FIN 513: Homework #8

Due on Tuesday, April 26, 2018

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Problem 1

(a) Assume that the amount of principal is equal to 1. Since the swap fee is payable at the end of each quarter, and there is no time value of money, the value of fee for each swap maturing at t_i is evaluated as $0.25 \times \sum_{t \leq t_i}^{t_i} \varphi(1 - H_{0,t})$, where $t \in \{0.25, 0.50, 0.75, 1.00\}$, and φ is a swap fee. Since risk-free rate is zero, current value of corresponding protection leg is evaluated as $(1 - R) \times H_{0,t_i}$, where R is recovery rate. By equating both equations, each H_{0,t_i} is calcuated as $\frac{0.25 \times \varphi \sum_{t \leq t_{i-1}}^{t_{i-1}} (1 - H_{0,t}) + 0.25\varphi}{(1-R) + 0.25\varphi}$. By calculating iteratively, we can calculate H_{0,t_i}. Table 1 shows the result.

Duration of CDS	Fee φ (BP)	H_{0,t_i}
3 months	900	0.0533
6 months	800	0.0927
9 months	750	0.1278
12 months	700	0.1562

Table 1: Cumulative default density: H_{0,t_i}

(b) Since the remaining time-to-maturity is one year, the current value I have to pay is equal to $0.25 \times 200 \times \sum_{t_i} (1 - H_{0,t_i})$ times the notional, where $t_i \in \{0.25, 0.50, 0.75, 1.00\}$. It is calculated as 0.8925 million dollars. In contrast, the current value of protection is calculated as $(1 - R) \times H_{0,1}$ times notional, which is about 3.1238 million dollars. Therefore, the current value of my position is $3.1238 - 0.8925 = 2.2313$ million dollars.

Problem 2

(a) Since H_T follows exponential distribution, cumulative distribution of D is derived as follows.

$$Prob(D < u) = Prob(e^{-aH_T} < u)$$
$$= Prob(-aH_T < \log u)$$
$$= Prob(H_T > -\frac{1}{a} \log u)$$
$$= \int_{-\frac{1}{a} \log u}^{\infty} be^{-bx} dx$$
$$= -e^{-bx} \Big|_{-\frac{1}{a} \log u}^{\infty}$$
$$= u^{\frac{b}{a}}$$

By using the same method, cumulative distribution of L is derived as follows.

$$Prob(L < v) = Prob((1-R)e^{-aH_T} < v)$$

$$= Prob(\log(1-R) - aH_T < \log v)$$

$$= Prob\left(H_T > -\frac{1}{a}\log\frac{v}{1-R}\right)$$

$$= \int_{-\frac{1}{a}\log\frac{v}{1-R}}^{\infty} be^{-bx}dx$$

$$= -e^{-bx}\Big|_{-\frac{1}{a}\log\frac{v}{1-R}}^{\infty}$$

$$= \left(\frac{v}{1-R}\right)^{\frac{b}{a}}$$

$$v \in [0, 1-R]$$

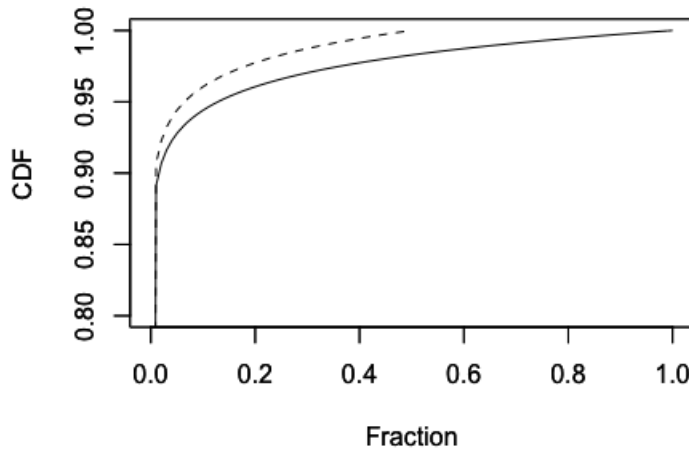Figure 1 shows cumulative distribution of D and L.

Figure 1: Cumulative distribution of D and L

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(b) Let x denote the lowest attachment point such that the probabilities of the tranche above that point experiencing any principal loss is p. Since cumulative distribution function is monotonically increasing, probability such that fraction of loss for CLO is greater than x is equal to p. (i.e. $Prob(L > x) = p$) Solving the equation, x is derived as follows.

$$Prob(L > x) = p$$

$$1 - \left(\frac{x}{1-R}\right)^{\frac{b}{a}} = p$$

$$x = (1-R) \times (1-p)^{\frac{a}{b}}$$

Table 2 shows the lower attachments for each tranche calculated by using the equation above. From the table about 50% of loans consists of AAA ratings.

