Fin 514: Financial Engineering II

Lecture 3: Practical issues with the binomial

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Outline

- Here we will consider some practical issues when valuing derivatives using a binomial tree.
- We will look at how to model three types of dividends: proportional, fixed and continuous.
- We will look at how to construct a tree and how to value European and American options with large numbers of time steps in Excel.
- Then we will build in call and conversion features and how to deal with coupon/interest payments.

Spreadsheet

- Let's think about actually implementing the binomial model with a reasonable number of steps.
- Call option:
  - Strike price $K = 40$
  - $T = 1$ year to expiration
  - Use $N = 50$ periods, $\Delta t = T/N = 1/50 = 0.02$
  - Currently $S_0 = 40$
  - $r = 0.04(4\%)$
  - $\sigma = 0.30(30\%)$

Dividends

- Dividends are a bit tricky. We assume that dividends can be modeled by three different approaches, listed from the easiest to model to the hardest.
  1. Continuous dividends, this is most suitable when the underlying asset is a Stock Index, a foreign exchange rate or a commodity that can be leased. It is also the easiest to model.
  2. Proportional discrete dividends. Here, dividends are only paid at discrete dates, typically every quarter but we assume that the dividend size is proportional to the stock price on the ex-dividend date. This is appropriate for single stock underlying assets and is simpler to model than...
  3. Fixed size discrete dividends. here dividends are only paid at discrete points in time but the size of the dividend is independent of the stock price, i.e fixed. This is the most realistic way of modeling single stock dividend payments, but causes problems for the binomial tree.

1. Continuous dividends

- Here the dividend affects the risk-neutral expected return of the stock, in that now,
$$E[S_T] = S_0 e^{(r-\delta)T}$$
and so all that needs to change is the calculation of the risk-neutral probabilities, so
$$q = \frac{e^{(r-\delta)\Delta t} - d}{u - d}$$
- The formulas for u and d can be any one of the standard functions.

2. Proportional Dividends

- This is best explained with an example. We assume:
  - Paid quarterly
  - Ex-dividend dates are times 0.125, 0.375, 0.625, 0.875.
  - Quarterly dividend is approximately 1% of stock price. In particular, on ex-dividend dates stock price drops from S to $S(1-D)=S(1-0.01)$ (This ensures that the tree recombines).

Implementation: Proportional dividends

- Across ex-dividend dates (By 'across' ex-dividend dates I mean that the ex-dividend date falls sometime between the two dates on the tree. For example, the ex-dividend date might be time 0.125, between the dates 0.12 and 0.14 of the tree.) we have

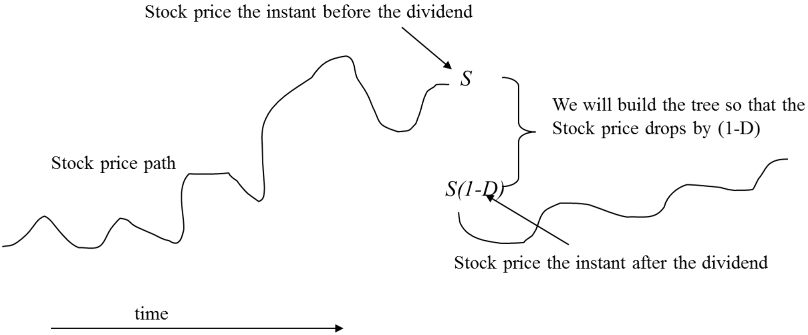
$$S \quad \begin{cases} (Se^{r\Delta t+\sigma\sqrt{\Delta t}})(1-D) = u(1-D)S = \hat{u}S \\ (Se^{r\Delta t-\sigma\sqrt{\Delta t}})(1-D) = d(1-D)S = \hat{d}S \end{cases}$$

$$\hat{u} = e^{r\Delta t+\sigma\sqrt{\Delta t}}(1-D) = u(1-D),$$
$$\hat{d} = e^{r\Delta t-\sigma\sqrt{\Delta t}}(1-D) = d(1-D)$$

Stock price movement across ex-dividend dates is like other dates, except stock price is reduced by the amount of the dividend.

