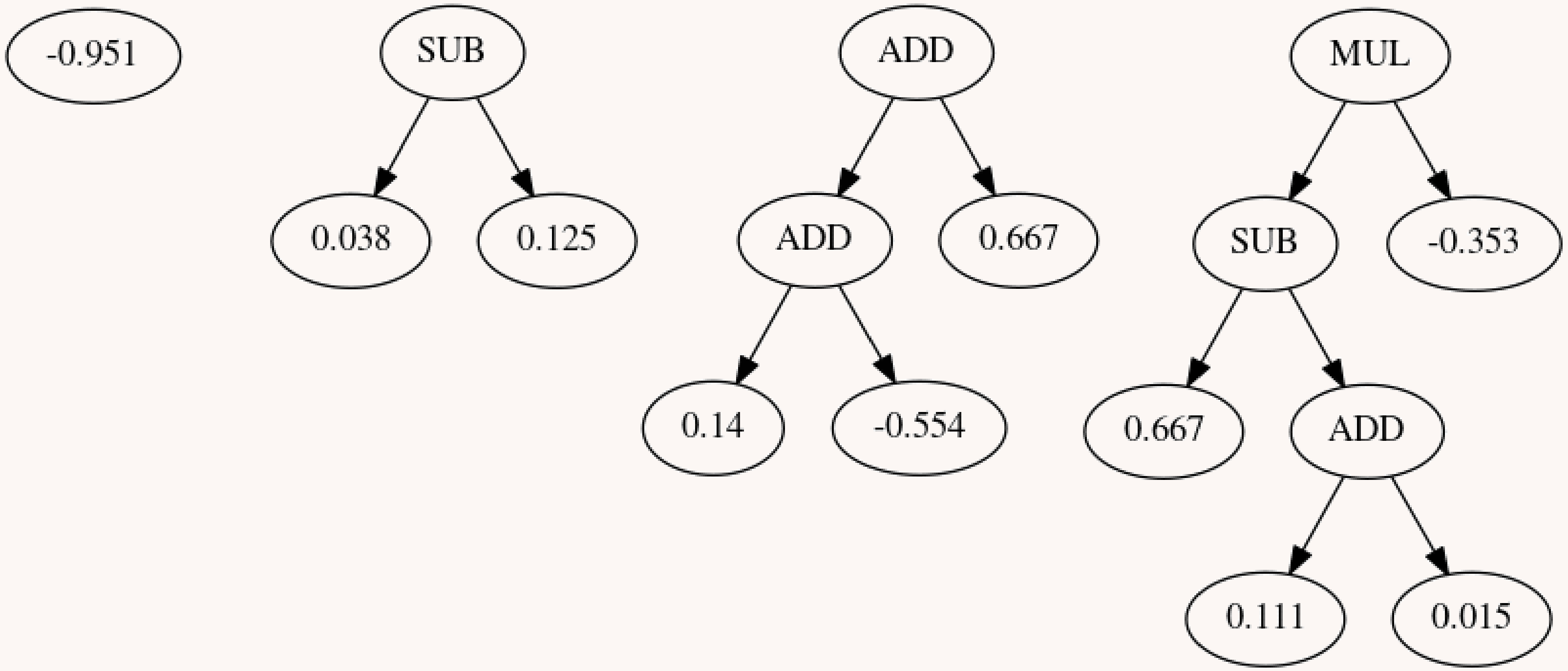
Back Ground 2019

- Applying Genetic Improvement to speed up existing GP system: COW62
  - To speed up eval of very bloated big 10^8 trees
- Only need binary trees (for now)
- Need lots of different big trees like those evolved by Genetic Programming (GP)
- Want random trees since:
  - Humungous GP trees take weeks to evolve
  - Without compression gigabytes each
- Existing (linear code) takes far too long.
- Code rewritten

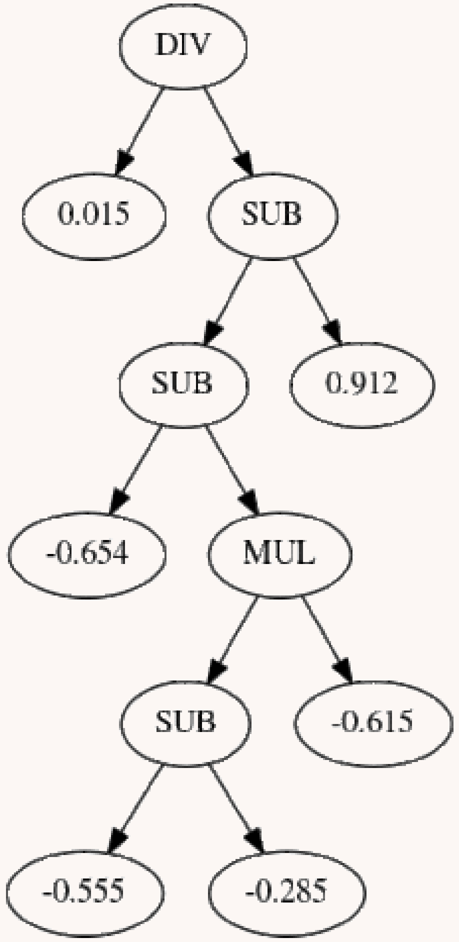
Evolved GP Trees

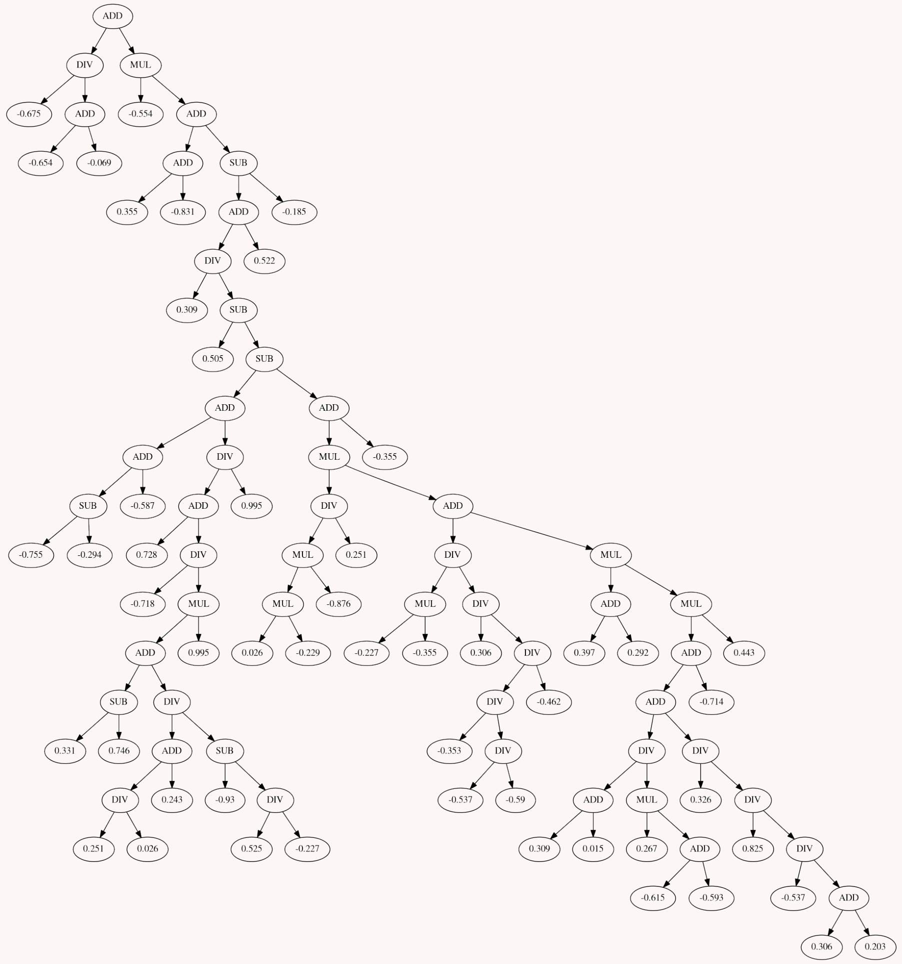
- Picture binary tree
- Picture GP population evolving

