FIN 513: Homework #8

Due on Tuesday, April 26, 2018

Wanbae Park

Problem 1

(a) Assume that the amount of principal is equal to 1. Since the swap fee is payable at the end of each quarter, and there is no time value of money, the value of fee for each swap maturing at t_i is evaluated as $0.25 \times \sum_{t \le t_i}^{t_i} \varphi(1 - H_{0,t})$, where $t \in \{0.25, 0.50, 0.75, 1.00\}$, and φ is a swap fee. Since risk-free rate is zero, current value of corresponding protection leg is evaluated as $(1 - R) \times H_{0,t_i}$, where R is recovery rate. By equating both equations, each H_{0,t_i} is calcuated as $\frac{0.25 \times \varphi \sum_{t \le t_{i-1}}^{t_{i-1}} (1 - H_{0,t}) + 0.25\varphi}{(1-R) + 0.25\varphi}$. By calculating iteratively, we can calculate H_{0,t_i}. Table 1 shows the result.

Duration of CDS	Fee φ (BP)	H_{0,t_i}
3 months	900	0.0533
6 months	800	0.0927
9 months	750	0.1278
12 months	700	0.1562

Table 1: Cumulative default density: H_{0,t_i}

(b) Since the remaining time-to-maturity is one year, the current value I have to pay is equal to $0.25 \times 200 \times \sum_{t_i} (1 - H_{0,t_i})$ times the notional, where $t_i \in \{0.25, 0.50, 0.75, 1.00\}$. It is calculated as 0.8925 million dollars. In contrast, the current value of protection is calculated as $(1 - R) \times H_{0,1}$ times notional, which is about 3.1238 million dollars. Therefore, the current value of my position is $3.1238 - 0.8925 = 2.2313$ million dollars.

Problem 2

(a) Since H_T follows exponential distribution, cumulative distribution of D is derived as follows.

$$
\begin{aligned}
Prob(D < u) &= Prob(e^{-aH_T} < u) \\
&= \int_0^u e^{-ax} \times be^{-bx} dx \\
&= \int_0^u be^{-(a+b)x} dx \\
&= -\frac{b}{a+b} e^{-(a+b)x} \Big|_0^u \\
&= -\frac{b}{a+b} [e^{-(a+b)u} - 1]
\end{aligned}
$$

Because L is proportional to D, cumulative distribution of D is derived as $(1 - R) \times Prob(D < u) = -(1 - R)\frac{b}{a+b}[e^{-(a+b)u} - 1]$. Figure 1 shows cumulative distribution of D and L. From the figure, it can be discovered that fraction of borrowers default and proportion of loss converges to some value. As u increases, $e^{-(a+b)u}$ converges to zero, therefore D converges to $\frac{b}{a+b}$ and L converges to $(1 - R)\frac{b}{a+b}$.

Problem 2 continued on next page. . .

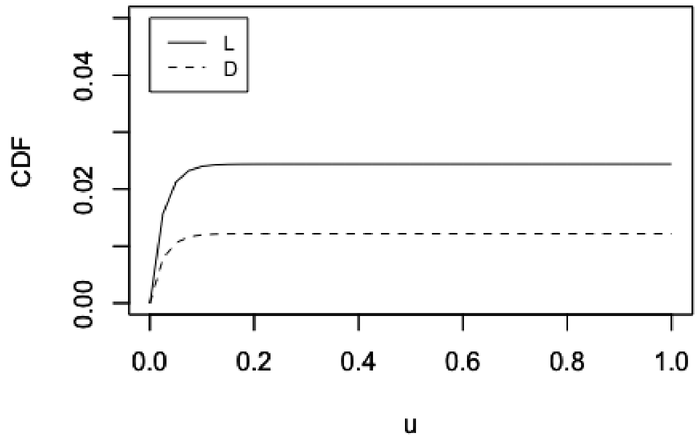

Figure 1: Cumulative distribution of D and L

(b)

(c)

Problem 3

(a)

(b)

Problem 4

Problem 5

(a)

(b)

(c)

(d)