Homework 4

1. *(Time decay.)* A straddle is just a portfolio of a call and a put (long both) at the same strike price and same expiration. Let the value of that portfolio be denoted Π.

   (a) Value a one year European straddle on the dollar/euro exchange rate with $S = K = 1.0$ (in dollars-per-euro, so the dollar is the domestic currency) using monthly time-steps and $r^{dollar} = .02, r^{euro} = .03$, (both annual, continuously compounded rates) and $\sigma = .16$.

   (b) From the tree you just built tree, you can also immediately get the values of Π for $S = 1.0$ with 10, 8, 6, 4, 2, and 0 months to expiration. Plot these values and use them to estimate

   $$\frac{\partial \Pi}{\partial t}$$

   as of each time-steps when $S = 1.0$. You can check your approximation by using the closed-form binomial formulas with smaller Δt.

   (c) Does your plot show that buying a straddle has a negative expected return? Explain.

2. *(Target range forwards.)* In 2008 and 2009 many Asian and Latin American corporate borrowers were fooled into overpaying for so-called "Target Range" forward contracts (TRFs).

   In these, they got the right to buy a foreign currency (e.g., the dollar) at each of a series of dates for a low price (the "reference level") until their cumulative profit reached a cap. Meanwhile, they also had the obligation to buy twice as many dollars at the same price at each date if the dollar fell in value below the reference level.

   Consider the currency option example used in the lecture notes (c.f. Lecture Note 4.2 page 9). Using the parameters of that problem, value a contract in which the payments dates are in 6 months and one year, the reference level is $K = 90$, and the cumulative profit cap is $C = 20$ yen.

   Specifically, if the dates in the contract are T_0, T_1, T_2, T_3, T_4, then this says that at T_2 you own the right to the payoff $\max[X_2 - K, 0]$ where $X_2 = \min[S_2, K + C]$. At T_4 you receive $\max[X_4 - K, 0]$ where $X_4 = \min[S_4, K + C - (S_2 - K), K + C]$. You also owe $2\max[K - S_2, 0]$ and $2\max[K - S_4, 0]$ at T_2 and T_4, respectively.

3. *(T/F.)* Agree or disagree with the following statement. Explain your reasoning.

   To derive options prices under the binomial model it is necessary to assume the options are not marked to market.

4. *(Restrictions on q.)* In class, we emphasized that the binomial parameter q isn't a real, physical probability of anything. It is just convenient to interpret it that way because then we can view options prices as discounted "expectations" of payoffs. The interpretation would seem even more natural if we also could show that, like real probabilities, q had to be between 0 and 1.

   (a) Prove that this is indeed the case by showing that if either bound is violated, one can construct an arbitrage. Specifically, let S_t be the price of a foreign currency whose one-period interest rate is r^f. The domestic rate is r^d. Show that if S moves on any binomial grid with the property that the implied $\bar{q}$ is bigger than one, you can find a one-period arbitrage without making any other assumptions about other assets. Also do the case $\bar{q} < 0$.

   (b) Your boss wants you to model the behavior of the *afghani/dinar* exchange rate on a monthly binomial grid. The Aghan 1-month rate is 2% and the Iraqi (dinar) rate is 8%. Both are simple rates per month (not annualized). (Let us assume these are truly riskless ,and borrowing and lending is possible at those rates.) The exchange rate y has an annual standard deviation of 10% . There is no reason to expect either currency to appreciate against the other: so assume the true probability of a down move in any month is 50%.

   What u and d can you use, or is this situation impossible? Explain.
   (Note: do not use the approximate relations on page 8 of Lecture Note 4.1 for this problem.)

5. (*Non-constant volatility.*) The current stock price of Hale Umbrellas is \$20.00. The volatility of the stock return on this company moves inversely with the stock price,

$$\sigma^2 = \frac{5}{S}.$$

   For example, the current standard deviation is $50\% = \sqrt{5/20}$ per year.

   (a) Construct a two-period tree with yearly time steps so as to model this volatility evolution. If the probability of an up-move in your tree is 55%, what is the exact volatility (i.e. standard deviation) of the stock's two-year continuously compounded return? What is it in annualized terms?

   (b) The riskless rate (simple) is 1% per year. Estimate the value of a two-year European call option with a strike price of \$20 assuming Hale will not pay any dividends. Would the value of the call be higher or lower if the volatility was instead constant, and equal to the annual average value computed in (a)?