FIN 513: Homework #1

Due on Thursday, January 25, 2018

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Problem 1

(a) Agree. Generally, low-growth stock gives higher dividend than high-growth stock. Since high-growth company needs more capital than low-growth company, the amount of retained earning might be larger than that of low-growth company. Therefore, the futures price of high-growth stock will be a less discount over the spot price.

(b) Disagree. Regardless of possibility of bankrupt, a portfolio which replicates payoff of futures can be constructed. Therefore, we do not have to care about bankrupt when pricing futures.

(c) Agree. When constructing replicating portfolio which replicates the dynamics of value of derivatives to value the derivatives relatively, we assume self-financing. that means all dividends are reinvested. If price of stocks does not drop by the amount of the dividend per share, the value of position of replicating portfolio would be greater than the value of stock, and it contradicts the meaning of replicating portfolio.

Problem 2

Let r_L^D denote dollar rates for lending, r_L^E denote euro rates for lending, r_B^D and r_B^E denote dollar and euro rates for borrowing, respectively. Let S_t^B and S_t^O denote bid and offer exchange rate, respectively. In the following way, we can replicate long position and short position of dollar/euro forward.

(a) 1) *Replicate Long Position*

i. Borrow $S_t^O \frac{1}{(1+0.5r_L^E)^{2(T-t)}}$ dollars at rate r_B^D

ii. Exchange $S_t^O \frac{1}{(1+0.5r_L^E)^{2(T-t)}}$ dollars to $\frac{1}{(1+0.5r_L^E)^{2(T-t)}}$ euros, then invest for time $T-t$ at rate r_L^E.

At time T, the value of strategy (a) becomes $-S_t^O(\frac{1+0.5r_B^D}{1+0.5r_L^E})^{2(T-t)}$ dollars and the value of strategy (b) becomes 1 euro which perfectly replicates long position of forward contract. Since the initial values of two strategies are identical, the forward price $F_{t,T}$ should be equal to $S_t^O(\frac{1+0.5r_B^D}{1+0.5r_L^E})^{2(T-t)}$ dollars.

2) *Replicate Short Position*

i. Borrow $\frac{1}{(1+0.5r_B^E)^{2(T-t)}}$ euros at rate r_B^E

ii. Exchange $\frac{1}{(1+0.5r_B^E)^{2(T-t)}}$ euros to $S_t^B \frac{1}{(1+0.5r_B^E)^{2(T-t)}}$ dollars, then invest at rate r_L^D.

At time T, the value of strategy (a) becomes -1 euro, which is identical to short position of forward contract. Therefore, if there is no arbitrage in market, the forward price should be equal to the value of strategy (b) at time T, which is $S_t^B(\frac{1+0.5r_L^D}{1+0.5r_B^E})^{2(T-t)}$

Since $r_L^D < r_B^D$, $r_L^E < r_B^E$ and $S_t^B < S_t^O$, the value of replicate portfolio of short position is less than that of long position. Therefore, the upper and lower bound of the contract is $S_t^O(\frac{1+0.5r_B^D}{1+0.5r_L^E})^{2(T-t)}$ and $S_t^B(\frac{1+0.5r_L^D}{1+0.5r_B^E})^{2(T-t)}$, respectively. Table 1 shows that the numerical results using parameters on the homework sheet.

T	*Lower Bound*	*Upper Bound*
1 yr	1.4913	1.4940
5 yr	1.4833	1.4993
10 yr	1.5218	1.5526

Table 1: Lower bound and upper bound of forward price

(b) Since hedging a long position is equivalent to replicating short position, it is required to buy $\frac{1}{(1+0.5r_B^E)^{2(T-t)}}$ times notional amount of euros to hedge it. Therefore, the amount to buy spot euro is:

$$10,000,000 \times \frac{1}{(1+0.5\times 0.0325)^{20}} = 7,244,173.2450$$

Problem 3

Assume that the fair price of single price is x, then x should be larger than 308 and smaller than 313. Consider the following strategy.

(a) Get a long position on the original contract.

(b) Get a short position on the new contract.

Since the cash flow at time $t = 1.5$ is known at $t = t_0$, we can reinvest the net cash flow$(x - 308)$ at $t = 1.5$ to $t = 2$ by using the following strategy.

