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Lecture Note 10: Valuation of Nonhedgeable Risk

As our study of credit derivatives and asset backed finance has shown, many financial engineering problems require valuation and risk assessment of claims tied to *nonhedgeable* risks. We have observed many times that no-arbitrage arguments are not strong enough to allow us to put prices on exposures that are not some how already replicable in the market. But that does not mean our *techniques* cannot be useful.

As this note will explain, the problem of valuation of nonhedgeable risks can actually be formally encompassed in a methodology that has the same structure as no-arbitrage models.

After developing this formalism, we will look at some applications that take us to the limits of financial engineering.

Outline:

I. Utility Theory and Subjective State Prices

II. Example: "Real" Options

III. Example: Restricted Stock

IV. Example: Rainfall Insurance

V. Summary

I. Utility Theory

- How much would you pay for a security that will pay off \$100 if you get a score above 90 on the final exam in this course?

- Earlier in the term, we discussed pricing derivatives by synthesizing their cash-flows out of very precise claims – butterflies – that we imagined we could trade.
  - ▶ Each butterfly was a zero-or-one payoff tied to the occurrence of a specific event.
  - ▶ We've noticed that there are some literal examples of such contracts traded (e.g., sporting/event betting; binary options).
  - ▶ In fact, we showed that we can *make* such payoffs when the event in question is the price outcome of an asset that has traded calls and puts.
  - ▶ More generally, we deduced that any derivatives pricing theory is equivalent to a specification of the prices of all the butterflies. This is the same as specifying a risk neutral distribution.

- For any event A_T at time T, denote the butterfly payoff as $1_{\{A_T\}}$.

- Now I am asking, for a particular event, how would you price its butterfly?

- The way economists approach the problem is to try to model this intuitive idea:

  You will be willing pay up to an amount P such that your loss in happiness due to giving up P dollars today is equal to the expected increase in happiness that the payoff would bring you.

- To operationalize this idea we need to describe what makes you happy.
  - ▶ As you know, one way to do this is to characterize your *utility function*, which describes the flow of happiness you get from your consumption today.
    - ∗ Here you should understand the word "consumption" to be a general descriptions of all the things that you value: food, health, respect, etc.
  - ▶ Then your *value function* is your expected life-time utility, which incorporates how future prospects are combined with immediate benefits.
    - ∗ Let me assume that we can decribe how you feel today about future utility at T with a fixed utility function, U, times a factor $e^{-\phi(T-t)}$ representing your impatience.
    - ∗ Then the value function J is related to the utility function and current and future consumption C by this expression:

$$J(t) = \mathrm{E_t}\left[\sum_{s=0}^{\infty} e^{-\phi(s-t)}\ U(C_s)\right].$$

- Thus giving up a unit of C today – holding every thing else equal – lowers your value function by $U'(C_t)$.
  - ▶ Note that your *marginal utility*, $U'(C)$ should be a declining function of C: You value an extra unit more when you have fewer of them.

- If we want to express marginal utility in terms of money, we can multiply $U'(C)$ by P^C, where P^C denotes the consumption-goods-per-dollar (or the inverse of the nominal price index for the basket of "goods" you consume).

- Now, returning to the pricing problem, the economic intuition I outlined above would translate into the statement that your value, V_A, (in dollars) of the claim in question would have to equate

$$V_A \, U'(C_t) \, P_t^C$$

with

$$e^{-\phi(T-t)} \, \mathrm{E_t} \left[1_{\{A_T\}} \, U'(C_T) \, P_T^C \right]$$

where T is the date of the event A.
  - ▶ Equaling these is equivalent to saying that V_A is the price such that you could not make yourself better off – that is, increase your value function – by buying or selling an extra unit of the claim at that price.
  - ▶ Even though we are not assuming you *can* by or sell the claim, this is a consistent definition of the value you *would* attach to it if you could do so.
  - ▶ Economists would call it your *shadow price*.

- If this utility theory set-up seems to you to be an oversimplification of how people actually value things, it might not seem so unrealistic as a model of how *institutions* approach such problems.
  - ▶ After all, institutions are usually created with some sort of objective function in mind.
  - ▶ And we might view their "consumption" as simply their total profit or wealth.
  - ▶ Most of the time in financial engineering we are interested in solving problems for institutions.
- Actually many of the conclusions I am going to draw today can also be expressed in terms of other theories of decision making (including behavioral ones).

- Now go back to the general derivatives problem we tackled with butterflies in the fourth week of the term.
- We deduced that, if we had *any* payoff $g(X_T)$ tied to the outcomes of any random variable X_T and we had the market's butterfly prices b_x associated with every event $1_{\{X_T=x\}}$, then the price, V^g, of a claim to g has to be equal to

$$\int_x g(x)\, b_x\, dx.$$

- What we just expressed above was that the *subjective* butterfly price – your valuation for the outcome $\{X_T = x\}$ – should satisfy the condition:

$$\tilde{b}_{\{X_T=x\}} \, \Lambda_t = \mathrm{E_t}\left[\Lambda_T 1_{\{X_T=x\}}\right]$$

where I am defining Λ_s to be $e^{-\phi(s)} \; P_s^C \; U'(C_s)$.

