University of Illinois, Urbana-Champaign *Professor Tim Johnson*
FIN 513 - Spring 2018 *tcj@illinois.edu*

Lecture Note 6.2: Credit Default Swaps

Introduction:

Much attention has been paid since 2008 to the role of *credit derivatives* in the financial system. Today we will begin to examine how credit derivatives work. In this note we'll examine the simplest ones – credit default swaps – and think about how to value and hedge them.

In the last note, we saw how no-arbitrage methods could be used to price risky debt itself. Now we are dealing with derivatives whose underlying is that debt.

While no-arbitrage theory will show us how to price CDSs, we can always turn the game around and view the CDSs as primary securities. Then we can turn back to last week's topic and use these tools to value and hedge risky bonds.

Outline:

I. An Overview of the Market.

II. Pricing Credit Default Swaps.

III. Two Ways of Looking at the Models

IV. Summary

I. Single-Name Credit Derivatives

- In our study of swaps, we have already briefly encountered credit default swaps.
  - ▶ Recall that this is basically to an insurance policy that allows the owner to deliver a defaulted bond of some entity X and receive the the face value (par or 100%) of that bond.
    - ∗ Note that it is NOT identical to a put with strike price 100. You cannot receive payment unless X formally defaults. (With an ordinary put, you might want to exercise it even if the bonds just trade down in price.)
    - ∗ Like an insurance policy, a *traditional* CDS is purchased with a fee, which is payable quarterly up until the nearer of the default event or some termination date T. The fee is quoted as an annualized percent of the face value.
      - Starting April, 2009 (June, 2009 in Europe) CDS markets began adopting a different convention: to fix the fees at standard amounts (25bp, 100bp, 500bp, 10000bp) and then to negotiate an up-front fee to clear the market.
      - We will discuss the reasons for this below.
      - Market makers are still mostly quoting fees under the zero-value convention. So for this note, I'll usually assume this.

- There are now CDS markets for thousands of reference entities, including private companies and countries.
  - Typical trades are for face values of \$5 or \$10 million.
  - Coverage is usually for $T = 5$ years. For the most active names, an entire *term structure* of maturities from 6 months to 30 years is traded.
  - The market has also adopted the convention that – unless specifically requested – CDS maturity dates will be taken to be either March 20, June 20, September 20 or December 20.
    - So a "5-year" contract entered into on April 20 would actually give protection for 5 years and 2 months.
  - Also the standard assumption is that these 4 dates are when the fee payments will be made each year.

- In 2009, ISDA estimated that there were 31 *trillion* dollars in principal amount of CDS insurance outstanding.
  - By comparison the capitalization of the U.S. stock market was about 22 trillion.
  - The size of total mortgage debt was 7 trillion, and there were 5 trillion outstanding in government bonds.

- Of course these comparisons are somewhat misleading. As we have discussed, the notional outstanding amounts are very different from market values.

- Moreover, these numbers include many non-netted and redundant exposures that in theory could be "compressed".

  - In fact, participants are working hard to streamline contracts so that more netting and compression will be possible to lower the outstanding amount.

  - This was the primary motivation behind the convention changes mentioned above.

▶ CDS contracts now all specify *cash settlement* in which a reliable post-default price, R, is solicited from the market via an auction, open to anyone, for the underlying deliverable debt.

- Insurance sellers must then pay all buyers 100-R.

- The auctions are run electronically on a platform called Creditex (see `www.creditfixings.com`).

- R is referred to as the *recovery amount*.

- Another important feature to note about this type of insurance is that the contract typically allows the buyer to deliver any of several debt obligations of the insured amount and receive par.
  - For example, it could specify any senior unsecured bond.
  - This gives the buyer a cheapest-to-deliver option. He/she will obviously choose to deliver the *most junior* security allowed with the longest maturity and lowest coupon , since these will be the ones worth least.
    - Then the payoff is $\max_i(100 - R^{(i)})$, where i indexes all the firm's eligible debt claims.
  - The contract does specify what the currency of the obligation must be.
- For a given reference entity, there could be both senior and subordinated CDS as well as CDS specifically covering *bank loans*.
  - These are called LCDS.

- As with any insurance, it is very important to iron out in advance the precise details of what events trigger payment.
  - The formal definition of a credit event can still differ from contract to contract.

