# FIN 513: Homework #6

Due on Tuesday, March 13, 2018

**Wanbae Park**

# Problem 1

(a) Since yield-to-maturity is a discount rate which discount "promised" payoff to current price, $F$ is represented as $F = \frac{1}{1+y}$.

(b) Under risk-neutral measure, all expected return is risk-free. Therefore, $F$ is represented as $F = \frac{p \times 1 + (1-p) \times 0}{1+R} = \frac{p}{1+R}$.

(c) Since expected return of the bond is $e$, and there is true survival probability $\pi$, $F$ is represented as $F = \frac{\pi \times 1 + (1-\pi) \times 0}{1+e} = \frac{\pi}{1+e}$.

(d) Since $F = \frac{p}{1+R} = \frac{\pi}{1+e}$, $\frac{\pi}{p} = \frac{1+R}{1+e}$. Furthermore, because the bond is risky, $e$ must be greater than $R$, therefore the inequality $\frac{\pi}{p} = \frac{1+R}{1+e} < 1$ holds.

(e) From (d), risk-neutral survival probability is always greater than true survival probability. Since given true probability is 97% and it is almost 100%, risk-neutral survival probability might be 100%. It means that under risk-neutral measure, expected payoff of bond is almost equal to 1. Therefore, yield-to-maturity is almost same as risk-free rate, and the bond price is almost equal to face value discounted at risk-free rate. Furthermore, since expected return of asset is 12%, and firm's asset is not risk-free, risk-free rate must be less than 12%. Therefore, bond price must le larger than $\frac{1}{1.12}$.

# Problem 2

Under Merton model, stock price is considered as a call option of firm value, with strike price is face value of a debt. Therefore, stock volatility can be calculated as follows.

$$S = VN(d_1) - Fe^{-r(T-t)}N(d_2)$$

$$\frac{\partial S}{\partial V} = N(d_1) \quad \text{which is delta of call option.}$$

$$\Rightarrow \textit{stock volatility} = N(d_1)\frac{V}{VN(d_1) - Fe^{-r(T-t)}N(d_2)}\sigma_V$$

$$\text{where} \quad d_1 = \frac{\log(V/F) + (r + \frac{1}{2}\sigma_V^2)(T-t)}{\sigma_V\sqrt{T-t}} d_2 = d_1 - \sigma_V\sqrt{T-t}$$

$$F : \text{Face value of debt}$$

By using the formula, stock volatilities were calculated with respect to stock prices from 0 to 40. The range of stock prices corresponds to the range of firm value from 11 to 110. Figure 1 shows the stock volatility as a function of stock price. As shown in figure, stock volatility decreases as stock price increases. Furthermore, under constant elasticity of variance model(CEV Model), since stock price process is defined as $\frac{dS}{S} = \mu dt + \frac{\omega}{\sqrt{S}}dW$, the corresponding variance of stock price is $\frac{\omega}{\sqrt{S}}$. Figure 2 shows volatility as a function of stock price under CEV model. As shown in figure, volatility also has a decreasing feature as stock price

Problem 2 continued on next page. . .

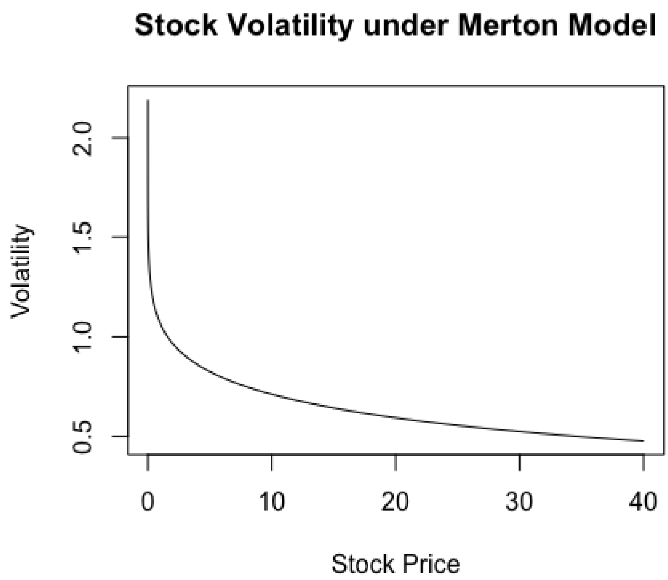


Figure 1: Stock Volatility under Merton Model

increases. However, since there is stock price in denominator term, if stock price gets closer to zero, volatility increases sharply.

