FE 514 Project 1

Product #4 Callable Buffered Range Accrual Securities

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1. Data collection, Parameter estimation and Choice of N
   A. Data collection(All data is from **Bloomberg.)**

a. Russell 2000 Index Volatility

Expiry Date	Strike Price	Implied Volatility	Source
12.30.2022	100%	20.18%	Russell 2000 Index European Call option
12.29.2023	100%	20.18%	Russell 2000 Index European Call option

RTY Index	90) Asset	91) Actions	92) Views	93) Settings	Volatility Surface
RUSSELL 2000 INDEX 1548.90 USD	CUSTOM(INFOLAB9	Sett	As of	23-Feb-2018	17:27
1) Vol Table	2) 3D Surface	3) Term	4) Skew		
Moneyness	Listed	16) Edit	Fwd	Dates	Strikes

Expiry	Exp Date	ImpFwd	90.0%	95.0%	97.5%	100.0%	102.5%	105.0%	110.0%	120.0%	130.0%	150.0%	175.0%	20
			1394.0	1471.5	1510.2	1548.9	1587.6	1626.3	1703.8	1858.7	2013.6	2323.4	2710.6	30
Sep-18	21 Sep 2018	1555.75	21.24	19.19	18.21	17.29	16.40	15.58	14.20	12.85	12.85	12.85	12.85	12
Sep-18	28 Sep 2018	1555.41	21.28	19.26	18.31	17.39	16.52	15.71	14.34	14.34	14.34	14.34	14.34	14
Dec-18	21 Dec 2018	1559.43	21.11	19.43	18.64	17.88	17.15	16.46	15.23	13.50	13.50	13.50	13.50	13
Dec-18	31 Dec 2018	1558.91	21.02	19.39	18.61	17.85	17.13	16.44	15.22	13.64	13.64	13.64	13.64	13
Jun-19	21 Jun 2019	1567.62	21.14	19.82	19.19	18.58	17.99	17.43	16.41	14.81	14.81	14.81	14.81	14
Dec-19	20 Dec 2019	1576.47	21.30	20.25	19.73	19.23	18.74	18.26	17.36	15.79	14.64	14.64	14.64	14
Dec-20	18 Dec 2020	1598.15	21.81	20.98	20.58	20.18	19.80	19.43	18.71	17.41	16.30	16.30	16.30	16
Dec-20	31 Dec 2020	1598.93	21.81	20.98	20.58	20.18	19.80	19.43	18.71	17.41	16.30	16.30	16.30	16
Dec-21	31 Dec 2021	1620.81	21.81	20.98	20.58	20.18	19.80	19.43	18.71	17.41	16.30	16.30	16.30	16
Dec-22	30 Dec 2022	1643.51	21.81	20.98	20.58	20.18	19.80	19.43	18.71	17.41	16.30	16.30	16.30	16
Dec-23	29 Dec 2023	1667.41	21.81	20.98	20.58	20.18	19.80	19.43	18.71	17.41	16.30	16.30	16.30	16
Dec-24	31 Dec 2024	1692.70	21.81	20.98	20.58	20.18	19.80	19.43	18.71	17.41	16.30	16.30	16.30	16
Dec-25	31 Dec 2025	1718.78	21.81	20.98	20.58	20.18	19.80	19.43	18.71	17.41	16.30	16.30	16.30	16
Dec-26	31 Dec 2026	1746.10	21.81	20.98	20.58	20.18	19.80	19.43	18.71	17.41	16.30	16.30	16.30	16
Dec-27	31 Dec 2027	1774.47	21.81	20.98	20.58	20.18	19.80	19.43	18.71	17.41	16.30	16.30	16.30	16

