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Lecture Note 8: Counterparty Risk

Now that we have tools for handling financial engineering problems with multiple sources of risk, let's return to the world of credit derivatives and examine how we might analyze counterparty risk.

In earlier lectures we observed that counterparty risk in a swap contract, for instance, was difficult to analyze because it depended on *both* the probability of counterparty default *and* on the likely value of the swap in the states of the world where the counterparty does default. Thus it is a multidimensional problem.

To start, we can use the tools we have developed in recent weeks to address the problem for certain cases. Then we will introduce some new modeling tools. This will show us general ways to value counterparty risk in derivatives.

Last, our models will show us something quite unexpected: how to value the risk of *our own default.*

Outline:

I. Counterparty risk in structural models

II. Reduced-form default models

III. Example: wrong-way risk in CDS

IV. CVA

V. DVA vs CVA

VI. Summary

I. Structural default models.

- As we discussed earlier in the course, counterparty risk is a major headache for valuing OTC derivatives.
  - ▶ Any security that is not centrally cleared (or collateralized) exposes you to it.

- Setting the stage, suppose we have a (European) derivative paying off $C(X_T)$ at T where X_t is the vector of relevent state variables.
  - ▶ Once we have diffusion models of X, we know the PDE that the price of the derivative obeys.
  - ▶ Then we invoke Feynman-Kac to tell us how we should modify dX to a model under which expected discounted payoffs solve the PDE:

$$V_1(X_t) = e^{-r(T-t)}\ \mathrm{E}^{\star}[C(X_T)].$$

    (Today I'm just going to assume the riskless rate r is constant for simplicity.)

  - ▶ On the other hand, I'm not assuming $C(X_T) > 0$. This could be a derivative that requires us to pay something in some states.

- Our goal for the day is to compute the *difference* between that computation and the same expectation taking into account how the payoffs will differ if our counterparty defaults.

- So, if our counterparty is denoted Y, and if the recovery rate if Y defaults is $R < 1$, then the value should really be written

$$V_2(X_t) = e^{-r(T-t)}\ \mathrm{E}^\star[C(X_T) 1_{\{Y \text{ doesn't default at } T\}}$$
$$+C(X_T) 1_{\{Y \text{ defaults at } T\}}\ 1_{\{C(X_T)<0\}}$$
$$+R\ C(X_T) 1_{\{Y \text{ defaults at } T\}} 1_{\{C(X_T)>0\}}].$$

  - ▶ Recall that the assumption is that if we have a negative value when the countepart defaults, we still have to pay that liability in the bankruptcy settlement process.

- Now add and subtract $C(X_T) 1_{\{Y \text{defaults at } T\}} 1_{\{C(X_T)>0\}}$ inside the expectation, and we can write $V_2(X_t)$ as

$$e^{-r(T-t)}\ \mathrm{E}^\star[C(X_T)] + (R-1)\ e^{-r(T-t)} \mathrm{E}^\star[C(X_T) 1_{\{Y \text{defaults at } T\}} 1_{\{C(X_T)>0\}}]$$

- The first term is the value without counterparty risk, $V_1(X_t)$. And so the extra term is the contribution from this risk. We can express this as

$$V_2(X_t) = V_1(X_t) + (R-1)\ e^{-r(T-t)}\ \mathrm{E}^\star[\max(C(X_T), 0)\ 1_{\{Y \text{ defaults at } T\}}] \tag{1}$$

- Computing this extra option-like term is our job for today.
  - ▶ In the more general case where the contract can have cash-flows at intermediate dates, we have to integrate the whole expression over time from t to T.

- So to do that calculation, we need to specify a model of what determines whether or not Y has defaulted at each date.

- A natural place to start is to return to the structural models we studied earlier.

- For example, suppose Y has stock price whose dynamics obeys a CEV model.
  - ▶ The risk-neutralized specification would be

$$dS_t^Y = r\, S_t^Y\, dt + \sigma_0^Y \sqrt{S_t^Y} \sqrt{S_0^Y}\, dW^Y .$$

  - ▶ We can then model bankruptcy as equivalent to the event that $S^Y = 0$.