Tranches	Probabilities	Attachments(Lower)
AAA	0.05%	0.490
AA	0.20%	0.462
A	1%	0.334
BBB	2%	0.223
BB	7.50%	0.022
Unrated		0.000

Table 2: Attachment of each tranche

(c) Let N denote the notional amount of CLO as a whole. Then, face value of each tranche is equal to proportion of each tranche times N. Assuming interests are paid annually, interest payment for each tranche is calculated. Table 3 represents annual interest payment for each tranche. Since we assumed

Tranches	Proportion	Interest	Payment
AAA	0.51	LIBOR + 0.25%	$-0.51 \times N \times (LIBOR + 0.25\%)$
AA	0.03	LIBOR + 0.5%	$-0.03 \times N \times (LIBOR + 0.5\%)$
A	0.13	LIBOR + 1.25%	$-0.13 \times N \times (LIBOR + 1.25\%)$
BBB	0.11	LIBOR + 2.5%	$-0.11 \times N \times (LIBOR + 2.5\%)$
BB	0.20	LIBOR + 4%	$-0.20 \times N \times (LIBOR + 4\%)$
Sum	0.98		$-0.98 \times N \times LIBOR + N \times 1.38\%$

Table 3: Annual payment of each tranche

that the bottom level tranche is entirely wiped out, and the loss is amortized equally, issuer will get $0.98 \times N \times (LIBOR + 4\%)$ amount of interest payment at each year. Therefore, the interest margin is calculated as $-0.98 \times N \times LIBOR + N \times 1.38\% - 0.98 \times N \times (LIBOR + 4\%) = N \times 2.54\%$. Therefore, interest margin for this scenario is 2.54% per year.

Problem 3

(a) Assume the swap matuares at T. In order to derive $V(S_t, t, \varphi_0)$, it needs to derive each leg of swap first. Since bank pays φ dollars continuously, present value of the leg is evaluated as $\varphi \int_t^T e^{-r_d s} ds = \varphi(-\frac{1}{r_d})e^{-r_d s}|_t^T = \frac{\varphi}{r_d}e^{-r_d t}(1 - e^{-r_d(T-t)})$. Similarly, present value of the foreign currency leg is evaluated

as $S_t \int_t^T e^{-r_f s} ds = S_t(-\frac{1}{r_f})e^{-r_f s}|_t^T = \frac{S_t}{r_f}e^{-r_f t}(1 - e^{-r_f(T-t)})$. Equating both equations, φ_0 is calculated as follows.

$$\frac{\varphi_0}{r_d}(1 - e^{-r_d T}) = \frac{S_0}{r_f}(1 - e^{-r_f T})$$

$$\Rightarrow \varphi_0 = \frac{S_0 r_d}{r_f}\left(\frac{1 - e^{-r_f T}}{1 - e^{-r_d T}}\right)$$

$$= \frac{0.01}{0.05}\left(\frac{1 - e^{-0.05 \times 5}}{1 - e^{-0.01 \times 5}}\right)$$

$$= 0.9071$$

(b) Since there is no collateral, we can assume that there is no recovery if the counterparty is default. Therefore, CVA of the contract is derived as follows.

$$CVA = \int_0^T e^{-r_d t}1_{\{c/p \text{ defaults at } t\}}\max(V_t, 0)dt$$

$$= \int_0^T e^{-r_d t}\lambda_t e^{-\int_0^t \lambda_u du}\max\left(\frac{S_t}{r_f}e^{-r_f t}(1 - e^{-r_f(T-t)}) - \frac{\varphi_0}{r_d}e^{-r_d t}(1 - e^{-r_d(T-t)}), 0\right)dt$$

In order to execute simulation, I discretized the integral as follows.

$$CVA = \sum_{i=0}^M \left[e^{-r_d(i\Delta t)}\lambda_i e^{-\sum_{j=0}^i \lambda_j \Delta t}\right.$$

$$\left. \times \max\left(\frac{S_i}{r_f}e^{-r_f(i\Delta t)}(1 - e^{-r_f(T-i\Delta t)}) - \frac{\varphi_0}{r_d}e^{-r_d(i\Delta t)}(1 - e^{-r_d(T-i\Delta t)}), 0\right)\Delta t\right]$$

$$M\Delta t = T$$

$$\lambda_i = \lambda_{i-1} \times \exp[(b_\lambda^Q - 0.5\sigma_\lambda^2)\Delta t + \sigma_\lambda \sqrt{\Delta t}\phi_1]$$

$$S_i = S_{i-1} \times \exp[(r_d - r_f - 0.5\sigma_S^2)\Delta t + \sigma_S \sqrt{\Delta t}\phi_2]$$

$$\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}\right)$$

By simulating 5000 times with $M = 450$ for $\rho = 0, 0.1, 0.2 \ldots 1.0$, and using given parameters, CVA was calculated at each correlation. Figure 2 shows CVA at each correlation. From the figure, we can discover that CVA increases as correlation increases. Since the bank earns more money if exchange rate increases, if default intensity is highly correlated with exchange rate, expected amount of position value when counterparty defaults will be higher. That's why CVA increases as correlation increases. Since CVA at $\rho = 0.5$ is calcuated as 0.789921%, if notional value of swap is 1 billion, CVA is calculated as about 7.899 million dollars.