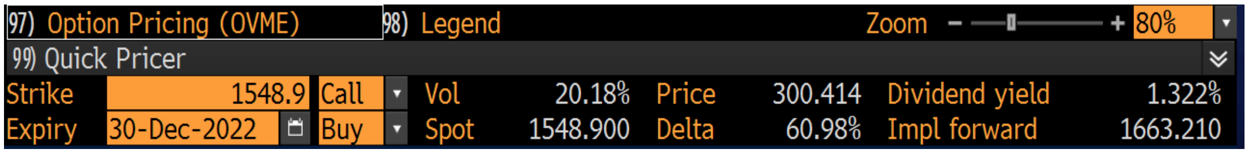
b. U.S Interest Rates

Maturity Date	Market Rate	Zero Rate	Discount	Source
02.27.2023	2.723596573	2.73164	0.873141464	SWAP

Period	Continuous Compounding Rate
02.23.2018 - 02.27.2023	2.713%

c. Dividend Yield

Russell 2000 Index Continuous Dividend Yield	1.322%



d. Business Days (from the pricing date, 02.23.2018)

Coupon Observation Dates	Redemption Dates	To Coupon Observation End-Dates (days)	To Redemption Dates (days)	Normalized time to Coupon Observation End-Dates	Normalized time to Redemption Dates
03.23.2018	03.28.2018	17	20	0.068	0.08
04.23.2018	04.26.2018	37	40	0.148	0.16
05.23.2018	05.29.2018	59	62	0.237	0.248
06.25.2018	06.28.2018	81	84	0.325	0.336
⋮	⋮	⋮	⋮	⋮	⋮
01.23.2023	01.26.2023	1225	1228	4.912	4.912
02.23.2023	02.28.2023	1247	1250	5.000	5.000

B. Parameter estimation

a. T and dt

We count 1 year as 250 days, so 5 year becomes 1250 days. Also, we reflect actual U.S business days since the underlying stock price moves only in business days. In order to plug-in binomial model, we set the maturity T equal 5.0 and every step(dt) in the model equal 0.004 ($\frac{1}{250}$).

b. Interest rates

We use 5-year discount factor(02.23.2018 - 23.27.2023) calculated from U.S swap rates by Bloomberg

c. Volatility

We use the implied volatility which Bloomberg derived from market price of 5 years Russell 2000 index ATM call option and the implied volatility is almost the same as 20.18% in the long term.

d. Dividend Yield

We use continuous dividend yield which Bloomberg gives.

e. Stock price

Exact Russell 2000 index level in the issue date does not matter in this security because barrier level is proportional(determined by 80% of the issue date)

f. Business days

We use a Bloomberg function in excel to calculate business days between issue date and Coupon Observation Period End-Dates("A") and between issue date and Contingent Coupon Payment Dates / Redemption Dates("B"). To be specific, Bloomberg function in Excel is =BCountPeriods(ISSUEDATE, A or B, "CDR", "US").

C. Choice of N

When it comes to choosing N, the most important thing is to match the day deciding early redemption with a node on the tree. Therefore, we decided N as 1250 which is the total business day of this product.

We learned from previous lecture that CRR method has a problem with non-linear error which does not decrease linearly as step goes up. We could raise the number of step to adjust the problem. However, as the result of the adjustment, there could be more error due to mismatch between early redemption decision dates and nodes in the binomial model. For instance, the first early redemption date is 1.08 at nodes. If we set N as not multiples of 1250, but other number, it is unlikely to match 1.08 exactly in the stock price tree. Hopefully, we think that dt(0.004) is small enough to converge when N is 1250.

2. Valuation Method

We used Cox-Ross-Rubinstein(CRR) method to value the security. Although there are some errors(non-linearity error, odd-even error) in this method, it is quite simple and easily applicable to some complicated securities. Time horizon of the tree is from pricing date to maturity date(1250 business days).

In terms of valuation, it is important to include path dependent features in our trees. The amount of coupon for this note depends on the number of business day where the spot price is greater or equal to the barrier level, so adding a dimension with respect to price at each state at coupon observation date might be a possible way. However, if we use this method, it needs to consider all possible path for each price, so computation cost will be too large: $O(2^n)$. Hence, alternative method is needed.