* Usually it is straightforward: if X fails to perform any of the required covenants in any of its debt contracts, that's a default – on all of them.

* This includes failure to pay or repudiation or moratorium.

* Things can get tricky, however, if X undertakes a *debt restructuring*. This is often quasi-voluntary, i.e., it is the result of negotiation between X and its bondholders. But it has the result of harming those bondholders. They ultimately receive less principle and interest than they were originally due.

* Under current ISDA documentation, restruturing may or may not count as a credit event. There are 4 main types of contract.

  XR Restructuring does not count as a default.

  CR Any restructuring counts as default.

  MR Any restructuring counts as default BUT in a restructuring only debts that mature within 2.5 years are protected.

  MM Any restructuring counts as default BUT in a restructuring only debts that mature within 5 years are protected.

* For North American names, XR is now the standard.

* For sovereigns it is usually CR.

- If X is acquired, the acquiror becomes the new reference entity.
  * Usually default by a "downstream" subsidiary of X is a credit event for X, but not default by an affiliate of common parent.
  * As far as I know, there is no clear way of determining what happens if a company splits apart into several successor companies!

- The 2009 agreements have established 5 global "Determination Committees" (for separate geographical regions) who decide exactly when a credit event occurred for a given reference entity.
  * Sometimes, it takes several weeks after the fact to fix the precise date.
  * Under the new market standards, if you have bought credit protection, you will receive payment on any default within a 60 day "look-back period."
    - So even if your contract expires at T, if it is determined at $T + 30$ that a default happened at $T - 10$, you are covered.

- Like most OTC contracts, CDS are <u>not</u> assignable without the permission of the original parties. However it is fairly common for traders to ask for such permission, which is called a *novation.*

  - ∗ If I have sold insurance on Ford to Citibank, and later I buy the same insurance from Deutsche Bank, I can send an electronic request to Citibank for a novation and they must respond within 2 hours.

  - ∗ If they agree, my trades are cancelled and Deutsche Bank becomes the seller of Ford insurance to Citibank. Otherwise I have two outstanding trades on my books.

  - ∗ When a novation occurs, I receive (or pay, if negative) the mark-to-market difference in values between the two swaps and Citi continues to pay the original rate to DB.

- Of course, netting out redundant positions happens automatically if both parties choose to use a *centeral counterparty* (CCP).

  - ∗ Starting in 2014, U.S. regulations made central clearing mandatory for trades in <u>baskets</u> of CDS, which, like stock indexes, are just portfolios of individual CDS.

  - ∗ Oddly, however, the individual CDS are not covered by the same regulations for now.

  - ∗ So use of a CCP for single-name CDS is voluntary.

- In addition, CFTC rules require index CDSs to trade on *swap execution facilities* just like interest rate swaps.
  - These are supposed to be open-access platforms for electronic trading.
  - Trades are supposed to be reported immediately to so-called "swap data repositories".
  - There are currently something like 10 firms with competing SEFs. Presumably only a few will survive.

- You may have already wondered *who insures the insurers?* What would be the point in buying insurance on a AAA company if it were written by a CCC company?
  - The market convention is to quote contracts assuming the seller is a LIBOR (i.e. AA) credit. Lesser rated players cannot sell unless some one else is guaranteeing their performance (or unless both sides agree on to use a CCP).
  - As with any other OTC derivative, the contracting parties can also stipulate some marking-to-market and collateralization provisions. And the issue is obviously less important for centerally cleared contracts.

* Notice that, even without collateralization, the buyer is *somewhat* protected against the seller's credit deteriorating: the buyer will stop paying the fee if seller defaults.

  - They will suffer no loss if they can replace their CDS with another one with a healthy counterparty.

  - The risk, then, is of a *simultaneous* impairment of both the seller and the reference entity.

  - This extra risk – which will be reflected in the price – can be important when both companies are in the same industry (e.g. finance).

* Of course, if an insurance writer has written so many contract that it is deemed *too big to fail*, then ultimately the governement may fullfil its obligations.

  - This happened for AIG.

* Moreover, during the height of the financial crisis, major governments themselves began providing credit insurance on the debt of banks and other finance companies.

