Fin 514: Financial Engineering II

Lecture 5: Path dependent options

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Outline

- I will now introduce some brief extensions to the binomial model.
- We will see a simple path dependency in a 'floating strike' option.
- Then we will try to value lookback options which demonstrates how to generally adapt the binomial to value options with path dependence.
- The second is to calculate implied volatility, which is very important from the practitioners point of view. Finally, we will very briefly discuss a different underlying process.

Quick note on discrete barrier/autocallable options

- You saw in problem set 3 that the most accurate option valuations are when there are nodes exactly on the **continuous** barrier and that the accuracy is very sensitive to the position of the nodes relative to the barrier.
- Recall that a barrier is only applied at certain points in time such as every week, month, quarter etc. In these cases there is no analytic formula for the value of the option and so a numerical method such as a binomial tree must be used (Kou (2008) discusses other approaches).
- The implementation of the discrete barrier is straightforward (see Problem Set 3) but the errors can also be substantial. Cheuk and Vorst (1997) (among others) suggest that for the smallest error when pricing discrete barrier options, **the barrier should be halfway between a layer of nodes** as we also found in Problem Set 3.

Floating strike options

- A reasonably common path dependent option in Venture Capital circles is a floating strike option. The idea being that upon an IPO the Venture Capitalist receives an at the money option with certain terms but that right now the strike price is unknown.
- The unknown strike price is an example of a path dependent feature because moving forward in time you can determine the payoff easily but moving backward in time you do not know the strike until further back in time.

Quiz

Which way does derivative valuation occur in a binomial tree?

- Forward
- Backward
- Neither
- Both

Floating strike options: tree set up

- Let's consider a simplified American floating strike call option. Assume options are issued at $t = 0$ on a non-dividend paying stock and that the strike is determined at $t = 1$ after which the options can be exercised at any time until final maturity at $t = T = 2$.
- For exposition, let's have a 50 step tree and so $\Delta t = 2/50 = 0.04$
- Valuing backward in time, to deal with the uncertain strike we need another dimension in the problem - the possible strike price, K.
- The possible strikes, K_l are all of the possible stock prices at $t = 1$,

$$K_l = S_0 u^l d^{25-l}$$

for $l = 0, \ldots 25$

Quiz

Will these options be exercised between $t = 1$ and $t = 2$?

- Yes, for large stock prices
- No, never
- Yes, at low stock price
- Yes, if the volatility is large enough

Floating strike options: valuation

- Now at maturity, the option value depends on the Stock price, $S_{N,i}$ and the possible strike, K_l. And so:

$$V_{N,i,l} = max\left(S_0 u^i d^{N-i} - K_l, 0\right)$$

for $i = 0, \ldots N$ and $l = 0, \ldots 25$.

- Between $t = 1$ and maturity ($i = 26, \ldots 49$) we need to consider all of the K values as well as checking for early exercise. So, for a given j, $i = 0, \ldots j$ and $l = 0, \ldots 25$:

$$V_{j,i,l} = max(S_0 u^i d^{j-i} - K_l, e^{-r\Delta t}(qV_{j+1,i+1,l} + (1-q)V_{j+1,i,l})$$

- Then, finally back at $t = 1$ or $j = 25$ we know that $K_l = S_{25,i}$ and so we need to only calculate for $i = 0, \ldots 25$ as $l = i$ and

$$V_{25,i} = max(S_0 u^i d^{25-i} - K_i, e^{-r\Delta t}(qV_{j+1,i+1,i} + (1-q)V_{j+1,i,i})$$

Floating strike options: valuation

- Finally, we need to value the option all the way back to $t = 0$ but here all uncertainty has been resolved and so, for $j = 0, \ldots, 24$.

$$V_{j,i} = e^{-r\Delta t}\left(qV_{j+1,i+1} + (1-q)V_{j+1,i}\right)$$

Lookback options: definition

- A lookback option is an option where the payoff is a function of the maximum value reached by the underlying asset over the lifetime of the option.
- If we denote this maximum value reached up until time t as $\hat{S}_t$, then the payoffs could be of the form $\max(\hat{S}_T - S_T, 0), \max(\hat{S}_T - K, 0)$ or $\max(K - \hat{S}_T, 0)$ where S_T is the asset price at the expiry of the option and K is an exercise price.
- We will see that the first type of payoff can be reduced to a two-dimensional problem as usual whereas the others will require constructing a three-dimensional tree (in $S, \hat{S}, t$).
- Eventually, we will see it is more natural to price such an option using Monte Carlo methods but these will create additional problems with accuracy and if we wish to add early exercise.
- Below will look at how to value an American lookback option using two different approaches.

