FIN 513: Homework #2

Due on Thursday, February 1, 2018

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Problem 1

(a) (1) *(Zero-coupon term structure)* Since the bonds are traded at par and these are riskless, zero-coupon term-structure can be obtained by just equating present value of their cash flow to their price(= 100). Therefore, the following equation holds.

$$100 = \sum_{t=1}^{T} \frac{\textit{Cash flows at time } t}{(1+r_{0,t})^t}$$

In order to obtain the whole term-structure, one should get $r_{0,t}$ first by equating the value of bond with maturity t to present value of its cash flow first, then $r_{0,t+1}$ can be obtained by using $r_{0,t}$. It is because it is necessary to use $r_{0,t}$ to discount cash flows of bond maturing at $t+1$. Therefore, calculate $r_{0,1}$ first.

$$100 = \frac{100 + 100 \times 0.0133}{1 + r_{0,1}} = \frac{100(1 + 0.0133)}{1 + r_{0,1}}$$

It is trivial that $r_{0,1} = 0.0133$. Then, let's use this result to calculate $r_{0,2}$.

$$100 = \frac{100 \times 0.0173}{1 + 0.0133} + \frac{100 + 100 \times 0.0173}{(1 + r_{0.2})^2}$$

Then it is calculated that $r_{0,2} = 0.01733$. Using this procedure ahead, zero-coupon term-structure from 1 year to 10 year can be obtained. Table 1 shows that the term structure of zero coupon bonds obtained by using this procedure.

Maturity(years)	$r_{0,t}(\%)$
1	1.330
2	1.733
3	2.162
4	2.577
5	2.752
6	3.196
7	3.435
8	3.459
9	3.474
10	3.562

Table 1: Term structure of zero coupon bonds

(2) *(Term-structure of one-year forward rates)* Let $r_{0,t,t+1}$ denote a forward rate from time t to $t+1$ determined at current time. Then $r_{0,t,t+1}$ can be obtained by comparing the following two strategies.

i. Invest \$1 to zero coupon bond maturing at time t, and when the bond matures, receive money and reinvest to zero coupon bond maturing at time $t+1$.

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ii. Invest \$1 to zero coupon bond maturing at time $t+1$.

Since the amount of investment at current time is equal for both strategies, by definition of forward rate, the following equation should hold.

$$(1+r_{0,t+1})^{t+1} = (1+r_{0,t})^t(1+r_{0,t,t+1})$$
$$\Rightarrow r_{0,t,t+1} = \frac{(1+r_{0,t+1})^{t+1}}{(1+r_{0,t})^t} - 1$$

Therefore, since the term structure of spot rate is given above, by using this formula term structure of 1-year forward rate can be obtained. Table 2 shows term structure of 1-year forward rate using the data given in assignment.

t	$r_{0,t,t+1}(\%)$
1	2.139
2	3.025
3	3.830
4	3.459
5	5.441
6	4.881
7	3.627
8	3.592
9	4.363

Table 2: Term structure of 1-year forward rate

(b) A fair swap rate s is a rate which makes the present value of fixed leg and that of floating leg equal. Therefore, by calculating present values of each leg separately and equating each other, fair swap rate can be obtained. Let X and T denote the notional principal of swap and the maturity of swap, respectively. Then using the following procedure, fair swap rate s can be obtained.

(1) *(Fixed leg)* Since there is no exchange of notional principal in IRS, present value of fixed leg can be obtained by solving following equation.

$$\sum_{t=1}^{T} \frac{sX}{(1+r_{0,t})^t}$$

(2) *(Floating leg)* Cash flows of floating leg can be replicated by using the following investment strategies.

i. Invest X amount of dollars to zero coupon bond maturing at time 1. When the bond matures, receive money and reinvest X amount of dollars to zero coupon bond maturing at time 2, and repeat the procedure until time $T-1$.

ii. Borrow zero coupon bond maturing at time T with face value X.

Using the strategies above, the initial value of floating leg is $X - \frac{X}{(1+r_{0,T})^T}$.

