FIN 513: Homework #1

Due on Thursday, January 25, 2018

Wanbae Park

Problem 1

(a) Agree. Generally, low-growth stock gives higher dividend than high-growth stock. Since high-growth company needs more capital than low-growth company, the amount of retained earning might be larger than that of low-growth company. Therefore, the futures price of high-growth stock will be a less discount over the spot price.

(b) Agree.

(c) Agree.

Problem 2

Let r_L^D denote dollar rates for lending, r_L^E denote euro rates for lending, r_B^D and r_B^E denote dollar and euro rates for borrowing, respectively. Let S_t^B and S_t^O denote bid and offer exchange rate, respectively. In the following way, we can replicate long position and short position of dollar/euro forward.

1) *Replicate Long Position*

   (a) Borrow $S_t^O \frac{1}{(1+0.5r_L^E)^{2(T-t)}}$ dollars at rate r_B^D

   (b) Exchange $S_t^O \frac{1}{(1+0.5r_L^E)^{2(T-t)}}$ dollars to $\frac{1}{(1+0.5r_L^E)^{2(T-t)}}$ euros, then invest for time $T-t$ at rate r_L^E.

   At time T, the value of strategy (a) becomes $-S_t^O(\frac{1+0.5r_B^D}{1+0.5r_L^E})^{2(T-t)}$ dollars and the value of strategy (b) becomes 1 euro which perfectly replicates long position of forward contract. Since the initial values of two strategies are identical, the forward price $F_{t,T}$ should be equal to $S_t^O(\frac{1+0.5r_B^D}{1+0.5r_L^E})^{2(T-t)}$ dollars.

2) *Replicate Short Position*

   (a) Borrow $\frac{1}{(1+0.5r_B^E)^{2(T-t)}}$ euros at rate r_B^E

   (b) Exchange $\frac{1}{(1+0.5r_B^E)^{2(T-t)}}$ euros to $S_t^B \frac{1}{(1+0.5r_B^E)^{2(T-t)}}$ dollars, then invest at rate r_L^D.

   At time T, the value of strategy (a) becomes -1 euro, which is identical to short position of forward contract. Therefore, if there is no arbitrage in market, the forward price should be equal to the value of strategy (b) at time T, which is $S_t^B(\frac{1+0.5r_L^D}{1+0.5r_B^E})^{2(T-t)}$

Since $r_L^D < r_B^D$, $r_L^E < r_B^E$ and $S_t^B < S_t^O$, the value of replicate portfolio of short position is less than that of long position. Therefore, the upper and lower bound of the contract is $S_t^O(\frac{1+0.5r_B^D}{1+0.5r_L^E})^{2(T-t)}$ and $S_t^B(\frac{1+0.5r_L^D}{1+0.5r_B^E})^{2(T-t)}$, respectively. The following table shows that the numerical results using parameters on the homework sheet.

T	*Lower Bound*	*Upper Bound*
1 yr	1.4913	1.4940
5 yr	1.4833	1.4993
10 yr	1.5218	1.5526

Problem 3

Assume that the fair price of single price is x, then x should be larger than 308 and smaller than 313. Consider the following strategy.

(a) Get a long position on the original contract.

(b) Get a short position on the new contract.

Since the cash flow at time $t = 1.5$ is known at $t = t_0$, we can reinvest the net cash flow$(x - 308)$ at $t = 1.5$ to $t = 2$ by using the following strategy.

(1) sell short $x - 308$ amount of zero coupon bond with maturity $t = 1.5$

(2) buy $\frac{(x-308)B_{0,1.5}}{B_{0,2}}$ amount of zero coupon bond with maturity $t = 2$.

By netting out the values of strategy (1) and (2), there is no initial amount of cash flow. Furthermore, the strategies also makes cash flow at $t = 1.5$ to be zero. Finally, at time $t = 2$, the net cash flow from the whole strategies is equal to $x - 313 + (x - 308)\frac{B_{0,1.5}}{B_{0,2}}$. Since there is no cash flow before $t = 2$, the net cash flow at $t = 2$ should be equal to zero, otherwise there exists arbitrage opportunities. Therefore, the following equation holds.

$$
\begin{aligned}
& x - 313 + (x - 308)\frac{B_{0,1.5}}{B_{0,2}} = 0 \\
& \Rightarrow B_{0,1.5}x + B_{0,2}x = 308B_{0,1.5} + 313B_{0,2} \\
& \Rightarrow (0.912 + 0.883)x = 308 \times 0.912 + 313 \times 0.883 \\
& \Rightarrow x = 310.4596
\end{aligned}
$$

Therefore, the fair price that a market maker would be willing to offer is 310.4596.

Prob. IV

Suspendisse vitae elit. Aliquam arcu neque, ornare in, ullamcorper quis, commodo eu, libero. Fusce sagittis erat at erat tristique mollis. Maecenas sapien libero, molestie et, lobortis in, sodales eget, dui. Morbi ultrices rutrum lorem. Nam elementum ullamcorper leo. Morbi dui. Aliquam sagittis. Nunc placerat. Pellentesque tristique sodales est. Maecenas imperdiet lacinia velit. Cras non urna. Morbi eros pede, suscipit ac, varius vel, egestas non, eros. Praesent malesuada, diam id pretium elementum, eros sem dictum tortor, vel consectetuer odio sem sed wisi.