FIN 513: Homework #6

Due on Tuesday, March 13, 2018

Wanbae Park

Problem 1

(a) Since yield-to-maturity is a discount rate which discount "promised" payoff to current price, F is represented as $F = \frac{1}{1+y}$.

(b) Under risk-neutral measure, all expected return is risk-free. Therefore, F is represented as $F = \frac{p\times 1+(1-p)\times 0}{1+R} = \frac{p}{1+R}$.

(c) Since expected return of the bond is e, and there is true survival probability π, F is represented as $F = \frac{\pi\times 1+(1-\pi)\times 0}{1+e} = \frac{\pi}{1+e}$.

(d) Since $F = \frac{p}{1+R} = \frac{\pi}{1+e}$, $\frac{\pi}{p} = \frac{1+R}{1+e}$. Furthermore, because the bond is risky, e must be greater than R, therefore the inequality $\frac{\pi}{p} = \frac{1+R}{1+e} < 1$ holds.

(e) From (d), risk-neutral survival probability is always greater than true survival probability. Since given true probability is 97% and it is almost 100%, risk-neutral survival probability might be 100%. It means that under risk-neutral measure, expected payoff of bond is almost equal to 1. Therefore, yield-to-maturity is almost same as risk-free rate, and the bond price is almost equal to face value discounted at risk-free rate. Furthermore, since expected return of asset is 12%, and firm's asset is not risk-free, risk-free rate must be less than 12%. Therefore, bond price must le larger than $\frac{1}{1.12}$.

Problem 2

Under Merton model, stock price is considered as a call option of firm value, with strike price is face value of a debt. Therefore, stock volatility can be calculated as follows.

$$
\begin{aligned}
&S = VN(d_1) - Fe^{-r(T-t)}N(d_2) \\
&\frac{\partial S}{\partial V} = N(d_1) \ \text{ which is delta of call option.} \\
&\Rightarrow \textit{stock volatility} = N(d_1)\frac{V}{VN(d_1) - Fe^{-r(T-t)}N(d_2)}\sigma_V \\
&\text{where } \ d_1 = \frac{\log(V/F) + (r + \frac{1}{2}\sigma_V^2)(T-t)}{\sigma_V\sqrt{T-t}} d_2 = d_1 - \sigma_V\sqrt{T-t} \\
&F : \text{Face value of debt}
\end{aligned}
$$

By using the formula, stock volatilities were calculated with respect to stock prices from 0 to 40. The range of stock prices corresponds to the range of firm value from 11 to 110. Figure 1 shows the stock volatility as a function of stock price. As shown in figure, stock volatility decreases as stock price increases. Furthermore, under constant elasticity of variance model(CEV Model), since stock price process is defined as $\frac{dS}{S} = \mu dt + \frac{\omega}{\sqrt{S}}dW$, the corresponding variance of stock price is $\frac{\omega}{\sqrt{S}}$. Figure 2 shows volatility as a function of stock price under CEV model. As shown in figure, volatility also has a decreasing feature as stock price

Problem 2 continued on next page. . .

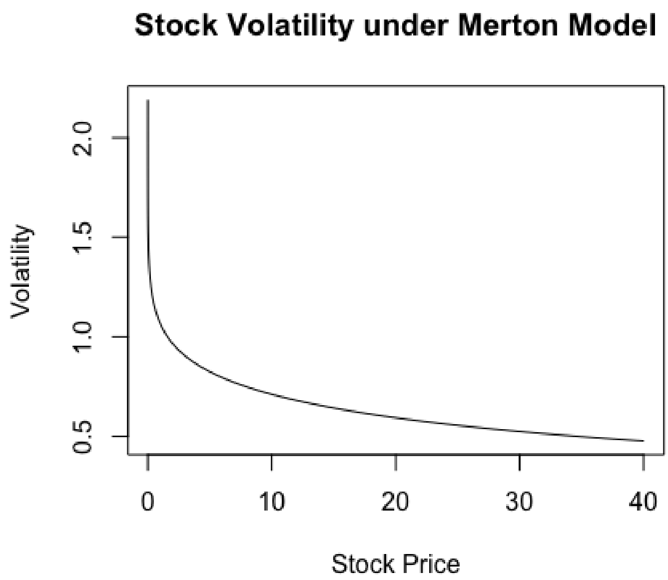

Figure 1: Stock Volatility under Merton Model

increases. However, since there is stock price in denominator term, if stock price gets closer to zero, volatility increases sharply.

