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Lecture Note 2.2: Interest Rate Swaps

A large segment of the derivatives market consists of swap agreements where the periodic payments are tied to short-term interest rates in the future. These claims are a crucial tool of financial engineering and risk management.

In this note, we'll study the most fundamental type of interest rate swap (IRS) in our no-arbitrage framework. As with commodity swaps, we'll first derive expressions for the valuation and pricing of IRS under perfect markets.

In fact, when we step back from the most idealized contracts and settings, it becomes impossible to exactly replicate/hedge the cash-flows of IRS. But that is part of the reason the market is so big: theses swaps are not always redundant, but are themselves primary securities that help complete the market.

Outline:

I. Standard interest rate swaps

II. Valuation when the floating rate is riskless

III. Risky floating rates

IV. Speculating with swaps: an example

V. Current market developments

VI. Summary

I. Standard interest rate swaps.

- You may already be familiar with how basic – "vanilla" – interest rate swaps work.
  - ▶ One party agrees to make fixed periodic payments – which are quoted as a fixed interest rate on a fixed notional principal amount.
  - ▶ The other pays a *variable* rate on that amount.
  - ▶ The swap is described by:
    - ∗ The notional amount;
    - ∗ The fixed rate;
    - ∗ The variable rate fixing dates;
    - ∗ The method of determination of the variable rate;
    - ∗ The money exchange dates, and instructions.
  - ▶ Often both cash-flow streams originate with floating- and fixed-rate bond issues by corporations.
    - ∗ Companies and banks often change their mind about whether or not they want fixed-rate financing
    - ∗ If two parties swap cash flows from bonds of the same notional amount, the principals at maturity exactly off-set (unlike in currency swaps).

- **Example of swap terms:**
  - 7-year \$100m swap of 3.1% fixed (quarterly compounded) against 90-day LIBOR.
  - Quarterly re-set; quarterly payment. Day count = actual/360.
  - LIBOR as per ICE web page at 12:00 GMT.
- Note that the variable cash-flow paid on any given date has been determined 90 days earlier (in this example).
- The 90-day period is called the *tenor* of the swap.

- Interest rate swaps play a huge role in the capital markets. They are used by thousands of companies, governments, banks, and pension funds as a way of managing mismatches that may occur between the interest rate exposure in their liabilities and that in their assets.

- There are all kinds of other more complicated IRS too, for example, ones with very complicated formulas for the floating payment.

- Luckily, basic interest rate swaps are not too hard to understand.
  - Don't get intimidated by the ugly notation!

II. Valuation: riskless floating rate.

- At first glance, interest rate swaps seem a lot more complicated than simple fixed price delivery contracts (like commodity swaps) since we don't know all the cash flows in advance.

- It turns out that replicating them isn't that tough.

- As usual, we'll assume there are riskless zero-coupon bonds of all maturities that can be traded (long or short) with no transactions costs.

- Let's first think about valuing a swap in which the floating rate is *the riskless interest rate* between payment periods.

- Do this by considering the values of the fixed and floating sides separately, and then subtracting.

- The fixed side is trivial (just like for interest-only currency swaps). If the notional amount of the swap is X and the fixed interest rate (the "swap rate") is s per year, this side is just worth

$$\text{fixed side value} = X\ s\ \sum_{t=1}^{T} B_{0,t}.$$

  (The formula assumes s is paid annually. If payment is quarterly, say, then replace s by $\frac{s}{4}$, etc.)

- The floating side is a little trickier. At future date t, this side gets an amount that I will denote $X\ R_{t-1,t}$ where R is the riskless simple, per-period interest rate (i.e. not annualized).

- Question: how can we replicate a payment stream of amounts that are *unknown today?*

- First answer: **dynamically!**

- Consider the cash-flows from a strategy that does the following transactions:

(A) Sell short X face-value of risk-free zero coupon bonds (treasury-bills) maturing at T.

(B) Buy X worth of one-period T-bills, i.e. maturing at t the first payment date.

(C) When we get to t, reinvest the same principle (X) in the *new* one-period T-bills, i.e. maturing at $t+1$.

- Then at each swap date, $t < T$, the cash-flow is just the one-period interest earned, $X\ R_{t-1,t}$.

- And at T we use our maturing one-period principle X to pay for the maturing T-period bills that we sold short in step 1.

- What is the cost of this strategy?

- The cost is just the net cash we take in today, which is

$$\text{floating side value} = X - XB_{0,T}.$$

  This is the cost of funding a dynamic strategy that exactly replicates the floating-side cash-flows. Hence if the floating side value is not this amount, we have an arbitrage.

