

FIN 513: Homework #1

Due on Thursday, January 25, 2018

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Problem 1

- (a) Agree. Generally, low-growth stock gives higher dividend than high-growth stock. Since high-growth company needs more capital than low-growth company, the amount of retained earning might be larger than that of low-growth company. Therefore, the futures price of high-growth stock will be a less discount over the spot price.
- (b) Agree.
- (c) Agree.

Problem 2

Let r_L^D denote dollar rates for lending, r_L^E denote euro rates for lending, r_B^D and r_B^E denote dollar and euro rates for borrowing, respectively. Let S_t^B and S_t^O denote bid and offer exchange rate, respectively. In the following way, we can replicate long position and short position of dollar/euro forward.

1) Replicate Long Position

- (a) Borrow $S_t^O \frac{1}{(1+0.5r_L^E)^{2(T-t)}}$ dollars at rate r_B^D
- (b) Exchange $S_t^O \frac{1}{(1+0.5r_L^E)^{2(T-t)}}$ dollars to $\frac{1}{(1+0.5r_B^E)^{2(T-t)}}$ euros, then invest for time $T - t$ at rate r_L^E .

At time T , the value of strategy (a) becomes $-S_t^O (\frac{1+0.5r_B^D}{1+0.5r_L^E})^{2(T-t)}$ dollars and the value of strategy (b) becomes 1 euro which perfectly replicates long position of forward contract. Since the initial values of two strategies are identical, the forward price $F_{t,T}$ should be equal to $S_t^O (\frac{1+0.5r_B^D}{1+0.5r_L^E})^{2(T-t)}$ dollars.

2) Replicate Short Position

- (a) Borrow $\frac{1}{(1+0.5r_B^E)^{2(T-t)}}$ euros at rate r_B^E
- (b) Exchange $\frac{1}{(1+0.5r_B^E)^{2(T-t)}}$ euros to $S_t^B \frac{1}{(1+0.5r_B^D)^{2(T-t)}}$ dollars, then invest at rate r_L^D .

At time T , the value of strategy (a) becomes -1 euro, which is identical to short position of forward contract. Therefore, if there is no arbitrage in market, the forward price should be equal to the value of strategy (b) at time T , which is $S_t^B (\frac{1+0.5r_L^D}{1+0.5r_B^E})^{2(T-t)}$

Since $r_L^D < r_B^D$, $r_L^E < r_B^E$ and $S_t^B < S_t^O$, the value of replicate portfolio of short position is less than that of long position. Therefore, the upper and lower bound of the contract is $S_t^O (\frac{1+0.5r_B^D}{1+0.5r_L^E})^{2(T-t)}$ and $S_t^B (\frac{1+0.5r_L^D}{1+0.5r_B^E})^{2(T-t)}$, respectively. The following table shows that the numerical results using parameters on the homework sheet.

T	<i>Lower Bound</i>	<i>Upper Bound</i>
1 yr	1.4913	1.4940
5 yr	1.4833	1.4993
10 yr	1.5218	1.5526

Problem 3

Assume that the fair price of single price is x , then x should be larger than 308 and smaller than 313. Consider the following strategy.

- (a) Get a long position on the original contract.
- (b) Get a short position on the new contract.

Since the cash flow at time $t = 1.5$ is known at $t = t_0$, we can reinvest the net cash flow $(x - 308)$ at $t = 1.5$ to $t = 2$ by using the following strategy.

- (1) sell short $x - 308$ amount of zero coupon bond with maturity $t = 1.5$
- (2) buy $\frac{(x-308)B_{0,1.5}}{B_{0,2}}$ amount of zero coupon bond with maturity $t = 2$.

By netting out the values of strategy (1) and (2), there is no initial amount of cash flow. Furthermore, the strategies also makes cash flow at $t = 1.5$ to be zero. Finally, at time $t = 2$, the net cash flow from the whole strategies is equal to $x - 313 + (x - 308)\frac{B_{0,1.5}}{B_{0,2}}$. Since there is no cash flow before $t = 2$, the net cash flow at $t = 2$ should be equal to zero, otherwise there exists arbitrage opportunities. Therefore, the following equation holds.

$$\begin{aligned}
 x - 313 + (x - 308)\frac{B_{0,1.5}}{B_{0,2}} &= 0 \\
 \Rightarrow B_{0,1.5}x + B_{0,2}x &= 308B_{0,1.5} + 313B_{0,2} \\
 \Rightarrow (0.912 + 0.883)x &= 308 \times 0.912 + 313 \times 0.883 \\
 \Rightarrow x &= 310.4596
 \end{aligned}$$

Therefore, the fair price that a market maker would be willing to offer is 310.4596.

Problem 4

- (a) If firm L did not lend out stocks, they will get 1,000,000 dollars for dividend and pay 300,000 dollars for tax. Therefore, the net cash flow from dividend is equal to 700,000 dollars. It means firm L would require at least \$700,000 plus loan fee for payment.
- (b) Consider the following agreement.

- (1) Firm L lend out 1,000,000 shares which is consistent with (a).
- (2) Firm H makes a repurchase agreement with firm E, which means firm H lend out 1,000,000 shares to firm E before dividend is paid, and get back after dividend is paid. This is possible since dividend schedule is certain.

In this case, firm L does not have to pay tax because DRD is not applied. Unless the firm H pays premium larger than \$300,000, this contract is makes more profit for all firms. Firm E does not expose on any risk with respect to Google stock, firm L stays long and firm H stays short.