Constructing the tree

- Pricing the lookback option involves constructing a tree for the underlying asset movement and then valuing the option at each time step (working backward) for all of the possible maximum values. These maximum values essentially add a third dimension to the problem.
- Let's assume that we are using the CRR method, where $u = e^{\sigma\sqrt{\Delta t}}$ and $d = 1/u$, so at time step j the possible stock prices are $S_0 u^{2i-j}$ for $0 \leq i \leq j$. This will simplify what comes next.
- The set of possible maximum values is obtained from looking at the path followed by $S_{j,i}$ in the binomial tree. In particular, the possible stock prices we can reach are given by $\hat{S}_k = S_0 u^k$ where $-N \leq k \leq N$, so this gives us a set of possible maxima.
- This set can be reduced somewhat as we already know the value of $S_{0,0}$ and so the maximum value reached must be greater than or equal to this value, so now possible maxima are $\hat{S}_k = S_0 u^k$ where $0 \leq k \leq N$.

Constructing the tree

- At expiry, for each up jump i, you need to value the option for all possible, achievable maximum values k, $0 \leq k \leq N$, we will call this $V_{j,i,k}$.
- To illustrate this consider a simple tree, with $N = 3, S_{0,0} = 50, \sigma = 0.4, r = 0.1, \delta = 0$, $T = 0.25, u = 1.1224, d = 0.8909, q = 0.5073$.
- The next diagram shows that value of the option at maturity if the final payoff is $\max(\hat{S}_T - S_T, 0)$.

The tree at maturity, $T = 0.25, j = 3$

	k = 0	k = 1	k = 2	k = 3
i = 3	$S_{3,3}$ = 70 $\hat{S}_0$ = 50 $V_{3,3,0}$ = N/A	$S_{3,3}$ = 70 $\hat{S}_1$ = 56.12 $V_{3,3,1}$ = N/A	$S_{3,3}$ = 70 $\hat{S}_2$ = 62.99 $V_{3,3,2}$ = N/A	$S_{3,3}$ = 70 $\hat{S}_3$ = 70 $V_{3,3,3}$ = 0
i = 2	$S_{3,2}$ = 56.12 $\hat{S}_0$ = 50 $V_{3,2,0}$ = N/A	$S_{3,2}$ = 56.12 $\hat{S}_1$ =56.12 $V_{3,2,1}$ = 0	$S_{3,2}$ = 56.12 $\hat{S}_2$ = 62.99 $V_{3,2,2}$ = 6.87	$S_{3,2}$ = 56.12 $\hat{S}_3$ = 70 $V_{3,2,3}$ = N/A
i = 1	$S_{3,1}$ = 44.55 $\hat{S}_0$ = 50 $V_{3,1,0}$ = 5.45	$S_{3,1}$ = 44.55 $\hat{S}_1$ = 56.12 $V_{3,1,1}$ = 11.57	$S_{3,1}$ = 44.55 $\hat{S}_2$ = 62.99 $V_{3,1,2}$ = N/A	$S_{3,1}$ = 44.55 $\hat{S}_3$ = 70 $V_{3,1,3}$ = N/A
i = 0	$S_{3,0}$ = 35.36 $\hat{S}_0$ = 50 $V_{3,0,0}$ = 14.64	$S_{3,0}$ = 35.36 $\hat{S}_1$ = 56.12 $V_{3,0,1}$ = N/A	$S_{3,0}$ = 35.36 $\hat{S}_2$ = 62.99 $V_{3,0,2}$ = N/A	$S_{3,0}$ = 35.36 $\hat{S}_3$ = 70 $V_{3,0,3}$ = N/A