- By the definition of conditional probability, the second term here is also

$$\mathrm{E_t}\left[\Lambda_T | X_T = x\right] \quad \mathrm{Prob}(X_T = x)$$

- So we have

$$\tilde{b}_{\{X_T=x\}} = \mathrm{E_t}\left[\frac{\Lambda_T}{\Lambda_t} | X_T = x\right] \quad \mathrm{Prob}(X_T = x).$$

- Thus we can express the subjective valuation of the claim to g as

$$V_t^g = \int_x g(x) \; \tilde{b}_x \; dx = \mathrm{E_t}\left[g(X_T) \; \frac{\Lambda_T}{\Lambda_t}\right],$$

which also says

$$\Lambda_t \; V_t^g = \mathrm{E_t}\left[g(X_T) \; \Lambda_T\right]. \tag{1}$$

- This equation encapulates our goals for the day. It expresses somebody's subjective valuation in terms of their *marginal utility adjusted* discounted expected payoffs.

- What we need to do next is to show how we might use this idea.

- First notice that the right hand side of (1) is a conditional (time-t) expectation of a time-T event. Call it h_t.
  - Now a basic fact from probability theory says that *you don't expect your own expectations to change systematically in one direction or the other*.
  - (If you expected your expectation to change, it wouldn't be your expectation!)

  That means that – as a stochastic process – h_t has zero drift.

- What is the drift of the left side of (1)?
  - Well, we need to decribe what we think drives changes to V and Λ.
  - For argument's sake, let us express Λ as a diffusion process:

$$\frac{d\Lambda}{\Lambda} = \mu_\Lambda \ dt + \sigma_\Lambda \ dW^\Lambda.$$

  - Next, write the process for the state variable X as

$$dX = m_X \ dt + b_X \ dW^X.$$

  - And then apply Ito's lemma to $V_t = V(X, t)$.

$$dV = [\frac{\partial V}{\partial t} + \frac{\partial V}{\partial X} \ m_X + \frac{1}{2}\frac{\partial^2 V}{\partial X^2} b_X^2] \ dt + \frac{\partial V}{\partial X} \ b_X \ \ dW^X.$$

  (Of course, it is possible that V also depends on other state variables. In that case, we would use the multivariate version of Ito here.)

- Now apply Ito's lemma again – this time to the process $h(\Lambda, V) = \Lambda\, V$.

$$dh = V\ d\Lambda + \Lambda dV\ \ + 1 \cdot \mathsf{Cov}[dV, d\Lambda]\ dt$$

where 1 is $\partial^2 h / \partial V \partial \Lambda$.

  - Using my definitions above, the instantaneous covariance term is $\rho_{X,\Lambda}\ \sigma_\Lambda\ \Lambda b_X\ \partial V / \partial X$.

- Now we have two expressions for the drift of h, which we can set equal to each other.

- The fact that the drift of h equals zero says (after cancelling a Λ that appears in each term):

$$\frac{1}{2} b_X^2 \frac{\partial^2 V}{\partial X^2} + [m_X + \rho_{X,\Lambda}\sigma_\Lambda b_X] \frac{\partial V}{\partial X} + \mu_\Lambda\ V + \frac{\partial V}{\partial t} = 0 \quad (2)$$

- Now you can see why I introduced all of this formalism.

  We are back to our no-arbitrage PDE.

- The equation (2) that characterizes the subjective valuation of the claim is of the exact same form as the PDE that characterizes derivatives' no-arbitrage prices.

- The subjective nature of the valuation just means that we have to recognize two alterations relative to the no-arbitrage PDE.

(1) The "subjective interest rate" is $\tilde{r} \equiv -\mu_\Lambda$.

This is the expected rate of *decline* of your marginal utility. If you expect to be much happier tomorrow, your marginal utility will be lower and you may be willing to pay a lot to borrow against that and consume today.

(2) The "subjective price of X risk" is $\tilde{\lambda}^X \equiv -\rho_{X,\Lambda}\sigma_\Lambda$.

This is bigger for risk factors that move opposite to your marginal utility. If a security pays off in states where you will be happy anyway, then holding it only increases your risk.

- But the really important observation is that we know how to solve PDEs like this.

  Feynman-Kac tells us everything!

  - We subjectively risk-adjust the dynamics of the X process.
  - Use the subjective interest rate to compute discount factors.
  - Then average the discounted payouts.

- Thus – given some information about someone's utility function – we can use the techniques of financial engineering to solve for their valuation.

II. "Real" Options

- Ever since 1973, people have been tempted to use options pricing formulas to evaluate the options features of business enterprises.
  - ▶ Managers of companies have the option to undertake new projects or stop old ones, depending on business conditions.
  - ▶ These are options on real – i.e. non-financial – investment opportunities.
- The trouble is, the underlying risk of many projects are mostly non-hedgeable. So it is misleading to think that no-arbitrage pricing can tell you their "correct" value.
  - ▶ For example, the net margins of a plant might depend on costs of labor, competitors' strategies, brand strength, and so on.
- However, it can still be very helpful to quantify the effects of options on project management, like determining the optimal times to exercise them.
  - ▶ Our framework allows us to do this once we take a stand on the utility of the agent who is doing the valuation.
- For example, suppose we work for a public company and we view our investors as being well-diversified in the sense that our particular decisions will not affect their consumption (and hence marginal utility) process in any significant way.

- ▶ And, to be concrete, suppose we model their marginal utility as $\Lambda_t = e^{-\phi t} C_t^{-\gamma}$, where C stands for average household consumption in the U.S. (or in the developed world).
- ▶ Here γ is called the coefficient of relative risk aversion. It might be a number like 7.
- ▶ I will let you verify (via Ito's lemma) that if C follows a geometric Brownian motion, then we get

$$\tilde{r} = \phi + \gamma\, \mu_C - \frac{1}{2}\gamma(\gamma+1)\, \sigma_C^2$$

- Now imagine we are operating a plant that produces revenues, θ_t, but involves fixed costs, m, per unit time.
  - ▶ Clearly it can turn into a losing operation if the revenues fall too low.
  - ▶ When should we abandon it (or shut it down)?
- Let us suppose that if we want to shut down, we can sell the machinery (or land, or patents, etc) for a known price $\underline{P}$.
  - ▶ Then the question is: how valuable is this disposal option and when would we exercise it.
  - ▶ Effectively, we have a claim to θ plus a *perpetual* American put on it.
- Following our program, let us compute the valuation, V, for an investor with the Λ given above.

