FIN 513: Homework #6

Due on Tuesday, March 13, 2018

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Problem 1

Assume that face value of the bond is equal to 1.

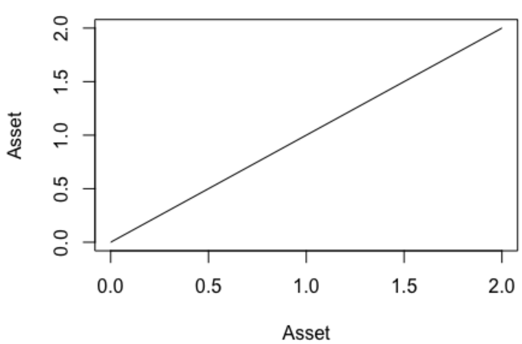
(a) Since yield-to-maturity is a discount rate which discount "promised" payoff to current price, F is represented as $F = \frac{1}{1+y}$.

(b) Under risk-neutral measure, all expected return is risk-free. Therefore, F is represented as $F = \frac{p\times 1+(1-p)\times 0}{1+R} = \frac{p}{1+R}$.

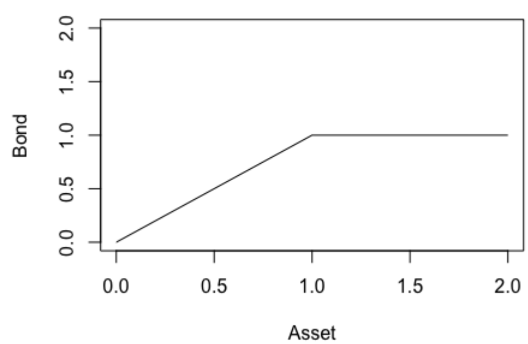
(c) Since expected return of the bond is e, and there is true survival probability π, F is represented as $F = \frac{\pi\times 1+(1-\pi)\times 0}{1+e} = \frac{\pi}{1+e}$.

(d) Since $F = \frac{p}{1+R} = \frac{\pi}{1+e}$, $\frac{\pi}{p} = \frac{1+e}{1+R}$. Furthermore, because the bond is risky, e must be greater than R, therefore the inequality $\frac{\pi}{p} = \frac{1+e}{1+R} > 1$ holds.

(e) As shown in Figure 1, payoff of a bond has a upper bound unlike payoff of asset. In other words, although asset value becomes larger than face value of bond, payoff of bond does not increases as asset value increases. Therefore, true expected return of a bond would be lower than expected return of the



(a) Asset



(b) Bond

Figure 1: Payoff of asset and bond

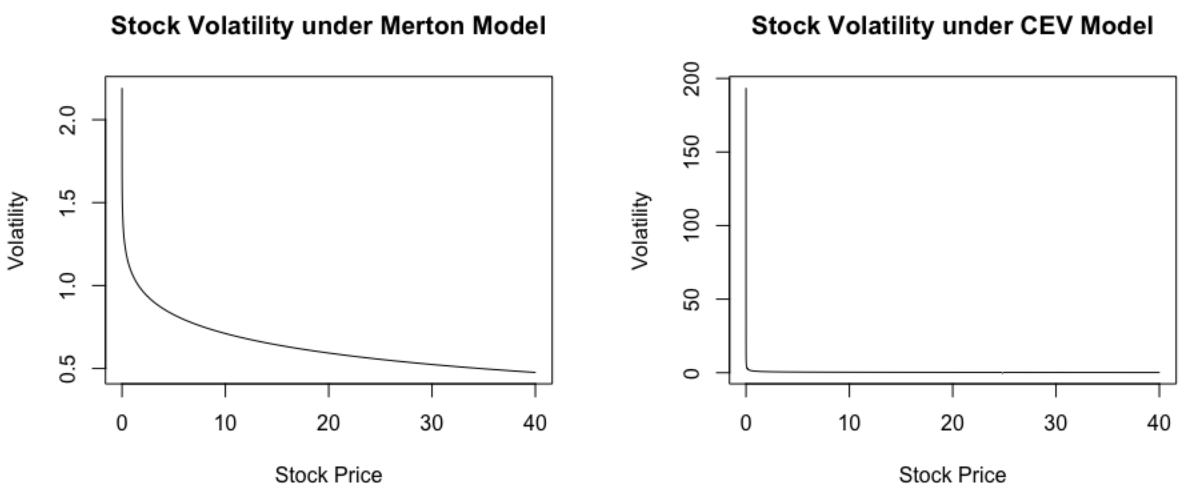
asset, which is 12%, hence by (c), price of bond would be larger than $\frac{\pi}{1+\mu} = \frac{0.97}{1.12}$.