## Problem 3

(a) Using the formula in question 2, volatilty multiplier was calculated for each volatility and face value. Figure 3 and 4 show volatilty multiplier with each face value with respect to volatility of firm value. As shown in figure, volatility multiplier remains constant if volatility of firm value is low, but as volatility of firm value increases, volatility multiplier starts to decrease after a threshold at some level. It means that after the level, marginal increase of stock volatility becomes less than the amount of increases in volatilty of firm value. Moreover, it can be found that the level of threshold is higher when face value of debt is lower, and volatility multiplier is higher when face value of debt is higher.

(b) If idiosyncratic risk is added, then $\sigma_V$ increases, but $\pi_V$ does not increase. Therefore, $\pi_S = \pi_V \frac{\sigma_S}{\sigma_V}$ decreases since volatility multiplier decreases as $\sigma_V$ increases, and $\pi_V$ remains constant. If $\sigma_V$ is low, changes in excess return of stock would not be too much because volatilty multiplier seems almost constant if $\sigma_V$ is less than threshold level.

(c) From answer to (b), excess return of stock decreases as idiosyncratic risk in firm value increases. From firm's perspective, less cost of equity capital is better to sell equity in future. Therefore, managers will prefer to increase idiosyncratic risk in firm value.

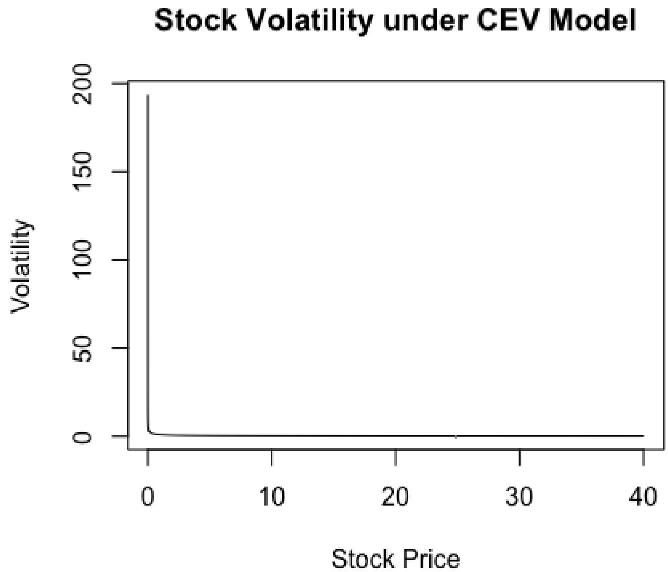


Figure 2: Stock Volatilty under CEV Model

# Problem 4

(a) Under Leland model, the optimal bankruptcy covenant is derived as $V_B = \frac{X}{1+X}\frac{\Gamma}{r}(1-\Omega)$, where $X = \frac{2r}{\sigma^2}$. From this result, we can analyze comparative statics of the Leland model.

(i) From the equation above, it can be showed that higher tax rate makes $V_B$ lower since all other parameter is positive.

(ii) In order to investigate the effect of changes in risk-free rate, we can differentiate $V_B$ with respect to risk-free rate, $r$.

$$\begin{aligned} V_B &= \frac{X}{1+X}\frac{\Gamma}{r}(1-\Omega) \\ &= \Gamma(1-\Omega)\frac{2r/\sigma^2}{1+2r/\sigma^2}\frac{1}{r} \\ &= 2(1-\Omega)\Gamma\frac{1}{\sigma^2+2r} \\ \Rightarrow \frac{dV_B}{dr} &= 2(1-\Omega)\Gamma \times (-\frac{1}{(\sigma^2+2r)^2} \times 2) \\ &= -4(1-\Omega)\Gamma\frac{1}{(\sigma^2+2r)^2} < 0 \end{aligned}$$

Therefore, higher risk-free rate makes optimal covenant level lower.