- If we assume that θ obeys a geometric Brownian motion, then its *subjective* risk-adjusted drift is

$$\tilde{\mu} = \mu_\theta - \gamma \rho_{C,\theta} \sigma_\theta \sigma_C$$

because the volatility of $C^{-\gamma}$ is $-\gamma\ \sigma_C$.

  - So to implement our valuation we just need to know is the correlation of the revenue stream with consumption.

- Next, we have to add the payout stream $(\theta - m)$ to our PDE (2) above, so that it becomes

$$\frac{1}{2}\theta^2\sigma_\theta^2\frac{\partial^2 V}{\partial \theta^2} + \tilde{\mu}\frac{\partial V}{\partial \theta} - \tilde{r}\ V + \frac{\partial V}{\partial t} + (\theta - m) = 0$$

(This is just like adding the coupon term on the PDE for valuing a bond.)

- Like the Leland model of perpetual debt, there is no time-dependency in this security. So we can get rid of the $\frac{\partial V}{\partial t}$ term.

- Also, like the Leland model, the equation can be satisfied by a solution of the form $V(\theta) = A_X\theta^X + A_1\theta + A_0$ where the coefficients are just constants.

- I will leave it to you to verify that if you plug in this guess, you will find that the PDE is satisfied only if we have

$$\begin{aligned} -\tilde{r}A_0 - m &= 0 \\ -\tilde{r}A_1 + \tilde{\mu}A_1 + 1 &= 0 \\ \frac{1}{2}\sigma_\theta^2\, X(X-1)A_X + \tilde{\mu}XA_X - \tilde{r}A_X &= 0 \end{aligned}$$

- ▶ The first two equations give us the solutions for A_0 and A_1.
- ▶ After cancelling the A_X terms, the third equation is a quadratic equation for X, whose solution is

$$X = \frac{-b \pm \sqrt{b^2 + 2\tilde{r}\sigma_\theta^2}}{\sigma_\theta^2}$$

where $b = \tilde{\mu} - \frac{1}{2}\sigma_\theta^2$.

- ▶ Here we will choose the negative of the two roots because the positive root will cause the solution to grow faster than linearly for large positive values of θ, which cannot be the correct answer.

- That still leaves the parameter A_X which will allow us to satisfy the abandonment condition.
  - ▶ Suppose we choose to abandon the project when $\theta = \underline{\theta}$.
  - ▶ Then we must have $V(\underline{\theta}) = \underline{P}$, or

$$\underline{P} = A_X \underline{\theta}^X + \frac{\underline{\theta}}{\tilde{r} - \tilde{\mu}} + \frac{m}{\tilde{r}}$$

which immediately gives us A_X.

  - ▶ You can clearly see how the subjective terms directly affect the solution for V via A_0, A_1, A_X, and X.

- That concludes our valuation problem. Now we can return to the interesting policy problem: *when to abandon?*

- Answer: Pick the value of $\underline{\theta}$ that maximizes V today.

- All we have to do is set $\partial V / \partial \underline{\theta} = 0$ and solve for $\underline{\theta}$. Since the A_0 and A_1 terms don't involve $\underline{\theta}$, that leaves the condition

$$0 = \theta^X \ \ \frac{\partial A_X}{\partial \underline{\theta}}$$

- Please verify that this yields

$$\underline{\theta} = \frac{-X}{1 - X} \, (\tilde{r} - \tilde{\mu}) \, (\underline{P} - \frac{m}{\tilde{r}}).$$

- This completely characterizes the solution and policy.
  - ▶ We could now substitute this $\underline{\theta}$ back into our solution for A_X and have everything in terms of the underlying parameters σ_θ^2, $\tilde{r}, \tilde{\mu}$, and m and $\underline{P}$.

- We could use this to answer numerous questions about how the abandonment decision depends on the fixed-costs, the salvage value, or volatility and growth rate of our revenue θ.

- If we wanted to see how much value the abandonment option contributes, we could see what the project would be worth if we set $\underline{P} = 0$.
  - ▶ For some projects we could imagine that there were actually abandonment penalties, for example, fees for terminating contracts early, laying off workers, and disposing of waste.
  - ▶ We could model that by putting $\underline{P} < 0$.

- Real options models are actually used by many companies, particularly in the natural resource and pharmaceutical industries.
  - For example, decisions on when to start or stop operations of a large refinery, or when to dig a gold mine.
  - Drug trials (and other R&D ventures) can also be viewed as multi-stage real options.

- The technique is not telling us the no-arbitrage value of the venture. Rather, it is showing us how to map our investor's risk aversion and impatience into a valuation framework that is internally consistent for them.

III. Example: Restricted Stock

- Consider the problem we studied earlier in the term of modelling how employees value stock options in compensation contracts.

- If they face no barriers to hedging (and no vesting provisions), then their valuation will be the same as the market's valuation: both should be determined by absence of arbitrage.

- But this is usually not the case.
  - ▶ In practice, employees may not be able to sell their firms' stock short, for either explicit contractual reasons or just because it looks bad.