Binary GP Trees



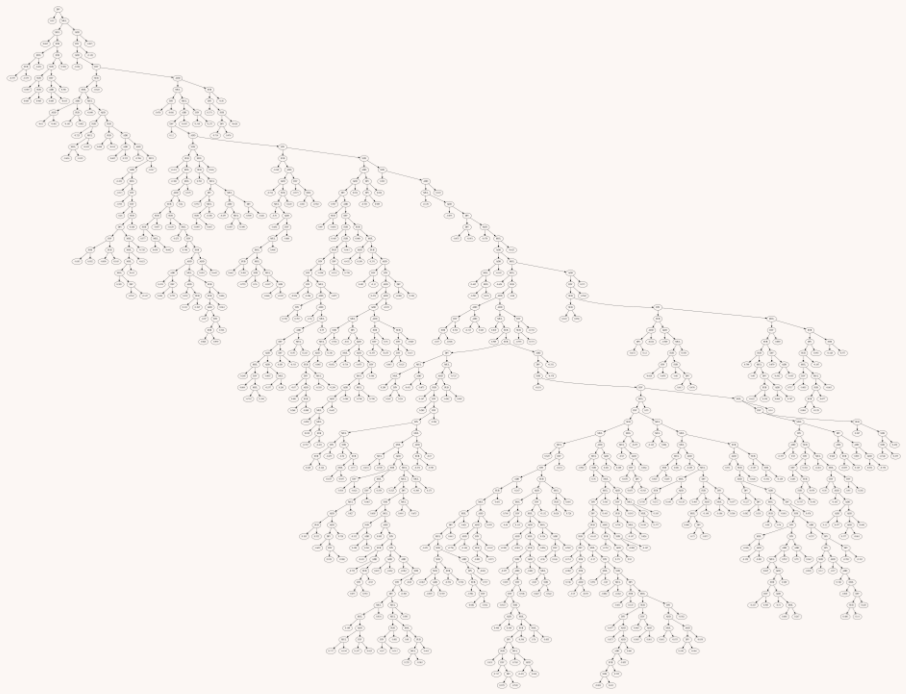
Binary GP Trees





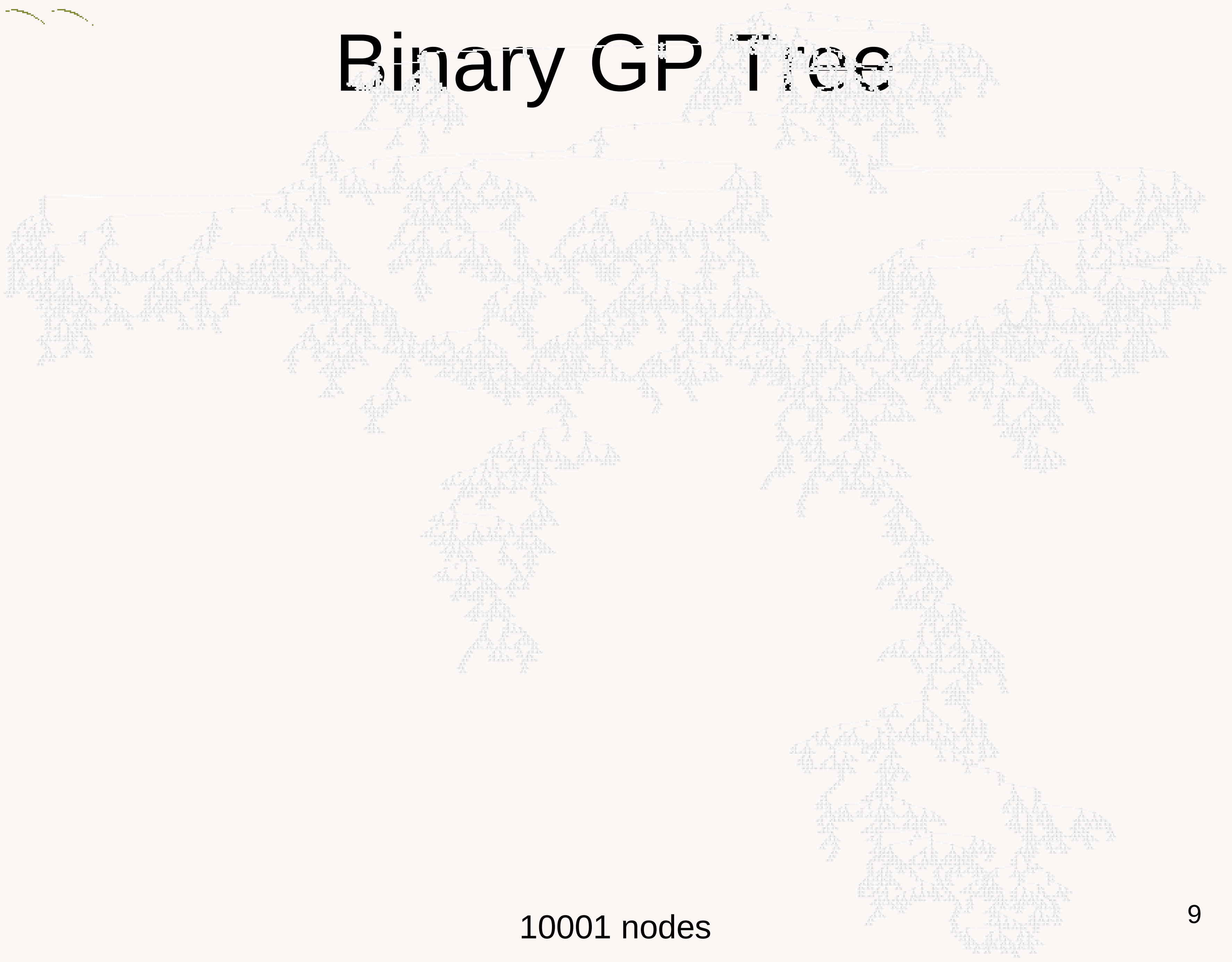
101 nodes

Binary GP Tree

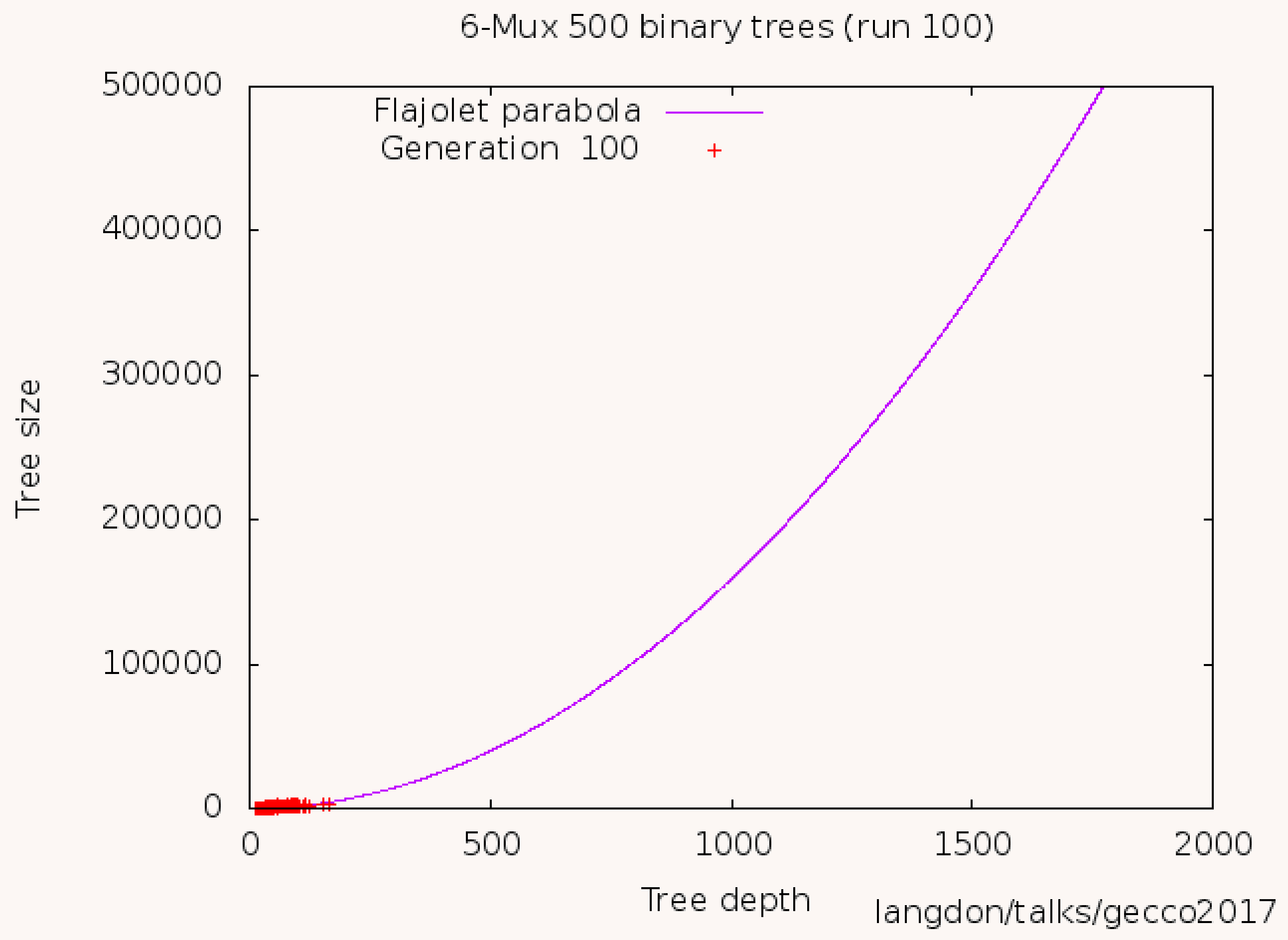


1001 nodes

Binary GP Tree


10001 nodes

GP population Evolving



video

Mathematics of Large Trees