We discovered that it is equivalent to pay coupon continuously instead of pay coupon only at payment date in terms of valuation so that it is possible to value the security as continuous barrier option as follows.

$$V_{N,j} = \begin{cases} Face\ value\ + daily\ coupon, if\ S_{i,j} \geq Barrier \\ Face\ value \qquad\qquad\qquad\qquad\qquad otherwise \end{cases}$$

$$V_{i,j} = \begin{cases} qV_{i+1,j+1} + (1 - q)V_{i+1,j} + e^{-r\times(t^*-t_i)} \times daily\ coupon, & if\ S_{i,j} \geq Barrier \\ qV_{i+1,j+1} + (1 - q)V_{i+1,j} & otherwise \end{cases}$$

$$t^*: next\ coupon\ payment\ date$$

The trick of this method is converting coupons paid at coupon payment date to those at paid daily. Using this method, we do not have to consider path dependent feature in terms of valuation because during backward reduction, all coupons will be eventually summed up at last step. It can also include accrued coupon from final observation date to maturity date by taking the final node of tree to be maturity date. Of course, the value of notes at intermediate step might be wrong, but we can get fair value of notes at final step of backward reduction. Furthermore, since business days slightly differ at every month, daily coupon payments will slightly differ at every month. We took the difference of coupon into account by using the following formula.

$$Daily\ coupon\ = Face\ value\ \times coupon\ rate\ \times \frac{1}{12} \times \frac{1}{Number\ of\ business\ days\ of\ the\ month}$$

The other important feature of this note is call feature. Issuer can call the notes at several predetermined dates. The call features can be included in the tree by comparing exercise value and continuation value at each callable date. Continuation value can be calculated relatively easily using the procedure mentioned above, but deriving exercise value seems difficult because of path dependency of the note. However, by adding an assumption that decision for call is made at call date, call features can relatively easily implemented as follows.

$$Value\ at\ callable\ date\ =\ min(Face\ value, Continuation\ value)$$

The formula above seems weird since exercise value does not consider index value path before the callable dates, but in terms of decision making for call, the formula holds since the continuation value also does not consider path before the callable dates, and path before the callable dates will be taken into account by continuing backward reduction. Although the value at callable date is not the fair value at that time, finally we can get fair value of the security by using the method.

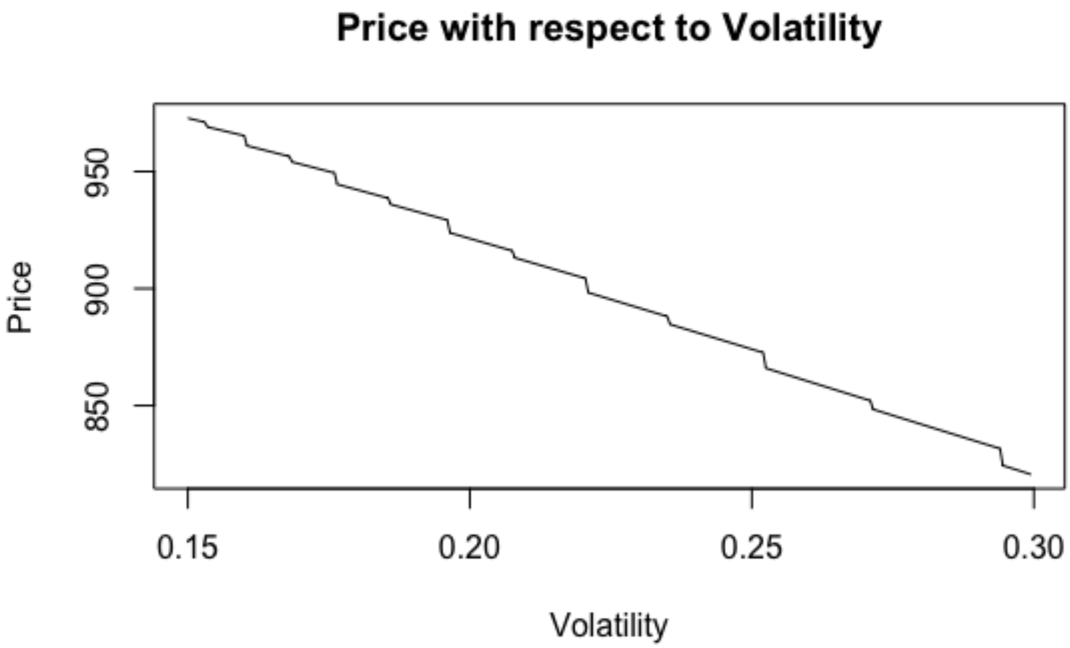
However, since the actual call decision should be made at least 3 days before the redemption date, the method we used will make some errors because we assumed decision is made at redemption date, but we expect the errors will not be large because 3 days are relatively short term.