- This means they are being forced to bear the risks of exposure to the firm's stock. Hence their subjective valuation of this extra exposure will affect their willingness to accept it.
  - ▶ Intuitively, restricted stock exposures may skew their personal wealth portfolio in a suboptimal way.

- Managers need to understand this valuation in order to decide if it makes sense to give the employees the restricted stock as part of their compensation.
  - ▶ Perhaps in some cases it makes more sense just to give them cash.

- Our theory can shed some light on the robustness of no-arbitrage valuation to the assumption that there are available traded securities for eliminating risk.

- In fact, we can learn a lot just from the problem of valuing *restricted stock*, i.e. even with no option features at all in the compensation contract.
  - Assume there are no indirect ways to hedge it either (no futures, options, etc).
  - From the employee's point of view, the stock price risk is non-hedgeable.
  - (Let's assume, for expostional purposes, that you can't even partially hedge it e.g. with other stocks in the same industry.)

- Imagine you are a manager and you are forced to hold, say, α shares in your firm's stock.
  - Intuitively, we know you will value it less than the market overall would.
  - The question is: how much?
- We can answer this question in our subjective valuation framework, given some utility specification.
- Consider the case of valuation with *exponential utility:* $U(C) = -\exp(-\gamma C)/\gamma$.
- Imagine that this is your utility function, and that you have outside wealth W invested in traded assets, as well as α shares of your company's stock.

- For simplicity, suppose the stock's dividend stream Y obeys

$$dY = \mu_Y \; dt \; + \; \sigma_Y \; dZ^Y.$$

This is an *arithmetic Brownian motion* which means there could be some losses that have to be funded (Y could go negative).

  ▶ (I am using dZ for the random part because I will use W to stand for wealth below.)

- Following our logic above, we want to compute

$$G(Y_t) = \mathrm{E_t}\left[\int_t^{\infty} e^{-\phi(s-t)} \; e^{-\gamma(C_s - C_t)} \; Y_s \; ds\right]$$

which represents our subjective valuation of the dividend stream. This is what the stock is worth to someone who can never trade it.

- Above, we got a PDE for payoffs at a specific date, like

$$g(T, Y_t) = \mathrm{E_t}\left[e^{-\phi(T-t)} \; e^{-\gamma(C_T - C_t)} \; Y_T\right]$$

which we showed had to have zero drift.

- Now we are valuing a sequence of payoffs: $G = \int_t^{\infty} \; g(s) \; ds$.
  ▶ The drift of G is then just $-g(t)$ or $-Y_t$.

- As before, we can derive the valuation PDE for G by expanding out the drift of $G(Y_t)$ using Ito's lemma.

- But now we need to recognize that G also depends on the employee's consumption process, which will depend on his/her wealth, which depends on G.

- Actually, it is not hard to show that a person with exponential utility will always consume the fraction ϕ times his/her total wealth.
  - ▶ So I will use this fact, and write $G = G(Y, W)$ where W stands for the non-restricted ("outside") wealth.

- We do not have to specify the wealth process completely. It is enough to know that the manager has an investment of π in other risky wealth (e.g. the market portfolio) and $W - \pi$ in bonds.
  - ▶ Write the dynamics of his risky investment as $\pi(\mu_m \, dt \ + \ \sigma_m \, dZ^m)$.
  - ▶ In keeping with the assumption of no hedging ability, we'll assume the correlation between dZ^m and dZ^Y is zero.
  - ▶ Outside wealth evolution then follows

$$dW = (\pi\mu_m + r(W - \pi) + \alpha Y - C)dt + \pi\sigma_m \, dZ^m$$

    where the drift terms impose the budget constraint that income minus consumption equals the net gains that you carry forward over time.

- Returning to the consumption process, the fact I cited above can now be written as:

$$C_t = \phi(W_t + \alpha G(Y_t, W_t)) + C_0.$$

where C_0 is a constant that won't affect our problem.

- Our solution for G will reflect the fact that more restricted stock implies more risky consumption – hence more risky marginal utility.
  - This will increase the discount we apply to the valuation of that stock.

- As a last simplifying assumption, let me impose that our manager is no more impatient or patient than the economy as a whole, so that $\phi = r$.

- Now with this set-up, I can proceed to write down the valuation PDE for G after we compute the subjective interest rate and the covariance of our payoffs with our marginal utility.
  - To anticipate, we will discover that we will be able to solve the PDE by finding a G function that is *not a function of wealth*.
  - So if you trust me that it will work, I will proceed with the assertion that $G = G(Y_t)$ and there will be a lot less algebra!

- Our marginal utility process now is

$$\Lambda_t = e^{-rt} \; e^{-\gamma C_0} \; e^{-\gamma r[W_t + \alpha G(Y_t)]}$$

- Our subjective interest rate is minus the drift of $d\Lambda/\Lambda$. From Ito, this is

$$\begin{aligned} \tilde{r} &= r + r\gamma\mu_W + r\gamma\alpha G_Y \mu_Y \\ &- \frac{1}{2}\sigma_Y^2[(r\gamma\alpha G_Y)^2 - r\gamma\alpha G_{YY}] - \frac{1}{2}\sigma_W^2 (r\gamma)^2. \end{aligned}$$

(I'm using subscripts on G to denote its derivatives.)