Working back through the tree

- Then as you move back in the tree, at times j in upstates i, we need to compare continuation values these to early exercise values $\hat{S}_k - S_{j,i}$, $\hat{S}_k - K$ etc.
- The continuation value requires a little more thought, again at a given time step j we will need option prices at all up steps i for all possible maxima that can have occurred prior to $S_{j,i}$.
- At the very least, the maximum must be at least as large as either the initial stock price or the current stock price, $S_{j,i} = Su^i d^{j-1} = Su^{2i-j}$ and so $max(0, 2i - j) \leq k \leq N$. W

Working back through the tree

- We can simplify this further as the stock can only have come down from a stock price $j-i$ up jumps higher, or it can be the current maximum. So this gives,

$$
\begin{aligned}
\max(S_0, S_{j,i}) \leq & \quad \hat{S}_k & \leq \max(S_{j,i}u^{j-i}, S_{j,i}) \\
\max(S_0u^0, S_0u^i d^{j-i}) \leq & \quad S_0u^k & \leq \max(S_0u^i d^{j-i}u^{j-i}, S_0u^i d^{j-i}) \\
\max(S_0u^0, S_0u^{2i-j}) \leq & \quad S_0u^k & \leq \max(S_0u^i, S_0u^{2i-j})
\end{aligned}
$$

and so our range of k will be

$$\boxed{\max(0, 2i-j) \leq k \leq \max(i, 2i-j)}$$

Quiz

If $N = 10$ and $j = 6, i = 4$ what is the largest value of k for which we have to consider a maximum?

- 0
- 4
- 6
- 8
- 10

Quiz

If we were to write $S_0 u^k = S_0 u^i d^{j-i}$ what i does you answer above correspond to?

- 0
- 1
- 2
- 3
- 4
- 5

Choices of k when $N = 3$

In our simple 3 step tree we can see how the range of k is determined:

j	i	Min k	Max k
0	0	0	0
1	0	0	0
1	1	1	1
2	0	0	0
2	1	0	1
2	2	2	2
3	0	0	0
3	1	0	1
3	2	1	2
3	3	3	3

Working back through the tree

- So now, we can develop our pricing methodology for an American option.
- First calculate the continuation value. At time j for $0 \leq i \leq j$ for $\max(0, 2i - j) \leq k \leq \max(i, 2i - j)$,

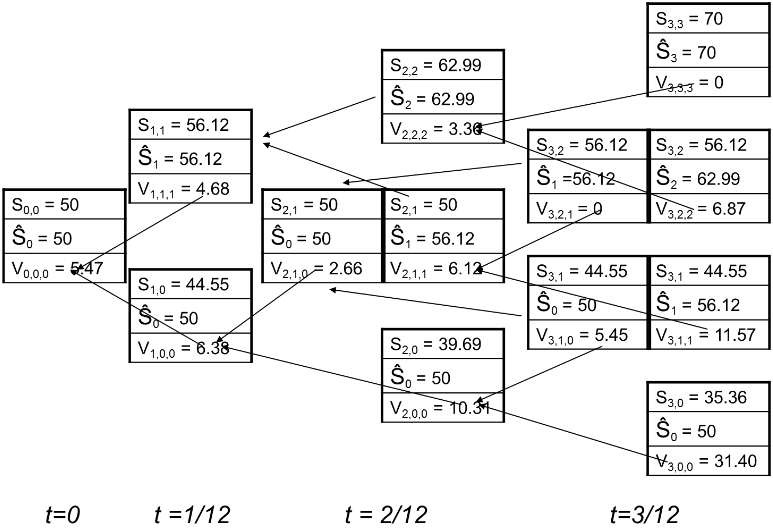
$$CV_{j,i,k} = e^{-r\Delta t}(qV_{j+1,i+1,k*} + (1-q)V_{j+1,i,k})$$

  where $k* = max(k, 2(i+1) - (j+1))$ allows for the fact that if the stock price increases then we will have a new possible maximum, $S_{j+1,i+1}$.
- This then gives the equation:

$$V_{j,i,k} = \max\left(e^{-r\Delta t}(qV_{j+1,i+1,k*} + (1-q)V_{j+1,i,k}), S_0 u^k - S_{j,i}\right)$$

  that we use all the way back to value the option right now, $V_{0,0,0}$.
- The next slide shows all of the calculations in our $N = 3$ example

