# FIN 513: Homework #8

Due on Tuesday, April 26, 2018

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# Problem 1

(a) Assume that the amount of principal is equal to 1. Since the swap fee is payable at the end of each quarter, and there is no time value of money, the value of fee for each swap maturing at $t_i$ is evaluated as $0.25 \times \sum_{t \le t_i}^{t_i} \varphi(1 - H_{0,t})$, where $t \in \{0.25, 0.50, 0.75, 1.00\}$, and $\varphi$ is a swap fee. Since risk-free rate is zero, current value of corresponding protection leg is evaluated as $(1 - R) \times H_{0,t_i}$, where $R$ is recovery rate. By equating both equations, each $H_{0,t_i}$ is calcuated as $\frac{0.25 \times \varphi \sum_{t \le t_{i-1}}^{t_{i-1}} (1 - H_{0,t}) + 0.25\varphi}{(1-R) + 0.25\varphi}$. By calculating iteratively, we can calculate $H_{0,t_i}$. Table 1 shows the result.

| Duration of CDS | Fee $\varphi$ (BP) | $H_{0,t_i}$ |
|---|---|---|
| 3 months | 900 | 0.0533 |
| 6 months | 800 | 0.0927 |
| 9 months | 750 | 0.1278 |
| 12 months | 700 | 0.1562 |

Table 1: Cumulative default density: $H_{0,t_i}$

(b) Since the remaining time-to-maturity is one year, the current value I have to pay is equal to $0.25 \times 200 \times \sum_{t_i} (1 - H_{0,t_i})$ times the notional, where $t_i \in \{0.25, 0.50, 0.75, 1.00\}$. It is calculated as 0.8925 million dollars. In contrast, the current value of protection is calculated as $(1 - R) \times H_{0,1}$ times notional, which is about 3.1238 million dollars. Therefore, the current value of my position is $3.1238 - 0.8925 = 2.2313$ million dollars.

# Problem 2

(a) Since $H_T$ follows exponential distribution, cumulative distribution of $D$ is derived as follows.

$$
\begin{aligned}
Prob(D < u) &= Prob(e^{-aH_T} < u) \\
&= Prob(-aH_T < \log u) \\
&= Prob(H_T > -\frac{1}{a} \log u) \\
&= \int_{-\frac{1}{a}\log u}^{\infty} be^{-bx} dx \\
&= -e^{-bx} \Big|_{-\frac{1}{a}\log u}^{\infty} \\
&= u^{\frac{b}{a}}
\end{aligned}
$$

Problem 2 continued on next page. . . 

By using the same method, cumulative distribution of $L$ is derived as follows.

$$\begin{aligned}
Prob(L < v) &= Prob((1-R)e^{-aH_T} < v) \\
&= Prob(\log(1-R) - aH_T < \log v) \\
&= Prob(H_T > -\frac{1}{a}\log\frac{v}{1-R}) \\
&= \int_{-\frac{1}{a}\log\frac{v}{1-R}}^{\infty} be^{-bx}dx \\
&= -e^{-bx}\Big|_{-\frac{1}{a}\log\frac{v}{1-R}}^{\infty} \\
&= \left(\frac{v}{1-R}\right)^{\frac{b}{a}}
\end{aligned}$$

$$v \in [0, 1-R]$$

Figure 1 shows cumulative distribution of $D$ and $L$.