Cominatorics of large trees hard but well studied

Robert Sedgewick and Philippe Flajolet [analysis of algorithms]

Number of binary trees is Catalan number N = (size-1)/2

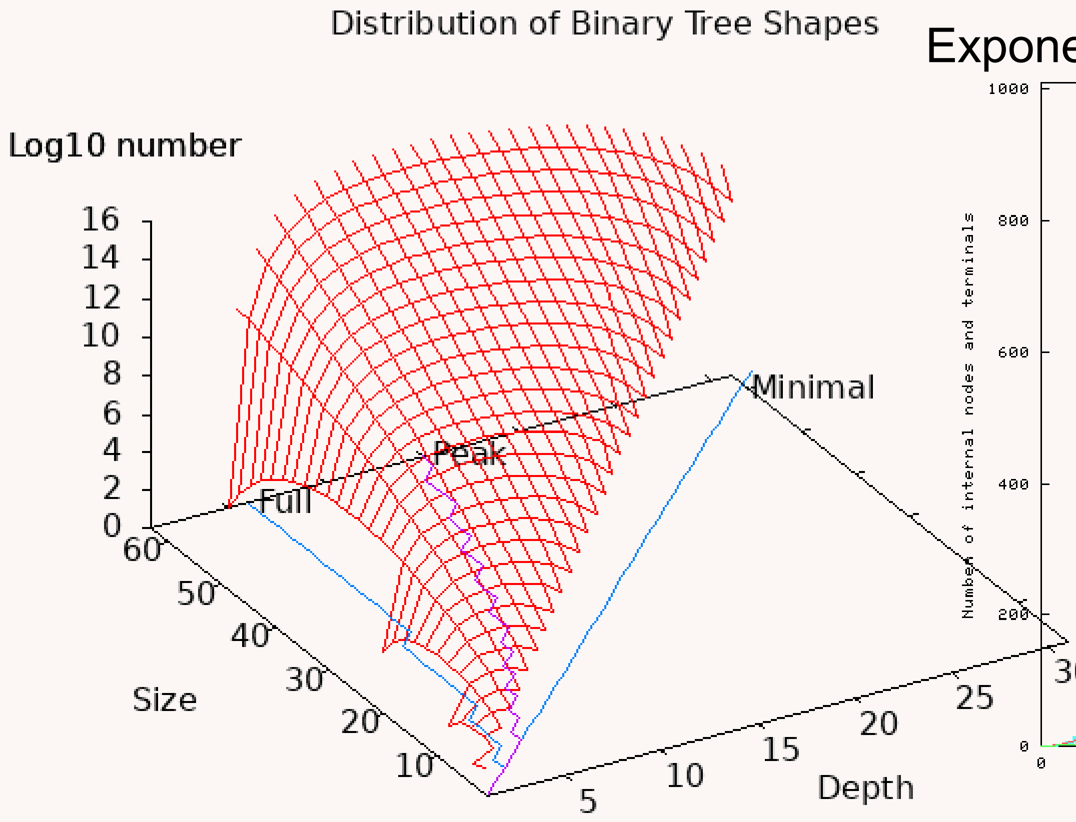
$$T_N = \frac{1}{N+1}\binom{2N}{N} = \frac{4^N}{\sqrt{\pi N^3}}\left(1 + O(\frac{1}{N})\right).$$