Problem 2

Under Merton model, stock price is considered as a call option of firm value, with strike price is face value of a debt. Therefore, stock volatility can be calculated as follows.

$$
\begin{aligned}
&S = VN(d_1) - Fe^{-r(T-t)}N(d_2) \\
&\frac{\partial S}{\partial V} = N(d_1) \quad \text{which is delta of call option.} \\
&\Rightarrow \textit{stock volatility} = N(d_1)\frac{V}{VN(d_1) - Fe^{-r(T-t)}N(d_2)}\sigma_V \\
&\text{where} \quad d_1 = \frac{\log(V/F) + (r + \frac{1}{2}\sigma_V^2)(T-t)}{\sigma_V\sqrt{T-t}} \\
&\qquad\qquad d_2 = d_1 - \sigma_V\sqrt{T-t} \\
&F : \text{Face value of debt}
\end{aligned}
$$

By using the formula, stock volatilities were calculated with respect to stock prices from about 0 to 40. The range of stock prices corresponds to the range of firm value from 11 to 110. Furthermore, under constant elasticity of variance model(CEV Model), since stock price process is defined as $\frac{dS}{S} = \mu dt + \frac{\omega}{\sqrt{S}}dW$, the corresponding variance of stock price is $\frac{\omega}{\sqrt{S}}$. Figure 2 shows the stock volatility as a function of stock price. As shown in figure, stock volatility decreases as stock price increases on both models. However, since there



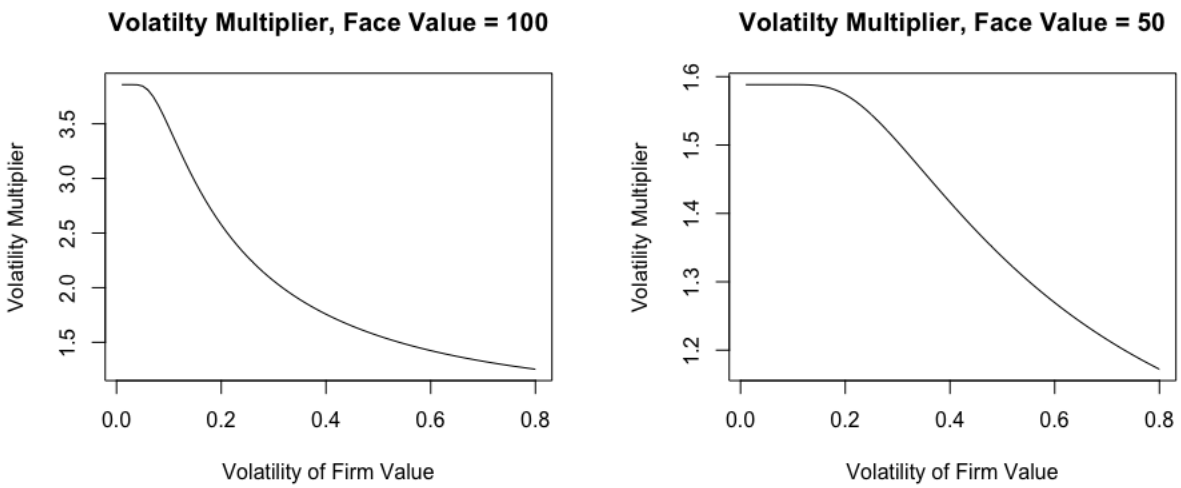
(a) Merton model (b) CEV model($\omega = 1$)

Figure 2: Stock volatility of Merton and CEV Model

is stock price in denominator term, if stock price gets closer to zero, volatility increases sharply on CEV model, whereas volatility increases smoothly on Merton model.