Problem 3

(a) Using the formula in question 2, volatilty multiplier was calculated for each volatility and face value. Figure 3 and 4 show volatilty multiplier with each face value with respect to volatility of firm value. As shown in figure, volatility multiplier remains constant if volatility of firm value is low, but as volatility of firm value increases, volatility multiplier starts to decrease. It means that from some level, marginal increase of stock volatility becomes less than the amount of increases in volatilty of firm value.

(b)

(c)

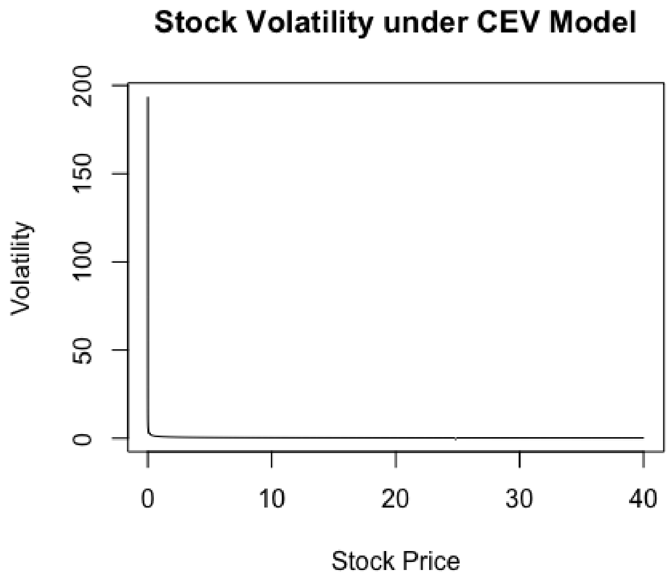

Figure 2: Stock Volatilty under CEV Model

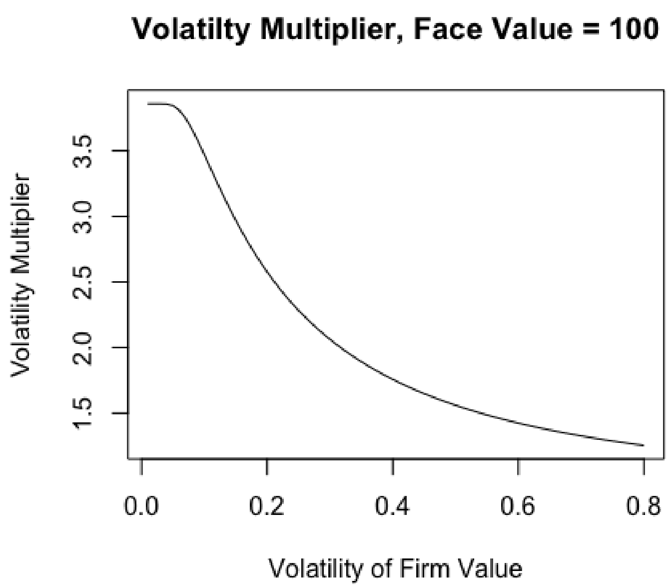

Figure 3: Volatility multiplier with face value 100

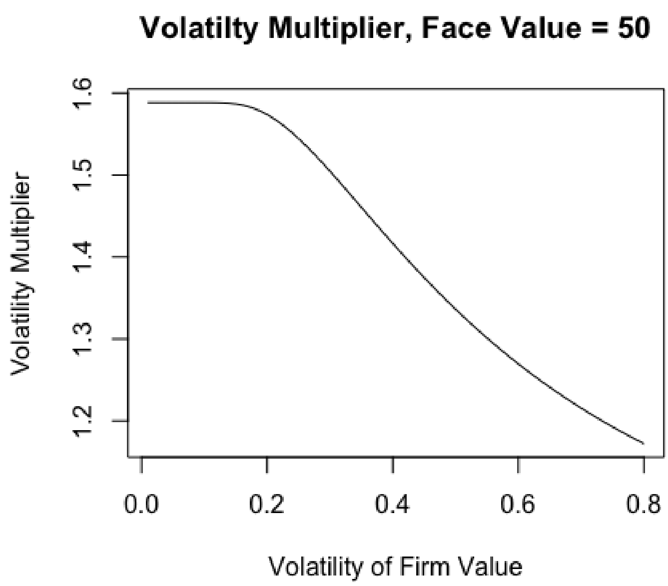

Figure 4: Volatility multiplier with face value 50