

## Questions about Revenue Puts

Read the RISK article on the Compass site, which describes the recent use of revenue puts by generators of electric power. This case asks you to analyze a simplified version of the product described in the article by employing a pricing model. The model is described in the file `Powerplant_model.pdf`.

Assume you work for an energy firm that is considering building a power plant that generates electricity from natural gas. The revenue for your plant will be the operating profit net of the costs of the natural gas input. If  $P^E$  is the electricity price, and  $P^G$  is the gas price, and  $h$  is the conversion factor for your plant, then your revenue, when you choose to operate, is equal to the “spark spread” which is  $S = P^E - h \cdot P^G$ , times your plant’s capacity,  $Y$ . Your plant’s “heat index” is  $h$ .

The underlying for the derivative contract also reflects your decision about whether or not to operate. Assume that turning on and off the plant is costless so that you will operate the plant at full capacity any time  $t$  that  $S_t > 0$ . Then your revenue is  $R_t = \max(Y S_t, 0)$  and the payoff from the revenue put for day  $t$  is

$$\max[K - R_t, 0] \quad \text{or} \quad \max[K - \max(Y S_t, 0), 0]$$

where  $K > 0$  is the strike price. The revenue is measured daily, and the payoff function is evaluated and paid out (if positive) every day. (This is the same as if you had one put option for each day.)

Prepare a report addressing the following questions.

- **Question 1.** As background, explain why power producing companies might want to hedge. If there are two power companies with identical technology and one hedges its output by purchasing derivatives – that are fairly priced – and the other one doesn’t hedge, how (or under what conditions) can this create any additional value for the owners of the first firm? For the specific firms described in the article, why might the costs of hedging exceed the benefits?
- **Question 2.** Use the stochastic specification and parameters given in the model document to describe the operating risk that the company faces over the next five years (if it does not hedge). Report the expected total revenue, and plot the distribution of total revenue (show a histogram). Report the 5th and 10th percentile of the distribution. Report the expected number of days for which the plant will not operate. Report the expected number of days for which the revenue is less than 40,000 dollars per day.

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- **Question 3.** Assume that the market prices of risk for gas and electricity are constants and estimate them using the your model and the data given for forward prices.

Now use your estimated risk prices to compute the no-arbitrage price for a 5-year revenue put whose strike price is  $K = 40,000$  dollars per day. Explain all the steps in your valuation, and report your numerical standard error.

Also compute the put's *true* expected payoff under your model. Is it more or less than the no-arbitrage price? Explain why.

- **Question 4.** As discussed in the article, the payout of the revenue put is not based on your plant's actual revenue, but is instead based on a theoretical (“idealized”) plant that operates according to mathematical rules like the ones described above. The article describes several ways in which the true revenue can differ from the model, thus lessening the effectiveness of the hedge. One example given is that plant operating decisions (to run or not run) have to be made with imperfect information, whereas the idealized revenue is computed with hindsight, i.e., perfect information. To see how much this could matter, compute the value of a put in which the operating decision is corrupted by the following “error” process.

Assume that every day for which the true value of  $S$  is positive there is a probability that you do not run the plant, and that every day for which  $S$  is negative there is a probability that you do run the plant. These operating errors are independent and identically distributed. The probability of a suboptimal decision (of either type) on day  $t$  is given by

$$0.05 e^{-0.7|S_t|}.$$

So now your revenue is  $\tilde{R}_t = Y S_t 1_{run,t}$ , where  $1_{run,t}$  is an indicator variable equal to one on any day that the plant is operating and zero otherwise. How much more would the put be worth if it paid  $\max[K - \tilde{R}_t, 0]$  every day? Should the hindsight advantage of the dealer be a significant factor in the decision whether to buy the put?