As mentioned above, a fair swap rate s makes the present value of both position equal. Therefore, the following equation holds.

$$\begin{aligned}\sum_{t=1}^{T} \frac{sX}{(1+r_{0,t})^t} &= X - \frac{X}{(1+r_{0,T})^T} \\ \Rightarrow s &= \frac{1 - \frac{1}{(1+r_{0,T})^T}}{\sum_{t=1}^{T} \frac{1}{(1+r_{0,t})^t}} \\ &= \frac{1 - B_{0,T}}{\sum_{t=1}^{T} B_{0,t}}\end{aligned}$$

Since $1 - B_{0,T} = \sum_{t=1}^{T}(B_{0,t-1} - B_{0,t}) = \sum_{t=1}^{T} B_{0,t}(\frac{B_{0,t-1}}{B_{0,t}} - 1) = \sum_{t=1}^{T} B_{0,t}R^f_{0,t-1,t}$, the equation $s = \frac{\sum_{t=1}^{T} B_{0,t}R^f_{0,t-1,t}}{\sum_{t=1}^{T} B_{0,t}}$ also holds. Using the equation, fair swap rates of any maturity can be calculated if sufficient data is given. Table 3 shows term-structure of fair swap rate using given data.

Maturity(year)	Swap Rate(%)
1	1.330
2	1.730
3	2.150
4	2.550
5	2.720
6	3.130
7	3.350
8	3.380
9	3.400
10	3.480

Table 3: Term structure of fair swap rate

(c) Since I am a floating payor, the value of position is worth value of fixed leg minus value of floating leg. Let s^*, X denote a swap rate and notional pricipal of the contract, respectively. Then the value of my position is $Xs^* \sum_{t=1}^{9} B_{0,t} - X(1 - B_{0,9})$. Therefore, my position is worth $100 \times 0.0430 \sum_{t=1}^{9} B_{0,t} - 100(1 - B_{0,9}) =$ 7.004 million dollars. If the swap rate increases by 1 basis point(i.e. if the swap rate were 4.31%), then by using the same formula, the value of my position would be worth 7.082 million dollars, which is 0.078 million dollars greater than the original value. In contrast, if the swap rate decreases by 1 basis point(i.e. if the swap rate were 4.29%), then my position value would be worth 6.926 million dollars, which is 0.078 million dollars less than the original value.

Problem 2

(a) Both companies will prefer the following contracts rather than the original contracts.

(1) *(Company A)*

i. Make a loan with fixed rate 12.0%

ii. Make a swap contract with paying LIBOR and receiving 12.0%.

(2) *(Company B)*

i. Make a loan with floating rate LIBOR + 0.6%.

ii. Make a swap contract with paying 12.1% and receiving LIBOR.

By using the strategies above, the net rate where company A has to pay is 12.0% + LIBOR - 12.0% = LIBOR, which is less than the offered floating rate. Furthermore, the net rate company B has to pay is LIBOR + 0.6% + 12.1% - LIBOR = 12.7%, which is also less than the offered fixed rate. Acting as an intermediary, the bank receives (LIBOR - 12.0%) + (12.1% - LIBOR) = 10 basis points. Therefore, since every firm makes more profit by making the contracts above, all of them will prefer these contracts rather than the original contracts.

(b) X and Y will both prefer the following contracts rather than the original one.

(1) *(Company X)*

i. Borrow the amounts of Yen required at rate 4.0%

ii. Make a currency swap contract with paying 5.0% interest of Yen, and receiving 8.5% interest of U.S dollars.

(2) *(Company Y)*

i. Borrow the amounts of U.S dollars required at rate 8.4%

ii. Make a currency swap contract with paying 8.2% interest of U.S dollars, and receiving 5.2% interest of Yen.

If both firms agree to make the contracts above, they would receive more profit. The effective cost of funding U.S dollars for company X is 4.0 + 8.5 - 5.0 = 7.5%, and the effective cost of funding Yen for company Y is 8.4 - 8.2 + 5.2 = 5.4%. Both firms have lower costs if they agree to make the contracts. In addition, the bank acting as an intermediary receives (8.5 - 8.2) + (5.2 - 5.0) = 50 basis points per annum. This can be possible since company X has a comparative advantage of borrowing Yen, and company Y has the advantage of borrowing U.S dollars.