- Likewise, we could back up a step further and model Y's asset value V_t^Y as a diffusion that goes bankrupt when it hits some level V_B^Y.
  - ▶ We could even potentially model the recovery rate to be a diffusion.

- Of course, the structural approach is no help if the counterparty is not a traded firm.
  - ▶ But they presumably do have some debt outstanding – or else they would never default!

- So what about a third approach where we view the value of their bonds as the driving diffusions?
  - ▶ This might be sensible if the bonds were actually readily tradeable.

- With any of these approaches, we can then just simulate the diffusion process for Y along with the other variables X and compute our adjusted value V_2 above.
  - ▶ Conceptually, there is nothing particularly difficult here.
  - ▶ We would only have to add one key ingredient: the correlations between dY's diffusion and the elements of dX.
  - ▶ This correlation (or correlations) would then determine the risk-neutral *joint* probability of the counterparty defaulting and losing money on the bet on X.

- In some applications, the structural approach is a good way of computing counterparty risk adjustments. However it has a number of limits. In particular, we might be concerned about two implementation problems.

  No tradeable hedging asset. If there isn't a liquid market for a hedging security, then the no-arbitrage argument breaks down. Also we need to have good data for it in order to estimate its correlations with other factors.

  Default surprises. Structural models all assume default events are not surprises in the sense that the diffusions hit the bankruptcy states continuously. This seems especially problematic if, in reality, we can't see the diffusion itself *or* the barrier with any precision.

- These considerations – as well as some computational issues we will encounter later – have motivated practitioners to explore an alternative to structural modelling.

II. Reduced-form default models.

- The so-called "reduced-form" modelling approach attempts to capture the risks of default by simply describing the probability of default at each instant (rather than describing, say, all the assets and liabilities).
  - ▶ One way to view it is a model of a state-variable A_t that equals one on every day t until default, and then drops to zero – i.e., and indicator variable for the firm being alive at t.
  - ▶ As we have learned from our study of credit models, we don't really care about anything but this state indicator.

- Suppose, at every date t there is an <u>independent</u> coin flip, I_t, with probability p_t that determines whether or not bankruptcy occurs for the reference entity that we are interested in.
  - ▶ Then the probability of survival to time t_K is

$$\mathrm{E_t}\left[\Pi_{k=1}^{K}(1-I_{t_k})\right]=\Pi_{k=1}^{K}\ \mathrm{E_t}\left[(1-I_{t_k})\right]=\Pi_{k=1}^{K}(1-p_{t_k})$$

  - ▶ In continuous time, we can describe the evolution of the probabilities by an intensity process λ_t such that $p_t \approx \lambda_t\ dt$ or $(1-p_t)=e^{-\lambda_t\ dt}$. Then

$$\Pi_{u=t}^{T}\ (1-p_{t_k}) \rightarrow \mathrm{E_t}\left[e^{-\int_t^T\ \lambda_u\ du}\right].$$

(Beware: λ doesn't stand for a market price of risk now.)

▶ If λ is a deterministic function, the expectation here is just $\exp(-\bar{\lambda}(T-t))$ where

$$\bar{\lambda} = \frac{1}{T-t} \int_t^T \lambda(u)\, du.$$

- This default intensity model is consistent with a continuous-time model of the state variable A_t which has jumps:

$$\frac{dA}{A} = -dJ_t.$$

- Here J_t is a *Poisson process* with intensity λ.
  ▶ The Poisson process starts at 0 and jumps up to 1 in time interval dt with probability $\lambda\, dt$. Such a jump then drives the indicator to zero. And it stays there.

- This is the first model we have seen that explicitly incorporates jumps.
  ▶ Notice that here the jump size is not a random variable (it's always 1.0). Also, we are only interested in the *first* jump because after that the entity is dead.

- Yes, jumps can be incorporated into the no-arbitrage pricing framework!
  ▶ If we have two securities, $c(J,t)$ and $p(J,t)$, exposed only to jump risk, then we can create an instantaneously hedged portfolio that holds one unit of c and sells short $(c(1,t) - c(0,t))/(p(1,t) - p(0,t))$ units of p. Then it will not change in value when J jumps from 0 to 1. So it must earn the riskless rate.