- What about the covariance of $d\Lambda_t/\Lambda_t$ with dG? Again, Ito tells us this is

$$-\alpha r\gamma G_Y \sigma_Y^2.$$

- So putting all the terms together the PDE says

$$\frac{1}{2}\sigma_Y^2 G_{YY} + [\mu_Y - \alpha r\gamma G_Y \sigma_Y^2]G_Y - \tilde{r}G + Y = 0$$

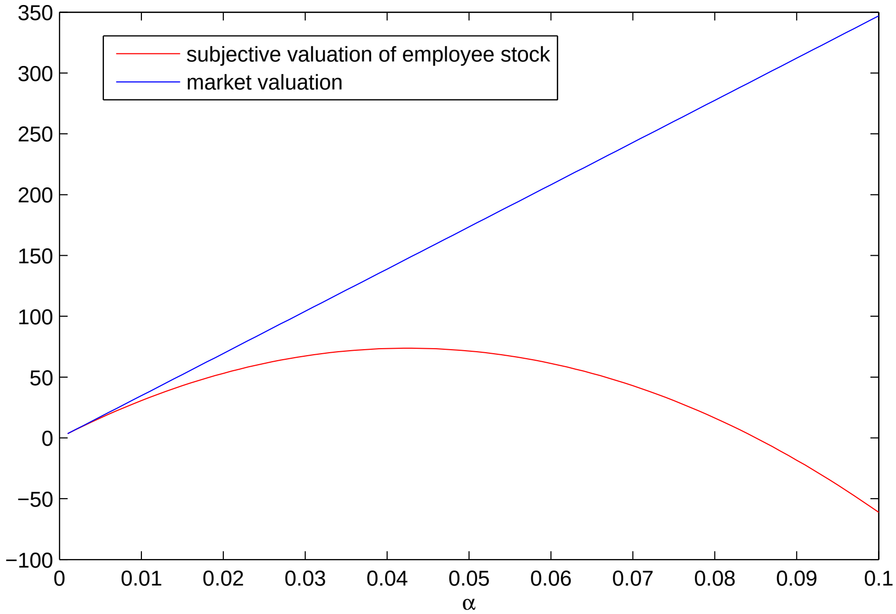
- Now there is still a lot of complication inside the $\tilde{r}$ term. So this looks like a very messy equation.
  - ▶ Remarkably, it's not!
  - ▶ In fact, we can satisfy it by a trial function of the form $G = bY + d$ where b, d are constants.
  - ▶ If the constant $b = 1/r$, then the W and Y terms in μ_W cancel and only constants are left. Then the whole expression for $\tilde{r}$ becomes constant because G_Y and G_{YY} are also constant.
  - ▶ So the ugly terms in the differential equation are all actually not so bad. I will let you fill in the details. We have one remaining free parameter, d, that we can choose to set all the rest of the terms to zero.

- The end result is that the employee's valuation of the firm is

$$G(Y) = \frac{Y}{r} + \frac{\mu_Y}{r^2} - \alpha\frac{\gamma}{r^2}\frac{\sigma_Y^2}{2}.$$

- For this model, the manager's valuation of the whole firm declines linearly with the stake he/she is forced to hold.

- Of course, this also says that the valuation of the manager's holding, αG, declines *quadratically* with the size of that holding, α.



Parameter values $Y = 100, \mu_Y = 10, \sigma_Y = 20, \gamma = 1, r = .05$

- In this setting, clearly the firm should think twice about making large stock grants to employees.
  - If they think they are manufacturing some benefit (loyalty, retention) which is an increasing function of α, there is only so far they should go.
  - The entire quadratic term represents a deadweight loss – a destruction of value – relative to the open market valuation of the same shares!

IV. Rainfall Insurance.

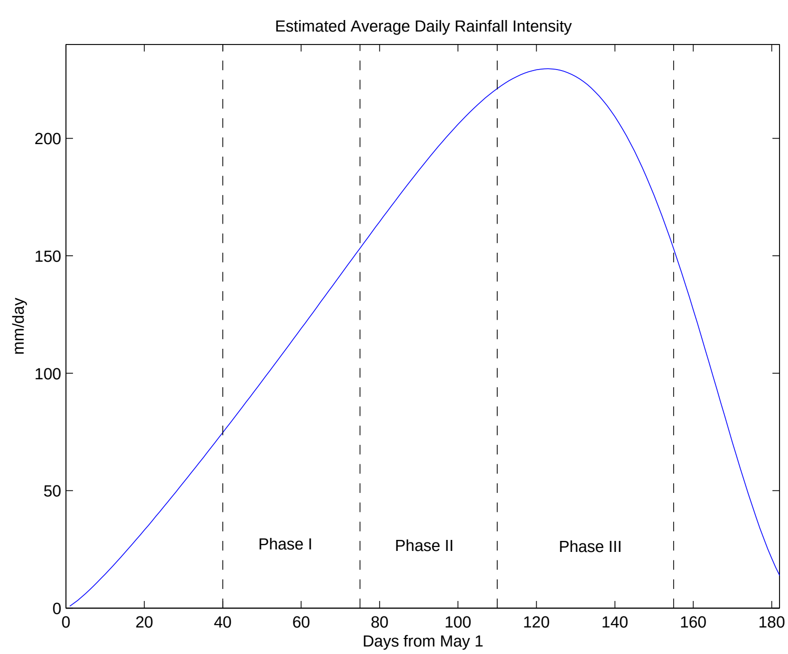
- Consider the objectives in the BASIX case in rural India in the Spring of 2004.
  - ▶ Social entrepreneurs want to create economical ways for very poor farmers to have access to insurance against terrible monsoons (i.e., not enough rain).
  - ▶ There are no existing markets in rural India that enable hedging of this risk.
- Among the issues is understanding how much the clients should be willing to pay.
  - ▶ Development economists suspect that such products could improve farmers' welfare if they understood them.
- At the same time, governments may believe it will also improve <u>other people's</u> welfare if farmer's had insurance.
  - ▶ Without it, the government has to come up with aid funds in bad years – which other people have to pay for.
    - ∗ Droughts also cause childhood malnutrition which can cause long-term losses in productivity and health.
  - ▶ Hence, the social cost of providing coverage may be less than its monetary cost.
- Let's examine the product from both sides of the transaction.

