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Lecture Note 1.2: Futures versus Forwards

Last time we took a detailed look at forward contracts. Today we consider their close cousin: futures contracts. There are many similarities. But the differences are very important to note too.

The distinction between forwards and futures highlights the institutional differences between exchange-traded, centrally-cleared trading and over-the-counter trading. But this distinction is eroding under regulatory pressure, as we will see.

Outline:

I. Features of Futures
- **(A)** Exchange Trading
- **(B)** Clearing
- **(C)** Marking-to-market
- **(D)** Cash Settlement

II. Futures vs. Forwards
- **(A)** Pricing
- **(B)** Hedge Ratios

III. Recent Developments

IV. Summary

I. Contract Features.

- Futures contracts are like forward contracts in that they are an agreement between two parties to buy (sell) something at a pre-specified date for a pre-agreed price.
  - However, whereas forward contracts are privately arranged between, say, two corporations or a corporation and a bank (i.e., between two professional players), futures contracts are:
    - Standardized and exchange-traded;
    - Cleared via a single counterparty;
    - Marked to market;
    - Potentially settled via cash rather than physical delivery.
  - The main goal of the contract design is: to enable numerous unacquainted agents to trade the same contract with minimal credit/legal risk.
    - That's the market imperfection that futures resolve.
- Let's think about each one of these features in some more detail.
- The exchange specifies a standard set of delivery dates, and other legal details for all contracts.
  - Then all trades are reported and the market is transparent.
  - Rather than bilateral negotiation, prices are set by all the public orders. Anyone can participate.

- The "clearing house" is a highly-capitalized, safe firm that assumes the other side of every trade.
  - Suppose you and I decide to trade 10000 bushels of soybeans for July delivery at a price of $f_0 = \$6.65$ per bushel, and that you are selling and I am buying.
  - Then, technically, that <u>one</u> trade is split into <u>two</u> trades: the clearing house agrees to buy from you and sell to me at that price .
    - The clearing firm becomes the **central counterparty** (CCP).
  - Neither of us has to worry about the other's creditworthiness before (or after) doing the trade.
    - Of course everybody has to worry about the CCP's credit.
    - Ususally its owners (members of the exchange) are required to give it more capital if it ever gets into trouble.
  - If, later, I unwind my position by going short, I don't have to do the trade with my original counterparty: my two trades (long and short) automatically **net out**.
    - This does not happen with forwards.
    - And it only works if the contracts are standardized, with the same deliverable good and same T.

- What is marking-to-market (MtM)?
  - ▶ Every day the clearing house uses the closing price to value everybody's open positions AND *winners get paid (and losers pay) immediately*.
  - ▶ So, continuing the above example, if July soybeans close today at $f_1 = 6.92$, then you have to pay the clearing house 0.27 X 10000 = 2700.
    - ∗ The settlement price of your position is also re-set to 6.92.
    - ∗ And this revaluation will continue for every day that you have an open position.
    - ∗ So if tomorrow's close is f_2, then you will have cash-flow of $-(f_2 - 6.92) \times 10000$ then.
  - ▶ This requirement is the clearing house's way of making sure that it is never exposed to too much risk of its customers defaulting.
    - ∗ If a customer defaults and they close out his position, the CCP can lose at most one-day's profit/loss.
  - ▶ Notice that, with MtM, your cash-flows are *the same as if you literally liquidated your positions each day*, and then re-established them immediately at the same price.

- When you first take a position in futures, the clearing house will *also* require you to hold some cash with them, called **initial margin** as collateral.
  - For example, the margin might be 5% of the notional value of the position.
  - This money is still yours (and sometimes it will even earn interest), so posting intial margin is not a cost to you.
  - But the clearing house has the right to seize it if you fail to pay your mark-to-market losses.
- Another name for those mark-to-market cash-flows is **variation margin**.
  - Unlike initial margin, these payments are actual gains/losses.

- Notice that marking to market may be *very* inconvenient if you are a farmer with a short position hedging your crop who *will have the soybeans* at harvest time but doesn't necessarily have any more cash today if prices go up.