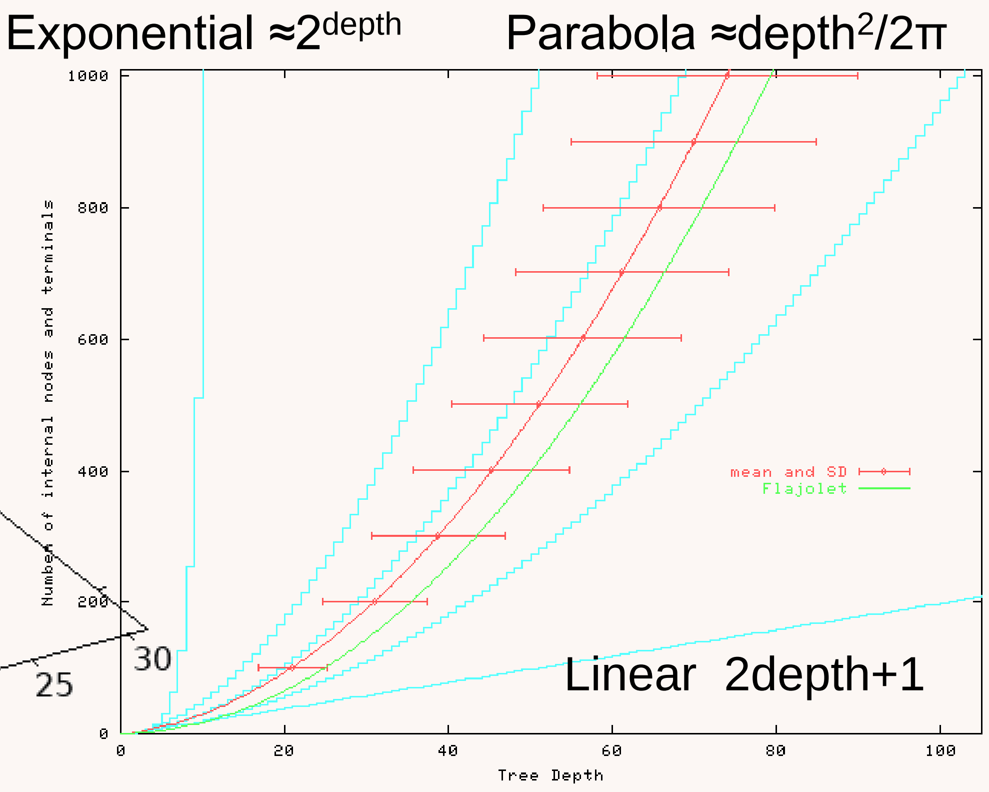
Large N limit

$$2\sqrt{\pi N} + O(N^{1/4+\epsilon})$$

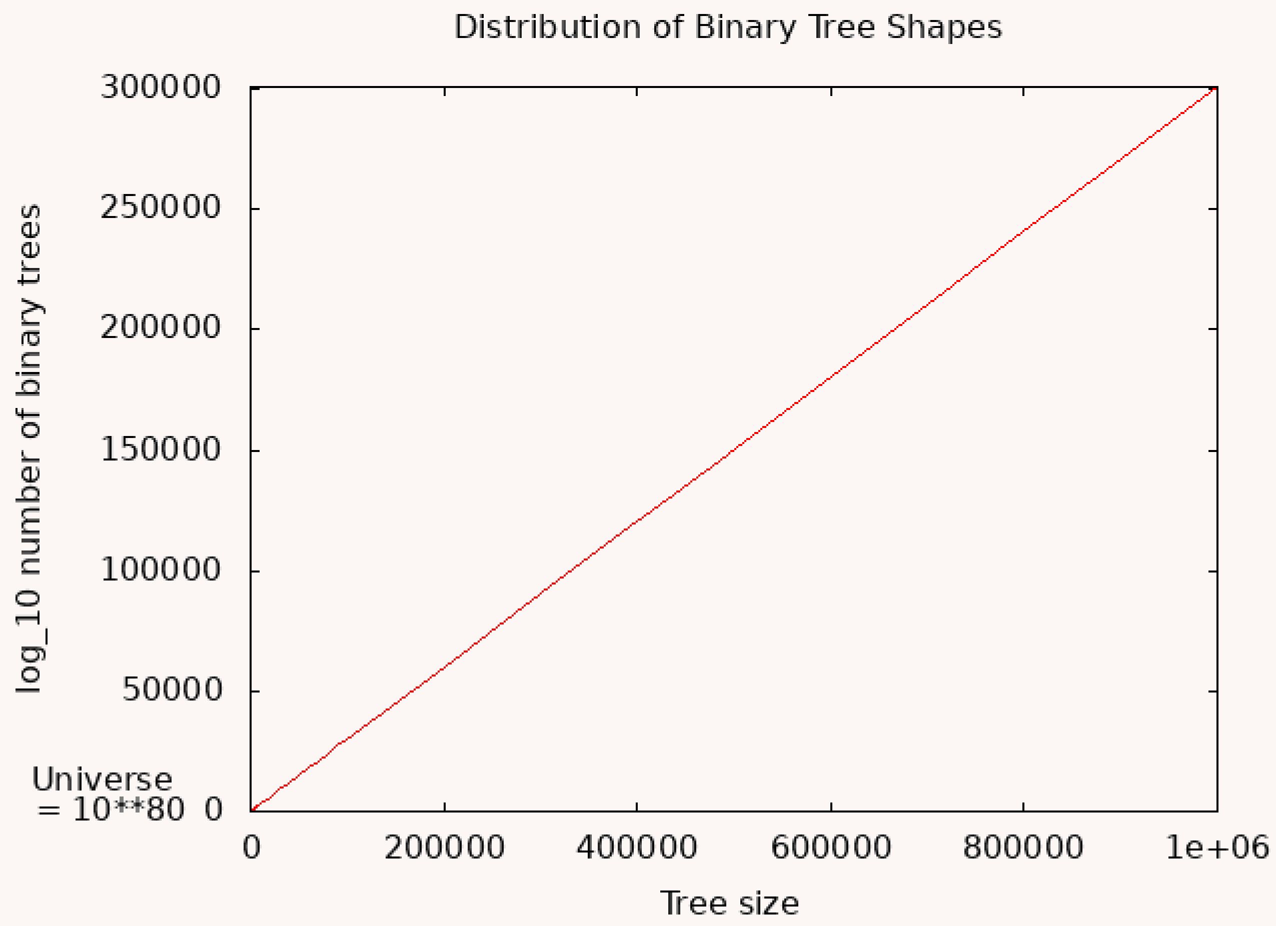
$$\approx \sqrt{}(2\pi\, size)$$

Distribution of binary trees





Number of possible binary trees



Large Evolved GP Trees

- Genetic Programming trees not random but random shape.
- Random trees good enough to test and time each new mutated eval() function.

Sampling Large Trees

- Sampling trees is hard
- Mathematical cheat: sample random permutation of size n, translate chosen permutation into tree
- Proof that sample permutation uniformly at random, so have sampled random trees uniformly at random.

1997 (Small) random Trees

- Original implementation followed Iba’s technical report ETL-TR-95-35 and existing random permutation C code.
- Not efficient. Linear time O(n) but not fast. Instead deal with hard maths by keeping code modular.
- Also needed to deal with functions with up to 4 arguments (arity 0..4)

Sampling Permutation for Binary Tree

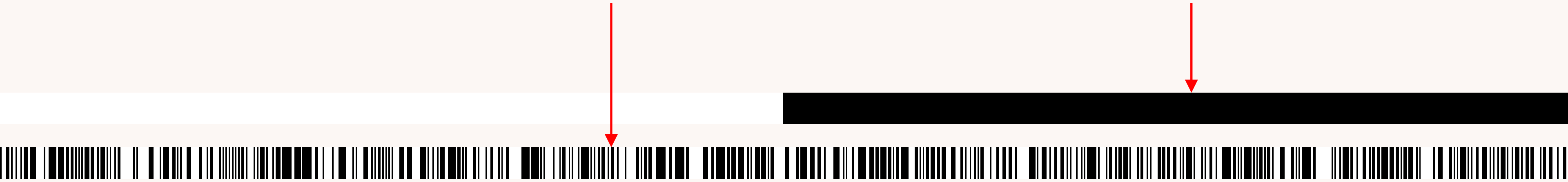
- Original code slow because general
- Simplify for binary tree
  - n/2 functions, n/2+1 leafs
- Start with empty sequence `int dyck[n]`
- Deterministically load n/2 node(a=2) and n/2+1 leafs (arity=0).