- This logic – plus the version of Ito's lemma for jump processes – leads to a no-arbitrage differential equation, as you would expect.

- Now a generalized Feynman-Kac theorem tells us that we can solve the equation to find $c = c(0,t)$ by *adjusting the jump intensity from* λ *to* λ^Q, and computing c's expected discounted cash-flows under this altered model.

- Here, $\pi = \lambda^Q - \lambda$ is the market price of jump risk. It is equal to the risk premium on an asset with unit exposure to J risk.

  * If it is positive, then, the pricing measure is more pessimistic: risk-neutral jumps happen more often than true ones.

- We have seen previously that we can infer a risk-neutral default distribution, $1 - H(t,T) = \text{Prob}^Q\{\text{survival to } T\}$ from market prices of single-name CDS – plus some assumption about the recovery rate.

- We can then use this to extract a λ_u^Q for every date $u \leq T$ from that inferred distribution via

$$1 - H(t,T) = \text{Prob}^Q\{\text{survival to } T\} = e^{-\int_t^T \lambda_u^Q \, du}.$$

- Market data – credit spreads – provide us with the term-structure of *expected* risk-neutral default intensities to each horizon.

- However in general these will not be constant.
  - In other words $\lambda_T^Q = \lambda_T^Q(t)$ will change over time.
  - That is natural if the true intensity is also changing.
  - If it were not then credit spreads would never change!
- We can handle this by letting λ itself obey its own random process. Such *stochastic intensity* models are a key building block in credit analysis.
  - These are also called *doubly stochastic models*.
- Once we introduce stochastic lambdas, bonds (or any claim subject to default risk) are functions of <u>two</u> separate random processes: the lambda itself AND the jump process.
  - Each of these is subject to its own risk-neutralization.
- Now when we return to the problem of counterparty risk modelling, the question arises: if we are using a reduced-form model of default, how do we capture the *correlation* between the credit events that we are worried about and the other state variables (X above) that effect the claim that we have?
- There are a number of possibilities. Here are two.

  Correlatation of intensities. If default *probability* changes systematically with other diffusive state variables, we can capture this just by making their Brownian shocks correlated.

In particular, if there are *multiple* default process that we care about, they might naturally have common factors driving their variation.

Simultaneous jumps. We can make the default *events* potentially correlated with other variables in the economy that *also* might jump. For example, a sudden steep devaluation of a currency might cause bankruptcies. In this case, our survival variable migh be modeled as

$$dA/A = -dJ_t^{(1)} - dJ_t^{(2)} \ldots - dJ_t^{(n)},$$

where any one of the events $1 \ldots n$ would kill the reference entity.

Another natural application is if many firms share a common jump term, then it captures a systematic bankruptcy event.

- These two approaches are not mutually exclusive. For example, one can also model the possibility of *default contaigon* by making the intensity processes for some firms jumps if any of them defaults.
  - For example, if one process jumps (one firm defaults), that could raise the intensity of all the other jumps in the same industry.

- Let's illustrate how this can work for a particular type of counterparty risk where there are multiple default risks.

III. Example: Wrong-way risk in CDS.

- Consider the problem of valuing a CDS written on entity X by entity Y when (1) Y can go bankrupt, and (2) Y and X can go bankrupt together.

- The risk of a swap counterparty going backrupt precisely when you need him to be healthy is called *wrong-way* risk in the CDS market.

- With default intensity models we can easily quantify the effects (1) and (2) in come cases.
  - ▶ Specifically, let's see what happens if the risk-neutral default intensities are constant.
  - ▶ In that case, using the notation of Lecture 6.2, the default density h of each entity is just an exponential function.
  - ▶ If X has risk-neutral intensity λ^X, then the probability of default by time T is

$$H^X(t,T) = (1 - e^{-\lambda^X(T-t)})$$

    and similarly for Y.

- To compute the adjusted CDS fee, we equate the value of the protection leg and the fee leg, recognizing that <u>both</u> payments are contingent on Y being alive.