- **Conclusion:** Net value of the swap to the fixed payor is

$$V = 1 - B_{0,T} - s \sum_{t=1}^{T} B_{0,t}$$

(times the notional amount X).

- Also note that

$$\begin{aligned} 1 - B_{0,T} &= \sum_{t=1}^{T} [B_{0,t-1} - B_{0,t}] \\ &= \sum_{t=1}^{T} B_{0,t} \left[\frac{B_{0,t-1}}{B_{0,t}} - 1 \right] \\ &= \sum_{t=1}^{T} B_{0,t} \; R^f_{0,t-1,t}. \end{aligned}$$

- The last line uses the definition of the one-period *forward rate*.
  - ▶ $R^f_{0,t-1,t}$ is then the (simple) riskless rate we could lock in today between dates $t-1$ and t.
  - ▶ It is not hard to show that the no-arbitrage price of a forward contract to buy a bond at t_1 which matures at t_2 is $B^f_{0,t_1,t_2} = \frac{B_{0,t_2}}{B_{0,t_1}}$.
  - ▶ The forward interest rate is then related to the forward bond price by

$$B^f_{0,t_1,t_2} = \frac{1}{1 + R^f_{0,t_1,t_2}}$$

- Using the forward rate expression for the valuation of our floating side, the swap valuation formula is then

$$V = X[\sum_{t=1}^{T} B_{0,t}\, R^f_{0,t-1,t} - s \sum_{t=1}^{T} B_{0,t}] = X[\sum_{t=1}^{T} B_{0,t}(R^f_{0,t-1,t} - s)].$$

- **We can view interest rate swap values as weighted sum of the spreads of the fixed over the floating <u>forward</u> rates.**

- If we set $V = 0$ and solve for the fair swap rate s we get

$$s = \frac{\sum_{t=1}^{T} R^f_{0,t-1,t}\, B_{0,t}}{\sum_{t=1}^{T} B_{0,t}}.$$

- Just like for commodity swaps, **the fair rate of an interest rate swap is a weighted average of today's forward rates.**

- The above derivation was the first time we have used a *dynamic replication* strategy to price a derivative.

- Interestingly, we could have derived the same formulas by staying within the static replication framework we used to value commodity swaps.
  - That is, we can still view IRS as an agreement by one side to deliver a fixed amount of a commodity in exchange for a fixed amount of cash.

- The trick is realize that the "commodity" is a bond!
  - Specifically, it is a risk-free zero-coupon bond which matures a fixed amount of time after the delivery date.
- I will leave it as an exercise for you to show that you can exactly replicate the floating-side cash flows if you can trade forward contracts on these bonds.

- There is one other type of simple IRS whose cash-flows look a little different, and so a slightly different valuation formula applies.
- These are swaps tied to the very shortest term interest rate – the one-day rate for borrowing and lending reserves among banks.
  - This interest-rate – called Fed-Funds rate in the U.S. and EONIA in Europe – is not truly riskless because banks can default. But in practice even defaulting banks pay back their interbank reserve loans.
- A swap referencing to the overnight rate as the floating side is called an *overnight indexed swap* or OIS.
- These also tend to be short-term swap contracts, with maturities as short as one week.

- The convention for OIS is that, instead of making a payment every single day, the floating rate payor instead *compounds* all of the overnight interest payments (as if the money were reinvested each day) and then makes a single payment at the end.
  - So, for example, for a contract with $T = 1$ month, at time T the fixed receivor gets

    $$s \; - \; [\Pi_{t=0}^{T-1}(1 + R^{FF}_{t,t+1}) - 1].$$

    where R^{FF} is each day's realized one-day (not annualized here) interbank rate and s is the T-period simple rate (also not annualized).
  - The value of s in the market is called the OIS rate.
- As an easy exercise, you can derive the no-arbitrage value of s assuming there is no default risk.

III. Floating rates with credit risk.

- Historically, almost all interest rate swaps used LIBOR, or commercial paper, or other risky rates for their floating side.
  - ▶ This is because the swaps originate via risky corporations (especially banks) adjusting their funding profiles.
  - ▶ So the floating rate that matters to them is their own borrowing cost, which reflects their risk of default.
- The most popular swaps are still tied to either the 90-day borrowing rate for large banks (LIBOR, EURIBOR, etc).
  - ▶ These are risky rates because banks can (and do) default on these loans.
- How does the credit risk in the floating rate affect the determination of the no-arbitrage swap rate?
  - ▶ This is a surprisingly difficult question.
- Suppose you have promised to pay the floating side of a 1yr-LIBOR swap for T years.
- One way to replicate these payments (and to hedge your obligation) would be the same dynamic strategy we used above: simply invest the notional amount in 1-year LIBOR deposits and roll them over, while also borrowing against the terminal repayment of principal.