Problem 4

I disagree. RMBS itself is not a main reason for subprime crisis. The main reason was re-securitized product. Even if the RMBS consists a pool of subprime ARMs, as long as it has other pools, probability of
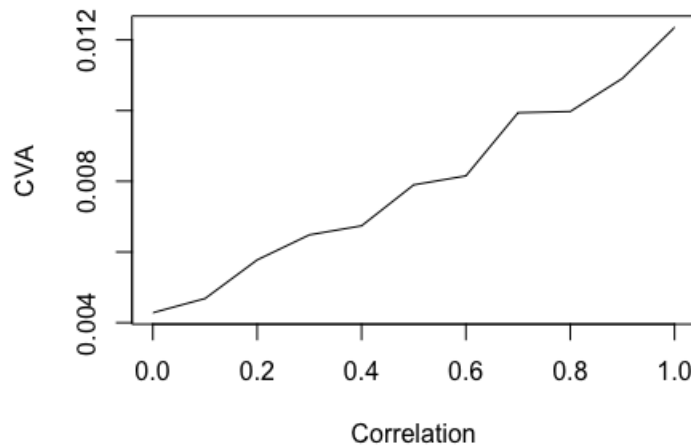
Figure 2: CVA at each correlation

loss for AAA tranche itself is not quite large. Actually, about 80% of pool was AAA tranche at that period. Therefore, although cumulative default rate on the pool reached 40%, investors would not have experienced extreme loss.

Problem 5

(a) For each $r \in \{0.01, 0.02 \ldots 0.16\}$, NPV was calculated by simulating 10,000 paths. Principal amount was assumed to be 100. Figure 3 shows NPV of MBS when $b = -30$, and $b = 0$, respectively. We can find out that NPV decreases as initial interest rate increases as future cash flows is highly discounted.

(b) From Figure 3, it can be discovered that NPV when $b = 0$ is always higher then when $b = -30$, and it has relatively linear shape though NPV when $b = -30$ is relatively concave. Figure 4 shows the absolute value of difference between NPV when $b = -30$ and $b = 0$, we can find that difference has a convex shape, which occurs because NPV when $b = -30$ has a concavity.

(c) In this case, factors which affects NPV of MBS are amount of prepayment and discount factor. Prepayment decreases future interest payments of the pool, and discount factor affects time value of money. They have trade-off because prepayment decreases future cash flow, but it is discounted less. When interest rate is low, prepayment has a negative effect because interest payments decreases, and discount factor affects a little. However, if interest rate is high, prepayment has a positive effect to NPV because time value of money dominates NPV. From this perspective, if π is sensitive to interest rate, it has negative effect because higher sensitivity means higher prepayment if interest rate is low, and lower
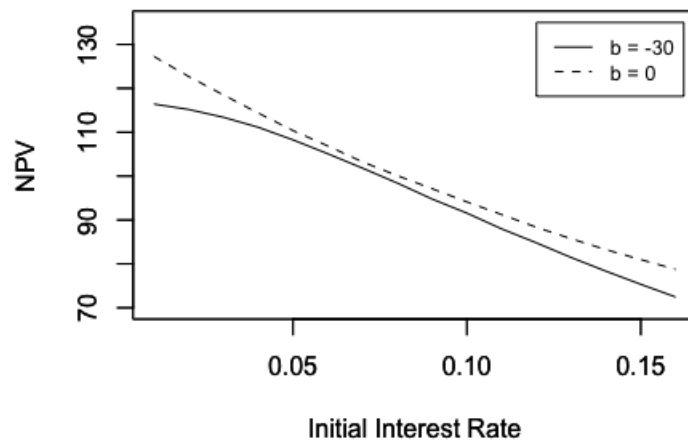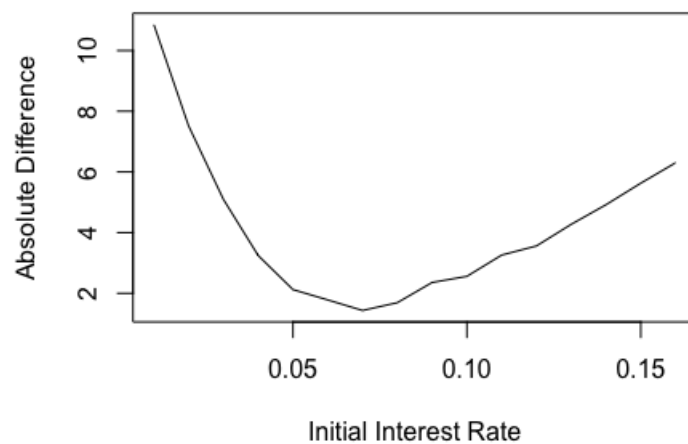
Figure 3: NPV at each r

Figure 4: Absolute difference of NPV

prepayment if interest rate is higher. Also, from this reason concavity of NPV occurs, and if sensitivity gets higher, NPV will be more concave.

(d) It needs to sell more the bond in (b) if price goes down. From the perspective of hedging interest rate risk, price of MBS goes down if interest rate gets higher. From Figure 3, we can easily find that the bond (a) gets more sensitive to interest rate if its price goes down, whereas sensitivity of bond (b) is relatively stable. It means that in order to match sensitivity of long position and short position, it needs to sell more bond (b) if price goes down.