(A) Valuation from the Issuer's Point of View.

- To start, we would need to estimate the true dynamics of rainfall.
  - ▶ Case gives realized distribution of cumulative rain for each phase in some recent data.
  - ▶ Not hard to design a stochastic specification of daily rain with mean and volatility detrministically switching over the different phases.
    - ∗ Example: assume percentage deviations from historical mean, $M(t)$, follow GBMP.

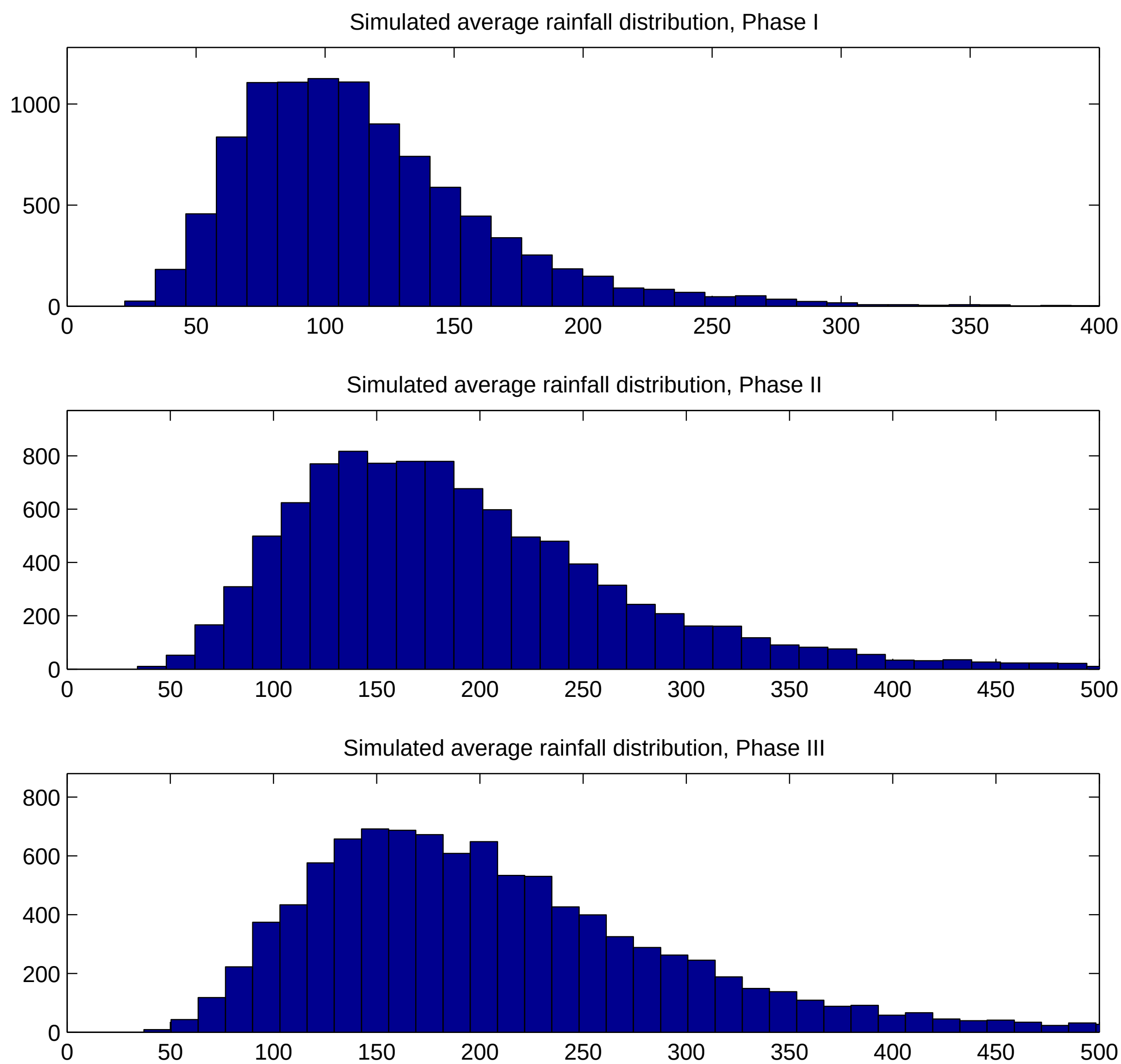
$$rainfall_t = P_t \; M_t \qquad\qquad \frac{dP}{P} = \sigma_t \; dW_t$$

      - Estimate mean daily intensity, M_t, to match Phase means given in Exhibit 6.



∗ Shocks in “Phase 0” have high volatility as information is rapidly revealed.

∗ Volatility declines in later periods, as monsoon strength becomes determined.