▶ The CDS market continues to evolve rapidly. It has proven extremely valuable to many participants. But more needs to be done so that regulators can accurately judge where all the risk ends up.

- A second important class of credit derivatives was designed specifically to address the issue of couterparty risk. These are credit-linked notes.
  - ▶ These are bonds issued by entity Y which don't pay back their principal if the reference entity X defaults before maturity.
    - ∗ Or, more directly, Y may have the right to deliver bonds of X instead of money at time T.
  - ▶ Notice that the buyer, Z, of such a bond is a *seller* of credit insurance. The compensation will be reflected in a high coupon rate.
  - ▶ But NOW there is no reason for Y to worry about Z's credit. Y got Z's cash up front.
  - ▶ Of course, Z may now demand a slightly higher coupon because of the risk that Y will not perform.
  - ▶ This is the same trick re-insurers use when trying to buy catastrophe coverage from many small investors.
    - ∗ They sell bonds (with high coupon rates) that they do not re-pay if a covered event occurs.

- Besides CDS and CLN, there are several other single-name credit derivatives that are less common, but also interesting.

  Binary Event Contracts. These pay out either 0 or 1 depending upon whether a credit event occurs. Recently, the CBOE has listed contracts of this type.

  Recovery Swaps. These are like forward contracts, not swaps. We agree today on a "fair" forward recovery amount, F, and then *if* a credit event happens, the short-side pays the long-side $R - F$. Together, these last two types of contracts split a CDS into its two component types of risk.

  Ratings-Trigger Swaps. These contracts allow the deliverable bonds of the reference entity to be delivered (for par) on any downgrade by a rating agency. Similarly, equity-trigger swaps define the relevant event to be a fall (to some prespecified level) in the stock price. These could have been useful to, e.g., Bear Stearns employees.

  Dividend Swaps. These are like return swaps: one side pays a fixed fee and receives the total payouts to shareholders – including payments for tender offers – from a specific company or basket of stocks. They can be used to protect against the risk of an LBO or special dividend that would affect a company's leverage.

  Credit-Spread Swaps. Here one side agrees to pay the difference between the yield on two bonds (or indices) such as the Moody's Aaa - Baa yield, or the the spread between particular Brazilian and U.S. Government bonds.

II. Pricing Credit Default Swaps

- We have already introduced structural models of credit risk, which led us to the insight that *that bankruptcy risk is hedgeable*, at least under certain assumptions.

- Let's think about what this implies about the valuation of credit derivatives.

- First, it means a risky zero coupon bond must be priced as its expected discounted value, using the risk-neutral probabilities of all possible payoffs.

- Suppose the future values of the firm are V_T and the risk-neutral probability of each outcome is $q(V_T)$ and the firm is bankrupt if $V_T \leq V_B$.
  - ▶ Then, because the possible payoffs are so simple, the pricing formula has an easy interpretation:

$$\widehat{B}_t(V_t) = B_{t,T} \cdot \sum_{V_T} F_T(V_T) \cdot q(V_T) = B_{t,T} \cdot \sum_{V_T > V_B} 1 \cdot q(V_T)$$

  - ▶ Here $B_{t,T}$ is the risk-free bond, and I'm assuming you get nothing for the bond if the firm has gone bankrupt.

- This says something remarkable. If $R = 0$,

  The risky bond price is just the risk-neutral probability that the firm survives (times B)

- Likewise a security that pays off one at T **only** when the firm defaults before T is worth

$$B_{t,T} \cdot \sum_{V_T \leq V_B} 1 \cdot q(V)$$

i.e *the risk-neutral probability that the firm dies* (times B).

- That last security is like a binary-event CDS.

- Now consider how a CDS fee, φ, is related to probabilities.

  - We already saw that, if you were to pay a one-time fee for a one-year contract, then, assuming $R = 0$,

    $$\varphi_1 = \text{Risk-Neutral Probability(bankruptcy next year)}$$

    is the no-arbitrage value of that year's fee. (if the fee is payable at the end of the year.)

  - Similarly, if you agree today to buy one-year protection starting one year from now, the fair fee would be

$$\varphi_2 = \text{R-N Probability(bankruptcy year-after-next }\underline{\text{given}}\text{ survival this year)}$$

  - In general, φ_n will be the *conditional* probability of death in the nth year, given survival through $n-1$ years.