- Randomise sequence, `dyck`
- New: use Knuth shuffle

Sampling Permutation for Binary Tree

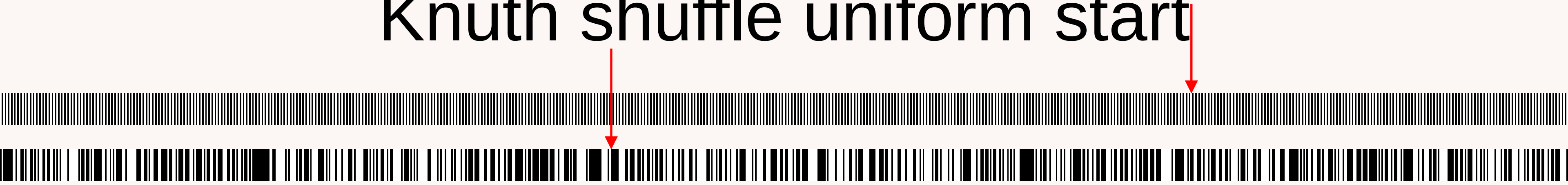
- Use Knuth shuffle to randomise sequence
- Prove that Knuth shuffle samples uniformly at random, hence have random permutation, hence will have random tree
- With real PRNG, need to help Knuth?
- Alternate 0,2 initial sequence (don't start with all 0 at one end). Then shuffle.
- Number of random trees far bigger than number of PRNG sequences.

Randomising sequence for real

Knuth shuffle unbalanced start



Knuth shuffle uniform start



Permutation into Tree via onedom()

- Sampling trees is hard
  - Odd language dyck sequence, 1-dominated
- For binary trees, sequence in n/2+1 square
  - Add leaf move horizontally.
  - Add function move vertically.
  - All random permutations move from start to End (albeit by different routes, next slide).
- Exactly one rotation makes route a valid tree.
- Routine onedom (1-dominated) converts random permutation into corresponding random tree.

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Onedom() permutation into tree

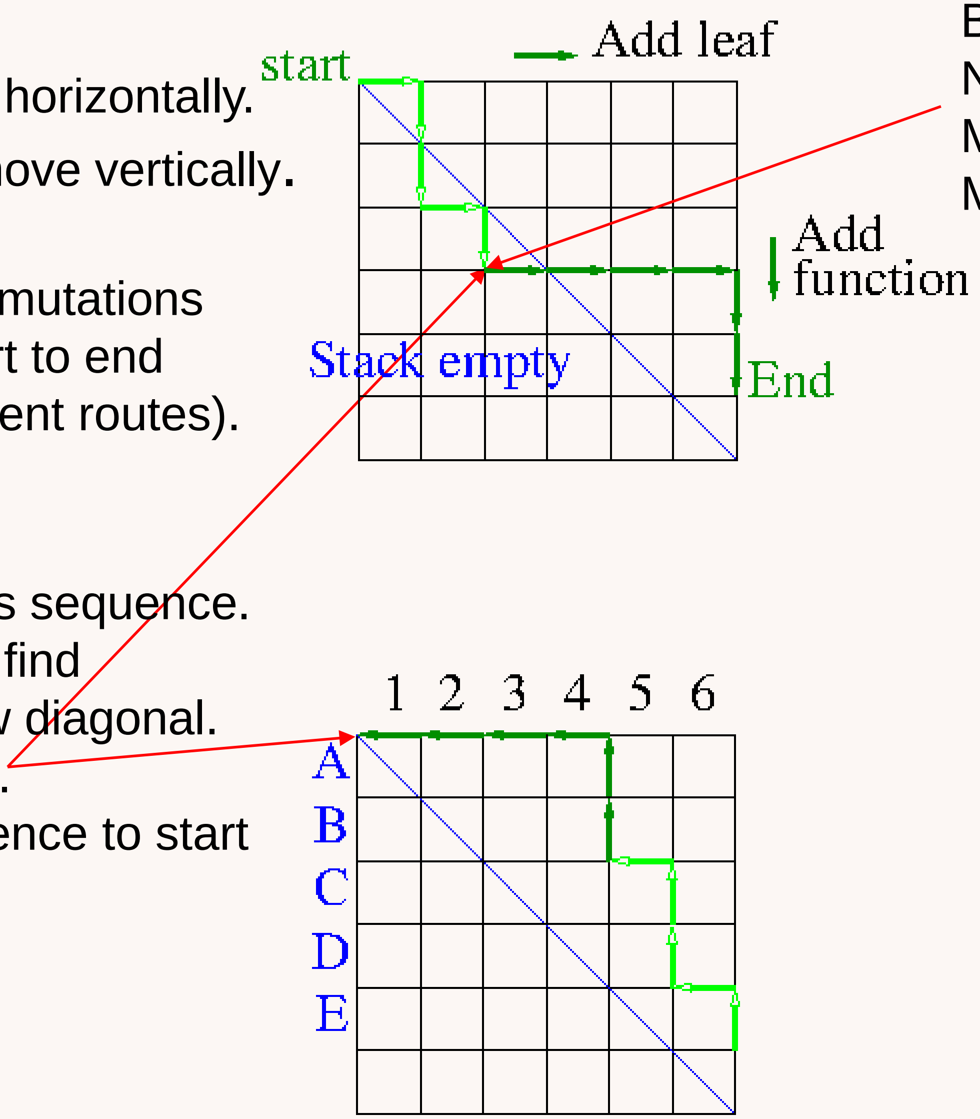
Add leaf move horizontally.