- If the two default events are independent, then the joint probabilities of each entity dying or surviving is just the product of their individual probabilities.

- So the protection leg is worth

$$(1 - R^X) \int_t^T e^{-r(s-t)}\; h^X(t,s)\; (1 - H^Y(t,s))\, ds$$

$$= \lambda^X (1 - R^X) \int_t^T e^{-r(s-t)}\; (1 - H^X(t,s))\; (1 - H^Y(t,s))\, ds$$

- But the fee leg is worth

$$\varphi \int_t^T e^{-r(s-t)}\; (1 - H^X(t,s))\; (1 - H^Y(t,s))\, ds$$

- The integral terms exactly cancel, and the fair fee is just $\lambda^X(1 - R^X)$ – which is the same as if Y were riskless!

• This seems strange – because this simple example missed something.

• The calculation neglected the potential *loss* to the protection holder that could result from Y's death while X is alive.

• Why could there be a loss?
  - Recall that if Y defaults, we have a claim on Y equal to the replacement value of the CDS.
  - But if that value is negative, the bankruptcy court has a claim on us.
  - We computed previously that, in terms of the time-s CDS fee, our contract's value at time s would be

$$V(\varphi_s; \varphi) = (\varphi_s - \varphi) \int_s^T e^{-r(u-s)}\, (1 - H^X(s,u))\, du.$$

- So if Y is bankrupt and they are net winners, we lose the swap but have to pay its value. So the net economic effect on us is zero.
- But if we are winners we lose something worth V but only gain a claim worth $R^Y V$.
- Hence our valuation should include another term like the one we wrote as equation (1) above:

$$\int_t^T e^{-r(s-t)}\ h^Y(t,s)\ (R^Y - 1)\max[V(\varphi_s;\varphi), 0]\, ds \quad (2)$$

  where $V()$ is expression above for the mark-to-market value of the CDS on X with no counterparty risk.

  * Of course, if φ_s is a random variable, this whole thing has to be inside a (risk-neutral) expectation.

- The assumption I made about constant default intensities means that in the future the cost of protection on X will also be $\lambda^X(1-R)$.
  - So in this example, I'm justified in ignoring the extra term: my assumptions imply V will necessarily always be zero.

- Now let's put in common defaults.

- If we still assume constant intensities then the recovery term will still be absent, but now we will see an effect of wrong-way risk.

- Suppose that there are three separate Poissons: idiosyncratic ones for X and Y (with intensities λ^{X^i} and λ^{Y^i}) and a common systematic one with intensity λ^Z.
  - ▶ So the probability of X defaulting in dt is $(\lambda^{X^i} + \lambda^Z)\, dt$.
  - ▶ Each marginal default event is still exponentially distributed.
- Now the protection leg is worth

$$(1 - R^X) \int_t^T e^{-r(s-t)}\ h^{X^i}(t,s)\ (1 - H^Y(t,s))\, ds$$

  i.e., for each s, the probability of an idiosyncratic X jump when Y is still alive. Or, here,

$$\lambda^{X^i}(1 - R^X) \int_t^T e^{-r(s-t)}\ e^{-(\lambda^{X^i}+\lambda^{Y^i}+\lambda^Z)(s-t)}\, ds$$

- But the fee leg is also worth

$$\varphi \int_t^T e^{-r(s-t)} \quad e^{-(\lambda^{X^i}+\lambda^{Y^i}+\lambda^Z)(s-t)}\, ds$$

  because the exponential term is the probability that none of the processes have jumped by time s
- So again the integrals cancel and the fair fee is $\lambda^{X^i}(1 - R^X)$.
  - ▶ The default correlation has lowered the fee by λ^Z
  - ▶ So if Citibank and Deutsche Bank each have default intensities of 60 bp per year, but the RN probability of a joint default is 25 bp per year, then you would only pay DB a fee of 35 bp for protection on CB instead of 60 (assuming zero recovery).

- The calculations here illustrate how one party in a swap would embed the other's counterparty risk into the swap fee that he/she would be willing to pay.

