Finance 513 - Spring 2018 | *Professor Johnson*
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Homework 8

1. (*Risk-neutral default distribution.*) Current market prices for short-dated CDS on the government of Ukraine senior unsecured US dollar debt are shown in the table below as a function of the length of coverage.

Duration of CDS:	fee φ (BP):
3 months	900
6 months	800
9 months	750
12 months	700

The swap fees are also in dollars and are payable quarterly in arrears (i.e. at the end of the covered quarter), and all rates are quoted on an annualized, quarterly-compounded basis.) If default happens within any quarter, the cash-flows from the CDS are paid at the end of the quarter. Assume there is no counterparty risk in these contracts, and that the recovery rate of Ukraine's senior unsecured dollar debt in default will be 60% with certainty. Finally, assume that the riskless interest rate for $t \leq 1$ year is zero.

(a) Use the CDS fees to compute the cumulative default density H_{t,t_i} for each quarterly maturity $t_i = 0.25, 0.50, \ldots, 1.0$ year.

(b) Suppose one year ago you had gone long Ukraine protection (bought a CDS) on 50 million USD notional for 2 years at a fee of 200 BP. How much is your position worth today?

2. (*Tranching.*) You work for an asset-backed finance department at a major investment bank. You have purchased a pool of 10000 automobile loans which all mature in $T = 4$ years. There is no amortization (pre-payment) of principal before maturity. At T, (but not before then) a fraction, D, of borrowers will default, and only 50% of the principal from them will be recovered.

Suppose the fraction D is given by $e^{-a\ H_T}$ where H_T measures the health of the economy. Assume H_T is exponentially distributed, i.e., has density $f(x) = be^{-b\ x}$.

(a) Derive an expression for the cumulative distribution of D, i.e. $Prob(D < u)$ for any u between 0 and 1. Also compute the cumulative distribution of the percentage, $L = (1 - R)D$, of principal lost at T. Plot both distributions for the case $a = 40, b = 1$.

(b) Assuming $a = 40, b = 1$, find attachment points for a CLO with tranches, rated AAA, AA, A, BBB, BB and a bottom (unrated) tranche as follows. Working from the top down, you choose the lowest attachment point such that the probabilities of the tranche above that point experiencing any principal loss are (in order) 0.05%, 0.20%, 1%, 2%, and 7.5%. (These probabilities are assumed to be sufficient to achieve the desired ratings.)

(c) Assume that, to be sold for their face value, the top five tranches must pay interest rates of LIBOR+0.25%, LIBOR+0.5%, LIBOR+1.25%, LIBOR+2.5%, and LIBOR+4%. Suppose that the underlying loans pay interest of LIBOR+4%. If you keep the bottom tranche and assume it is entirely wiped out (ammortizing the loss equally over 4 years), what is the interest margin (net profit to you) on this transaction?

3. (*CVA.*) You are a swap dealer at a large bank and one of your clients is a sovereign entity, i.e. an arm of the government of country X. Country X refuses to collateralize or centerally clear any of its swaps. They want to enter into an interest-only currency swap with you in which they deliver one unit of their currency each period and you pay them φ dollars.

   For simplicity, assume the domestic rate (dollars) is a constant $r_d = 0.01$ and the foreign rate is constant $r_f = 0.05$. (Note that we are assuming the existence of a riskless debt market in both countries, even if the government itself cannot issue riskless debt.)

   Also for simplicity assume the cash-flows are paid continuously: they pay you 1 dt foreign currency units per period, and you pay them $\varphi\ dt$ dollars.

   Now assume that under the risk-neutral measure the default intensity of country X follows

$$d\lambda/\lambda = b_\lambda^Q\ dt + \sigma_\lambda\ dW^\lambda \quad \text{or} \quad d\log\lambda = (b_\lambda^Q - 0.5\sigma_\lambda^2)\ dt + \sigma_\lambda\ dW^\lambda$$

   and also the spot exchange rate, S, under the RN measure obeys

$$d\log S = (r_d - r_f - 0.5\sigma_S^2)\ dt + \sigma_S\ dW^S$$

   in dollars per foreign currency unit.