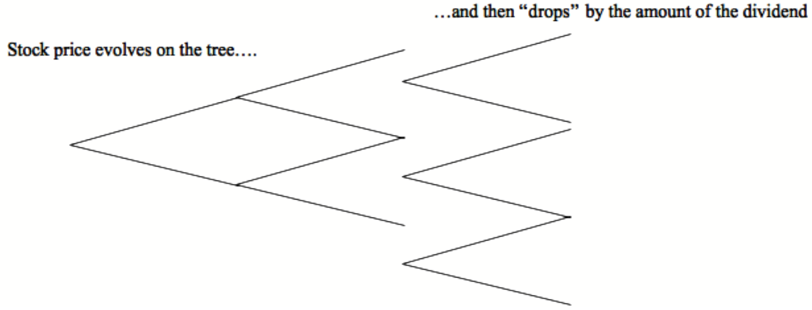
Price path with Proportional dividends

Stock price drops on ex-dividend date:



Price path on binomial tree

Make a binomial tree that 'drops' on ex-dividend date, where the ex-dividend date occurs in the step before the drop



Quiz

Does the tree recombine? That is an up movement before the dividend followed by a down the same as a down movement before followed by an up after?

- No
- Yes
- Only for special parameter choices

R-N probabilities for Proportional dividends

- let's just verify that we do not need to adjust our risk-neutral probabilities of up and down movements when we have proportional dividends.
- For periods with no dividend the risk-neutral probabilities are

$$q = \frac{e^{r\Delta t} - d}{u - d}, \quad 1 - q = \frac{u - e^{r\Delta t}}{u - d}$$

- Across ex-dividend dates, they are actually still the same:

$$\begin{aligned} q &= \frac{e^{r\Delta t}(1-D) - \hat{d}}{\hat{u} - \hat{d}} = \frac{e^{r\Delta t} - d}{u - d} \\ 1 - q &= \frac{\hat{u} - e^{r\Delta t}(1-D)}{\hat{u} - \hat{d}} = \frac{u - e^{r\Delta t}}{u - d} \end{aligned}$$

3. Fixed dividends

- These are the most problematic because they do not lead to a recombining tree. They are however, the most realistic when the underlying is a single stock.
- Let's now imagine that the dividend is \$1 rather than 1% and is paid at $t = 0.12$. A down movement before the dividend followed by an up takes us to,

$$(dS_t - 1)u = udS_t - u$$

whereas an up movement before and a down movement afterwards takes us to,

$$(uS_t - 1)d = udS_t - d$$

and so these do not take us to the same place and the tree does not recombine.

Apple dividend history

Apple Inc. Dividend Date & History

$95.59* 1.54 ↓ 1.59%

*Delayed - data as of Jan. 19, 2016 14:15 ET - Find a broker to begin trading AAPL now

CME SSFN TWTR AAPL EFA

Save Stocks

The Dividend History page provides a single page to review all of the aggregated Dividend payment information. Visit our Dividend Calendar: Our partner, Zacks Investment Research, provides the upcoming ex-dividend dates for the next month.

Ex/Eff Date	Type	Cash Amount	Declaration Date	Record Date	Payment Date
11/5/2015	Cash	0.52	10/27/2015	11/9/2015	11/12/2015
8/6/2015	Cash	0.52	7/21/2015	8/10/2015	8/13/2015
5/7/2015	Cash	0.52	4/27/2015	5/11/2015	5/14/2015
2/5/2015	Cash	0.47	1/27/2015	2/9/2015	2/12/2015
11/6/2014	Cash	0.47	10/20/2014	11/10/2014	11/13/2014
8/7/2014	Cash	0.47	7/22/2014	8/11/2014	8/14/2014
5/8/2014	Cash	3.29	4/23/2014	5/12/2014	5/15/2014
2/6/2014	Cash	3.05	1/27/2014	2/10/2014	2/13/2014
11/6/2013	Cash	3.05	10/28/2013	11/11/2013	11/14/2013
8/8/2013	Cash	3.05	7/23/2013	8/12/2013	8/15/2013
5/9/2013	Cash	3.05	4/23/2013	5/13/2013	5/16/2013
2/7/2013	Cash	2.65	1/23/2013	2/11/2013	2/14/2013
11/7/2012	Cash	2.65	10/25/2012	11/12/2012	11/15/2012
8/9/2012	Cash	2.65	7/24/2012	8/13/2012	8/16/2012
11/21/1995	Cash	0.12	--	11/21/1995	--
8/16/1995	Cash	0.12	--	8/16/1995	--
5/26/1995	Cash	0.12	--	5/26/1995	--
2/13/1995	Cash	0.12	--	2/13/1995	--