The tree


$S_{3,3} = 70$
$\hat{S}_3 = 70$
$V_{3,3,3} = 0$
$S_{2,2} = 62.99$
$\hat{S}_2 = 62.99$
$V_{2,2,2} = 3.36$
$S_{1,1} = 56.12$
$\hat{S}_1 = 56.12$
$V_{1,1,1} = 4.68$
$S_{3,2} = 56.12$
$\hat{S}_1 = 56.12$
$V_{3,2,1} = 0$
$S_{3,2} = 56.12$
$\hat{S}_2 = 62.99$
$V_{3,2,2} = 6.87$
$S_{0,0} = 50$
$\hat{S}_0 = 50$
$V_{0,0,0} = 5.47$
$S_{2,1} = 50$
$\hat{S}_0 = 50$
$V_{2,1,0} = 2.66$
$S_{2,1} = 50$
$\hat{S}_1 = 56.12$
$V_{2,1,1} = 6.12$
$S_{1,0} = 44.55$
$\hat{S}_0 = 50$
$V_{1,0,0} = 6.38$
$S_{3,1} = 44.55$
$\hat{S}_0 = 50$
$V_{3,1,0} = 5.45$
$S_{3,1} = 44.55$
$\hat{S}_1 = 56.12$
$V_{3,1,1} = 11.57$
$S_{2,0} = 39.69$
$\hat{S}_0 = 50$
$V_{2,0,0} = 10.31$
$S_{3,0} = 35.36$
$\hat{S}_0 = 50$
$V_{3,0,0} = 31.40$
$t=0$
$t=1/12$
$t=2/12$
$t=3/12$

Working back through the tree

- For example

$$
\begin{aligned}
V_{2,2,2} &= \max\left(e^{-r\Delta t}(qV_{3,3,3} + (1-q)V_{3,2,2}), S_0u^2 - S_{2,2}\right) \\
&= \max\left(e^{-0.1/12}(0.5073 \times 0 + (1-0.5073) \times 6.87), 62.99 - 62.99\right) \\
V_{2,1,0} &= \max\left(e^{-r\Delta t}(qV_{3,2,1} + (1-q)V_{3,1,0}), S_0u^0 - S_{2,1}\right) \\
&= \max\left(e^{-0.1/12}(0.5073 \times 0 + (1-0.5073) \times 5.45), 50.00 - 50.00\right) \\
V_{2,1,1} &= \max\left(e^{-r\Delta t}(qV_{3,2,1} + (1-q)V_{3,1,1}), S_0u^1 - S_{2,1}\right) \\
&= \max\left(e^{-0.1/12}(0.5073 \times 0 + (1-0.5073) \times 11.57), 56.12 - 50.00\right) \\
V_{2,0,0} &= \max\left(e^{-r\Delta t}(qV_{3,1,0} + (1-q)V_{3,0,0}), S_0u^0 - S_{2,0}\right) \\
&= \max\left(e^{-0.1/12}(0.5073 \times 5.45 + (1-0.5073) \times 14.64), 50.00 - 39.69\right) \\
\vdots &= \vdots
\end{aligned}
$$

Dimension reduction

- The previous solution to the lookback problem was not ideal for two reasons:
    - First, keeping track of the maximums adds another dimension to the problem - this makes it much more time consuming to run.
    - Second, it is annoying to program and keep a track of all those maxima.
- In the case where the payoff is of the form: $\max(\hat{S}_T - S_T, 0)$ then there is a simple change of variables that reduces the dimension of the problem (see Cheuk and Vorst, 1996, on COMPASS).
- If $\hat{S}_t$ is the highest value of S achieved up until time t, then introduce a variable $Y_t = \hat{S}_t / S_t$.