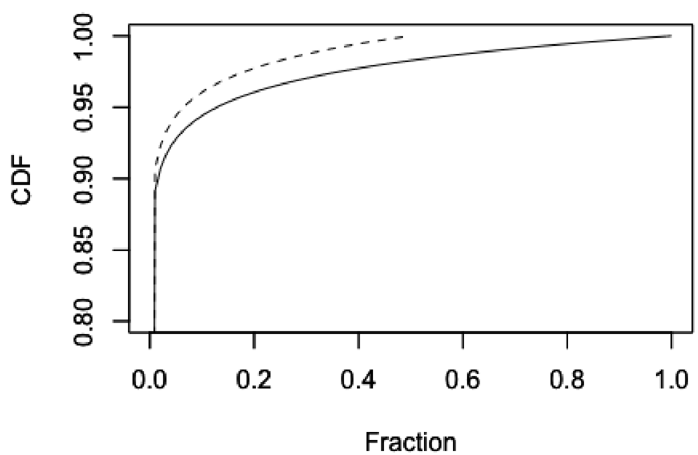


Figure 1: Cumulative distribution of $D$ and $L$

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(b) Let $x$ denote the lowest attachment point such that the probabilities of the tranche above that point experiencing any principal loss is $p$. Since cumulative distribution function is monotonically increasing, probability such that fraction of loss for CLO is greater than $x$ is equal to $p$. (i.e. $Prob(L > x) = p$) Solving the equation, $x$ is derived as follows.

$$Prob(L > x) = p$$

$$1 - \left(\frac{x}{1-R}\right)^{\frac{b}{a}} = p$$

$$x = (1-R) \times (1-p)^{\frac{a}{b}}$$

Table 2 shows the lower attachments for each tranche calculated by using the equation above. From the table about 50% of loans consists of AAA ratings.

| Tranches | Probabilities | Attachments(Lower) |
|---|---|---|
| AAA | 0.05% | 0.490 |
| AA | 0.20% | 0.462 |
| A | 1% | 0.334 |
| BBB | 2% | 0.223 |
| BB | 7.50% | 0.022 |
| Unrated | | 0.000 |

Table 2: Attachment of each tranche

(c) Let $N$ denote the notional amount of CLO as a whole. Then, face value of each tranche is equal to proportion of each tranche times $N$. Assuming interests are paid annually, interest payment for each tranche is calculated. Table 3 represents annual interest payment for each tranche. Since we assumed

| Tranches | Proportion | Interest | Payment |
|---|---|---|---|
| AAA | 0.51 | LIBOR + 0.25% | $-0.51 \times N \times (LIBOR + 0.25\%)$ |
| AA | 0.03 | LIBOR + 0.5% | $-0.03 \times N \times (LIBOR + 0.5\%)$ |
| A | 0.13 | LIBOR + 1.25% | $-0.13 \times N \times (LIBOR + 1.25\%)$ |
| BBB | 0.11 | LIBOR + 2.5% | $-0.11 \times N \times (LIBOR + 2.5\%)$ |
| BB | 0.20 | LIBOR + 4% | $-0.20 \times N \times (LIBOR + 4\%)$ |
| Sum | 0.98 | | $-0.98 \times N \times LIBOR + N \times 1.38\%$ |

Table 3: Annual payment of each tranche

that the bottom level tranche is entirely wiped out, and the loss is amortized equally, issuer will get $0.98 \times N \times (LIBOR + 4\%)$ amount of interest payment at each year. Therefore, the interest margin is calculated as $-0.98 \times N \times LIBOR + N \times 1.38\% - 0.98 \times N \times (LIBOR + 4\%) = N \times 2.54\%$. Therefore, interest margin for this scenario is 2.54% per year.

## Problem 3

(a) Assume the swap matuares at $T$. In order to derive $V(S_t, t, \varphi_0)$, it needs to derive each leg of swap first. Since bank pays $\varphi$ dollars continuously, present value of the leg is evaluated as $\varphi \int_t^T e^{-r_d s} ds = \varphi(-\frac{1}{r_d})e^{-r_d s}|_t^T = \frac{\varphi}{r_d}e^{-r_d t}(1 - e^{-r_d(T-t)})$. Similarly, present value of the foreign currency leg is evaluated as $S_t \int_t^T e^{-r_f s} ds = S_t(-\frac{1}{r_f})e^{-r_f s}|_t^T = \frac{S_t}{r_f}e^{-r_f t}(1-e^{-r_f(T-t)})$. Equating both equations, $\varphi_0$ is calculated as follows.

$$\begin{aligned}
\frac{\varphi_0}{r_d}(1-e^{-r_d T}) &= \frac{S_0}{r_f}(1-e^{-r_f T}) \\
\Rightarrow \varphi_0 &= \frac{S_0 r_d}{r_f}\left(\frac{1-e^{-r_f T}}{1-e^{-r_d T}}\right) \\
&= \frac{0.01}{0.05}\left(\frac{1-e^{-0.05\times 5}}{1-e^{-0.01\times 5}}\right) \\
&= 0.9071
\end{aligned}$$

