FIN 513: Homework #2

Due on Thursday, February 1, 2018

Wanbae Park

Problem 1

(a) (1) *(Zero-coupon term structure)* Since the bonds are traded at par and these are riskless, zero-coupon term-structure can be obtained by just equating present value of their cash flow to their price($=100$). Therefore, the following equation holds.

$$100=\sum_{t=1}^{T}\frac{\textit{Cash flows at time t}}{(1+r_{0,t})^t}$$

In order to obtain the whole term-structure, one should get $r_{0,t}$ first by equating the value of bond with maturity t to present value of its cash flow first, then $r_{0,t+1}$ can be obtained by using $r_{0,t}$. It is because it is necessary to use $r_{0,t}$ to discount cash flows of bond maturing at $t+1$. Therefore, calculate $r_{0,1}$ first.

$$100=\frac{100+100\times 0.0133}{1+r_{0,1}}=\frac{100(1+0.0133)}{1+r_{0,1}}$$

It is trivial that $r_{0,1}=0.0133$. Then, let's use this result to calculate $r_{0,2}$.

$$100=\frac{100\times 0.0173}{1+0.0133}+\frac{100+100\times 0.0173}{(1+r_{0.2})^2}$$

Then it is calculated that $r_{0,2}=0.01733$. Using this procedure ahead, zero-coupon term-structure from 1 year to 10 year can be obtained. Table 1 shows that the term structure of zero coupon bonds obtained by using this procedure.

Maturity(years)	$r_{0,t}$(%)
1	1.330
2	1.733
3	2.162
4	2.577
5	2.752
6	3.196
7	3.435
8	3.459
9	3.474
10	3.562

Table 1: Term structure of zero coupon bonds

(2) *(Term-structure of one-year forward rates)* Let $r_{0,t,t+1}$ denote a forward rate from time t to $t+1$ determined at current time. Then $r_{0,t,t+1}$ can be obtained by comparing the following two strategies.

i. Invest \$1 to zero coupon bond maturing at time t, and when the bond matures, receive money and reinvest to zero coupon bond maturing at time $t+1$.

ii. Invest \$1 to zero coupon bond maturing at time $t+1$.

Since the amount of investment at current time is equal for both strategies, by definition of forward rate, the following equation should hold.

$$
\begin{aligned}
(1+r_{0,t+1})^{t+1} &= (1+r_{0,t})^t(1+r_{0,t,t+1}) \\
\Rightarrow r_{0,t,t+1} &= \frac{(1+r_{0,t+1})^{t+1}}{(1+r_{0,t})^t} - 1
\end{aligned}
$$

Therefore, since the term structure of spot rate is given above, by using this formula term structure of 1-year forward rate can be obtained. Table 2 shows term structure of 1-year forward rate using the data given in assignment.

t	$r_{0,t,t+1}(\%)$
1	2.139
2	3.025
3	3.830
4	3.459
5	5.441
6	4.881
7	3.627
8	3.592
9	4.363

Table 2: Term structure of 1-year forward rate

(b) A fair swap rate s is a rate which makes the present value of fixed leg and that of floating leg equal. Therefore, by calculating present values of each leg separately and equating each other, fair swap rate can be obtained. Let X and T denote the notional principal of swap and the maturity of swap, respectively. Then using the following procedure, fair swap rate s can be obtained.

(a) *(Fixed leg)* Since there is no exchange of notional principal in IRS, present value of fixed leg can be obtained by solving following equation.

$$\sum_{t=1}^{T} \frac{sX}{(1+r_{0,t})^t}$$

(b) *(Floating leg)*

(c)