- I used the joint-default assumption (with constant intensity) to model correlated defaults for simplicity.
  - If, instead, the default risks are linked through correlated intensity processes, then we have a more complicated joint default density $h^{X,Y}(t,s)$ that we would typically need to evaluate via simulation.
  - And, for that computation, (or any case with non-constant intensities) we would also have to confront that replacement value expression (2) that we were able to ignore here.

IV. CVA.

- Sophisticated institutions take into account the connection between their credit exposures and their positions in all their pricing and hedging decisions.

- In fact, global bank regulations require large banks to actively monitor their derivatives counterparty exposure and to reserve capital against it – or else hedge it.

- So all the major banks have desks that attempt to compute the so-called **credit value adjustment** (CVA – also sometimes called counterparty value adjustment) to each trading desks' exposure to each counterparty
  - ▶ Theoretically, CVA is defined as the difference between the value of the portfolio and the value it would have if the counterparty were riskless.
  - ▶ In fact, this is precisely the thing in equation (1).
  - ▶ It can be computed contract-by-contract. But since Basel rules allow netting of exposures, it is usually computed on a portfolio basis.

- Of course, for many bank exposures – loans – it is easy to compute: it's just the value of the credit spread on the loan – the loans' value minus an equivalent riskless one.
  - ▶ (Regulators are willing to let you consider the government riskless for CVA purposes.)
  - ▶ In this case the counterparty risk IS the only risk of the position.

- Derivatives are more interesting.

- A classic example occurred with the demise of the hedge fund LTCM in 1998.
  - They had sold a lot of stock index volatility, which then went very high.
  - As it went up, their counterparties were feeling very rich.
  - Until they realized that if it went up a lot more they would be very poor because it would kill LTCM who wouldn't be able to pay them anything.
  - So to hedge the risk of further rises in volatility....the counterparties had to buy more!

- Say we have bought from LTCM a derivative which will pay us σ_T for some stock index at T. (This is just a simple way of illustrating an exposure: it could be to any factor.)

- Our CVA is the economic loss we suffer if they ever default.

- Assuming no recovery, if they default at time τ the replacement value of our position at τ is our loss.

- So if $V(\tau) \geq 0$ is the replacement value of our position, then our CVA at t would be

$$\mathrm{E}_{\mathrm{t}}^{\mathrm{Q}}[\int_t^T B_{t,\tau} 1_{\{\text{LTCM dies at } \tau\}} V(\tau) d\tau]$$

- If LTCM's credit risk is independent of V (and B), then I can bring the expectation inside the integral and factor it into separate terms.
  - And the expectation of the default indicator is just the risk-neutral default density.
  - Also we can write $V(\tau) = B_{\tau,T}\mathrm{E}^{Q}_{\tau}[\sigma_T]$ and the CVA becomes

$$\int_t^T h(\tau)\ \mathrm{E}^{\mathrm{Q}}_{\mathrm{t}}[B_{t,\tau}\ B_{\tau,T}\ \mathrm{E}^{Q}_{\tau}[\sigma_T]]\ d\tau.$$

    And by the law of iterated expectations, this is

$$\int_t^T h(\tau)\ d\tau\ \ \mathrm{E}^{\mathrm{Q}}_{\mathrm{t}}[B_{t,T}\sigma_T] = H(t,T)\ V(t)$$

- Now to add back the effect of correlation, we could bring in stochastic intensity.
- Suppose that, under the risk-neutral measure, their lambda obeyed

$$d\lambda = \kappa^{\lambda}(\bar{\lambda} - \lambda)\,dt\ +\ b^{\lambda}\,dW^{\lambda}$$

- Then we could see what happens if dW^{λ} is highly correlated with the innovations to volatility dW^{σ}.
- Then we can't factor the expectation in the CVA calculation.
- We will have to go use Monte Carlo to evaluate it.
  - Notice that this means simulating a <u>trivariate</u> process: λ, σ, and the Poisson process whose intensity is λ.