- ▶ Again, the replication cost would be $1 - B_{0,T}^{ED}$ (multiplied by the notional amount), where the superscript ED stands for Eurodollar rate – which is the same thing as LIBOR.

- But notice some assumptions have crept in here.
  - ▶ I'm investing in bank deposits at the LIBOR rate. What if the bank I'm investing in goes bankrupt? (There's a reason why they can't borrow at the riskless rate!)
    - ∗ Notice this is distinct from the risk of my *counterparty* going bankrupt....
    - ∗ ...unless the bank I invest with is my counterparty.
  - ▶ I also assumed that I could sell short a T-period LIBOR deposit, or borrow money from a bank at the LIBOR rate to date T.
    - ∗ If I do this with the same bank that I invested in each period, then this would hedge my default risk on my investment.
    - ∗ Note that I too have to be a LIBOR-rated entity to borrow at this rate.

- Conversely, suppose a counterparty promises you an annual stream of 1yr-LIBOR payments for T years. How could you hedge this? And how much cash would the hedging transactions produce today?

- You could do the opposite of the above: borrow each period at the LIBOR rate, and invest $B^{ED}_{0,T}$ to fund your terminal principal repayment.
  - ▶ Again, think of this as selling short a one-period risky bond each period and holding on to a T=period bond.
  - ▶ Again, you have the risk that the T-period bond defaults, which is ok only if the one-period bond you are short is of the same entity (so that your principal is hedged).
  - ▶ And, once more, those one-period bonds only truly offset your counterparty's promise if they are issued by him.

- Summarizing: exact replication of the floating side only works under some strict conditions.
  - ▶ But if it does then it also takes care of counterparty risk.
  - ▶ And the value is $(1 - B^{ED}_{0,T})$.

- If we have to assume the floating payor is a LIBOR-rated entity (e.g., a AA-rated bank) then what if we make the same assumption about the <u>fixed</u> payor?

- Then the promise of a LIBOR credit entity to pay an amount s at each future date clearly has value

$$s \cdot \sum_{t=1}^{T} B_{0,t}^{ED}$$

since, if we can transact in risky zero-coupon bonds of the counterparty, this is exactly what it would cost us to replicate the payments.

  ▶ Again, the formula is only valid if we can do the replication.

  ▶ If we can't literally trade counterparty deposits, we could realize the same value if we could purchase default insurance on each payment, since the cost of this insurance would be the same as counterparty's credit spread.

    * More about that later.

- Now let's put the two sides of the swap together.

  ▶ First recall that

$$1 - B_{0,T}^{ED} = \sum_{t=1}^{T} \left[B_{0,t-1}^{ED} - B_{0,t}^{ED} \right]$$

- ▶ We can re-write the floating side in terms of forward rates:

$$\begin{aligned}\sum_{t=1}^{T}\left[B_{0,t-1}^{ED}-B_{0,t}^{ED}\right] &= \sum_{t=1}^{T}\frac{B_{0,t-1}^{ED}-B_{0,t}^{ED}}{B_{0,t}^{ED}}\cdot B_{0,t}^{ED}\\ &= \sum_{t=1}^{T}\left[\frac{B_{0,t-1}^{ED}}{B_{0,t}^{ED}}-1\right]\cdot B_{0,t}^{ED}\\ &= \sum_{t=1}^{T}R_{0,t-1,t}^{f,ED}\cdot B_{0,t}^{ED}\end{aligned}$$

- ▶ In the last equality, I am defining $R_{0,t-1,t}^{f,ED}$ – the LIBOR forward rates – in direct analogy with the riskless case.

- ▶ As with T-bills, the forward rates are the LIBOR rates you could lock-in today, and the ratio of zero-coupon bond prices is the no-arbitrage value of a forward contract to deliver a LIBOR deposit in the future.

- ▶ Now subtract the value of the fixed side from this last expression.

- The difference between the fixed and floating values is

$$V_0(s)=\sum_{t=1}^{T}(R_{0,t-1,t}^{f,ED}-s)B_{0,t}^{ED}$$

just like the formula for T-bill swaps.

- The zero-value swap rate today likewise is

$$s_0^{ED} = \frac{\sum_{t=1}^{T} R_{0,t-1,t}^{f,ED} \; B_{0,t}^{ED}}{\sum_{t=1}^{T} B_{0,t}^{ED}}.$$

  The risky swap rate is still an average of risky forward rates.

