FIN 513: Homework #8

Due on Tuesday, April 26, 2018

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Problem 1

(a) Assume that the amount of principal is equal to 1. Since the swap fee is payable at the end of each quarter, and there is no time value of money, the value of fee for each swap maturing at t_i is evaluated as $0.25 \times \sum_{t \leq t_i}^{t_i} \varphi(1 - H_{0,t})$, where $t \in \{0.25, 0.50, 0.75, 1.00\}$, and φ is a swap fee. Since risk-free rate is zero, current value of corresponding protection leg is evaluated as $(1 - R) \times H_{0,t_i}$, where R is recovery rate. By equating both equations, each H_{0,t_i} is calcuated as $\frac{0.25 \times \varphi \sum_{t \leq t_{i-1}}^{t_{i-1}} (1 - H_{0,t}) + 0.25\varphi}{(1-R) + 0.25\varphi}$. By calculating iteratively, we can calculate H_{0,t_i}. Table 1 shows the result.

Duration of CDS	Fee φ (BP)	H_{0,t_i}
3 months	900	0.0533
6 months	800	0.0927
9 months	750	0.1278
12 months	700	0.1562

Table 1: Cumulative default density: H_{0,t_i}

(b) Since the remaining time-to-maturity is one year, the current value I have to pay is equal to $0.25 \times 200 \times \sum_{t_i} (1 - H_{0,t_i})$ times the notional, where $t_i \in \{0.25, 0.50, 0.75, 1.00\}$. It is calculated as 0.8925 million dollars. In contrast, the current value of protection is calculated as $(1 - R) \times H_{0,1}$ times notional, which is about 3.1238 million dollars. Therefore, the current value of my position is $3.1238 - 0.8925 = 2.2313$ million dollars.

Problem 2

(a) Since H_T follows exponential distribution, cumulative distribution of D is derived as follows.

$$\begin{aligned}
Prob(D < u) &= Prob(e^{-aH_T} < u) \\
&= Prob(-aH_T < \log u) \\
&= Prob(H_T > -\frac{1}{a} \log u) \\
&= \int_{-\frac{1}{a}\log u}^{\infty} be^{-bx} dx \\
&= -e^{-bx} \Big|_{-\frac{1}{a}\log u}^{\infty} \\
&= u^{\frac{b}{a}}
\end{aligned}$$

Problem 2 continued on next page...

By using the same method, cumulative distribution of L is derived as follows.

$$
\begin{aligned}
Prob(L < v) &= Prob((1-R)e^{-aH_T} < v) \\
&= Prob(\log(1-R) - aH_T < \log v) \\
&= Prob(H_T > -\frac{1}{a}\log\frac{v}{1-R}) \\
&= \int_{-\frac{1}{a}\log\frac{v}{1-R}}^{\infty} be^{-bx}dx \\
&= -e^{-bx}\Big|_{-\frac{1}{a}\log\frac{v}{1-R}}^{\infty} \\
&= \left(\frac{v}{1-R}\right)^{\frac{b}{a}}
\end{aligned}
$$

$$v \in [0, 1-R]$$

Figure 1 shows cumulative distribution of D and L.

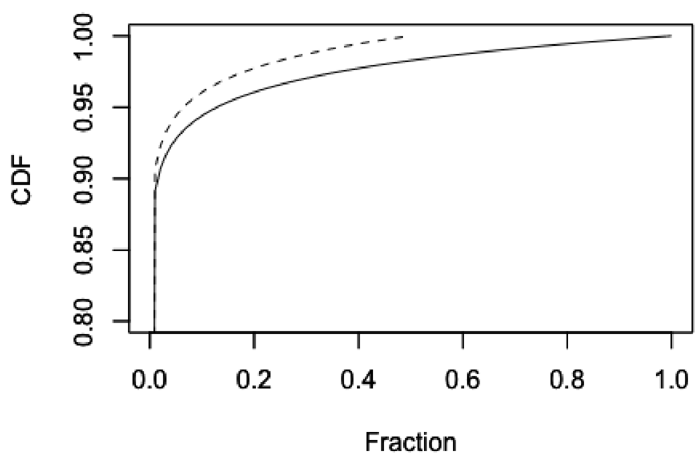

Figure 1: Cumulative distribution of D and L

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(b) Let x denote the lowest attachment point such that the probabilities of the tranche above that point experiencing any principal loss is p. Since cumulative distribution function is monotonically increasing, probability such that fraction of loss for CLO is greater than x is equal to p. (i.e. $Prob(L > x) = p$) Solving the equation, x is derived as follows.

$$Prob(L > x) = p$$

$$1 - \left(\frac{x}{1-R}\right)^{\frac{b}{a}} = p$$

$$x = (1-R) \times (1-p)^{\frac{a}{b}}$$

Table 2 shows the lower attachments for each tranche calculated by using the equation above. From the table about 50% of loans consists of AAA ratings.

Tranches	Probabilities	Attachments(Lower)
AAA	0.05%	0.490
AA	0.20%	0.462
A	1%	0.334
BBB	2%	0.223
BB	7.50%	0.022
Unrated		0.000

Table 2: Attachment of each tranche

(c) Let N denote the notional amount of CLO as a whole. Then, face value of each tranche is equal to proportion of each tranche times N. Assuming interests are paid annually, interest payment for each tranche is calculated. Table 3 represents annual interest payment for each tranche. Since we assumed

Tranches	Proportion	Interest	Payment
AAA	0.51	LIBOR + 0.25%	$-0.51 \times N \times (LIBOR + 0.25\%)$
AA	0.03	LIBOR + 0.5%	$-0.03 \times N \times (LIBOR + 0.5\%)$
A	0.13	LIBOR + 1.25%	$-0.13 \times N \times (LIBOR + 1.25\%)$
BBB	0.11	LIBOR + 2.5%	$-0.11 \times N \times (LIBOR + 2.5\%)$
BB	0.20	LIBOR + 4%	$-0.20 \times N \times (LIBOR + 4\%)$
Sum	0.98		$-0.98 \times N \times LIBOR + N \times 1.38\%$

Table 3: Annual payment of each tranche

that the bottom level tranche is entirely wiped out, and the loss is amortized equally, issuer will get $0.98 \times N \times (LIBOR + 4\%)$ amount of interest payment at each year. Therefore, the interest margin is calculated as $-0.98 \times N \times LIBOR + N \times 1.38\% - 0.98 \times N \times (LIBOR + 4\%) = N \times 2.54\%$. Therefore, interest margin for this scenario is 2.54% per year.

Problem 3

(a)

(b)

Problem 4

Problem 5

(a)

(b)

(c)

(d)