Quiz

What is the intial value of Y, $Y_{0,0}$?:

- d
- u
- 1
- $S_{0,0}$

Dimension reduction

- Given this definition, we have $Y_{0,0} = 1$. Then we move forward to $j = 1$.
- An up movement in S causes $S_{0,0} \to uS_{0,0}$ and $\hat{S}_1 = uS_{0,0}$ thus

$$Y = \frac{uS_{0,0}}{uS_{0,0}}.$$

  So Y has not changed, we call this $Y_{1,0}$.
- A down movement causes $S_{0,0} \to dS_{0,0}$ and $\hat{S}_1 = S_{0,0}$ thus

$$Y = \frac{S_{0,0}}{dS_{0,0}} = \frac{1}{d} = u$$

  in the CRR model (a very convenient choice here), so a down movement in S causes an increase in Y, we call this $Y_{1,1}$.
- So, a tree for Y would start at 1 and an up jump in S leads to Y either staying at 1 (if all the jumps have been up) or decreasing to $Y/u = Yd$, whereas a down-jump in S leads to Y increasing to uY.

Option value function transformation

- thus the possible values of Y are given by

$$Y_{j,i} = u^i, i = 0, \dots j.$$

- Now let's consider the lookback option value as a function of Y rather than S. In standard notation we have

$$\begin{aligned} V(S_T, T) &= \max(\hat{S}_T - S_T, 0) \\ &= \max(Y_T S_T - S_T, 0) \\ &= \frac{1}{S_T} \max(Y_T - 1, 0) \end{aligned}$$

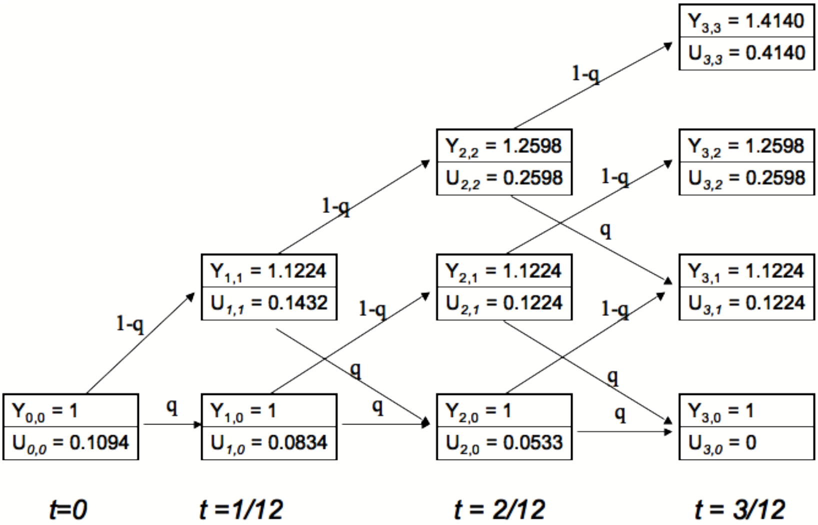
So, if we introduce a new function $U(Y_t, t) = \frac{V(S_t,t)}{S_t}$ then we have

$$U(Y_T, T) = \max(Y_T - 1, 0)$$

- In a binomial model this gives us,

$$U_{N,i} = \max(Y_{N,i} - 1, 0), i = 0, \dots N.$$

The reduced tree



Algorithm

- In the above example, we have switched the way in which the i counter works. Now i will denote up jumps in Y and not in S and so $Y_{2,2}$ means that Y has two up jumps - which actually corresponds to S having two down jumps!
- Note how we calculated $Y_{2,2} \to Y_{3,1}$ for example. Here $\hat{S}_{2,0} = S_{0,0}, S_{2,0} = S_{0,0}d^2, Y_{2,2} = u^2$, so $S_{3,1} = S_{0,0}d^2u = S_{0,0}/u$ so $Y_{3,1} = uS_{0,0}/S_{0,0} = u(= Y_{2,2}d)$.
- In this example $T = 0.25, N = 3, u = 1.1224$ and it is valuing an American option with payoff of maturity as $\max(\hat{S}_T - S_T, 0)$.
- This dimension reducing concept is challenging but the resulting algorithm is very simple - see overview slide!
- The value of U at expiry is $U_{N,i} = (\max(Y_{N,i} - 1, 0)$. Then to value back through the tree we simplify our standard expression (here i' denotes the regular i notation)

$$V_{j,i'} = \max\left[\hat{S}_{j,i'} - S_{j,i'}, e^{-r\Delta t}((1-q)V_{j+1,i'} + qV_{j+1,i+1'})\right]$$

Algorithm - for derivation not for use!

