FIN 514: Problem Set #3

Due on Sunday, February 18, 2018

Wanbae Park

Problem 1

Using the following Black-Scholes formula, the option price was calculated as 10.2479.

$$\textit{Put option price} = Ke^{-r(T-t)}N(-d_2) - Se^{-\delta(T-t)}N(-d_1)$$

$$d_1 = \frac{log(\frac{S}{K}) + (r - \delta + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = d_1 - \sigma\sqrt{T-t}$$

Then, using Cox, Ross and Rubinstein(CRR), Rendleman and Bartter(RB), Leisen and Reimer(LR) method each, put option value was calculated from $N = 50$ to $N = 1000$. Figure 1 shows the error of each method. As shown in figure, except LR method, there seems to exist some problems. In CRR method, it looks like

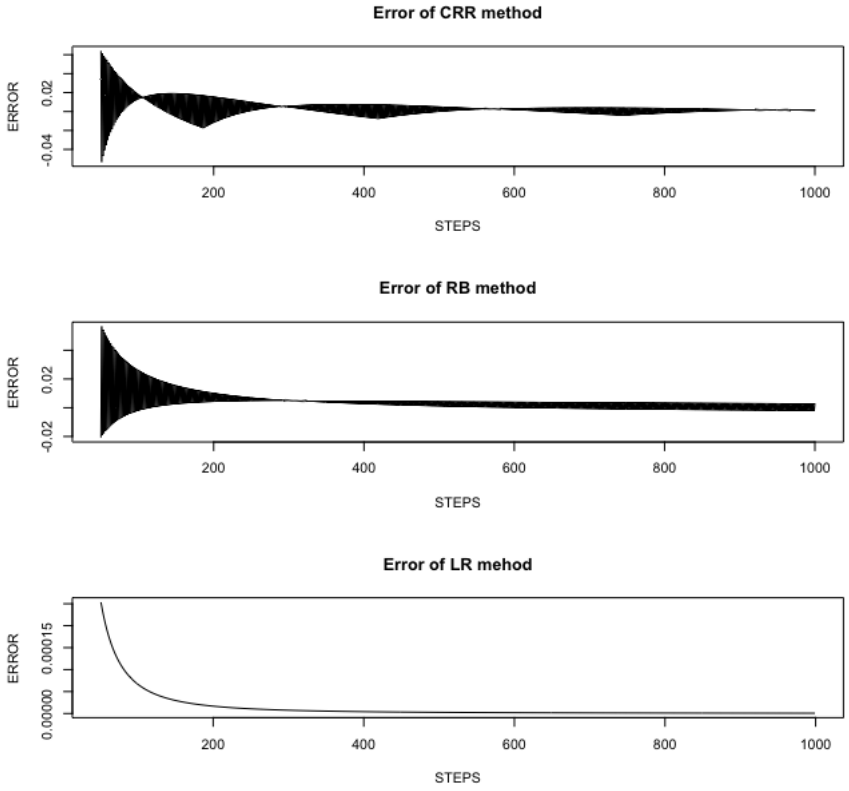

Figure 1: Error of each methods

that error is converging to zero, but it is not monotonic. It means it does not guarantee that applying more steps makes more accurate values. Regarding RB method, it seems better than CRR, but error is increasing from some points(about $N = 300$). The reason for this phenomenon is that option payoff is not linear shape. Since LR method solves this problem when N is odd, the shape of error in LR method seems monotonically decreasing to zero as N goes to some large value. The reason why monotonicity is important is that we can extrapolate values from two binomial trees to get more accurate values if monotonic error is guaranteed.

Problem 1 continued on next page...

Figure 2 shows the error after extrapolation($M = 2N$ is used when using CRR and RB method, $M = 2N - 1$ is used for extrapolation.). Since it is well-known that LR method has $O(1/n^2)$ errors, the extrapolation

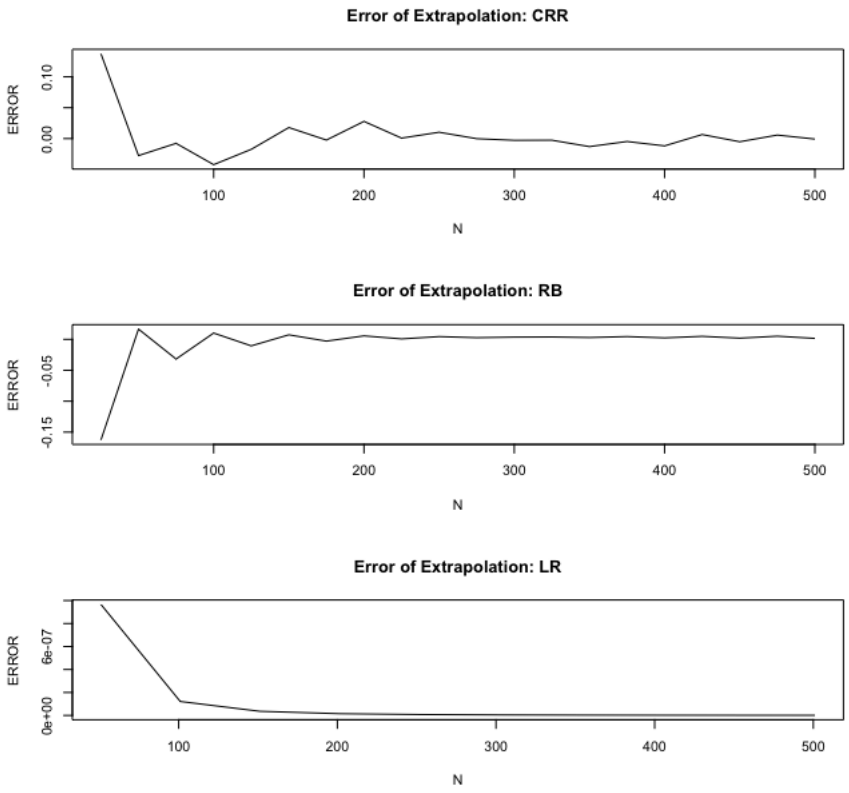

Figure 2: Error of each methods after extrapolation

procedure has changed from original one to followings.

$$V_{EXACT} \approx \frac{M^2 V_M - N^2 V_N}{M^2 - N^2} \quad \text{where } M \text{ and } N \text{ are odd numbers.}$$

As shown in Figure 2, the error of CRR and RB methods seems sawtoothing, but error of LR method is converging to zero monotonically. Furthermore, the accuracy of value is even worse at some points for CRR and RB method when extrapolation is applied. Before using extrapolation technique, the maximum error of CRR and RB method is about 0.05, but there are some points where error is about 0.1 after extrapolation. However, in LR method, the error after extrapolation is always smaller than before. That is why monotonicity is important when using extrapolation to get more accurate value.

Problem 2