Fixed dividends

- There are many solutions, I will show you what I believe to be the simplest that is still widely applicable for early exercise, barriers etc. It is formally described in Vellekoop and Nieuwenhuis (2006).
- Construct the stock price tree as usual with u, d, q described as for the non-dividend case. Value the option backwards in time until we hit an ex-dividend date, t_d.
- We will assume that we have a time step, $j = k$ say, that coincides with this ex-dividend date, stock prices here are given by $S_{k,i}$ where i denotes the number of up jumps in the tree. At this point the stock price should fall from $S_{k,i}$ to $S_{k,i} - D$. So, here we need to replace the option values $V(S_{k,i}, k\Delta t)$ (usually more simply written as $V_{k,i}$) with $V(S_{k,i} - D, k\Delta t)$ (we'll call this $V^*_{k,i}$).

Fixed dividends

- The simplest way to do this is to interpolate across the known values from the tree. For example if S_d is the largest value in the tree where $S_d < S_{k,i} - D$ and S_u is the smallest value in the tree where $S_u > S_{k,i} - D$ then we calculate $V(S_{k,i} - D, k\Delta t)$ (or $V^*_{k,i}$)

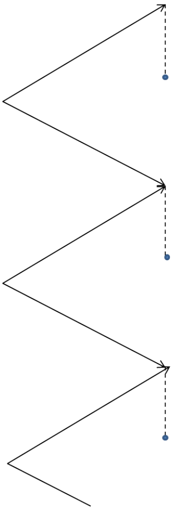
$$V^*_{k,i} = V(S_d, k\Delta t) + \frac{S_{k,i} - D - S_d}{S_u - S_d} \times (V(S_u, k\Delta t) - V(S_d, k\Delta t))$$

  then replace all of the $V_{k,i}$ values with $V^*_{k,i}$.

- Then calculate back through the tree applying this rule whenever there are dividend payments.

- If the option is American then if it is a call you will wish to exercise before the dividend payment and so $V_{k,i} = \max(S_{k,i} - K, V^*_{k,i})$ whereas for a put it will be after and so $V_{k,i} = \max(K - S_{k,i} - D, V^*_{k,i})$.

Fixed dividends



$S_{k,10}$ $V_{k,10} = e^{-r\Delta t}(qV_{k+1,11} + (1-q)V_{k+1,10})$

$S_{k,10} - D$ $V^*_{k,10} = V_{k,9} + (S_{k,10} - D - S_{k,9}) \times (V_{k,10} - K_{k,9})/(S_{k,10} - S_{k,9})$

$S_{k,9}$ $V_{k,9} = e^{-r\Delta t}(qV_{k+1,10} + (1-q)V_{k+1,9})$

$S_{k,9} - D$ $V^*_{k,9} = V_{k,8} + (S_{k,9} - D - S_{k,8}) \times (V_{k,9} - K_{k,8})/(S_{k,9} - S_{k,8})$

$S_{k,8}$ $V_{k,8} = e^{-r\Delta t}(qV_{k+1,9} + (1-q)V_{k+1,8})$

$S_{k,8} - D$ $V^*_{k,8} = V_{k,7} + (S_{k,8} - D - S_{k,7}) \times (V_{k,8} - K_{k,7})/(S_{k,8} - S_{k,7})$

One more thing...

- Of course for the lowest value of i, $i = 0$ there will be no S_d value.
- In this case, we set $S_d = 0$ as $V(0,t)$ is always known, as $S_t = 0$ for all t until maturity.
    - e.g for a Call. $V(0,t) = 0$
    - For a put, $V(0,t) = Ke^{-r(T-t)}$ for European and $V(0,t) = K$ for an American.
    - For other products it is usually easy to calculate.