(1) sell short $x - 308$ amount of zero coupon bond with maturity $t = 1.5$

(2) buy $\frac{(x-308)B_{0,1.5}}{B_{0,2}}$ amount of zero coupon bond with maturity $t = 2$.

By netting out the values of strategy (1) and (2), there is no initial amount of cash flow. Furthermore, the strategies also makes cash flow at $t = 1.5$ to be zero. Finally, at time $t = 2$, the net cash flow from the whole strategies is equal to $x - 313 + (x - 308)\frac{B_{0,1.5}}{B_{0,2}}$. Since there is no cash flow before $t = 2$, the net cash flow at $t = 2$ should be equal to zero, otherwise there exists arbitrage opportunities. Therefore, the following equation holds.

$$x - 313 + (x-308)\frac{B_{0,1.5}}{B_{0,2}} = 0$$

$$\Rightarrow B_{0,1.5}x + B_{0,2}x = 308B_{0,1.5} + 313B_{0,2}$$

$$\Rightarrow (0.912 + 0.883)x = 308 \times 0.912 + 313 \times 0.883$$

$$\Rightarrow x = 310.4596$$

Therefore, the fair price that a market maker would be willing to offer is 310.4596.

Problem 4

(a) Since the loan has a fee of 25 basis points, lending 1,000,000 shares of stocks with loan fee is equivalent to lending 997,500 shares of stocks. If firm L did not lend out stocks, they will get 997,500 dollars for dividend and pay $997,500 \times 0.3 \times 0.3 = 89,775$ dollars for tax. However, if the firm lend out stocks, they should pay $1,000,000 \times 0.3 = 300,000$ dollars for tax. Therefore, firm L would require at least $300,000 - 89,775 = 210,225$ dollars from firm H.

(b) Consider the following agreement.

(1) Firm L lends 1,000,000 shares to broker.

(2) Broker sells equal amount of shares to firm E, and make forward contracts to repurchase after dividends are paid. It is possible since dividend schedule is certain.

(3) Firm E lends out 1,000,000 shares to firm H.

(4) At maturity of forward contract, broker repurchase stocks and get back to firm L immediately.

In this case, since the firm E has an ownership to stocks, E does not have to pay tax. Therefore, if the dividends are appropriately distributed for all parties, all firms would get more profit than (a).

Problem 5

(a) Assume that the contract is closed at time T. The contract is equivalent to a contract in which person who is in long position would get a stock, and would pay c and interest when the position is closed. Since we assumed that interest is debited or credited continuously, the total payment is equal to $ce^{r(T-t)}$. Like forward contract, we can replicate same payoff by taking long position to a stock, and borrowing the amount of money to buy a stock. At time T, the value of long position to a stock would be S_T and the value of short position to bond would be $S_t e^{r(T-t)}$. If there is no arbitrage in market, the value of short position to bond should be equal to the total payment in CFD. Therefore, the following equation should hold.

$$ce^{r(T-t)} = S_t e^{r(T-t)}$$

Therefore, the fair price of CFD is $c = S_t$. Otherwise, there would be arbitrage opportunities.

(b) Let t_0 and t_2 be closing times and t_1 be open time. Assume that $t_0 < t_1 < t_2$. An arbitrage opportunity exists by using the following strategy.

(1) Take a long position on CFD.

(2) Take a short position on a underlying stock.

(3) Invest S_{t_0} at rate r with maturity t_2.

Problem 5 continued on next page. . .

(4) Close every position at time t_2.

Since the interest is charged only for the position held overnight, the amount of interest at time t_2 would be $S_{t_0}e^{r(t_1-t_0)}$, and that is important key to make an arbitrage opportunity. Table 2 describes the payoff from the strategy. Since we already assumed that $t_2 > t_1$, the net payoff $S_{t_0}(e^{r(t_2-t_0)-r(t_1-t_0)})$ is greater than zero. Therefore, by using the strategy above, we can make arbitrage profit.

Position/Time	t_0	t_1	t_2
Long on CFD			$S_{t_2} - S_{t_0} - S_{t_0}(e^{r(t_1-t_0)} - 1)$
Short on Stock	S_{t_0}		$-S_{t_2}$
Invest to Bond	$-S_{t_0}$		$S_{t_0}e^{r(t_2-t_0)}$
Net Payoff	0		$S_{t_0}(e^{r(t_2-t_0)-r(t_1-t_0)})$

Table 2: Payoff structure of the strategy