    * If each year's survival is independent, each conditional death probability is the same as the first one: $\varphi_n = \varphi_1$.

  - But now recognize that an N-year CDS is the same as buying a basket of N of these sequential one-year contracts, but with a single, average, fee for all N years.

- ▶ This tells us that φ must be roughly *the average risk-neutral bankruptcy probability per year over the next N years.*

- An important lesson from this discussion is that, in a sense, pricing credit derivatives is easier than a lot of other problems we've come across:
  - ▶ *We don't need the entire risk-neutral distribution. of V_T.*
  - ▶ All we need are the bankruptcy probabilities.

- Mathematically, we can summarize the information we need in one function: *the risk-neutral default density*.
  - ▶ Let me call this object $h(s;t)$. It is defined to be the (RN) probability (as of time t) that the firm dies in the interval $[s, s+ds]$
  - ▶ So the total (RN) probability of death by time T is just $\int_t^T h(s;t)\,ds$, which I'll call $H(T;t)$, the cumulative default distribution.
  - ▶ In terms of h and H we can also define the conditional risk-neutral default density, f, via

$$f(s;t) \equiv h(s;t)/(1-H(s;t)) \quad \text{or} \quad h(s;t) = f(s;t)\,(1-H(s;t)).$$

That is, the probability that the firm dies in the interval $[s, s+ds]$ is the probability that it survives up to s multiplied by the conditional probability of dying, given that it has survived that long.

- f is sometimes also called the **forward default intensity**, in analogy with forward interest rates.

• With these functions, we can be a little more precise in our pricing formula for a CDS.

- To start, with this notation, the value of a risky zero-coupon, zero-recovery bond maturing at s, must be

$$\widehat{B}_{t,s} \equiv e^{-r(s-t)}\,(1 - H(s;t)) = B_{t,s}\,(1 - H(s;t))$$

  where I'm assuming r is constant for simplicity.

- Next, what's the value of the insurance payoff to a CDS? If it pays off 1 at the instant of default (if that happens before T), its price must be

$$\int_t^T 1\ e^{-r(s-t)}\ h(s;t)\,ds.$$

  * This just sums (discounted-payoff-times-probability) over all possible default times until T.

  * Replace 1 by $1 - R$ if you don't want to assume the recovery is zero.

- Now think about the price of the fee payments: what's the value of the constant stream φ to be paid <u>until</u> default?

* The instantaneous fee paid from $[s, s{+}ds]$ is worth $\varphi\, e^{-r(s-t)}\, ds$ – if it happens.
* We need to multiply that times the survival probabilty $1 - \int_t^s h(u;t)\, du = 1 - H(s;t)$ to get its value today (at t).
* Hence, adding up all the fee payments, the total fee stream to T is worth

$$\int_t^T \varphi\, e^{-r(s-t)}\, (1 - H(s;t))\, ds.$$

▶ So if we come across a a CDS whose fee has been set to some number Φ, its net value (to the fee payor) is just the difference:

$$V(\Phi; h) = \int_t^T e^{-r(s-t)}\, h(s;t)\, ds. - \Phi \int_t^T e^{-r(s-t)}\, (1 - H(s;t))\, ds.$$

* You could use this formula to figure out the fair up-front payment on a CDS whose "coupon" level was set arbitrarily say to $\Phi = 100$bp.

▶ Or, setting $V = 0$, the fair zero-cost fee must be

$$\varphi = \frac{\int_t^T e^{-r(s-t)}\, h(s;t)\, ds}{\int_t^T e^{-r(s-t)}\, (1 - H(s;t))\, ds}$$

or

$$\frac{\int_t^T e^{-r(s-t)}\, (1 - H(s;t))\, f(s;t)\, ds}{\int_t^T e^{-r(s-t)}\, (1 - H(s;t))\, ds}$$

or

$$\frac{\int_t^T \widehat{B}_{t,s}\, f(s;t)\, ds}{\int_t^T \widehat{B}_{t,s}\, ds}$$

which, in discrete time, would be

$$\frac{\sum_{s=t}^T \widehat{B}_{t,s}\, f(s;t)}{\sum_{s=t}^T \widehat{B}_{t,s}}$$

which I hope begins to look familiar!