(iii) The optimal covenant level does not include bankruptcy cost. It means that $V_B$ is independent of bankruptcy cost, $\alpha$.

(b) Assuming that manager of each country choose optimal coupon in order to maximize firm's equity value.

Problem 4 continued on next page. . . 

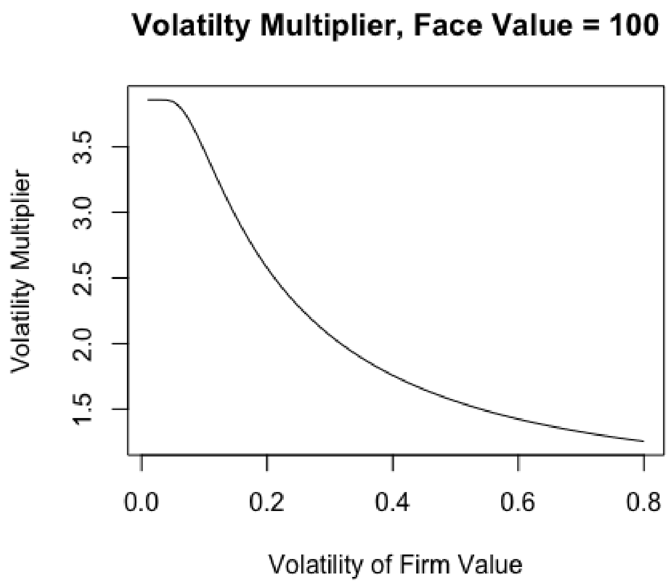


Figure 3: Volatility multiplier with face value 100

Under Leland model, equity value is represented as $S = V_0 - (F - F^{TS} + F^{BC})$, where $F, F^{TS}, F^{BC}$ denotes value of debt, tax shield, bankruptcy cost, respectively. Plug the equation (5), (6), (7) in lecture note to the equation above, firm's stock price is calculated as follows.

$$
\begin{aligned}
S &= V_0 - \left(\frac{\Gamma}{r} + \left[(1-\alpha)V_B - \frac{\Gamma}{r}\right]\left(\frac{V_B}{V_0}\right)^X - \Omega\frac{\Gamma}{r}\left[1 - \left(\frac{V_B}{V}\right)^X\right] + \frac{\alpha}{V^X}V_B^{1+X}\right) \\
&= V_0 - \left(\frac{\Gamma}{r} + \left[V_B - \frac{\Gamma}{r}\right]\left(\frac{V_B}{V_0}\right)^X - \Omega\frac{\Gamma}{r}\left[1 - \left(\frac{V_B}{V}\right)^X\right]\right)
\end{aligned}
$$

From the equation above, we can find that stock price is independent of bankruptcy cost. Therefore, regardless of choice in $\Gamma$, covenant level does not changes as bankruptcy cost changes, if others are constant.

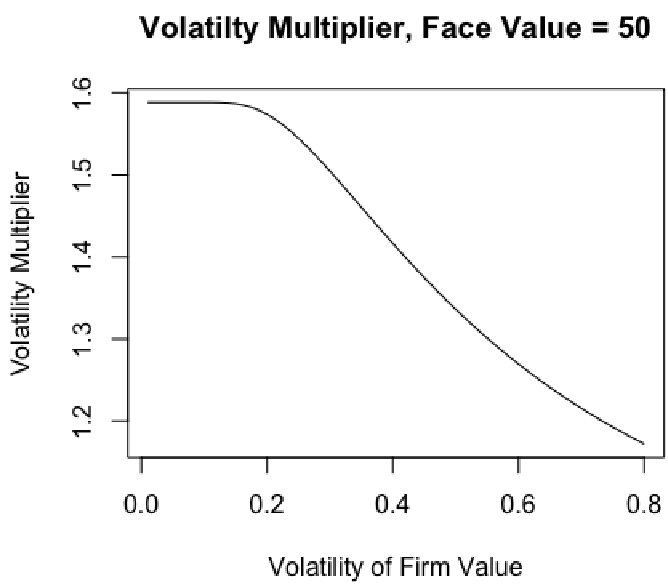


Figure 4: Volatility multiplier with face value 50