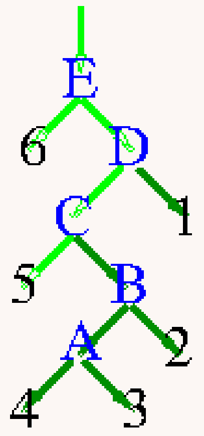
Add function move vertically.

All random permutations move from start to end (albeit by different routes).

Onedom scans sequence. For each (x,y) find distance below diagonal. Keep last max. Reorder sequence to start at max.

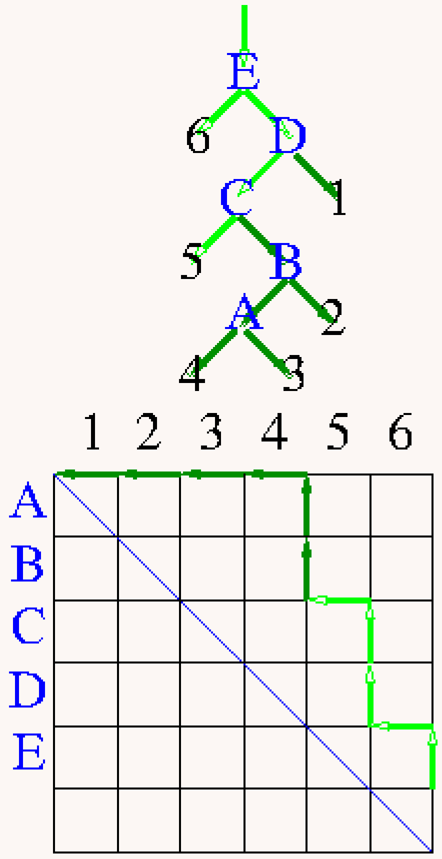
Below diagonal. Not a valid tree. More functions than leafs. More pops than pushes.

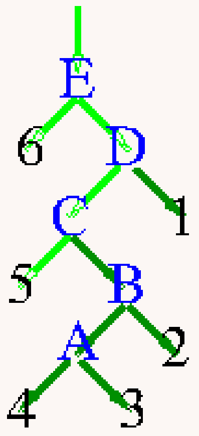




Permutation into Tree

- It turns out to be easier to trace route backwards
- From end: Add E,6,D,C,5,B,A,4,3,2,1





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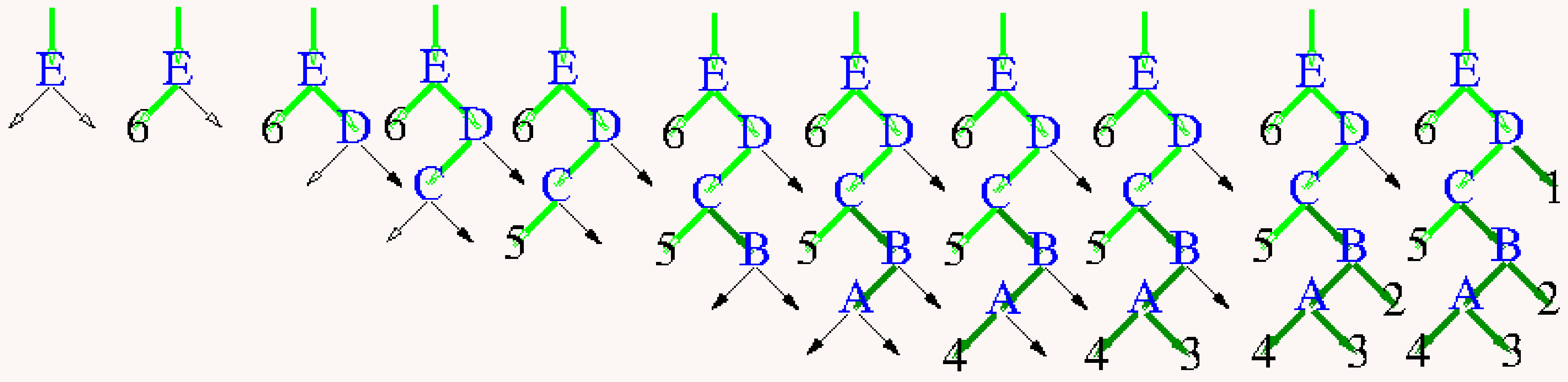

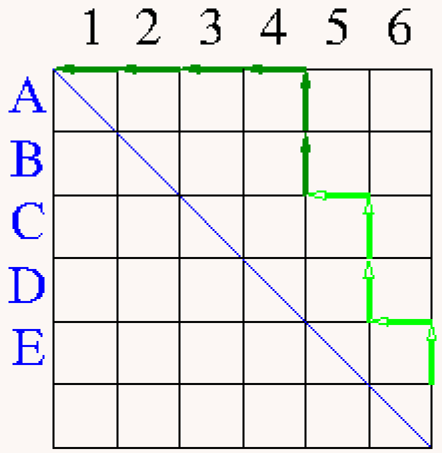

Tree into GP Program

- Label tree with GP functions and leafs
  - – If function (2) chose random function
  - – If leaf (0) chose leaf at random
- Return GP tree and its depth

Depth an additional benefit

- Some data structures in eval() depend on depth of tree
- The distribution of random trees is known in advance so can conservatively set tree depth from known size of tree.
  - depth < mean + multiple of standard deviation
  - depth < $10 \times \sqrt{(size)} + 100$
- Estimated tree depth much smaller than worst case (size/2) but still wasteful.
- New code returns exact depth.