▶ Actually, there's one simplification we can exploit: conditional on the λ paths, the individual default realizations are independent. So

$$\mathrm{E}_{\mathrm{t}}^{\mathrm{Q}}[1_{\{\text{c/p dies at } \tau\}} V(\tau)] = \mathrm{E}_{\mathrm{t}}^{\mathrm{Q}}[\lambda_\tau e^{-\int_t^\tau \lambda_u \, du} V(\tau)].$$

▶ In other words, given a realization of λ, the probability of death at τ is just λ_τ times the conditional probability of survival before τ.

▶ We can compute the right side by just simulating λ and σ.

▶ On the other hand, I have been implictly assuming that interest rates are uncorrelated with λ and σ.

  * If that's not true, then we have another process in the system.

  * And, remember, we have $DF_{t,\tau} = e^{-\int_t^\tau r_u \, du}$ inside the expectation.

- This example was just dealing with a derivative whose value is nonnegative.

- The most general computation of CVA is for derivatives that can be either assets or liabilities, like an interest rate swap or a CDS.

- Now the economic loss in the case of counterparty default is zero if the position is in the counterparty's favor.

- S0 we get back to expression (1) from earlier:

$$\mathrm{E}_{\mathrm{t}}^{\mathrm{Q}}[\int_{t}^{T} DF_{t,\tau}\ 1_{\{\text{c/p dies at } \tau\}}\ \max[V(\tau), 0]\ d\tau]$$

  - ▶ If the recovery rate is non-zero, we need to put a $(1 - R)$ back inside too.
  - ▶ We could even have the recovery risk be random – if it was a hedgeable risk and we knew its market price of risk.

- Because of the $\max[\ \ ]$ function in the expression, our liability is effectively the sum of a bunch of options (with strike price zero) on V.
  - ▶ This is exactly what it is under independence.

- In implementing this calculation, as you can tell, you need to know a formula for $V(X_t)$ (where X is the set of state variables affecting the contract).
  - ▶ As a practical matter, if V's value in the future itself has to be computed by Monte Carlo, the procedure becomes computationally untenable.

- As I mentioned, CVAs are now routinely being computed and the CVA desk will charge the trading desk this amount. Hence dealers attempt to get their counterparties to pay it.

- Collateralization takes away a lot – but not all – CVA.

- Suppose the net mark-to-market value is posted to a collateral account at time intervals of length Δ.
- Then inside the CVA integral, we would replace $\max[V(\tau), 0]$ with $\max[V(\tau) - V(\tau - \Delta), 0]$ – which will be a much smaller number.

- Finally, think about hedging CVA.
- In theory this can be done *either* by hedging the market risk *or* by hedging the credit risk.
- As my LTCM example illustrated, you could hedge it by adjusting your position in V.
  - But it will mean buying more as default becomes more likely.
  - With correlation, this is when the price of V is going up.
- Alternatively, you could hedge by trading in CDS of your counterparty.
  - Now you will increase the amount of protection you need to buy as V rises.
- Notice that in both cases the *size* of the hedge with respect to one risk depends on the *amount* of the *other* risk.

V. DVA.

- If I promise a swap counterrparty a fixed stream of payments φ, assuming zero recovery, they will value that as
$$\varphi \int_t^T e^{-r(s-t)} \left(1 - H^I(t,s)\right) ds$$
where I denotes me.
- They will view the H^I part of this expression as the CVA of my promise.
  - ▶ If it gets bigger, they lose money.
  - ▶ It's a real economic loss even if they don't recognize it.
- So where did the money go?
  - ▶ **Answer:** to me!
- I am short a promise that declined in value. Therefore I gained exactly as much as the counterparty lost.
- This is not an abstract accounting idea.
  - ▶ When companies (or nations) buy back their own bonds at a discount to their issuance price, they get to keep the difference.
  - ▶ It's real cash.
  - ▶ Therefore as prices decline, they should recognize a gain in value.