- Just like the swaps we analyzed in week 3, *the fair fee for a default swap is a weighted average of forward default probabilities* where the weights are the risky zero-coupon bond prices to each date.

- We can also use this to re-express the value of an existing CDS with fee Φ in terms of the current fair fee, φ:

$$V(\Phi;\varphi) = (\varphi - \Phi) \int_t^T e^{-r(s-t)}\,(1 - H(s;t))\, ds.$$

  * This is the mark-to-market (MtM) formula for CDSs.
  * If you observed an up-front (price,coupon) quote (V, Φ), you could also use this equation to solve for the no-cost "conventional quote", φ.

- One more observation: note that we can also re-write the integral term in the above expression as

$$(\varphi - \Phi) \int_t^T e^{-r(s-t)} \left(1 - H(s;t)\right) ds$$

$$= \frac{(\varphi - \Phi)}{r} \left[1 - e^{-r(T-t)}\right] - (\varphi - \Phi) \int_t^T e^{-r(s-t)} H(s;t) \, ds$$

where the first term is the value of a <u>certain</u> unit payment stream until T.

  * The second term subracts off the value of *the remaining fees* $\int_s^T (\varphi - \Phi) \, e^{-r(u-s)} \, du$ that you don't get in case of default at s.

  * That is,

$$\int_t^T e^{-r(s-t)} H(s;t) \, ds = \int_t^T e^{-r(s-t)} h(s;t) \left[\int_s^T e^{-r(u-s)} \, du\right] ds$$

  by an application of Fubini's theorem.

  * All this math has a very simple meaning. Going long a CDS at φ and short at Φ is NOT worth the present value of the difference in fees until T.

    - If the company defaults at s, the fees from s to T will never be paid.

    - (Notice: this is not about counterparty risk!)

Example:

- How much money do you make if you go long a 5-year CDS on Delta Airlines for 50 BP and then bad news comes out so the rate jumps to 550 BP?
  - NOT 500 BP compounded for 5 years.
  - If you off-set your position immediately, you only make the spread until default.
  - That's what the MtM expression quantifies.

- If we go back and put in a payment of $(1-R)$ for the insurance rate then the MtM formula doesn't change. The formula for the fair fee becomes

$$\varphi = \frac{(1-R)\int_t^T e^{-r(s-t)}\, h(s;t)\, ds}{\int_t^T e^{-r(s-t)}\, (1-H(s;t))\, ds}$$

  - But we do have to be careful about substituting in the $\widehat{B}$s.
  - A zero coupon bond with non-zero recovery has price

$$\widetilde{B}_{t,T} = B_{t,T}\,(1-H(T;t)) + R\int_t^T e^{-r(s-t)}\, h(s;t)\, ds.$$

- A useful application of these formulas is to take the market fees as given and *impute* the risk-neutral density $h(s;t)$ from them.

- To see how, write our last formula in discrete time, and with explicit reference to the time horizon.

$$\varphi_{t,T} = \frac{(1-R)\sum_{s=t+1}^{T} B_{t,s}\, h(s;t)}{\sum_{s=t+1}^{T} B_{t,s}\,(1-[\sum_{u=t+1}^{s} h(s;t)])}$$

  - Now if I have a one-period-ahead (say one quarter) CDS, then $T = t+1$ and this just says

$$\varphi_{t,t+1} = \frac{(1-R)B_{t,t+1}\, h(t+1;t)}{B_{t,1+1}\,(1-h(t+1;t)])}$$

    which is easy to solve for $h(t+1;t)$.

  - Then, write down the analogous equation for $\varphi_{t,t+2}$. Then it is likewise just a linear equation for $h(t+2;t)$, given that you already got $h(t+1;t)$.

  - And so on.

III. Two Uses of the Models.

- As that last application suggests, a very common use of the CDS market is to view it as providing primary pricing information – i.e., not view them as "derivatives" at all.

- Using their information, one can apply no-arbitrage pricing to all the pieces of a firm's capital structure (bonds, loans, etc) without need of an explicit structural model.
  - ▶ This can be an advantage since it avoids the necessity of making the assumptions required by the structural models, and avoids some of the tricky issues of estimation and calibration they require.
  - ▶ Effectively, ther term structure of CDS spreads identifies the risk-neutral default density, which is sufficient for a lot of valuation problems.