Does it work

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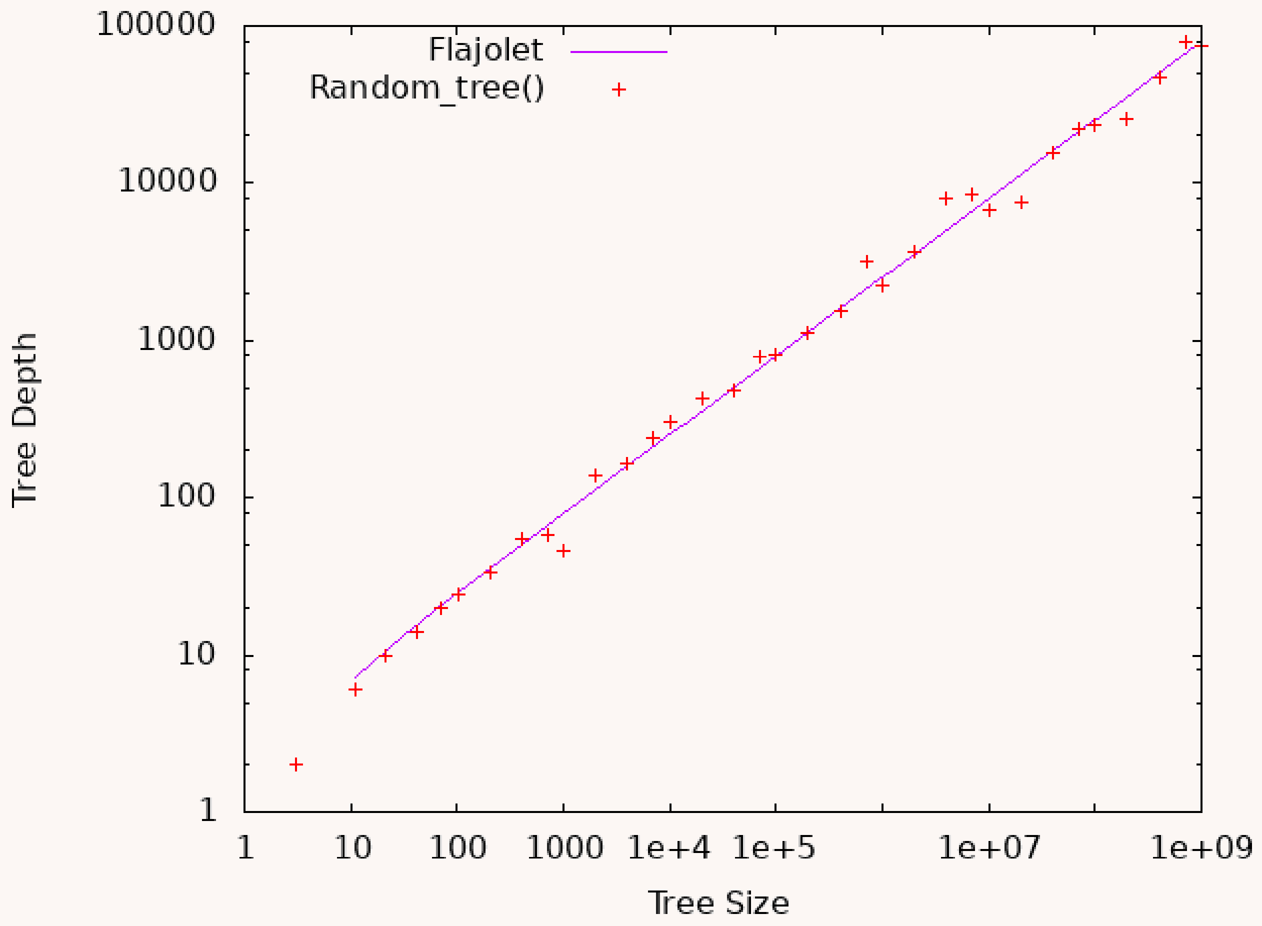
Large Random Trees



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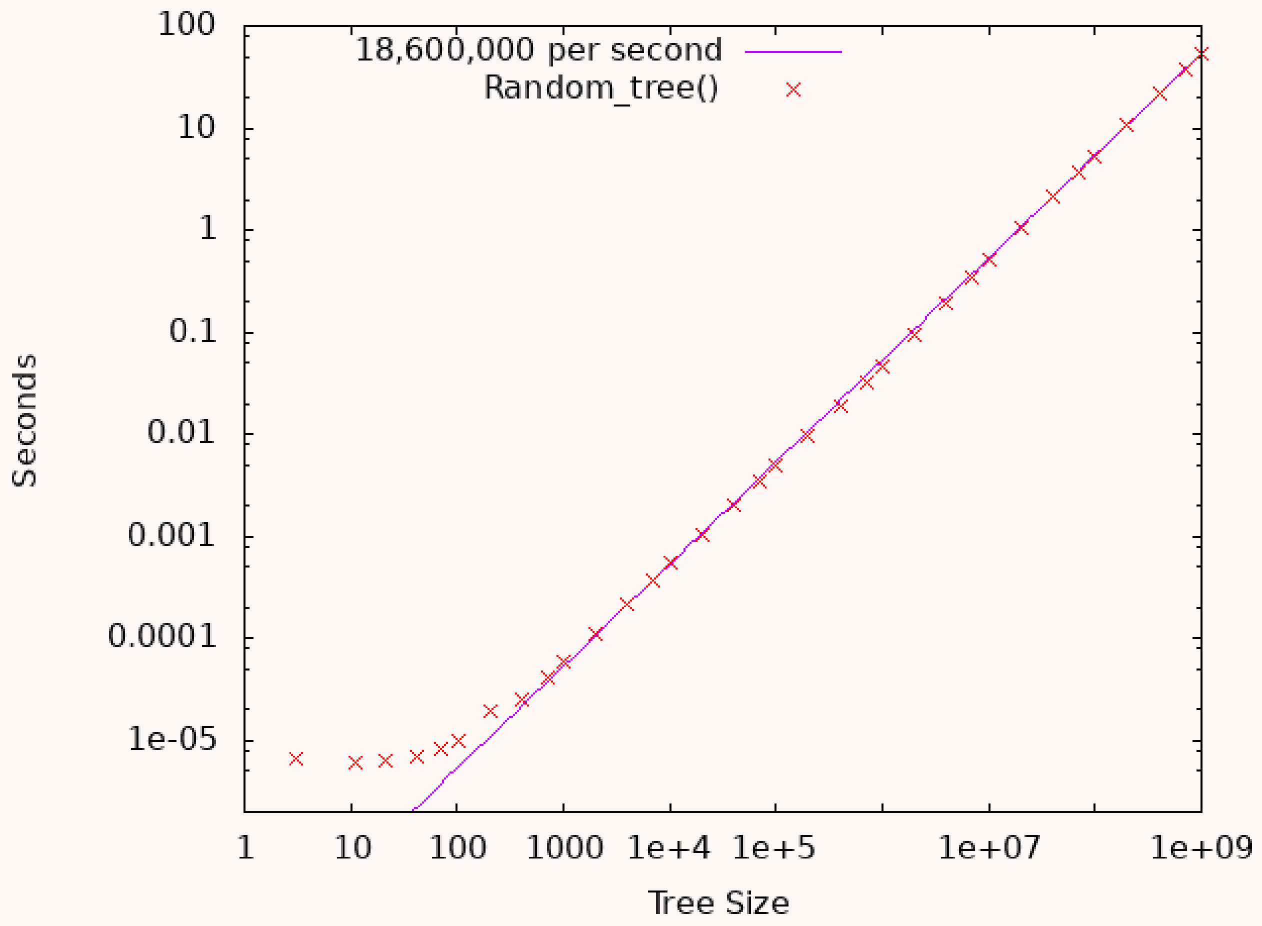
Speed of Large Random Trees



