Homework 6

1. (*Expected returns vs yields.*) This is an easy problem to make sure you are comfortable with different terms we have been using in talking about risky debt.

   Let F be the price of a one-year zero-coupon bond of some risky issuer. Assume that if the issuer defaults you get nothing.

   (a) Write F as a function of y, the annual (simple) yield-to-maturity.

   (b) Write F as a function of R the risk-free rate, and p the risk-neutral survival probability.

   (c) Write F as a function of e the bond's expected return, and π, the true survival probability.

   (d) In terms of these quantities, what can you say about the ratio π/p?

   (e) Suppose the true survival probability is 97% and the expected return on the firm's assets is $\mu = 12\%$. What can you say about the bond price?

2. *(Stock volatility in the Merton model.)*

   Consider the Merton model for a firm which has one share of stock and one zero-coupon bond which matures in 5 years (face value 100). Assume that $r = 6\%$ (annualized, continuous compounding) and the assets have volatility $\sigma_V = 20\%$. This question investigates the sensitivity of the stock's price to V shocks.

   Recall that the volatility is whatever multiplies the dW term in the model for dS/S. In fact, Itô's lemma tells us

$$\frac{dS}{S} = (\text{stuff})\, dt \ + \ \left[\frac{\partial S}{\partial V}\,\frac{V}{S}\,\sigma_V\right] dW.$$

   So the stock volatility is the function in square brackets in front of dW. Since the model decribes S as a function of V according to the Black-Scholes formula, you know the slope (delta) that appears in this expression, which is also a function of V

   Plot the stock volatility as a function of the level of the stock price – not of V – over the range 0 to 40. *Hint: this will correspond to a V range of about 0 to 100.*

   How does your curve compare to the plot one would get using the constant elasticity of variance model for S?

3. *(Asset risk and equity risk.)*

   In the previous problem, you computed σ_S in the Merton model. This question analyzes that function further. Continue to assume $T = 5$, $r = 6\%$, and set $V = 100$.

   (a) Plot the volatility multiplier σ_S/σ_V for values of σ_V ranging from 1% to 80%. Do this for debt of face value 100 and also 50.

(b) Now recall that the firm's expected excess return on equity (or its equity risk premium) is $\pi_S = \pi_V \ \sigma_S/\sigma_V$, where π_V is the risk premium of the underlying assets. Suppose the manager of the firm can add idiosyncratic risk to the firm, which raises σ_V but does not change π_V. According to your answer to (a), what will happen to π_S? Equivalently, what does this say about the relationship between idiosyncratic volatility and expected stock returns?

(c) If managers want to sell more equity in the future, their cost of equity capital will be $r + \pi_S$. In this case, will managers prefer to increase or decrease idiosyncratic risk?

4. *(Comparative statics of the Leland model.)*

Consider two countries in which the Leland model holds. They may differ according to their risk-free rate r, their tax rate Ω, or their bankruptcy costs α. In which country would you expect to see more <u>lenient</u> bankruptcy covenants? In other words, which will have <u>lower</u> V_B?

(a) Assuming we are comparing two firms with the same coupon interest Γ and the same asset risk σ, will V_B be lower for:

(i) The country with a higher or lower tax rate (when r and α are the same)?

(ii) The one with a higher or lower risk-free rate (when Ω and α are the same)?

(iii) The one with a higher or lower bankruptcy costs (when r and Ω are the same)?

(b) Now assume the comparison is being made between firms with the same asset value V_0 and σ, but where each one has chosen Γ optimally. Will V_B be higher or lower for the country with higher α, when r and Ω are the same?