FIN 513: Homework #5

Due on Tuesday, March 6, 2018

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Problem 1

Let V and S denote sum of values of options and stock price respectively, then we can denote portfolio of the market maker as $\Pi = V - \Delta S$. By Ito's lemma, the following equation follows.

$$d\Pi = \frac{\partial V}{\partial t}dt + \frac{\partial V}{\partial S}dS + \frac{1}{2}\frac{\partial^2 V}{\partial S^2}dS^2 - \Delta dS$$

Since the portfolio has zero delta, $d\Pi = \frac{\partial V}{\partial t}dt + \frac{1}{2}\frac{\partial^2 V}{\partial S^2}dS^2 = \frac{\partial V}{\partial t}dt + \frac{1}{2}S^2\sigma^2\frac{\partial^2 V}{\partial S^2}dt$ holds. Since the portfolio became riskless, by no arbitrage principle, its return must be equal to risk-free rate as follows.

$$d\Pi = \frac{\partial V}{\partial t}dt + \frac{1}{2}S^2\sigma^2\frac{\partial^2 V}{\partial S^2}dt = r\Pi dt$$

Since parameters are given as $\frac{\partial^2 V}{\partial S^2} = -1.725$, $S = 143$, $r = 0.05$, $\sigma = 0.7$, plugging them into the equation above, we can obtain the following result.

$$\begin{aligned}&\frac{\partial V}{\partial t} + \frac{1}{2} \times (143)^2 \times (0.7)^2 \times (-1.725) = 0.05 \times 30,000,000\\ &\Rightarrow \frac{\partial V}{\partial t} = 1,508,642.26\end{aligned}$$

Assuming a year is equal to 365 days, if the stock price is unchanged, the expected value of the positions is approximately $\Pi + d\Pi = \Pi + \frac{\partial V}{\partial t}dt = 30,000,000 + 1,508,642.26 \times \frac{1}{365} = 30,004,133.3$. The value might not be exact since the whole procedure was implemented on continuous time framework which is not exactly consistent to this problem. However, since $dt = \frac{1}{365}$ is small enough, errors can be ignored.

Problem 2