- About cash settlement:
  - On the very last day of a contract's life, somebody who is long receives the actual good and pays the previous day's marking price. That's called <u>physical settlement</u>.
  - If the spot price is S_T, then the economic value of this last cash-flow is $S_T - f_{T-1}$, which would be realized by selling the physical good after it is delivered.
  - A very natural and easy modification would be to eliminate the need for the physical transaction and simply pay the long position $S_T - f_{T-1}$ in money. (And minus that to the short side, of course.) That's <u>cash settlement</u>.
  - This makes the last day just like every other day in the contract's life, with the spot price replacing the futures closing price for marking-to-market purposes.
  - Since most people trade these instruments for their economic exposure properties (and not to actually receive/deliver the underlying product) this saves lots of hassle.
  - There is no reason why you couldn't specify cash settlement for a forward contract to work basically the same way.
    - Such contracts are called **non-deliverable forwards** or NDFs.

- One you understand the mechanism of cash-settlement the door is open to an entirely new class of contracts: **non-physical underlyings**.
  - ▶ We can trade in a contract on any index or statistic!
  - ▶ Such a statistic might be:
    - ∗ Daily temperature at the Tokyo airport.
    - ∗ Fed Funds rate.
    - ∗ Number of seats won by a particular party in an election.
    - ∗ Next month's inflation rate.
    - ∗ The total payouts to all home-owner insurance policies in Florida.
  - ▶ If the statistic, X_T, can be determined in a reliable and fair way, then we will just agree that the final cash-flow will be $X_T - f_{T-1}$ on date T.
  - ▶ Notice that there is no way you could have physical settlement for any of the above examples. There is no "good" to be delivered.

- Examples of cash settlement:

Eurodollar futures: The eurodollar contract is used to hedge movements in LIBOR, the 90-day interbank dollar rate in London. On the settlement day of the futures contract, the exchange takes the average rate offered by a sample of large banks at noon.

▶ This contract may not be around very much longer.

Stock index futures: Contracts on indexes like the Dow-Jones 30 or the S&P 500 are used to bet on the overall movement in the market, without having to trade the component stocks. On settlement date, the exchange computes the final price of each stock in the index (usually averaged over some period like the last hour), and then weights these according to the index weights.

Freight forwards: Shipping companies can hedge fluctuations in the costs of operating "Supramax" cargo ships on particular routes. The Baltic Shipping Exchange publishes indexes every month of actual costs on 50 different routes. These are used to settle *forward freight agreements.*

Q: Suppose you have cash-settlement of a currency contract. Would the no arbitrage formula that we developed last time still apply? In other words, could we still do the cash-and-carry arbitrage if the contract was mispriced?

A: It depends! For the cash-settlement to be *economically* the same as physical settlement, it has to be the case that the reference settlement price that is used really IS the price at which someone could sell/buy a position in the deliverable good. If it's not, then someone attepting to do the arbitrage would not have a riskless position. They would be said to face *settlement risk.*

- Notice the potential for abuse here.

- If you have a big derivatives position that will be settled using some spot market reference quote, you will have an incentive to try to *artificially manipulate* the spot market if possible.

  - In a thin spot market, you might be willing to lose a small amount of money bidding it up too high if you were long a lot of futures.

  - This practice happens all the time, and regulators are constantly battling aginast it.

II. Futures vs. Forwards.

- We have discussed the contractual and institutional differences between forwards and futures.

- Now we need to think about whether those differences have important consequences for pricing and hedging?

(A) Pricing.

- **Question:** If the spot price (of something) is S and the forward price is F, what would be the futures price f such that you are indifferent between going long or short a <u>futures</u> contract at f?

- An obvious guess is $F = f$ since then going long a forward and short a future involves no cash flow now and no position in the underlying at T.

- What about intermediate cash flows?

  ▶ If you are long a forward and short a future and spot goes up, you will make money at T but you will have to pay out money *today*.

  ▶ Does this timing difference "wash-out" (it could work for you or against you)?

  ▶ Or does it make the long side of the future systematically better than the short side (or vice versa)?

- Just to make sure you get the intuition, Suppose there are three days left until T and you observe $f_{T-3,T} > F_{T-3,T}$ for the exact same underlying good.
  - Is there any reason why this would *not* present an arbitrage opportunity?
  - Think about the total cash-flows to buying the forward and selling the future. Here are the profits to each leg:

Date	Forward	future
T-3	0	0
T-2	0	$f_{T-3,T} - f_{T-2,T}$
T-1	0	$f_{T-2,T} - f_{T-1,T}$
T	$S_T - F_{T-3,T}$	$f_{T-1,T} - S_T$

    If you just add up the futures column, you get $f_{T-3,T} - S_T$ and if you add that to the forward P&L, then the S_T cancels and you have $f_{T-3,T} - F_{T-3,T}$ which we assumed is positive.