Parameters for our example

- For our example:
  - $T = 1$ year to expiration
  - Use $N = 50$ periods, $\Delta t = T/N = 1/50 = 0.02$
  - $r = 0.04(4\%)$
  - $\sigma = 0.30(30\%)$
- Thus

$$q = \frac{e^{r\Delta t} - d}{u - d} = 0.489395, \quad 1 - q = \frac{u - e^{r\Delta t}}{u - d} = 0.510605$$

Create stock price tree

- Worksheet Stock of workbook `ImpBinomial.xlsx` contains the stock price tree.
- We will count time steps using j, so that $t = j \times \Delta t$, i counts up jumps so that $S_{j,i} = S_0 u^i d^{j-i}$.
- Beginning is:

Formula in cell G48 is =F49*u

Cell F49

41.76697 $= u \times 40$

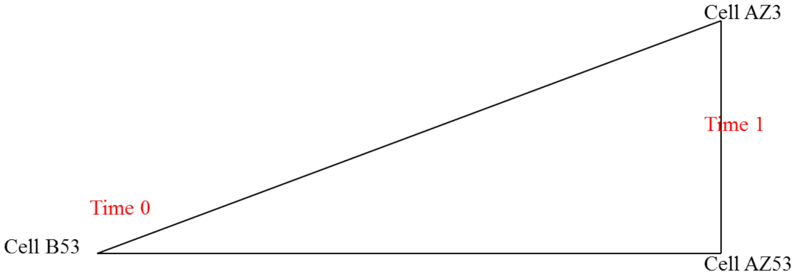
Tree starts in cell B53

5						49.65065
4					47.55016	45.61145
3				45.53853	43.68183	41.90084
2			43.612	41.83385	40.12821	38.4921
1		41.7669711	40.06405	38.43056	36.86367	35.36067
0	40	38.369123	36.80474	35.30414	33.86472	32.48399
Time	0	0.02	0.04	0.06	0.08	0.1
Period (t)	0	1	2	3	4	5

38.36912 $= d \times 40$

Create stock price tree

- 50-period tree uses 51 columns, 51 rows: the beginning is in cell B53 it ends in column AZ, rows 3 to 53. Note that the tree is a triangle in the worksheet:



Create stock price tree: proportional dividends

- Tricky part is across dividend dates (note that in spreadsheet, $D = div$). Look at time 0.125:

Formula in cell I46 is =H47*u*(1-div)

Cell H47

41.76697 $= u \times 40$

Tree starts in cell B53

7								53.59276
6							51.84393	49.23285
5						49.65065	47.6263	45.22764
4					47.55016	45.61145	43.75178	41.54826
3				45.53853	43.68183	41.90084	40.19246	38.1682
2			43.612	41.83385	40.12821	38.4921	36.9227	35.06312
1		41.7669711	40.06405	38.43056	36.86367	35.36067	33.91895	32.21065
0	40	38.369123	36.80474	35.30414	33.86472	32.48399	31.15956	29.59023
Time	0	0.02	0.04	0.06	0.08	0.1	0.12	0.14
Period (t)	0	1	2	3	4	5	6	7

38.36912 $= d \times 40$

Start at end (time 1) of option price tree

- Worksheet EurOption contains tree for European call option (again, a triangle).
- These are parts of columns AZ (time 1), rows 20 through 30 of stock and option tree.

Stock!AZ22 →

Part of stock tree at time 1

66.53945
61.1263
56.15352
51.58529
47.3887
43.53351
39.99195
36.7385
33.74973
31.00411
28.48184

Part of option tree at time 1

26.53945
21.1263
16.15352
11.58529
7.388697
3.533507
0
0
0
0
0

← =MAX(Stock!AZ22-K,0)

Formula in option tree refers to stock tree

Overall

- At end of the tree (time 1 or $j = N$), use rule $C_{N,i} = \max(S_{N,i} - K, 0)$, then use

$$C_{j,i} = e^{-r\Delta t}[qC_{j+1,i+1} + (1-q)C_{j+1,i}]$$

in the remainder of the tree.

	AW	AX	AY	AZ = time 1	
22	35.48585	32.38012	29.39969	26.53945	=MAX(Stock!AZ22-K,0)
	29.35269	26.49702	23.75645	21.1263	
	23.71847	21.09252	18.57231	16.15352	
	18.54262	16.12769	13.80991	11.58529	
	13.78783	11.56677	9.434938	7.388697	
27	9.419854	7.376884	5.415883	3.533507	
=EXP(-rate*Dt)*(p*AX27+(1-p)*AX28)	5.408294	3.529954	1.727898	0	
	2.15725	0.844948	0	0	
	0.413183	0	0	0	
	0	0	0	0	
32	0	0	0	0	

American options

- Worksheet AmerOption contains tree for American call option (again, a triangle).
- Start at end using formula for payoff $C_{N,i} = \max(S_{N,i} - K, 0)$
- Then move through option tree using rule:

$$C_{j,i} = \max[e^{-r\Delta t}[qC_{j+1,i+1} + (1-q)C_{j+1,i}], S_{j,i} - K]$$

where the first expression is the value if ‘left alive’ or not exercised and the second expression is the exercise value. Recall that

$$S_{j,i} = S_0 u^i d^{j-i}$$

American options

- At each point, investor will choose to exercise or not in order to maximize value of option.

