FIN 513: Homework #3

Due on Tuessday, February 13, 2018

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Problem 1

The put-call parity of American options on a currency is $e^{-r_f(T-t)}S - K \leq C - P \leq S - e^{-r_d(T-t)}K$ or $B^f S - K \leq C - P \leq S - B^d K$, where S and K denotes spot exchange rate and strike price as usual, r_d and r_f denotes domestic risk-free rate and foreign risk-free rate, respectively.

(Proof) Suppose not.

1. Suppose $C - P > S - e^{-r_d(T-t)}K$. Then an arbitrage opportunity exists by constructing the following portfolio.

   (a) Buy a put option and a unit of foreign currency.

   (b) Write a call option and borrow $e^{-r_d(T-t)}K$ amount of domestic currency on risk-free rate.

   All possible situations can be separated by two cases: early exercise of written option occurs and no early exercise of written option. Assuming that early exercise occurs at $t^* < T$, payoff of the portfolio is as follows.

   Case 1. Early exercise at $t^* < T$ occurs.

   (a) $e^{r_f(t^*-t)}S_{t^*}$

   (b) $-(S_{t^*} - K) - e^{-r_d(T-t^*)}K$

   The sum of payoff from two strategies is $(e^{r_f(t^*-t)} - 1)S_{t^*} + (1 - e^{-r_d(T-t^*)})K$, which is positive.

   Case 2. No early exercise.

   i. $S_T > K$

   (a) $0 + e^{r_f(T-t)}S_T$

   (b) $-(S_T - K) - K$

   ii. $S_T \leq K$

   (a) $(K - S_T) + e^{r_f(T-t)}S_T$

   (b) $0 - K$

   In this case, the portfolio also has a positive payoff regardless of spot exchange rate at maturity.

   Since the portfolio has a positive payoff at all possible situations, there must be a cost for implementing the strategies if there is no arbitrage opportunity. However, by the assumption, the initial cost for constructing portfolio is negative, so a contradiction occurs. Therefore, $C - P \leq S - e^{-r_d(T-t)}K$ must hold.

2. Suppose $C - P < e^{-r_f(T-t)}S - K$. Then there is also an arbitrage opportunity exists considering the following portfolio.

Problem 1 continued on next page...

(a) Buy a call option and invest K amount of domestic currencies on domestic risk-free rate.

(b) Write a put option and sell short a foreign risk-free zero coupon bond.

By using similar procedure above, existence of arbitrage can be derived.

Case 1. Early exercise at $t^* < T$ occurs.

(a) $e^{rt^*}K$

(b) $-(K - S_{t^*}) - e^{-r_f(T-t^*)}S_{t^*}$

In this case, the sum of payoff from two strategies above is $S_{t^*}(1 - e^{-r_f(T-t^*)}) - (e^{rt^*} - 1)K$, which is positive.

Case 2. No early exercise.

i. $S_T > K$

(a) $(S_T - K) + e^{rT}K$

(b) $0 - S_T$

ii. $S_T \leq K$

(a) $0 + e^{rT}K$

(b) $-(K - S_T) - S_T$

At the maturity, the portfolio has a positive value regardless of spot exchange rate at T.

Since the portfolio has a positive payoff at all possible situations, there is an arbitrage opportunity since we assumed that there is a negative initial cost for constructing this portfolio. Therefore, $C - P \geq e^{-r_f(T-t)}S - K$ must hold.

Combining all results above, the put-call parity mentioned above must hold. Otherwise, there would be an arbitrage opportunity.

Problem 2

(a)

(b)

(c)

(d)

(e)

(f)

Problem 2 continued on next page...

(g)

(h)

Problem 3

(a) Figure 1 represents plots of function f, g, h over the range -1 to 2.

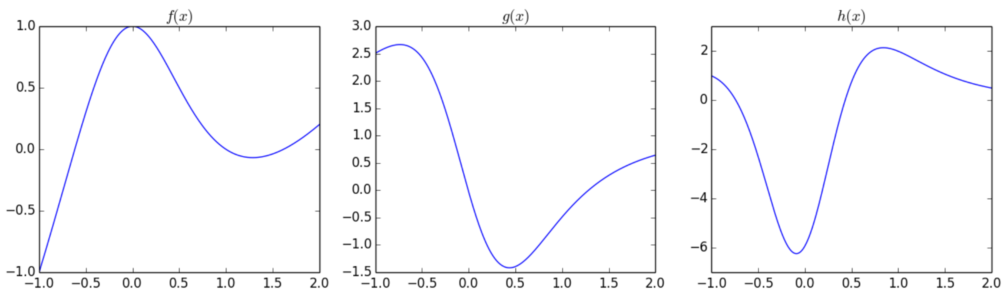

Figure 1: $f(x)$, $g(x)$, $h(x)$ (from left to right)

(b) From the figure 1, it seems that from $x = 0.5$(approximately) the function f is convex since its slope starts increase at this point. Actually, using some calculus, since $h(x) = -\frac{2(x^3-9x^2-3x+3)}{(x^2+1)^3}$, $f(x)$ is convex from $x = 0.4422$.

(c) It appears that from $x = -0.1$ to 0.8(approximately) the function looks convex. It is because function $h(x)$ starts to increase at -0.1 and stops increasing at 0.8.

(d) Visually, it seems that from $x = -0.5$ to 0.4 $h(x)$ looks convex.

Problem 4

(a)

(b)