- As we've seen, the CDS price will reflect the expected recovery rate on the *cheapest to deliver* or most junior debt.

- If we have such a bond, then we can literally combine it with a CDS to form a riskless bond.
  - ▶ This suggests another no-arbitrage relationship: *the CDS fee should equal the credit spread on the deliverable bond.*
  - ▶ Intuitively, the credit spread is the extra yield on the risk bond (over riskless ones). So if that yield doesn't equal the cost of default insurance there should be money to be made.

- In practice, some real-world details mess up this relationship a little.
  - The risky bond might be expensive to borrow.
  - The counterparty to the CDS might be risky.
  - There might not be a riskless bond which has the same price today and same maturity as the risky one.
  - The CDS does not cover the risk of the final coupon payment.
  - The CDS contains the extra delivery option, which could be valuable if some new – more junior – debt is issued before T.
- Despite these, arbitrageurs are usually able to enforce the intuitive relation to within 10-50 basis points.

- While taking the credit derivatives prices as given can be very helpful for a lot of problems, one might also view them as another security that we could price if we did have a structural model that we believed.
  - Or, we might believe there were potential mispricings in the CDS themselves.
- The distinction here is the same as the distinction between using call options to extract the risk-neutral density (via butterflies) in order to price other derivatives *versus* pricing options "from scratch" using a model of underlying dynamics.

- Remember, there are some important limitations to relying on extracted risk-neutral probabilities.
  - It requires access to a lot of high quality data all of the time.
  - And today's data tells you nothing about tomorrow's data.
- So this is where structural models come in.
  - As an example, recall from last time that there is an exact formula for the distribution of default times under Longstaff-Schwartz type models (where V_B is constant and V_t has constant σ)
  - From this H, you can get h and f by differentiation with respect to s. Then the formulas above give the implied CDS fees.
    - Moreover, we have a direct parameterization for how these densities will vary with the state variable V.
  - Here we are relying on the structural model to *provide* extra information about where the risk-neutral probabilities come from.
- In fact, a lot of money is devoted to *capital structure arbitrage* in which people attempt to value bonds, CDS, convertible, and options all within structural models, and then trade these against each other.
- Playing this game requires keeping a close eye on all the practical difficulties that the structural models don't know about.

IV. Summary

- This note was an overview of one of the most important classes of derivatives for risk management.
  - ▶ The particular contracts and institutional arrangements will continue to evolve rapidly with changes in regulation and technology.

- We covered the pricing and hedging of single-name credit default swaps.
  - ▶ We can view these as derivatives which depend only on the risk-neutral default intensity, which we can derive from structural models.

  - ▶ Or we can view them as primary securities which allow us to price and hedge risky corporate bonds, via using the implied risk-neutral default density.

  Later, we'll extend our analysis to pricing and hedging credit index products.

- Intuitively, credit default swaps fees are very close to the credit spread on a risky entity's debt.
  - ▶ They also correspond approximately to the risk-neutral annualized probability of default, times $(1-R)$, the loss-given-default.

Lecture Note 6.2: Summary of Notation

Symbol	Page	Meaning
R	p4	*post-default price of bonds of entity X*
$\widehat{B}_t(V_t)$	p13	*value of a 0-coupon, 0-recovery bond when firm's value is* V_t
$q(V_T)$	p13	*R-N probability (density) that firm value at* T *is* V_T
$B_{t,T}$	p13	*price of riskless 0-coupon bond maturing at* T
V_B	p13	*default boundary*
φ_n	p14	*fee for 1-year contract insuring against default during year* n *(to be paid at* $n-1$*)*
φ	p14	*fee for multi-year contract insuring against default until* T
r	p15	*instantaneous riskless rate*
$h(s;t)$	p15	*R-N default density conditional on survival from* t *to* s
$H(s;t)$	p15	*cumulative R-N default distribution*
$f(s;t)$	p15	$h/[1-H]$
$V(\Phi,h)$	p17	*value of a CDS with fee* Φ *when R-N density is* h
$V(\Phi,\varphi)$	p18	*value of a CDS with fee* Φ *when fair fee is* φ
$\widetilde{B}$	p20	*price of risky zero-coupon bond w/ recovery* R