Erasmus University Rotterdam

FEM21040

Mathematical Programming Assignment 1 - Column Generation

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Graded assignment 1 - Column generation

In this assignment a 'Bin packing problem' is considered. The first part will be dedicated to the notation of the problem and the fact that a relaxation can be made to the problem without changing the actual problem. Thereafter, the bin packing problem will be solved for the case in which there is a limiting number of packings to choose from. The last two parts are concerned with solving the pricing problem such that new 'optimal' packings can be added to the original problem.

A. The mathematical representation of the 'Bin packing problem' is given in equation 1. In this representation each item is included in the solution exactly once. A modification to this representation replaces this equality with a 'greater or equal than' sign. This means that if there are multiple packings which all pack a certain item, they are allowed to be in the same solution, such that this item is in the solution more than once. This does not cause any problems, since you can always exclude this item for all packings but one to get a feasible solution to the original problem. Furthermore, since we are solving a minimization problem, carrying redundant copies of a certain item will probably be very unfavourable, and will automatically be excluded from optimal solutions.

$$\begin{aligned} &\min \textstyle\sum_{p \in P} x_p \\ &\textstyle\sum_{p \in P} a_{ip} x_p = 1 \quad \forall i \in N \\ &x_p \in \{0,1\} \qquad\quad \forall p \in P \end{aligned} \tag{1}$$

B. Using the initial set of packings the solution of the RMP is given by a vector in which each entry i indicates the amount of packing x_i used in the optimal solution. Note that the solution is probably not feasible, since it is a solution to the LP relaxation. The vector x_p is given below.

$$\begin{aligned} x_p = [&0.01, 0.0, 0.07, 0.29, 0.46, 0.0, 0.35, 0.64, 0.36, 0.0, 0.64, 0.15, 0.85, 0.01, 0.24, 0.48, 0.08, 0.92, \\ &1.0, 1.0, 0.30, 1.0, 0.40, 0.15, 0.06, 0.54, 0.46, 1.0, 1.0, 0.0] \end{aligned}$$

The optimal objective value of the initial RMP is: 12.44

C. Sub question 'B' gives the optimal solution to the 'Restricted Master Problem' (RMP). This solution is based on a limited number of packings to choose from. Since far more possible packings can be generated, there is probably a packing out there which could lower the objective value of the 'Master Problem' (MP). Since this is very favourable in a minimization problem we are interested in a packing which is not included in the RMP but lowers the objective value and should therefore be added to the RMP.

This problem is solved in the 'Pricing model', in which there is a search for the packing with the most negative reduced cost. We find the reduced cost of the problem by first solving the LP relaxation of the MP (which is the RMP), described in model 2.

$$\begin{aligned} &\min \textstyle\sum_{p \in P} x_p \\ &\textstyle\sum_{p \in P} a_{ip} x_p \geq 1 \quad \forall i \in N \\ &x_p \geq 0 \qquad\qquad \forall p \in P \end{aligned} \tag{2}$$

If we let λ_i be the dual of constraint i in problem description 2, then the reduced cost is given by equation 3.

$$RC\left(x_p\right) = 1 - \sum_{i \in N} \lambda_i a_{ip} \tag{3}$$

We can drop the index p, because a_{ip} is going to be the decision variable, such that the reduced cost could also be described by equation 4.

$$RC = 1 - \sum_{i \in N} \lambda_i a_i \tag{4}$$

Minimizing the reduced cost is equal to maximizing the sum $\sum_{i \in N} \lambda_i a_i$. This is exactly the objective of the pricing problem. The maximization problem is subjected to constraints regarding the maximum volume a packing can have (denoted by V), and the complete pricing model can be described by model 5.

$$\begin{aligned} & \max \textstyle\sum_{i \in N} \lambda_i a_i \\ & \textstyle\sum_{i \in N} v_i a_i \leq V \\ & a_i \in \{0,1\} \qquad \forall i \in N \end{aligned} \tag{5}$$

The dual variables are extracted from the solution of the RMP. Using these dual variables as input for the pricing model, CPLEX is used for solving the pricing model (an 'Integer Problem').

D. The results of the first seven iterations of the 'Column generation algorithm' are given in table 1. For each iteration the objective value of the RMP, the new added packing, and the reduced cost of this new added packing are given.

Tabel 1: Most important results of the 'Column generation algorithm' per iteration

Iteration	Objective value RMP	Items optimal packing	Reduced cost
1	12.44	[9, 10, 15, 28]	-2.29
2	11.76	[25, 33, 35, 38]	-2.08
3	11.33	[13, 15, 25, 31]	-1.89
4	10.97	[17, 20, 33]	-1.53
5	10.96	[3, 6, 21, 33]	-1.12
6	10.91	[12, 17, 28]	-0.97
7	10.83	[2, 32, 34, 38]	-1.12