- Well, you can see that this isn't quite right: it ignores the fact that the futures cash-flows happen at different times, so we may be ignoring some kind of interest rate or reinvestment effects.

- But it's not obvious whether these effects would *systematically* favor the long side or the short side and thus change the conclusion that we ought to have $F = f$.

- In fact, usually, the timing difference has little value to either side.
  - ▶ It is not difficult to show that, if interest rates (and other carry costs) are predictable then we must have $f = F$ or there will be arbitrage.
    - ∗ Imagine that there are two days until T and we know both the (simple) one-day interest rate (call it $R_{T-2,T-1}$) AND the one day interest rate that will hold tomorrow (i.e. the forward rate $R_{T-2,T-1,T}$).
    - ∗ Suppose we have both a two-day forward and a two-day futures contract on a non-dividend paying stock, with $f_{T-2,T} > F_{T-2,T}$.
    - ∗ With a little imagination, you can construct an arbitrage! (And likewise if you assume $f_{T-2,T} < F_{T-2,T}$.)
    - ∗ Then, by backwards induction, you can extend the argument back to any earlier date to show that we must have $f_{T-k,T} = F_{T-k,T}$ for all k.
  - ▶ Furthermore, the conclusion will hold even with random carry costs *as long as the rates aren't correlated with the spot price.*
  - ▶ What would happen if they were?

- Take the case of (extreme) negative correlation.
  - ∗ Suppose every time the spot price went UP interest rates went DOWN.
  - ∗ Would you rather be long a forward or long a future?
    - There is still no reason to think that the mark-to-market cash-flows from a futures position will be more positive or negative.
    - BUT NOW the reinvestment of those cashflows DOES result in a systematic pattern.
    - Can you see which way it goes?
    - What's an example of an underlying asset that has this type of correlation?

- Even with this extreme correlation, the reinvestement effect will still usually be quite small. Hence:
- **Main Conclusion:** $f = F$ **holds unless**

  1. net carry costs from now till T are unpredictable, **and**

  2. the spot price S is highly correlated with those costs.
- Secondary conclusion:
  - ∗ $F > f$ if and only if $\rho(S_t, (r_t - d_t)) < 0$.

(B) Hedging.

- The **hedge ratio** or "delta" of one security with respect to another is just the increase (or decrease) in its value when the other one changes in value by $1 – holding everything else fixed.

- So if the value of a derivative is $V = V(X, t)$, where X is the value of the underlying, we write
$$\delta = \frac{\partial V}{\partial X} \approx \frac{\Delta V}{\Delta X}.$$

- If you have a formula for V, you can compute this sensitivity just by perturbing the input (X) by a little and seeing how much V changes (or by using calculus).

- Consider the delta of a forward contract with respect to its underlying (a stock, for instance) after it is created.

- First, **beware** a potential source of confusion:
  - ▶ When we refer to a "stock price", it means – of course – the value of one share of stock.
  - ▶ But we have been using "forward price" to mean the <u>settlement</u> price of a *new* forward contract whose <u>value</u> – we have seen – is zero by definition.
  - ▶ Now we're considering the value of an *old* forward contract, i.e. one whose *settlement* price was agreed upon some time in the past.

- ▶ So let that old "price" be K, and let the contract's *value* at time t be $V^K = V(S_t, K, r, \ldots)$.
- ▶ Obviously this doesn't have to be zero, if, for instance, the underlying has gone up a lot since the date at which the contract was originated.

- To calculate the value of an old forward contract with forward price K, let's imagine you off-set a (long) position by going short another forward at the current price $F_{t,T}$.
  - ▶ Then you have a riskless position which will pay you $(F - K)$ at T.
  - ▶ Since the new forward doesn't cost anything, the value of the old forward must be the (present) value of this position:

$$V^K = (F - K) \cdot e^{-r(T-t)}.$$

- Now plug in the expression for $F = S_t e^{+r(T-t)}$ that we derived, and get

$$V^K = S_t - K \cdot e^{-r(T-t)}.$$

(This is for zero dividends/convenience yield, etc. Make sure you can get the general expression yourself.)