(b) Since there is no collateral, we can assume that there is no recovery if the counterparty is default. Therefore, CVA of the contract is derived as follows.

$$\begin{aligned}
CVA &= \int_0^T e^{-r_d t} 1_{\{\text{c/p defaults at t}\}} \max(V_t, 0) dt \\
&= \int_0^T e^{-r_d t} \lambda_t e^{-\int_0^t \lambda_u du} \max\left(\frac{S_t}{r_f}e^{-r_f t}(1-e^{-r_f(T-t)}) - \frac{\varphi_0}{r_d}e^{-r_d t}(1-e^{-r_d(T-t)}), 0\right) dt
\end{aligned}$$

In order to execute simulation, I discretized the integral as follows.

$$\begin{aligned}
CVA = \sum_{i=0}^{M} \Bigg[ & e^{-r_d(i\Delta t)}\lambda_i e^{-\sum_{j=0}^{i}\lambda_j \Delta t} \\
& \times \max\left(\frac{S_i}{r_f}e^{-r_f(i\Delta t)}(1-e^{-r_f(T-i\Delta t)}) - \frac{\varphi_0}{r_d}e^{-r_d(i\Delta t)}(1-e^{-r_d(T-i\Delta t)}), 0\right)\Delta t\Bigg]
\end{aligned}$$

$$M\Delta t = T$$

$$\lambda_i = \lambda_{i-1} \times \exp[(b_\lambda^Q - 0.5\sigma_\lambda^2)\Delta t + \sigma_\lambda\sqrt{\Delta t}\phi_1]$$

$$S_i = S_{i-1} \times \exp[(r_d - r_f - 0.5\sigma_S^2)\Delta t + \sigma_S\sqrt{\Delta t}\phi_2]$$

$$\begin{pmatrix}\phi_1 \\ \phi_2\end{pmatrix} \sim N\left(\begin{pmatrix}0 \\ 0\end{pmatrix}, \begin{pmatrix}1 & \rho \\ \rho & 1\end{pmatrix}\right)$$

By simulating 5000 times with $M = 450$ for $\rho = 0, 0.1, 0.2 \ldots 1.0$, and using given parameters, CVA was calculated at each correlation. Figure 2 shows CVA at each correlation. From the figure, we can discover that CVA increases as correlation increases. Since the bank earns more money if exchange rate increases, if default intensity is highly correlated with exchange rate, expected amount of position value when counterparty defaults will be higher. That's why CVA increases as correlation increases. Since CVA at $\rho = 0.5$ is calcuated as 0.789921%, if notional value of swap is 1 billion, CVA is calculated as about 7.899 million dollars.

## Problem 4

## Problem 5

(a)

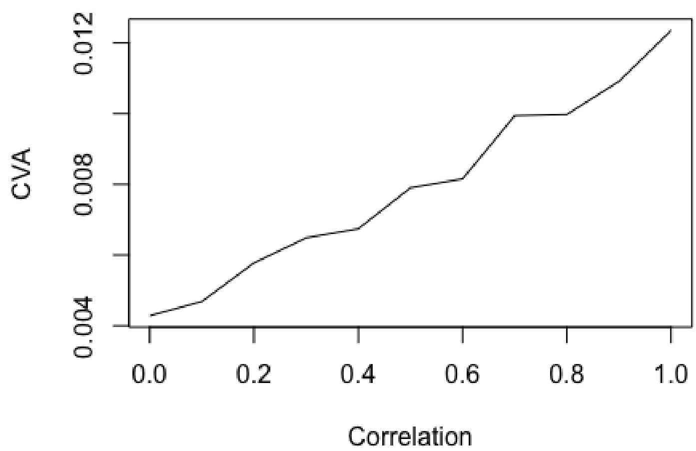


Figure 2: CVA at each correlation

(b)

(c)

(d)