UCL Computer Science Research Notes

- UCL CS RN more than 20 years history
- UCL CS Research Notes no longer visible to the world
- Hostile to non-student www presence

Conclusions

- Can generate large trees quickly
  - up to 2 billion
- Original system was linear O(n), still O(n) but now usable for big trees
  - big O notation only gets you so far
- Applied to do fitness testing of genetic programming system GPquick
  - next week COW62
- Scope for further speed up if wanted
- C++ code available via www

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GI 2020

Genetic Improvement of Software
http://www.human-competitive.org/
Workshop at ICSE 2020

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https://icse20-gi8.hotcrp.com/

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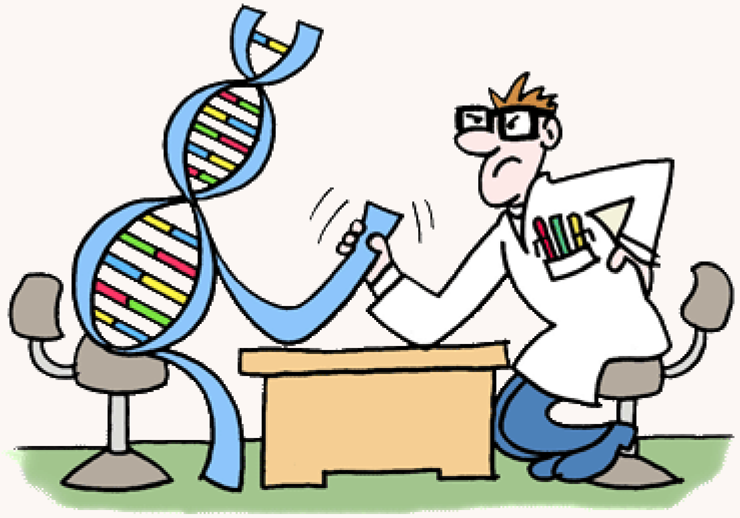

Humies
http://www.human-competitive.org/
Awards for Human-Competitive results
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Presentation at GECCO-2020 in Cancun, Mexico
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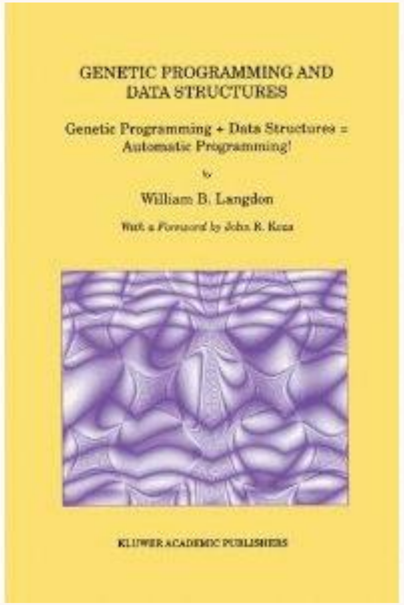


GENETIC PROGRAMMING AND DATA STRUCTURES

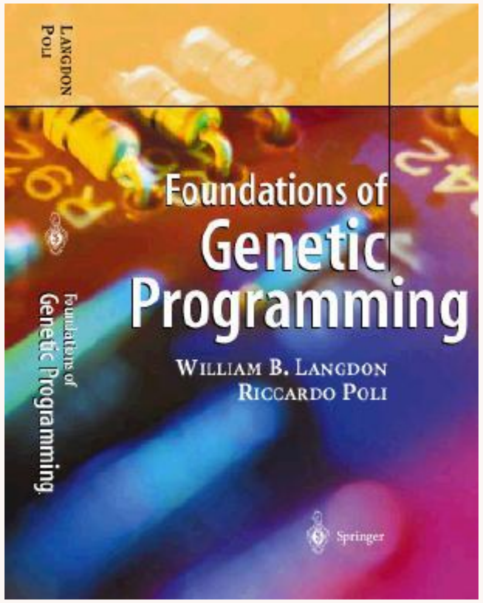
Genetic Programming + Data Structures = Automatic Programming!

by

William B. Langdon

With a Foreword by John R. Koza

KLUWER ACADEMIC PUBLISHERS



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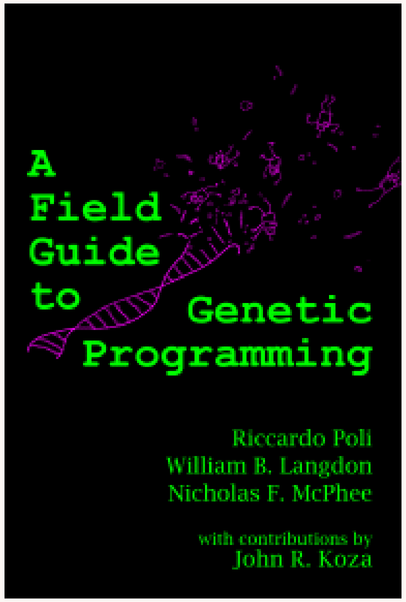
Foundations of Genetic Programming

WILLIAM B. LANGDON

RICCARDO POLI

Springer



A Field Guide to Genetic Programming

Riccardo Poli

William B. Langdon

Nicholas F. McPhee

with contributions by

John R. Koza

The Genetic Programming Bibliography

http://gpbib.cs.ucl.ac.uk/

13435 references, 12000 authors

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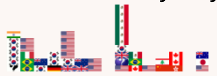

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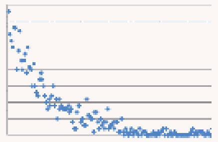

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