Problem 3

(a) Using the formula in question 2, volatilty multiplier was calculated for each volatility and face value. Figure 3 show volatilty multiplier with each face value with respect to volatility of firm value. As shown



(a) Face value: 100

(b) Face value: 50

Figure 3: Volatility Multiplier

in figure, volatility multiplier remains constant if volatility of firm value is low, but as volatility of firm value increases, volatility multiplier starts to decrease after a threshold at some level. It means that after the level, marginal increase of stock volatility becomes less than the amount of increases in volatilty of firm value. In other words, since risk of an asset is measured by volatility, if firm's total risk increases, then the proportion of equity risk among total firm risk decreases. Moreover, it can be found that the level of threshold is higher when face value of debt is lower, and volatility multiplier is higher when face value of debt is higher.

(b) If idiosyncratic risk is added, then σ_V increases, but π_V does not increase. Therefore, $\pi_S = \pi_V \frac{\sigma_S}{\sigma_V}$ decreases since volatility multiplier decreases as σ_V increases, and π_V remains constant. If σ_V is low, changes in excess return of stock would not be too much because volatilty multiplier seems almost constant if σ_V is less than threshold level. At first glance, it seems weird because in many finance context, risk premium increases as risk increases. Of course, risk premium increases as risk increases, however, it is important to note that the risk which makes risk premium increase is "systematic" risk, not idiosyncratic risk. In this case, the increased risk is idiosyncratic risk. It means that firm's total risk was increased, but systematic risk remains constant. As the proportion of equity risk among total firm risk decreases as firm risk increases, "systematic" equity risk decreases since total systematic risk remains constant. In other words, increase in idiosyncratic risk transferred systematic risk from equity

Problem 3 continued on next page. . .

holder to debt holder. Hence risk premium of equity decreases.

(c) From answer to (b), excess return of stock decreases as idiosyncratic risk in firm value increases. From firm's perspective, less cost of equity capital is better to sell equity in future. Therefore, managers will prefer to increase idiosyncratic risk in firm value.

Problem 4

(a) Under Leland model, since others are constant, the optimal bankruptcy covenant is derived as $V_B = \frac{X}{1+X}\frac{\Gamma}{r}(1-\Omega)$, where $X = \frac{2r}{\sigma^2}$. From this result, we can analyze comparative statics of the Leland model.

(i) From the equation above, it can be showed that higher tax rate makes V_B lower since all other parameter is positive.

(ii) In order to investigate the effect of changes in risk-free rate, we can differentiate V_B with respect to risk-free rate, r.

$$\begin{aligned} V_B &= \frac{X}{1+X}\frac{\Gamma}{r}(1-\Omega) \\ &= \Gamma(1-\Omega)\frac{2r/\sigma^2}{1+2r/\sigma^2}\frac{1}{r} \\ &= 2(1-\Omega)\Gamma\frac{1}{\sigma^2+2r} \\ \Rightarrow \frac{dV_B}{dr} &= 2(1-\Omega)\Gamma \times (-\frac{1}{(\sigma^2+2r)^2} \times 2) \\ &= -4(1-\Omega)\Gamma\frac{1}{(\sigma^2+2r)^2} < 0 \end{aligned}$$

Therefore, higher risk-free rate makes optimal covenant level lower.

(iii) The optimal covenant level does not include bankruptcy cost. It means that V_B is independent of bankruptcy cost, α.

(b) Since value of debt P is equal to $\frac{\Gamma}{r}$ under Leland model, if others are constant, and Γ is optimally chosen, the value of debt P is also optimally chosen, which maximizes the firm value. The optimal amount of debt is as follows.

$$\begin{aligned} &P^* = \frac{V_0}{n}\left[1 + X + \alpha X\frac{1-\Omega}{\Omega}\right]^{-1/X} \\ &V_B = nP^* \\ &0 < n < 1 \end{aligned}$$

To find how α affects V_B, we can differentiate V_B with respect to α. The result is as follows.

$$\frac{\partial V_B}{\partial \alpha} = -\frac{1}{X}V_0\left(1 + X + \alpha\frac{1-\Omega}{\Omega}\right)^{-\frac{1}{X}-1} \times \frac{1-\Omega}{\Omega}$$

Since $X > 0$, $\frac{1-\Omega}{\Omega} > 0$, $V_0 > 0$, and $\alpha > 0$, partial derivative of V_B with respect to α has negative value. It means that if bankruptcy cost increases, optimal covenant value decreases under assumption that others are constant.