- In fact, U.S accounting rules now do require banks to report this **Debit Value Adjustment** or DVA – the flip side of CVA.
- Some banks even hedge their DVA.
  - ▶ This makes sense: they will lose value if their credit improves.
  - ▶ So they should sell CDS on themselves! (...presumably collateralized)
- More generally, if we own a derivative whose replacement value is $V(X_t)$, then if we default at time τ our economic gain is the reduction in value on the liabilites that we do not repay:

$$(1 - R^I)\ \max[-V(X_\tau), 0]$$

  - ▶ Multiplying this quantity times the risk-neutral default indicator for us, and integrating the discounted risk neutral expectation over time yields the DVA.
- If both sides to a derivative trade recognize *both party's* default options, this would resolve a problem we noticed in Lecture 2.3, namely, that if each side tries to charge the other for the value of its CVA, then it becomes impossible to trade.
- By understanding DVA trade between two risky parties becomes possible.
- For example, consider a (continuous) swap where one party, X, pays a risky stream $C_t\ dt$ each period in exchange for the fixed amount $\varphi\ dt$ by party Y.

- Then both sides should agree that the value (to X) is

$$
\begin{aligned}
&\mathrm{E}_{\mathrm{t}}^{\mathrm{Q}}[\int_t^T DF_{t,\tau}\ \varphi\ 1_{\{\text{Both alive at } \tau\}}\ d\tau] \\
&-\mathrm{E}_{\mathrm{t}}^{\mathrm{Q}}[\int_t^T DF_{t,\tau}\ C_\tau 1_{\{\text{Both alive at } \tau\}}\ d\tau] \\
&-\mathrm{E}_{\mathrm{t}}^{\mathrm{Q}}[(1-R^Y)\ \int_t^T DF_{t,\tau}\ 1_{\{Y \text{ death at } \tau;\ X \text{ alive}\}}\ \max[V(\tau),0]\ d\tau] \\
&+\mathrm{E}_{\mathrm{t}}^{\mathrm{Q}}[(1-R^X)\ \int_t^T DF_{t,\tau}\ 1_{\{X \text{ death at } \tau;\ Y \text{ alive}\}}\ \max[-V(\tau),0]\ d\tau]
\end{aligned}
$$

  and that this is minus the value to Y.

  - Hence both sides should agree that it is fair to trade at a swap rate φ_0 that sets this quantity equal to zero.

  - Finding that rate still requires a calculation however. In the above expression, φ implicitly appears in the V function which represents the replacemtn value used to in default.

    * For example, if the default free liquidation value is used then, recalling Lecture 2.2, V is given by

$$
\mathrm{E}_\tau^Q[\int_t^T DF_{\tau,u}\ \varphi\ du] - \mathrm{E}_\tau^Q[\int_t^T DF_{\tau,u}\ C_u\ du]
$$

      which, under independence of C and interest rates, reduces to

$$
\varphi\ \int_t^T B_{\tau,u}\ du - \int_t^T B_{\tau,u}\ F_{\tau,u}\ du.
$$

- Notice that the extra DVA-CVA terms in my valuation expression are going to be negative for one of the parties and positive for the other.
  - So the implication is that one of the parties has actually net profited from counterparty risk.
  - This profit is then off-set by the net adjustment to φ.
- Also notice that all the default indicator functions depend on the joint survival density.
  - So to apply this formula we have to model our own default correlation with the counterparty.
  - So even if there is no wrong-way risk (no correlation of either default with the derivative's underlying), and no correlation with the discount factor, we still have a correalted-default calculation.
- DVA is still a controversial concept.
  - Many market participants feel that if it is recognized it could increase a firm's incentive to make itself more risky.

VI. Summary.

- Valuing counterparty risk for a derivatives contract is straightforward in principle, but a lot of work in practice.

- We introduced reduced-form default models as an alternative to structural models to capture the risk of a counterparty failing.
  - ▶ These involve an auxilary stochastic process of the jump events themselves.
  - ▶ If the jump risk is hedgeable, then we risk-neutralize this by adding the market price of jump risk to the intensity.
  - ▶ If the intensity itself is stochastic (and hedgeable) then risk-neutralizing it is a separate step.

- One key ingredient in the valuation of counterparty risk is the potential correlation – wrong-way risk – between the market variables in the contract and the credit of the counterparty.

- Even without this correlation, the asymmetry in the CVA calculation leads to option-like exposures.

- It is very much an open question whether firms should view themselves as exposed positively to the risk of *themselves* defaulting.

- This is at the cutting edge of financial engineering right now.