- As we did with commodity swaps, we can re-write the value (to the fixed payor) of a swap entered into at rate s_{old}

$$V(s_0; s_{old}) = \sum_{t=1}^{T} (R_{0,t-1,t}^{f,ED} - s_{old}) B_{0,t}^{ED} = \sum_{t=1}^{T} (s_0 - s_{old}) B_{0,t}^{ED}$$

- When we compare interest rate swap rates based on LIBOR with the riskless yield curve, we can actually learn something quite interesting about the economy.

- "Swap spreads" are defined to be the difference between risky swap rates and the yield-to-maturity on a riskless (par) bond with the same maturity.

- Using the formulas we derived, we can express this difference in a neat way.

- The first step is to realize that our weighted-average-spread formula for LIBOR swaps hardly changes at all if we change the weightings slightly:

$$s^{ED} = \frac{\sum_{t=1}^{T} R^{f,ED}_{0,t\text{–}1,t}\; B^{ED}_{0,t}}{\sum_{t=1}^{T} B^{ED}_{0,t}}$$
$$\approx \frac{\sum_{t=1}^{T} R^{f,ED}_{0,t\text{–}1,t}\; B^{TB}_{0,t}}{\sum_{t=1}^{T} B^{TB}_{0,t}}.$$

- Here TB denotes risk-free zero-coupon bond (T-bill) prices.
- We are NOT saying these are the same as the risky zero prices, just that the relative weightings will be quite close for both types of bonds.

• Next, the yield-to-maturity on, say, the "current" 10-year Treasury, y^{TB}_{10}, is also the coupon a *new* 10-year treasury would pay if it were to have a price of par (i.e. 100% of face value).

• This means
$$1 = y^{TB}_{T} \sum_{t=1}^{T} B^{TB}_{0,t} + B^{TB}_{0,T}$$
which says
$$y^{TB}_{T} = \frac{1 - B^{TB}_{0,T}}{\sum_{t=1}^{T} B^{TB}_{0,t}}.$$

• But if we play our game of writing the numerator in terms of forward rates again, this is
$$y^{TB}_{T} = \frac{\sum_{t=1}^{T} R^{f,TB}_{0,t\text{–}1,t}\; B^{TB}_{0,t}}{\sum_{t=1}^{T} B^{TB}_{0,t}}.$$

- This is the exact same as our formula for the riskless swap rate!

- Now subtract this from our (approximate) expression for the LIBOR swap rate:

$$s^{ED} - y_T^{TB} = \frac{\sum_{t=1}^{T} \left[R_{0,t-1,t}^{f,ED} - R_{0,t-1,t}^{f,TB}\right] \; B_{0,t}^{TB}}{\sum_{t=1}^{T} \; B_{0,t}^{TB}}.$$

- **The swap spread is a weighted average of LIBOR forward credit spreads.**

IV. Swap Trading: An Example

- In 1998 LTCM – a large hedge fund – lost over a billion dollars with this position:

  long (=receiving fixed) 5-year-forward British pound-LIBOR swaps.

  short gilts (British government bonds).

  short (= paying fixed) 5-year-forward Deutsche mark-FIBOR swaps.

  long bunds (German government bonds).

- **Question:** What bet were they making?

- To answer this, let's examine the pieces. Take just the UK side first.

- We now know that the value of a LIBOR swap is determined by an average of risky forward rates.

- So there are two ways paying floating can win.

  1. All rates could go down, or

  2. the risk-premium in LIBOR could decline.

- The fund managers were paying floating in pounds AND short gilts. The short gilt position was calibrated precisely to off-set the first exposure – i.e. to the level of rates.

- So we can conclude: <u>they were long swap spreads.</u>

The pound position was a bet that U.K. credit spreads would narrow.

- Since the swaps involved were forward, the bet was that this would happen *or be expected to happen* five years in the future.

- The Deutsche mark position was exactly the reverse bet – for Germany.

- This meant that the trade did not necessarily require either countrys' credit spreads to go up or down. Instead, the whole thing boils down to...

  speculating that U.K. and German credit spreads would converge.

- The firm believed that Britain would eventually join Euro, but didn't think there was much of an edge in just betting that rates themselves would converge.
  - However, once in the Euro, British LIBOR and Deutsche mark LIBOR would be the *same rate*, i.e. there would only be one interbank rate.
  - So only one credit spread for bank risk could prevail.

- As a footnote, the short gilt/bund position involved in this trade was actually accomplished via a separate swap: a *return swap*.
  - We'll talk about them next time.