3. Price and Discussion

A. Price

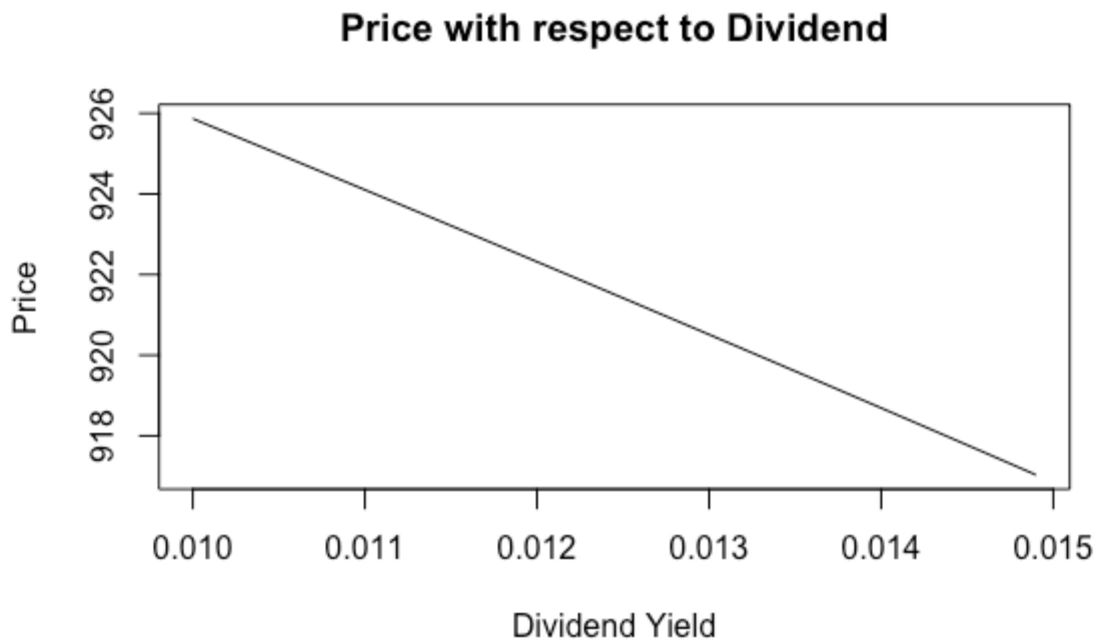
Using the method mentioned above, value of securities was calculated as 920.1126, which is lower than the price calculated by Morgan Stanley. There are some features which make deviation between our price and price calculated by Morgan Stanley.

There might be estimation error. Even though we have calculated fair value of the security, the value might not be the true value if the input parameters are not estimated correctly. Parameter which is the most difficult to estimate is volatility because volatilities from current time to maturity are not observed. Therefore, if price of security is sensitive to volatility, there might be some errors in calculated value. The following figure represents price with respect to volatility. We calculated price of securities by changing volatility from 15% to 30% by 5bps.



From the figure, price goes down from about 980 to 800 with respect to volatility which goes up from 15% to 30%. In other words, price is sensitive to volatility, hence estimating volatility correctly is important. It is because higher volatility makes higher possibility of hitting barrier. Furthermore, price curve in the figure does not seem smooth, but there are some kinked points. It is because non-linearity of the security. Since volatility affects the degree to which tree spreads up and down, and payoff drastically changes at the neighbor of barriers by non-linearity, there are some kinked point on the graph.

