FIN 513: Homework #2

Due on Thursday, February 1, 2018

Wanbae Park

Problem 1

(a) (1) *(Zero-coupon term structure)* Since the bonds are traded at par and these are riskless, zero-coupon term-structure can be obtained by just equating present value of their cash flow to their price(= 100). Therefore, the following equation holds.

$$100 = \sum_{t=1}^{T} \frac{\textit{Cash flows at time } t}{(1+r_{0,t})^t}$$

In order to obtain the whole term-structure, one should get $r_{0,t}$ first by equating the value of bond with maturity t to present value of its cash flow first, then $r_{0,t+1}$ can be obtained by using $r_{0,t}$. It is because it is necessary to use $r_{0,t}$ to discount cash flows of bond maturing at $t+1$. Therefore, calculate $r_{0,1}$ first.

$$100 = \frac{100 + 100 \times 0.0133}{1 + r_{0,1}} = \frac{100(1 + 0.0133)}{1 + r_{0,1}}$$

It is trivial that $r_{0,1} = 0.0133$. Then, let's use this result to calculate $r_{0,2}$.

$$100 = \frac{100 \times 0.0173}{1 + 0.0133} + \frac{100 + 100 \times 0.0173}{(1 + r_{0.2})^2}$$

Then it is calculated that $r_{0,2} = 0.01733$. Using this procedure ahead, zero-coupon term-structure from 1 year to 10 year can be obtained. Table 1 shows that the term structure of zero coupon bonds obtained by using this procedure.

Maturity(years)	$r_{0,t}$(%)
1	1.330
2	1.733
3	2.162
4	2.577
5	2.752
6	3.196
7	3.435
8	3.459
9	3.474
10	3.562

Table 1: Term structure of zero coupon bonds

(2) *(Term-structure of one-year forward rates)* Let $r_{0,t,t+1}$ denote a forward rate from time t to $t+1$ determined at current time. Then $r_{0,t,t+1}$ can be obtained by comparing the following two strategies.

i. Invest \$1 to zero coupon bond maturing at time t, and when the bond matures, receive money and reinvest to zero coupon bond maturing at time $t+1$.

ii. Invest \$1 to zero coupon bond maturing at time $t+1$.

Since the amount of investment at current time is equal for both strategies, by definition of forward rate, the following equation should hold.

$$\begin{aligned}(1+r_{0,t+1})^{t+1} &= (1+r_{0,t})^t(1+r_{0,t,t+1})\\ \Rightarrow r_{0,t,t+1} &= \frac{(1+r_{0,t+1})^{t+1}}{(1+r_{0,t})^t} - 1\end{aligned}$$

Therefore, since the term structure of spot rate is given above, by using this formula term structure of 1-year forward rate can be obtained. Table 2 shows term structure of 1-year forward rate using the data given in assignment.

t	$r_{0,t,t+1}(\%)$
1	2.139
2	3.025
3	3.830
4	3.459
5	5.441
6	4.881
7	3.627
8	3.592
9	4.363

Table 2: Term structure of 1-year forward rate

(b)

(c)