	AW	AX	AY	AZ = time 1
22	35.48585	32.38012	29.39969	26.53945
	29.35269	26.49702	23.75645	21.1263
	23.71847	21.09252	18.57231	16.15352
	18.54262	16.12769	13.80991	11.58529
	13.78783	11.56677	9.434938	7.388697
27	9.419854	7.376884	5.415883	3.533507
	5.408294	3.529954	1.727898	0
	2.15725	0.844948	0	0
	0.413183	0	0	0
	0	0	0	0
32	0	0	0	0

← =MAX(Stock!AZ20-K,0)

=max(EXP(-rate*Dt)*(p*AX27+(1-p)*AX28), Stock!AW26-K) →

Quiz

When will it be optimal to exercise the American call option

- Just before dividend payment
- Just after dividend payment
- Never

Option prices at time 0

- The prices of the European and American options turn out to be: European: 4.555653, American: 4.64552.
- The difference is the premium due to the early exercise feature of the American option.

Barrier options

- Another common type of option feature is a barrier or 'autocall' feature.
- A barrier feature is a **stock price** dependent event, where there is a condition that is applied if the stock price hits a particular level during or at a certain time period.
- The simplest type of barrier options to value are those which are canceled if the stock price goes above or below a particular level at any point in time. These are usually called continuous down-and-out and up-and-out options.
- For example, consider a down-and-out European call option on the same stock as above. Now we have a new condition that if the stock price ever falls below $30 then the option expires worthless.
- This type of option has an analytic formula and can also be valued easily using a binomial tree approach.

Barrier options: valuation

- The value of a European down-and-out call is given below, where δ is the continuous dividend yield and B is the barrier level:

$$
\begin{aligned}
V(S,0) &= S_0 e^{-\delta T} N(d_1) - K e^{-rT} N(d_2) - \left(\frac{B}{S_0}\right)^{1+2(r-\delta)/\sigma^2} S_0 e^{-dT} N(h_1) \\
&\quad + \left(\frac{B}{S_0}\right)^{-1+2(r-\delta)/\sigma^2} K e^{-rT} N(h_2)
\end{aligned}
$$

where d_1 and d_2 are the same as for European options and.

$$
h_1 = \frac{\ln\left(\frac{B^2}{KS_0}\right) + (r - \delta + \sigma^2/2)T}{\sigma\sqrt{T}}
$$

and

$$
h_2 = h_1 - \sigma\sqrt{T}
$$

Barrier options: valuation

- In the binomial tree then before maturity, the formula for a European down-and-out call option is:

$$C_{j,i} = \begin{cases} e^{-r\Delta t}[qC_{j+1,i+1} + (1-q)C_{j+1,i}] & \text{if } S_{j,i} \geq B \\ 0 & \text{if } S_{j,i} < B \end{cases}$$

- At maturity then,

$$C_{N,i} = \max(S_{N,i} - K, 0)$$

  as usual.

- In the worksheet, EuroDAO, we value a European down-and-out call option with $B = \$30$, we obtain a value of $C_{0,0} = 4.46084$.

Barrier options: Adaptations

- In practice it is rare to have barrier options that are continuous. More usually, the barrier is discretely observed in that the barrier level is only checked every month, quarter etc. This leads to a **discrete barrier option**. These do have solution methods other than trees but no analytic formulas (see Kou, 2008 on COMPASS).
- Discrete barrier options can be valued in the tree method very easily by only applying the barrier condition on the observation dates.
- Secondly, many barrier options are not of 'knock-out/knock-in' form but are in the form of 'autocall' features. An autocall feature says that if the stock price ever falls below/goes above a certain level (the barrier feature) then the option is canceled and the holder receives the call price - see this Term Sheet from 2014 class.
- This can easily be applied within the binomial tree, on **observation dates**:

$$C_{j,i} = \begin{cases} e^{-r\Delta t}[qC_{j+1,i+1} + (1-q)C_{j+1,i}] & \text{if } S_{j,i} \geq B \\ \text{Call price(j)} & \text{if } S_{j,i} < B \end{cases}$$

Barrier options: Adaptations

- It is also possible to adapt the analytic formula in the continuous case, but most autocall features are discrete.
- As you may imagine, the accuracy of binomial methods for barrier options is of more concern and we will discuss this in the next class.