- Noting that $uS_{j,i'}V_{j+1,i+1'} = U_{j+1,i-1}$ and $dS_{j,i'}V_{j+1,i'} = U_{j+1,i+1}$ this transforms to:

$$
\begin{aligned}
V_{j,i'} &= \max\left[\hat{S}_{j,i'} - S_{j,i'}, e^{-r\Delta t}((1-q)V_{j+1,i'} + qV_{j+1,i+1'})\right] \\
S_{j,i'}U_{j,i} &= S_{j,i'} \max\left[Y_{j,i} - 1, e^{-r\Delta t}((1-q)U_{j+1,i+1}d + qU_{j+1,i-1}u)\right], i = 1, \dots j \\
U_{j,i} &= \max\left[Y_{j,i} - 1, e^{-r\Delta t}((1-q)U_{j+1,i+1}d + qU_{j+1,i-1}u)\right], i = 1, \dots j
\end{aligned}
$$

and in the case where $i = 0$ or $i' = N$ we have

$$
\begin{aligned}
S_{j,N}U_{j,0} &= S_{N,i} \max\left[Y_{j,0} - 1, e^{-r\Delta t}((1-q)U_{j+1,1}d + qU_{j+1,0}u)\right], i = 0 \\
U_{j,0} &= \max\left[Y_{j,0} - 1, e^{-r\Delta t}((1-q)U_{j+1,1}d + qU_{j+1,0}u)\right], i = 0.
\end{aligned}
$$

Notes on Algorithm

- Note that due to the transformation you need to multiply the values of U by u and d depending upon whether the movement is up or down because the value of $S_{j,i}$ is changing as you move forward in the tree, affecting the scaling.
- Note that q is now the probability of a down movement in Y as this corresponds to the underlying asset price increasing.
- Also note that you use $U_{i+1,j-1}$ rather than $U_{i+1,j}$ (unless $j = 0$) this is because an up movement in S means that Y decreases from $Y = \hat{S}/S$ to $Y = \hat{S}/uS$.
- Finally, to obtain $V_{0,0}$ multiply $U_{0,0}$ by $S_{0,0}$

Algorithm: overview

- **Construct a tree in Y where**

$$Y_{j,i} = u^i, i = 0, \dots j.$$

- **At maturity, $j = N$**

$$Y_{N,i} = \max(Y_{N,i} - 1, 0), i = 0, \dots N.$$

- **For $j < N$ we have**

$$\begin{aligned} U_{j,i} &= \max\left[Y_{j,i} - 1, e^{-r\Delta t}((1-q)U_{j+1,i+1}d + qU_{j+1,i-1}u)\right], i = 1, \dots j \\ U_{j,0} &= \max\left[Y_{j,0} - 1, e^{-r\Delta t}((1-q)U_{j+1,1}d + qU_{j+1,0}u)\right], i = 0 \end{aligned}$$

- **And then the current option price is:**

$$V_{0,0} = S_0 U_{0,0}$$

Quiz

Do you think that the dimension reducing technique can be used for lookback options where:

- The payoff is $\max(\hat{S} - K, 0)$?
- The option is already in existence and already has a maximum greater than S_0?

Asian options

- An Asian option is an option where the payoff is a function of the average value of the underlying asset where the averaging happens at discrete points in time (every day, every month etc.)
- The average could be either arithmetic of geometric depending upon the terms of the contract. The payoffs could be $\max(A - K, 0)$, $\max(A - S_T, 0)$ etc. where A is the average value.
- The difficulty when valuing an Asian option is that unlike valuing a lookback option the average, A, may not exactly correspond to nodes in the tree. In addition the set of possible averages will grow exponentially with the number of steps in the tree.
- A way of overcoming this is to calculate the option values for certain values of A and then use interpolation to determine the option value for a general A as you move backward through the tree.

