FIN 513: Homework #4

Due on Tuesday, February 20, 2018

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Problem 1

Problem 2

Problem 3

False. It is not necessary to assume the options are not marked to market. The only assumption to derive options prices under binomial model is that underlying asset and riskless bond are tradable(both long and short) at each steps. Regardless of marking-to-market, it is possible to price option only if we can replicate payoff of options by trading underlying asset and riskless bond at each node.

Problem 4

a. Let $\bar{r}^d$ and $\bar{r}^f$ denote gross return of domestic riskless bond and foreign riskless bond, respectively. Then the risk-neutral probability $\bar{q}$ is calculated as $\bar{q} = \frac{\bar{r}^d/\bar{r}^f - d}{u-d}$.

i. *Assume* $\bar{q} > 1$. Since $u - d > 0$, $\bar{q} = \frac{\bar{r}^d/\bar{r}^f - d}{u-d} > 1$ is equivalent to $\frac{\bar{r}^d}{\bar{r}^f} - d > u - d \Rightarrow \frac{\bar{r}^d}{\bar{r}^f} > u > d$. Using this inequality, it is possible to make an arbitrage profit using the following strategies.

(1) Borrow an foreign currency and exchange to domestic currency.

(2) Invest domestic currency from exchange of foreign currency to riskless bond.

After one period, assume that the exchange rate becomes higher to $S_t \times u$. Then the investor have to pay $S_t u \bar{r}^f$ to the lender, and gets $S_t \bar{r}^d$ from the investment of domestic riskless bond. Therefore, the final payoff of the strategy is equal to $S_t \bar{r}^d - S_t u \bar{r}^f = S_t(\bar{r}^d/\bar{r}^f - u)$. Because we assumed that $\bar{r}^d/\bar{r}^f - u > 0$, and there is no initial cost to make the portfolio, the investor can make an arbitrage profit. Case in which exchange rate becomes $S_t \times d$ is analogous.

ii. *Assume* $\bar{q} < 0$. $\bar{q} = \frac{\bar{r}^d/\bar{r}^f - d}{u-d} < 0$ is equivalent to $\frac{\bar{r}^d}{\bar{r}^f} - d < 0 \Rightarrow \frac{\bar{r}^d}{\bar{r}^f} < d < u$. In this case, it is also possible to make an arbitrage profit using the following strategies.

(1) Borrow S_t amount of domestic currency and exchange into a foreign currency.

(2) Invest a foreign currency to riskless bond.

Then after one period, assuming the exchange rate becomes to $S_t \times d$, the investor will get $S_t d \bar{r}^f$ and have an obligation to pay $S_t \bar{r}^d$. Therefore, the final payoff the strategy is equal to $S_t d \bar{r}^f - S_t \bar{r}^d = S_t(d - \bar{r}^d/\bar{r}^f)$, which is positive. Therefore, the investor can make an arbitrage profit since there is no initial amount of investment. Case in which exchange rate becomes $S_t \times u$ is analogous.

b. First, let us assume that exchange rate y becomes $y \times u$ or $y \times d$ after one month. Assuming the expect return of investing in Afghani is zero, in order to match the expected return, the following equation should hold.

$$y = 0.5 \times yu + 0.5 \times yd$$

Cancelling y out on both sides and rearranging the terms, equation $d = 2 - u$ is obtained. Using the result, in order to match standard deviation, we can construct the following equation.

$$\begin{aligned}
&0.5 \times u^2 + 0.5 \times d^2 = 0.5 \times u^2 + 0.5 \times (2-u)^2 = \frac{0.1}{12} \approx 0.008 \\
&\Rightarrow u^2 - 2u + 2 - 0.008 = 0 \\
&\Rightarrow u = \frac{2 \pm \sqrt{4 - (1.992)^2}}{2} = 1.089 \text{ or } 0.910
\end{aligned}$$

Since $u > d$, $u = 1.089$ and $d = 2 - u = 0.910$. By using the given interest rate to calculate risk-neutral probability $\bar{q}$, it is calculated as $\bar{q} = \frac{1.02/1.08-0.910}{1.089-0.910} = 0.1924$, which is between 0 and 1, satisfies the condition derived in (a). Therefore, if the value of u and d calculated above are used for constructing tree, it is possible to match the moments of exchange rate.

Problem 5