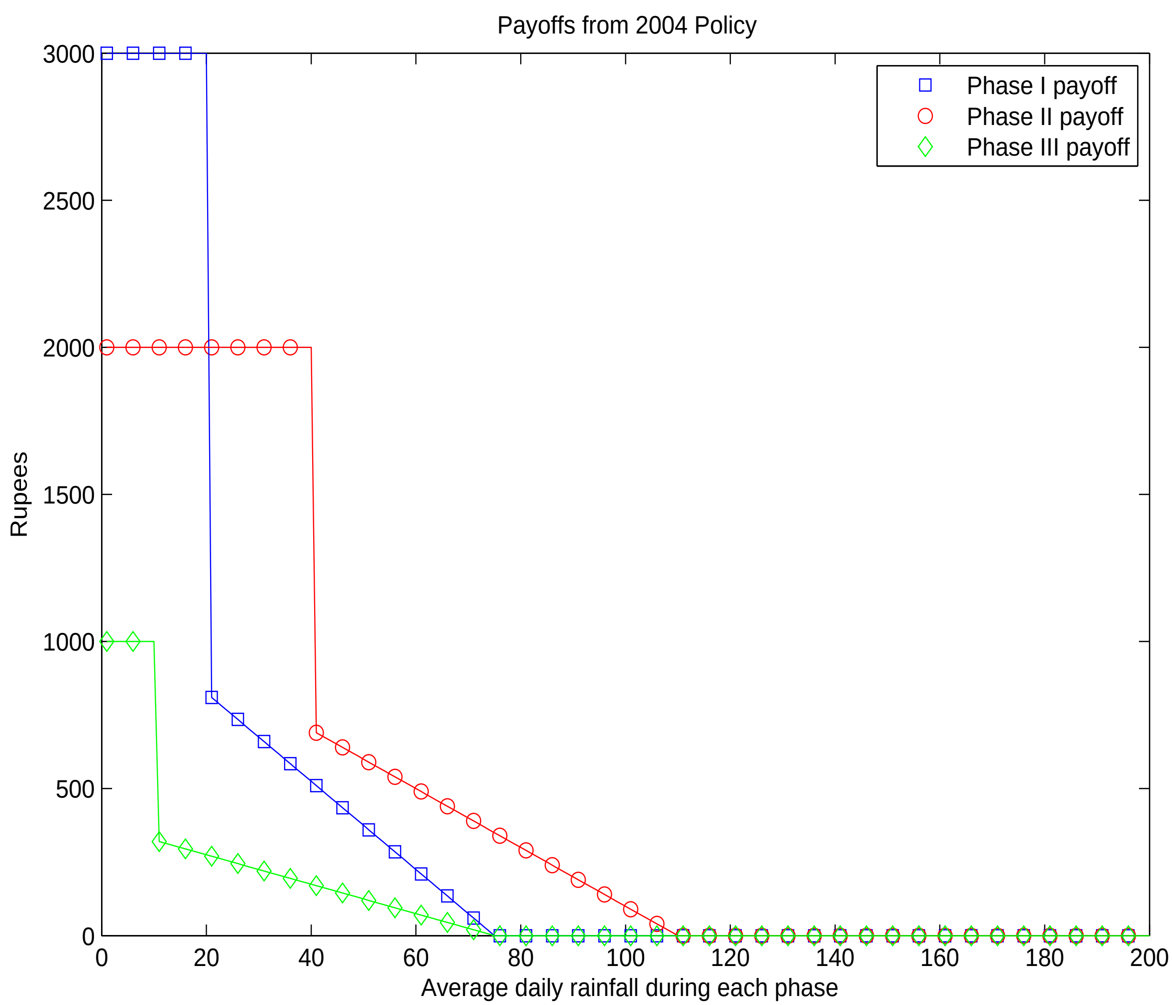


∗ Could add additional high-frequency mean reverting component to this process to capture local fluctuation.

- Of course, we should also use any additional knowledge about the *future* that might imply differences from the past.
  - Some research suggests climate change could imply more volatile monsoons.
  - No consensus on whether the *average* effect would be for more or less rain.

- Next step: How does rainfall translate into insurance payoff?

- Payoff is a linked series of 3 Asian precipice puts. Highly path dependent.



- **Q:** What is the logic behind such a complex structure?

- **Q:** If they want to provide insurance against crop failure, why tie payout to rainfall rather than crop yields?

- Next, assume BASIX (and market as a whole) neutral with respect to Mahbubnagar rainfall risk: $\lambda^R = 0$.
- Value insurance payoff as *true* expected payoff.
  - Company's cost of funds $= 9\%$ in INR.
  - Payment for each phase made 30 days after it ends.

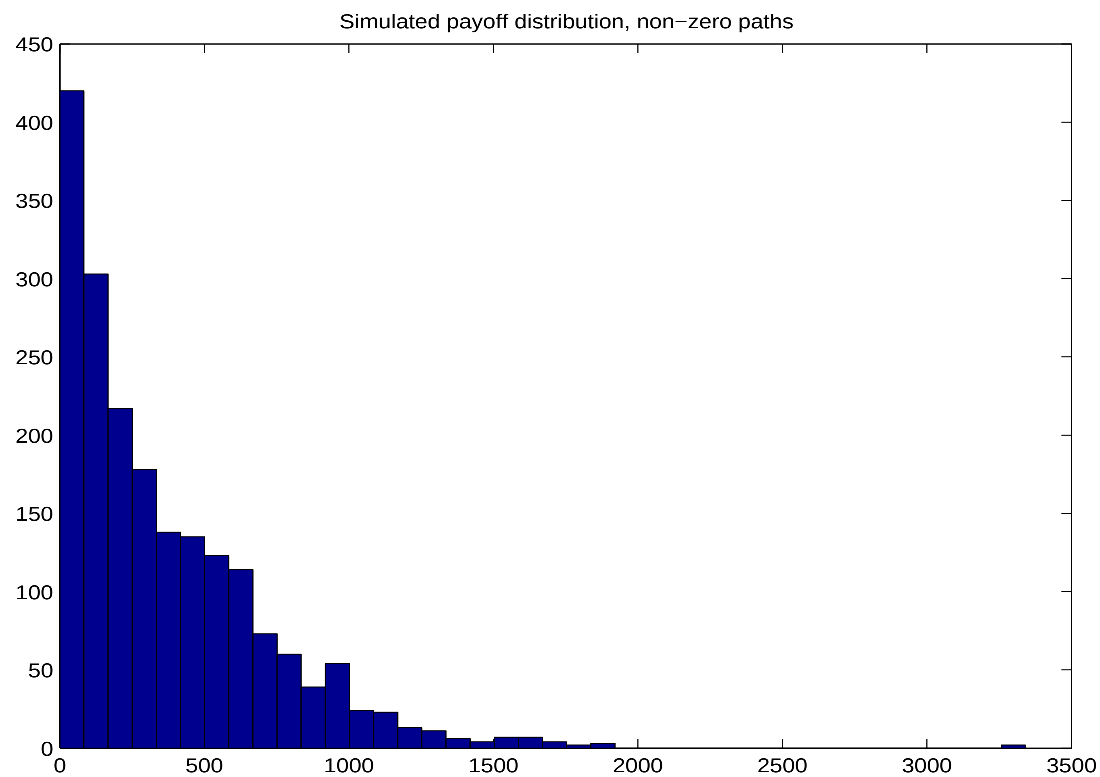
- Numerical evaluation of discounted expectation is straightforward.