Even though we estimated volatility correctly, the price might not equal to the true value of the security since we assumed that volatility is constant. Empirical studies showed that true volatility is dependent on stock level and time to maturity. If we ignore this fact, there could be a gap between true volatility and the constant volatility. We could use Local Volatility model to resolve this error. If the model is reflected, volatility becomes a function of stock level and time, so it can reflect non-constant volatility to model. Of course the model will become more complicated, but as the result of the adjustment, we could have more sophisticated and small error valuation model.



Current dividend yield is 1.322% and we can assume that index dividend yield does not fluctuate relatively, compared to individual stock dividend yield. To compare, we calculate prices by changing dividend yield from 1.00% to 1.50% by 1bp. Though dividend yield affects prices in the calculation of risk-neutral probability, its impact is less sensitive than volatility. And as mentioned above, an estimation of index dividend yield is fairly predictable. Therefore, we should spend more time to estimate an adequate volatility.

B. Discussion

a. Rationality of the price

We came up an idea to justify our price. Pricing the security is difficult because redemption dates and call decision dates are different. However, if we can find other security which is relatively easy and accurately to calculate price, and also can derive relative relationship with respect to price using no arbitrage principle, we can derive upper and lower bound of the security. We can think of upper and lower bound by considering whether or not security has call feature. Call feature makes security price low. If you can call everyday, the security becomes lower bound. On the other hand, if you do not have a call, it would be upper bound by no arbitrage. Below you can find out the prices

b. Moral hazard on valuation by financial institution

Definition of moral hazard is that a person who can hide or have more information tends to behave on their incentive. Due to regulation in financial market, financial institution ought to report detailed information on their security such as fair value, redemption condition and so on.

As shown in Volatility solution, the calculated fair value significantly depend on volatility. By selecting advantageous volatility model, financial institution could show the security is more valuable than true value. However, there is no standard for volatility. Which means, they have an incentive to distort the volatility by creating favorable circumstances in valuation model.

< APPENDIX >

<table>
<tr><th colspan="3">SUMMARY TERMS</th></tr>
<tr><td>Issuer</td><td colspan="2">Morgan Stanley Finance LLC</td></tr>
<tr><td>Underlying index</td><td colspan="2">Russell 2000® Index</td></tr>
<tr><td>Principal amount</td><td colspan="2">$1,000 per security</td></tr>
<tr><td>Issue price</td><td colspan="2">$1,000 per security</td></tr>
<tr><td>Estimated value on the pricing date</td><td colspan="2">Approximately $953.70 per security, or within $30.00 of that estimate.</td></tr>
<tr><td>Pricing date</td><td colspan="2">February 23, 2018</td></tr>
<tr><td>Original issue date</td><td colspan="2">February 28, 2018</td></tr>
<tr><td>Maturity date</td><td colspan="2">February 28, 2023</td></tr>
<tr><td>Optional early Redemption</td><td colspan="2">Beginning on February 28, 2019, monthly.
at discretion, notice at least 3 business days before the redemption date</td></tr>
<tr><td>Redemption payment</td><td colspan="2">$1,000 + any accrued and unpaid contingent monthly coupon</td></tr>
<tr><td>Contingent monthly coupon</td><td colspan="2">Unless the securities are previously redeemed, the contingent monthly coupon payable on the securities will be determined as follows:

At a rate of 5.00% per annum times N/ACT

“N” = the total number of index business days in the applicable coupon payment period on which the index closing value is greater than or equal to the barrier level (each such day, an “accrual day”)
“ACT” = the total number of index business days in the applicable coupon payment period.</td></tr>
<tr><td rowspan="3">Payment at maturity</td><td colspan="2">If the securities have not previously been redeemed, investors will receive on the maturity date a payment at maturity determined as follows:</td></tr>
<tr><td>If the final index value is greater than or equal to the barrier level</td><td>the stated principal amount and any accrued and unpaid contingent monthly coupon with respect to the final coupon payment period</td></tr>
<tr><td>If the final index value is less than the barrier level:</td><td>$1,000 x (final index vale / initial index value + buffer amount) + any accrued and unpaid contingent monthly coupon with respect to the final coupon payment period</td></tr>
</table>

Buffer amount	20%
Minimum payment at maturity	$200 per security
Barrier level	**80% of the initial index value**