FIN 513: Homework #7

Due on Tuesday, April 10, 2018

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Problem 1

(a) **Disagree.** Under CAPM, market price of risk for an arbitrary asset is defined as $\lambda_X = \rho_{X,M}\lambda_M$, where X is an arbitrary asset, and M denotes market. Let X denotes temperature, and if temperature increases are associated with lower stock prices, it means that correlation between temperature and stock price, $\rho_{X,M}$, is negative. Since market price of risk associated with stock price is positive, which associated with global warming will be negative, not positive.

(b) **Agree.** From the equation $\lambda_X = \rho_{X,M}\lambda_M$, market price of risk for an arbitrary asset is calculated as multiple of correlation coefficient between the asset and market and Sharpe ratio for the stock market. Since correlation coefficient is between -1 and 1, its absolute value must be smaller than the Sharpe ratio for the stock market.

Problem 2

(a) Since the market price of volatility risk is given as $\lambda(v) = \lambda_0 + \lambda_1 v$, risk-neutralized process of v is derived as follows.

$$\begin{aligned} dv_t &= \kappa(\bar{v} - v_t)dt - \lambda(v)s_v dt + s_v dW_t^v \\ &= (\kappa\bar{v} - \lambda_0 s_v - (\kappa + \lambda_1 s_v)v_t)dt + s_v dW_t^v \\ &= k^*(\bar{v}^* - v_t)dt + s_v dW_t^v \end{aligned}$$

where $\kappa^* = \kappa + \lambda_1 s_v$, and $\bar{v}^* = \frac{\kappa\bar{v} - \lambda_0 s_v}{\kappa^*}$. Since κ^* and $\bar{v}^*$ are constant, v_t follows Ornstein-Uhlenbeck process under risk-neutral measure.

(b) Let F denote forward price for the contract. If there is no arbitrage opportunity, F should satisfy the following equation.

$$\begin{aligned} \mathrm{E}^*[B_{0,T}(\sigma_T - F)] &= 0 \\ \Rightarrow F &= \mathrm{E}^*[\sigma_T] \end{aligned}$$

Where $B_{0,T}$ denotes discount factor from current date to the maturity and E^* denotes expectation operator under risk-neutral measure. It is known that a random variable which follows Ornstein-Uhlenbeck process is normally distributed, so $v_t = \log\sigma_t^2$ is normally distributed. Therefore, there is a closed form of $\mathrm{E}^*[\sigma_T]$. From the analogy of distribution in lecture note 7.2, v_T is distributed as $N(v_0 e^{-\kappa^* T} + \bar{v}^*(1 - e^{\kappa^* T}), \frac{s_v^2}{2\kappa^*}(1 - e^{-2\kappa^* T}))$. Since $\mathrm{E}^*[\sigma_T] = \mathrm{E}^*[e^{\frac{1}{2}v_T}]$, and $\frac{1}{2}V_T$ is distributed as $N(\frac{1}{2}[v_0 e^{-\kappa^* T} + \bar{v}^*(1 - e^{\kappa^* T})], \frac{s_v^2}{8\kappa^*}(1 - e^{-2\kappa^* T}))$, no arbitrage forward price is derived as follows.

$$F = \mathrm{E}^*[\sigma_T] = e^{\frac{1}{2}[v_0 e^{-\kappa^* T} + \bar{v}^*(1 - e^{\kappa^* T})] + \frac{s_v^2}{16\kappa^*}(1 - e^{-2\kappa^* T})}$$

(c) Letting $T \to 0$, since $e^{\kappa^* T} \to 1$, forward price F converges to $e^{\frac{1}{2}v_0}$, which is equal to σ_0.

Problem 3

Problem 4

Problem 5

(a)

(b)

(c)