FIN 513: Homework #3

Due on Tuessday, February 13, 2018

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Problem 1

The put-call parity of American options on a currency is $e^{-r_f(T-t)}S - K \leq C - P \leq S - e^{-r_d(T-t)}K$ or $B^f S - K \leq C - P \leq S - B^d K$, where S and K denotes spot exchange rate and strike price as usual, r_d and r_f denotes domestic risk-free rate and foreign risk-free rate, respectively.

(Proof) Suppose not.

1. Suppose $C - P > S - e^{-r_d(T-t)}K$. Then an arbitrage opportunity exists by constructing the following portfolio.

   (a) Buy a put option and a unit of foreign currency.

   (b) Write a call option and borrow $e^{-r_d(T-t)}K$ amount of domestic currency on risk-free rate.

   All possible situations can be separated by two cases: early exercise of written option occurs and no early exercise of written option. Assuming that early exercise occurs at $t^* < T$, payoff of the portfolio is as follows.

   Case 1. Early exercise at $t^* < T$.

       (a) $e^{r_f(t^*-t)}S_{t^*}$

       (b) $-(S_{t^*} - K) - e^{-r_d(T-t^*)}K$

       The sum of payoff from two strategies is $(e^{r_f(t^*-t)}-1)S_{t^*}+(1-e^{-r_d(T-t^*)})K$, which is positive.

   Case 2. No early exercise

       i. $S_T > K$

           (a) $0 + e^{r_f(T-t)}S_T$

           (b) $-(S_T - K) - K$

       ii. $S_T \leq K$

           (a) $(K - S_T) + e^{r_f(T-t)}S_T$

           (b) $0 - K$

       In this case, the portfolio also has a positive payoff regardless of spot exchange rate at maturity.

   Since the portfolio has a positive payoff at all possible situations, there must be a cost for implementing the strategies if there is no arbitrage opportunity. However, by the assumption, the initial cost for constructing portfolio is negative, so a contradiction occurs. Therefore, $C - P \leq S - e^{-r_d(T-t)}K$ must hold.

2. Suppose $C - P < e^{-r_f(T-t)}S - K$. Then there is also an arbitrage opportunity exists considering the following portfolio.

    (a) Buy a call option and invest K amount of domestic currencies on domestic risk-free rate.

    (b) Write a put option and sell short a foreign risk-free zero coupon bond.

Problem 2

(a)

(b)

(c)

(d)

(e)

(f)

(g)

(h)

Problem 3

(a)

(b)

(c)

(d)

Problem 4

(a)

(b)