Simple callable, convertible bond

- We can handle wide range of securities. Illustrate this using simple callable convertible bond:
  - Maturity = 1 year
  - Face = 100
  - Conversion ratio = 2 (each bond converts to 2 common shares)
  - Call feature: On or after time 0.5, the issuer may call the bond for its face value of 100 plus any accrued interest. But, after the bond is called, the owner may convert.
  - Coupon: The bond pays a quarterly coupon of 2.5%, or, \$0.625 per quarter, assume they are paid at $t = 0.25, 0.5, 0.75, 1$.
  - Other parameters same as for options examples ($S = 40, r = 0.04, \sigma = 0.30$, etc.)

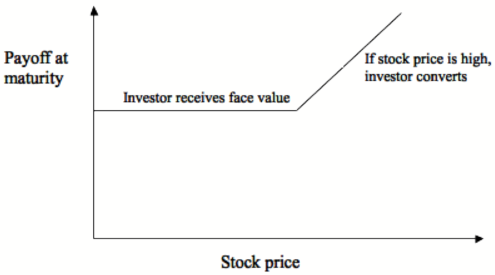
Quiz

What does a call feature do to a bond price?

- Increase it.
- Decrease it.
- Unclear.

Textbook convertible bond

The usual corporate finance picture of the payoffs of convertible bond is this:



This picture is not realistic, because actual convertible bonds are typically callable, and if the stock price goes up do not survive until the maturity date

Time 1 (maturity) is handled like expiration date of call options

- Worksheet ConvBond contains tree for convertible bond (again, a triangle).
- Start at end using formula $\max(2S_{T,i}, 100) + \$0.625$ as we must remember to add on the final coupon payment.
- These are parts of columns AZ (time 1), rows 22 through 32 of stock and conv. bond trees.

	Part of stock tree at time 1	Part of bond tree at time 1	
Stock!AZ22 →	66.55285	133.7039	← =MAX(Ratio*Stock!AZ22,Face)+$0.625
	61.13861	122.8776	
	56.16483	112.932	
	51.59568	103.7956	
	47.39824	100.625	
	43.54227	100.625	
	40	100.625	
	36.7459	100.625	
	33.75653	100.625	
	31.01035	100.625	
	28.48758	100.625	

Formula in conv. bond tree refers to stock tree
Ratio = 2,
Face = 100
Coupon = 2.5%

Note: If bond survives uncalled until maturity, owner gets to either convert and receive $2S$ or take face of 100 plus the coupon. But, bond will probably be called prior to maturity

How to include interest payments?

- Our approach for an option or bond is to compute the unexercised/unconverted value (typically called the continuation value) using the rule

$$V_{j,i} = e^{-r\Delta t}(qV_{j+1,i+1} + (1-q)V_{j+1,i})$$

- In words, value this period = discounted expected value of what we will have next period.
- For most periods, 'what we will have next period' is just the convertible bond, and we use formula:

$$V_{j,i} = e^{-r\Delta t}(qV_{j+1,i+1} + (1-q)V_{j+1,i})$$

- But for periods in which the convertible bond pays interest or a coupon, 'what we will have next period' is the exchangeable note, plus the interest payment.

How to include interest payments?

- For these periods, we use the formula:

$$V_{j,i} = e^{-r\Delta t}(qV_{j+1,i+1} + (1-q)V_{j+1,i}) + \text{PV of interest payment}$$

- For example, for the third coupon at $t = 0.75$, at $t = 0.74$ we have that

$$V_{j,i} = e^{-r\Delta t}(qV_{j+1,i+1} + (1-q)V_{j+1,i}) + e^{-r\times 0.01} \times \$0.625$$

Accrued interest

- To determine the call price we must also calculate the accrued interest on a given date, this is to compensate the holder of the bond for any coupon that they are owed.
- At a given time step $0 \leq j \leq N$ the accrued interest, $AI(j)$ is calculated by:

$$AI(j) = \text{Coupon size} \times \frac{j \times \Delta t - \text{time of last coupon}}{\text{time between coupons}}$$