- My result: value $= 68.1 \pm 1.1$ (with 40k paths).
- Company pricing product at 125.
  - They have to cover high sales and administrative cost.
  - Will customers want it?

(B) Valuation from Customer's Point of View.

- Farmers' *subjective* price of risk may differ from market's (as with any type of insurance).

- What does payout distribution look like?



- Very high probability (80%) of no payout at all.

- We need to know how badly farmers feel about those 10% of outcomes.

  ▶ In terms of pricing theory, need to know their λ^R.

- No direct way of estimating it.

- ▶ Although perhaps indirect evidence based on their unwillingness to purchase last year's insurance offering (according to case).

- It will certainly be positive: there will be strong negative correlation with their marginal utility.

- But it may not be as high as one would first expect.

  1. There are other insurance mechanisms, e.g., through government or extended family support.

  2. Also self-selection: people who are very risk averse to weather risk may not become farmers.

  3. Moreover, perhaps crop <u>prices</u> adjust upwards in bad harvest years, buffering their losses.

- Also, the exposure we are asking about is not the same as harvest risk.

  - ▶ Even though rainfall is measured locally, its realization may not correspond to the experience of each farmer – or correlate perfectly with his plant yield.

- Interestingly, there is evidence that payout realizations historically would have been significantly correlated with the Indian economy.

  - ▶ This reflects the importance of the monsoon to the entire country. Bad monsoons equal low GDP.

- ▶ It means that the farmers are likely to be poor when the rest of the country is least able to afford to bail them out.

- Re-run Monte Carlo pricing with different λ.

  - ▶ Can learn how risk averse customers need to be in order to make the product attractive to them.

- Also we do know that the farmers face a very high cost of funds: $\tilde{r} \approx 24\%$ or more.

  - ▶ This makes insurance less valuable, since up-front payment required.

  - ▶ On the other hand, may be neglecting positive externality if purchase of insurance lowers this rate.

Insurance Value
as a Function of Lambda

λ	V
0.0	64.9
0.2	74.2
0.4	84.5
0.6	96.4
0.8	109.4
1.0	123.7

- Robustness check: raising volatilities by 5 points does raise these values by around 15 points. Still need $\lambda > 0.7$
- Sharpe ratio this high $\rightarrow$ very high risk aversion.
  - ▶ Recall the stock market ratio is closer to 0.4.

- Behavioral approach to estimating risk aversion:

  How much would a farmer pay for a fair bet paying 1 if rainfall is above average and 0 if it is below?

  Or:

  If he demands a Sharpe ratio on this bet of 0.9, how much of a price discount below fair value (1/2) is required to make him willing to do it?

  - ▶ For binomial bet with 0/1 payoff and price p and true probability π, expected return is $(\pi - p)/p$; return variance is $\pi(1-\pi)/p^2$, so

$$\text{Sharpe ratio} = \frac{\pi - p}{\sqrt{\pi(1-\pi)}}$$

- ▶ If $\pi = 0.5$, this says $\lambda \approx (1 - 2p)$ or $p = (1 - \lambda)/2$
- ▶ For example, $\lambda = 0.9$ means only willing to pay 0.05 or $1/10$th of fair value for this bet!
- ▶ Not very likely farmers are that fearful.

- So **conclusion:** based upon price alone, it would not be surprising to see this product fail if priced at 125.

- What other factors might enter into a farmer's decision?
- Policy question: are there smarter ways we could design a policy so that more people insure themselves and rely less on governments?
- Our methodology shows us some of the factors that will affect this decision. So provides a good starting point.

V. Conclusion

- Today I have argued that financial engineering models can be used in a variety of settings that have nothing to do with "arbitrage" or even making money.

- Our exploration of valuation has taught us that there are two distinct things we can ask of a model.

  What is something worth to us subjectively ?

  What is something worth to the market?

  The two are only the same in a world of complete and perfect markets.

- In a utility based framework we can consistently employ the tools of no-arbitrage pricing even in problems involving risks that we cannot hedge.

- *Subjective valuations* and *subjectively optimal policies* with meaningful economic interpretation can still be defined and computed.

- Effective financial engineering may require us to tackle both types of questions.

- Financial engineering is not about a few magical formulas.

  ▶ It is about a few powerful principles.

- If we apply them with a clear understanding of their assumptions and limitations, they can be extremely powerful and useful.