- **Conclude:**

$$\frac{\partial V^K}{\partial S_t} = 1.$$

- Now let's calculate the delta of a futures contract similarly (with no payouts, as above).

- At inception, or at the last mark-to-market date, the value is zero, by definition. Call the future's "price" then f_0.

- At any time (until the next mark-to-market) after that, the contract could be offset in the market with a new contract at futures price $f_t = S_t e^{r(T-t)}$.

- The profit/loss would be realized at the end of the end of the day when both contracts are marked-to-market at some price f_1 to net:

$$(S_t e^{r(T-t)} - f_1) - (f_0 - f_1) = S_t e^{r(T-t)} - f_0.$$

- Thus the time-t value of the original contract, V^f is $S_t e^{r(T-t)} - f_0$.

- Hence,

$$\frac{\partial V^f}{\partial S_t} = e^{r(T-t)} > 1.$$

  (Changing S by a dollar changes $aS + b$ by a dollars.)

- **Conclusion: futures and forwards have different hedge ratios**.

- In fact, for a non-dividend paying stock (i.e. no payouts), we have just shown that the forward delta is one-for-one and the futures delta is always greater than this.
- So, when a given spot position is hedged with futures, one would go short slightly FEWER futures contracts.
  - This is called "tailing the hedge".
  - Intuitively, the futures price moves more than spot because of the implicit leverage in a futures position.
- If the underlying has payouts (e.g. a currency or a commodity) we will still get different deltas for forwards and futures, but either one might be bigger, depending on the sign of the net carry.

III. Recent Developments

- Since the financial crisis, regulators all over the world have been trying to make OTC markets behave more like exchange-traded ones.
  - ▶ They dislike the lack of transparency in OTC trading.
  - ▶ They also fear the hidden risks of bilateral clearing arrangements.

  In today's context, this means forwards are starting to look more and more like futures.

- While there is no requirement to centrally clear forward contracts (in which the CCP would mark the positions to market daily), there <u>are</u> now rules mandating that users of non-centrally-cleared derivatives of all types will have to post variation margin for them.
  - ▶ In this case, the two parties to a trade agree to name some third party service (a custodial bank) as the monitor of the trade, who will value it *every day*, and collect the money from the loser and pay it to the winner.
  - ▶ So this is economically the same as if a CCP was doing the marking to market.

- So when these new rules are in force, our conclusions about the price and hedge ratio for forwards that we derived are no longer correct because they assumed no cash-flows prior to T.
  - ▶ Instead, they must be treated like futures.

- Currently, the margin rules do not apply to "end users" of forwards, for example an airline hedging the price of jet fuel, and an import business hedging currency risk.

- But, as of 2017, they do apply to banks and most financial firms worldwide.

- As we study other derivatives markets through the semester, we will have to pay attention to what type of regulatory requirements for margining and/or clearing apply to each one.

IV. Summary

- Futures eliminate assignability and credit-risk problems with forwards; enable numerous unacquainted parties to transact with each other.

- Big difference: marking-to-market accelerates realization of gains and losses.
  - *Does not* – usually – have important valuation consequences.
    - With otherwise identical terms and predictable carry costs, forward and futures prices are equal.
  - *Does* have important consequences for hedge ratios.
    - The delta of forwards and futures with respect to spot may differ, and may not be equal to one.

- Understanding these distinctions is increasingly important as marking-to-market (or margining) is applied to more and more types of contracts.

Lecture Note 1.2: Summary of Notation

Symbol	Page	Meaning
f_n	*p4*	*futures price on day* n
S_T	*p6*	*spot market price of underlying asset at* T
X_T	*p7*	*value of arbitrary observable variable at* T
$f_{t,T}, F_{t,T}$	*p11*	*future and forward prices for the same good on date* t *for settlement at* T
$R_{T-2,T-1,T}$	*p12*	*today's one-day forward, one-day interest rate*
$\rho(S,(r-d))$	*p13*	*correlation between spot price and carry costs*
$V(X)$	*p14*	*value of some asset determined by a parameter* X
δ	*p14*	*the incremental change in* V *per unit change in* X
V^K	*p15*	*the value of an existing forward contract with delivery price* K *at time* t
$\frac{\partial V^K}{\partial S}$	*p15*	*the "delta" of a forward contract*
$\frac{\partial V^f}{\partial S}$	*p16*	*the "delta" of an equivalent futures contract*