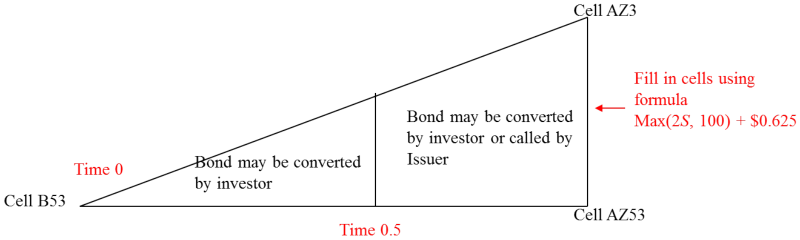
- For example at $j = 30$ the accrued interest is

$$AI(30) = \$0.625 \times \frac{0.6 - 0.5}{0.75 - 0.5} = \$0.25$$

Create convertible bond price tree

There are two regions to consider:

- After time 0.5 when bond may be either called or converted.
- Before time 0.5 when it may be converted by investor, but not called.
- We start with first region (before time 0.5) because it is easier.



Early part (time 0 to 0.5) of convertible bond tree

- Move through convertible bond tree using rule:

$$V_{j,i} = \max[e^{-r\Delta t}(qV_{j+1,i+1} + (1-q)V_{j+1,i}), 2S_{j,i}]$$

- At each point, investor will choose to convert or not in order to maximize value of convertible bond.We assume that upon conversion the holder does not receive any accrued interest (this is a common feature).
- Note that at $t = 0.24$ then we have to add in the coupon payment.

Remainder (time 0.5 to 0.98) of convertible bond tree

- Move through convertible bond tree using rule:

$$V_{j,i} = \min\left\{\max[e^{-r\Delta t}(qV_{j+1,i+1} + (1-q)V_{j+1,i}), 2S_{j,i}], \max[2S_{j,i}, 100 + AI(j)]\right\}$$

  where 100 is the face value of the bond - where it can be called. $2S$ is the conversion value, $AI(j)$ is the accrued interest, and $e^{-r\Delta t}(qV_{t+1,i+1} + (1-q)V_{t+1,i})$ is the value if not converted or called.

- At $t = 0.5$ we have to be a little careful what to do with the coupon, I would add it into the continuation value formula and leave the accrued interest as \$0.625 for calculating the call value, so

$$V_{j,i} = \min\left\{\max[e^{-r\Delta t}(qV_{j+1,i+1} + (1-q)V_{j+1,i}) + \$0.625, 2S_{j,i}], \max[2S_{j,i}, 100.625]\right\}$$

- At $t = 0.74$ then we also have to add in the coupon payment as above.

Advice: try to understand the rule

- Issuer gets to decide whether to call, and uses this option to **minimize** the value of the liability:

$$V_{j,i} = \min \{\text{Value if issuer does not call}, \text{value if issuer calls}\}$$

- If issuer does not call, investor has right to convert, and

$$\text{Value if issuer does not call} = \max[e^{-r\Delta t}(qV_{j+1,i+1} + (1-q)V_{j+1,i}), 2S_{j,i}]$$

- If issuer does call, investor has right to convert and receive $2S$, or just take the face value of 100 and the accrued interest.

$$\text{Value if issuer does call} = \max[2S_{j,i}, 100 + AI(j)]$$

- Putting it together:

$$V_{j,i} = \min \left\{ \max[e^{-r\Delta t}(qV_{j+1,i+1} + (1-q)V_{j+1,i}), 2S_{j,i}], \max[2S_{j,i}, 100 + AI(j)] \right\}$$

Remainder (time 0.5 to 0.98) of convertible bond tree

- Then move through bond tree using rule:

$$V_{t,i} = \min\left\{\max[e^{-r\Delta t}(qV_{t+1,i+1} + (1-q)V_{t+1,i}), 2S_{t,i}], \max[2S_{t,i}, 100 + AI(j)]\right\}$$

- The bond tree is given to you in tab 'ConvBond'.

Conclusions

- We have looked at lots of additions to the basic binomial model.We can now factor in all different types of dividends into our tree. The only problematic model is the fixed (cash amount) dividend payments.
- We can value American options as well as options where there are conversion, and/or call features factoring interest payments and accrued interest calculations if necessary.