V. Recent developments

- Even though interest rate swaps played no role at all in the 2007-2008 financial crisis, U.S. and European regulators have concentrated a lot of their efforts on transforming this market.

- As a result, the IRS market today operates very differently than it did even 5 years ago.
  - ▶ As of 2014, all ordinary IRS traded by U.S. entities are required to be cleared through a central counterparty, which then marks them to market daily.
    - ∗ SwapClear and the CME Group are the two biggest CCPs.

    Since the clearing firms are very safe entities, in theory, we should be able to ignore counterparty default risks for cleared swaps.

  - ▶ In addition, U.S. entities are no longer allowed to trade IRS by bilateral OTC negotiation.
    - ∗ Instead, they are required to use electronic platforms called "swap execution facilities" (SEFs) which are supposed to ensure that all interested participants have equal access to prices that are posted.
    - ∗ Tradeweb and Bloomberg have so far been the most successful platforms.
    - ∗ As of January 2018, similar requierments apply to European entities.

- All U.S. swap trades now need to also be reported daily to specialized data collection firms called "swap data repositories" (SDRs). These firms make the data available to the public.

- See for yourself:
  `http://www.swapsinfo.org`
  `https://fia.org/sef-tracker`

- Another important recent development is the phasing out of LIBOR as a benchmark.
  - You are probably aware that the process of determining LIBOR was historically subject to manipulation by banks, and was not based on observations of banks' true borrowing costs.
  - While the process of surveying banks has improved, many participants no longer trust LIBOR rates, particularly for longer loan horizons or for smaller currencies.
  - Regulators in the U.K. will not require banks to respond to the LIBOR surveys after 2020.
  - But what will replace it?

- ▶ The Federal Reserve and US industry groups are pushing for the use of "SOFR", which is the rate for repo loans, i.e., ones collateralizeed by government bonds.
  - ∗ These are very close to riskless loans.
  - ∗ On the other hand, there is not an active market for repo loans for terms beyond one week.

  If LIBOR goes away completely, there will also be a very difficult and messy job of converting old swaps that reference it to some new rate, which may require renegotiation on a case-by-case basis.

- Overall, there is a lot of important change happening in the infrastructure of the market for interest rate swaps.

VI. Summary.

- Even though interest rate swaps seem very different from commodities swaps, the valuation formulas *look the exact same*.

- Replicating the floating side of an interest rate swap can easily be done *dynamically* by rolling over one-period investments at the floating rate.

- The main assumption behind the formula: no counterparty risk.

- There are active liquid markets for standard swaps based on interbank floating rates for every major currency for maturities as long as 30 years.
  - ▶ These are extremely valuable tools for speculation and hedging. However, they may not themselves be pure derivatives.
  - ▶ When the floating rate is not the riskless rate replication of the swap is only possible if (a) the floating rate is the borrowing cost of the floating rate payor in the swap; and (b) one can trade (short-sell) bonds of that party or otherwise hedge his default risk.

Lecture Note 2.2: Summary of Notation

Symbol	Page	Meaning
s	*p4*	*fixed rate for an interest rate swap*
X	*p4*	*Notional amount of interest rate swap*
R_{t_0,t_1}	*p4*	*simple interest rate at* t_0 *for a payment at* t_1
$R^f_{t_0,t_1,t_2}$	*p6*	*simple (uncompounded) forward rate at* t_0 *for*
		an investment at t_1 *paying off at* t_2
$B^f_{t_0,t_1,t_2}$	*p6*	*forward price at* t_0 *of a zero-coupon bond deliverable at*
		t_1 *maturing at* t_2
V	*p6*	*value of a swap with fixed rate* s
R^{FF}	*p9*	*interbank (Fed Funds) interest rate*
Π	*p9*	*product (multiplication) operator*
$R^{ED}, R^{f,ED}, B^{ED}$	*p11*	*interest rates, forward rates, and zero-coupon bond prices*
		for a company of LIBOR (or Eurodollar) credit
s_0^{ED}	*p14*	*fair rate for a new LIBOR swap at* $t = 0$
$R^{TB}, R^{f,TB}, B^{TB}$	*p15*	*interest rates, forward rates, and zero-coupon bond prices*
		for a risk-free credit (eg treasury-bills)
$V(s_0; s_{old})$	*p15*	*value to fixed-payor of swap entered into at rate*
		s_{old} *when current rate is* s_0
y_T^{TB}	*p16*	*yield-to-maturity on a treasury bond maturing at* T
		$=$ *coupon rate for a treasury which is worth its face value*