   (a) Give an expression for the value $V(S_t, t; \varphi_0)$ to you the receivor of foreign currency of an interest-only swap to time T that would prevail at time t when the current spot rate is S_t and the fixed swap rate is φ_0 assuming no credit risk to either party. Assume $T = 5$ years and the current spot rate is $S_0 = 1.0$, then what value of φ_0 makes your valuation formula equal to zero.

   (b) Suppose that you do the 5 year swap with your counterparty at the rate you just calculated. Assume that the current risk-neutral default intensity is 0.005 and its risk neutral expected percentage growth rate and volatility are $b_\lambda^Q = 0.25, \sigma_\lambda = 0.50$. Also assume the spot currency volatility is 0.20.

      Compute and plot your CVA for this trade as a function of the correlation between dW^λ and dW^S using values between 0 and 1. What is the monetary value of this charge when the correlation is 0.5 if the dollar notional value of the swap is 1 billion dollars?

4. (*Subprime losses.*) Agree or disagree with the following statement, and explain your reasoning.

   Investors who bought the "super senior" AAA tranche of a residential mortgage backed security (RMBS) consisting of a pool of subprime ARMs originated in California in 2006 would have experienced extreme loss of principal in 2010 when the cumulative default rate on the pool reached 40% and the average recovery rate fell to 40%.

5. (*MBS.*)

Consider a mortgage backed security (MBS) that has no default risk because of government guarantees. Assume the mortgages in the pool each have the same principal amount and each has a 6 % fixed interest rate (in annual, monthly-compounded terms) and 30 years of life. This problem considers how the possibility of homeowner refinancing affects the NPV of the cash-flows from the pool and its sensitivity to interest rates.

Following the lecture notes, we will assume a prepayment function that has a deterministic component, a random component, and a component that is due to interest rates. Prepayments will be parameterized by the function $\pi(r, e, t)$ given by

$$\pi = \bar{\pi}(t) + b(r_t - c) + e_t$$

where b and c are constants, and $\bar{\pi}(t) = \bar{\pi}(n)$ is the function

$$-6 - \frac{12}{n} + .02 * (120 - n)$$

and $n = 1, \ldots 360$ is the month in the life of the mortgages. Assume $c = .06, b = -30$.

At each month, the total fraction of mortgages prepaid during that month is $Z \equiv e^{\pi}/(1 + e^{\pi})$. Thus the cash-flow in that month from prepayment is $Z * P * O$ where O is the fraction of remaining (non-prepaid) mortgages at the start of the period and $P = P_n$ is the remaining principal balance of each of these mortgages. (All the mortgages are assumed to start at the same date and thus have the same values of P at all subsequent dates.) After the prepayment in month n, we have $O_{n+1} = O_n(1 - Z_n)$.

We want to evaluate Z along a large number of simulated paths of the state variables e and r. In continuous time notation, we will assume

$$dr = \kappa_r(\bar{r} - r_t)\, dt + \sigma_r r_t \; dW^r$$

and

$$de = -\kappa_e e_t \, dt + s_e \; dW^e.$$

We will discretize these to Δt = 1 month=0.0833. Assume $\bar{r} = 0.05$, $\kappa_r \Delta t = .01$, $\kappa_e \Delta t = .015$, $\sigma_r = .2425$, $s_e = 0.83$ and that the two shocks are uncorrelated.

At each month, each mortgage currently outstanding makes fixed a scheduled payment of A. The total monthly cash-flow is then $AO + ZPO$. Use the formulas in the lecture notes to compute A as well as the ammortization schedule that determines P_n.

Tabulate the total NPV along each path by summing the product of month n's cash flow times the cumulative discount factor $DF_n =$

$$\Pi_{i=0}^{n-1} \left(\frac{1}{(1 + r_i \Delta t)} \right).$$

(a) For intital values of r_0 running from 1% to 16% plot the average NPV of the MBS computed across $M = 10k$ simulations.

(b) Now suppose there is a second bond with identical cash-flow characteristics but no sensitivity of prepayment to interest rates ($b = 0$). Compute and plot its NPVs for the same starting values of r.

(c) Can we interpret the difference between these two plots as the value of homeowners' option to refinance their mortgages?

(d) If you were long the bond in (a) and wanted to hedge your interest rate risk by selling the bond in (b), would you have to sell more or less as prices went down?