

Homework 1

1. (*Single-Stock Futures.*) The Intercontinental Exchange (ICE) lists futures on individual stocks. Assume all dividends and borrowing costs for these stocks are certain. Agree or disagree with the following statements:

- (a) Other things being equal, the futures price will be a higher premium over the spot price for a high-growth stock (like a young technology company) than for a low-growth stock (like a mature auto company).
- (b) To value the futures contract by no-arbitrage, we need to assume the stock will not go bankrupt before the settlement time T .
- (c) When the underlying stock pays a dividend, we expect to see its futures prices for all maturities drop by approximately the amount of the dividend per share the moment the shares go ex-dividend.

2. (*Forward pricing and hedging.*)

You are a market maker in dollar/euro forward prices at a large, highly rated bank. You can borrow and lend in each currency (risklessly) at the interest rates shown below.

T	dollar rates		euro rates	
	lend	borrow	lend	borrow
1 yr	0.25	0.30	0.70	0.80
5 yr	2.10	2.19	2.20	2.32
10 yr	3.40	3.45	3.10	3.25

The interest rates are in annualized bond-equivalent (semi-annually compounded) percentage. The spot exchange rate is 1.4995 bid - 1.5000 offer (in dollars/euro).

- (a) Give an expression for the upper and lower no-arbitrage prices for the forward exchange rate. Compute numerical values of your bid and offer prices for forward euros to each horizon, using the numbers shown. (Give answers to four decimal place accuracy.)
- (b) If a customer buys 10 million euros for 10-year forward delivery (at your offer price computed in part a), how many spot euros would you buy to hedge?

3. (*A 2-date Contract.*) Suppose you want to buy heating oil at two separate times in the future, the first being 18 months away, and the second 24 months. You observe the bond prices and forward prices in the market that are given below. For planning purposes, you want to lock in one single price you will pay for oil on both dates. What is the fair (no-arbitrage) price that a market maker would be willing to offer you for the type of contract you want?

Date	Bond	Fwd
$t - t_0$	$e^{-r_t(t-t_0)}$	F
1.5	0.912	308
2.0	0.883	313

4. (*Repo versus Stock Loan.*) In the U.S., corporations do not have to pay tax on 70% of the dividend income they get from stocks. This is called the “dividends-received deduction” or DRD.

However, if an institution lends out a stock and the stock pays a dividend and the stock borrower pays the lender the amount of the dividend, this payment does NOT count as a dividend for tax purposes and so does not get the DRD.

You work for a brokerage firm and you have arranged a stock loan between a hedge fund client, H (the borrower), and a taxable institution, L (the lender) of 1 million shares of Google stock. The loan has a fee of 25 basis points. The hedge fund intends to keep its short position in Google for at least 6 months. Google then declares a dividend of \$1 per share to be paid in 1 month. Client L has a marginal tax rate of 30%. So they will require a payment from H that is large enough to make them willing to not terminate the loan.

- (a) How much is the payment that L will require from H?
- (b) You have a third client, E, that is tax exempt, meaning it has a marginal tax rate of 0%. (For example this could be a municipal entity, a charity, or just a corporation with no profits in the current year.) Assume that you can trade in forward contracts for Google at the no-arbitrage price set by tax-exempt firms.

Design a transaction that can be executed right before the dividend between your firm and E, H, and L that (i) exposes neither your firm nor E to any risk with respect to Google stock; (ii) allows H to stay short and L to stay long; and (iii) saves everybody money – except the tax authorities – relative to the alternative in part (a).

5. (*Contracts For Difference.*) A very popular product in Europe, called a contract for difference (CFD), has similarities to both forwards and futures, but also some unique features. Like both, it is entered into initially with no up-front payment to either side. However,

- There is no terminal date; CFD positions stay open until an off-setting position is taken.
- Positions are marked-to-market daily *at the spot closing price*.
- Long positions are paid the amount of any dividend (when the underlying is a stock), and are charged “interest” on the notional value of the position every day. (Short positions have the opposite payouts.)

In some jurisdictions there are tax and regulatory differences as well (e.g. no stamp duty), which we will ignore for now.

Assume perfect markets: the interest rate used to compute the financing charge is r the riskless rate; there are no commissions, sort-sales charges, or bid/ask spreads.

- (a) Assume the underlying is a non-dividend-paying stock and that interest is debited/credited continuously. What is the no-arbitrage price, c , of the CFD in terms of S and r ?
- (b) Now suppose the interest rate is still r , but that it is ONLY charged for CFD positions held overnight. Furthermore suppose that your broker always sets the CFD price to the spot price: $c_t = S_t$ for all t . Does this present you with an arbitrage opportunity? Why or why not?