Mathematical Programming
Graded assignment 2 - Column generation

Please conform to the following instructions:

1. Hand in a report as pdf file.
   (a) Use the tile "GA2_Report_<your studentnumber>.pdf", (for example "GA2_Report_123456.pdf").
   (b) At the start of your report mention your name and student number.
   (c) The report must be clearly written, concise and complete.
   (d) Use between 1 and 3 pages.
2. Hand in the used code in a zip file.
   (a) Use the title "GA2_Code_<your studentnumber>.zip", (for example "GA2_Code_123456.zip").
3. Hand in your report and code through Canvas.

Assignment

In this assignment, we consider the multi-commodity flow problem. Let $G = (N, A)$ be a directed graph. Associated with each arc $(i, j) \in A$ is a flow capacity Q_{ij} and a cost c_{ij} per unit of flow. Let K be the set of commodities. Associated with each commodity $k \in K$ is an origin $O(k) \in N$, a destination $D(k) \in N$ and a flow demand $F(k) \geq 0$ that has to flow from origin to destination. The multi-commodity flow problem is to have all the demand for each commodity flow through the network from origin to destination, such that the total costs are minimized, while satisfying the capacity constraints on each arc.

Next, an LP formulation is provided for the multi-commodity flow problem. Let f_{ijk} be the decision variable indicating the amount of flow of commodity $k \in K$ on arc $(i, j) \in A$. The LP formulation is as follows.

$$\min \sum_{k \in K} \sum_{(i,j) \in A} c_{ij} f_{ijk} \tag{1}$$

$$\sum_{k \in K} f_{ijk} \leq Q_{ij} \quad \forall (i,j) \in A \tag{2}$$

$$\sum_{j \in N \setminus \{i\}} f_{ijk} - \sum_{j \in N \setminus \{i\}} f_{jik} = \begin{cases} F(k) & i = O(k),\ \forall k \in K \\ 0 & \forall i \in N \setminus \{O(k), D(k)\},\ \forall k \in K \\ -F(k) & i = D(k),\ \forall k \in K \end{cases} \tag{3}$$

$$f_{ijk} \geq 0 \qquad \forall (i,j) \in A,\ \forall k \in K \tag{4}$$

Here, (1) is the objective of minimizing the total costs. Constraints (2) ensure that the flow over each arc does not exceed the capacity. Constraints (3) ensure that for each commodity, the outflow of the origin and the inflow of the destination is $F(k)$, and that the inflow matches the outflow for all other nodes. Finally, constraints (4) are the non-negativity constraints.

a. Construct a Dantzig-Wolfe reformulation. Clearly describe the decomposition into activities, and which constraints are the linking constraints.

b. Can you tell upfront whether it is possible for the subproblem to be unbounded?

c. Describe how you initialize the Dantzig-Wolfe decomposition algorithm.

d. Describe which variable(s) your algorithm adds at every iteration.

e. Solve the instance `MCFInstanceSmall.txt`. At each iteration, report

1. the solution value of the restricted master problem,
2. the extreme point or direction of the variable that is added to the restricted master problem,
3. the reduced cost of the new variable.

Also report the final solution.

f. Solve the instance `MCFInstanceLarge.txt`. Provide a plot of the value of the restricted master problem at each iteration.
You may set a time limit of 10 minutes for your code to run. In case the optimal solution is not yet found within this time, you still need to provide the plot, but clearly state that the algorithm has not found the optimal solution within the time limit.