Quiz

Do you think that the binomial pricing method will be:

- More accurate for valuing Asians than lookbacks
- Less accurate for valuing Asians than lookbacks
- The same accuracy for both?

Calculating Implied volatility

- One of the most important calculations in option pricing is that of implied volatility.
- Implied volatility is important as typically the two parameters that are difficult to estimate are option price and volatility. However, when options are traded we have the price and so it is possible to determine the volatility implied by the price.
- When the option is European than this problem is solved by iteratively solving the Black-Scholes equation, rearranging to have this equation as the volatility as the subject.
- However, if the option is American then the Black-Scholes equation no longer holds so we have to use our binomial tree to estimate volatility.

Calculating implied volatility

- The iterative procedure works just as well for trees as for the Black-Scholes equation. The most obvious iterative method is Newton-Raphson which would work as follows:

$$\sigma^*_{i+1} = \sigma^*_i - \frac{V(\sigma^*_i)(\sigma^*_i - \sigma^*_{i-1})}{V(\sigma^*_i) - V(\sigma^*_{i-1})}$$

  where i denotes the number of iterations, σ^* is the estimate of implied volatility and $V(s)$ is the option value obtained from the tree for that σ. The iteration ceases when $V(\sigma^*_i) = V_{\mathrm{market}}$ which is the market value of the option.

- To start the iteration you need two choices of σ, these can pretty much be any reasonable guesses or perhaps the historic σ and the historic $\sigma + 0.05$.

Implied volatility trees

- It is also possible to adapt the binomial pricing approach so that the volatility used in the binomial tree matches the implied volatility and interest rates from the market. The best sources are Derman and Kani (1994, a version on COMPASS), Jackwerth, 1996, and Derman, Kani, and Chriss, 1996 (on COMPASS) for Trinomial trees.
- Here I present the general idea of the model to get you started.
- Assume that the tree has been correctly constructed up until time step $i = n - 1$. Then we need to step forward to the nodes at n.
- Within the implied tree we are allowed to select the possible values of the underlying, $n + 1$ values, at each of the nodes as well as q, the risk-neutral probability, n values, of getting there.
- This gives us $2n + 1$ degrees of freedom. The first n are used to make sure that the expected returns across the period are equal to the risk-free rate.

Implied volatility trees

- The remaining degrees of freedom are used to ensure that European options with exercise prices equal to the nodes in the tree expiring at time n are correctly priced within the model (i.e essentially matching the correct implied volatilities). This uses up n more degrees of freedom.
- The final degree of freedom ensures that the tree is centered around the current stock price.
- This procedure leads to very strange looking trees and occasionally to negative risk-neutral probabilities. The typical way to deal with these is to ignore them or to move them to a default value. See Derman and Kani.
- This use of volatilities over small time steps leads to a class of models known as **Local Volatility** models, which are closely linked to modeling a volatility surface, the classic book on this is Jim Gatheral's The Volatility Surface (and many of his talks available online).

Quiz

Why is it useful to have an implied volatility tree?

- It gives more accurate European/American option prices than given by the market?
- It allows you to check if the market prices are correct?
- It allows you to value more accurately other, exotic derivatives on the same underlying asset?

Other processes

- The most obvious area where it one needs to use a binomial tree to model a different underlying process is when valuing interest rate derivatives.
- Here the underlying is typically a bond or an interest rate, and so it is the interest rate which is of key importance. Rates are usually modeled using a mean reverting process.
- We will look at the most practically useful ones later in this course: the one factor Black, Derman,Toy model (on COMPASS) and the Hull White Model.
- Often, for a first approximation for different stochastic processes it is easier to use Monte Carlo methods rather than binomial trees. As we shall see shortly. Indeed for some the binomial tree will not recombine.

Overview

- We have seen a brief taster of extensions to the binomial trees, in particular we considered valuing a path dependent option (the lookback), looked out how, in principle, we could design a tree to fit the market data and also saw some references for constructing interest rate binomial trees.