Fin 514: Financial Engineering II

Course overview

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Financial Engineers and the crisis - a warning!

Mr. Markowitz doesn't excuse the financial engineers who bundled complex mortgage-based and other securities. They violated the first principle of his portfolio theory. "Diversifying sufficiently among uncorrelated risks can reduce portfolio risk toward zero," he says in an interview. "But financial engineers should know that's not true of a portfolio of correlated risks".
In traditional Markowitz-inspired investing, such as mutual funds and index funds, there is a discipline around variables such as asset classes and models of covariance. In contrast, collateralized mortgage obligations and related securities had no such discipline. These risks sank together. "Selling people what sellers and buyers don't understand," he says with understatement, "is not a good thing."

The Wall Street Journal

Thoughts from Emmanuel Derman

Physics and finance are only superficially similar. While theoretical physics captures the essence of the material world to an accuracy of 10 significant figures, theoretical finance is at best an untrustworthy, limited representation of the mysterious way in which financial value is determined. Yet Thomas Wilson, the chief insurance risk officer of the ING Group, wisely remarks: "A model is always wrong, but not useless." Despite the inadequacies of quantitative finance, we have nothing better. And, on the practical side, Andrew Sterge, the chief executive of AJ Sterge Investment Strategies, writes: "The greatest research in the world does no good if it cannot be implemented."

E. Derman in The Wall Street Journal

What is Financial Engineering?

- From wikipedia: “Financial Engineering [,] is a cross-disciplinary field which relies on computational intelligence, mathematical finance, numerical methods and computer simulations to make trading, hedging and investment decisions, as well as facilitating the risk management of those decisions.”
- From investopedia: “Financial engineers use various mathematical tools in order to create new investment strategies. The new products created by financial engineers can serve as solutions to problems or as ways to maximize returns from potential investment opportunities.”
- As well as developing new products, financial engineers also develop techniques to more accurately value existing products and to develop strategies for hedging existing products against all possible risks.

Why is it important?

- Financial engineering is important to investment banks, corporations and funds.
- Its first role is to accurately value complex financial projects or products. This is useful for investment banks or corporations issuing structured products or derivative claims, and for funds who with to invest in such products. Financial engineering can also be used to make corporate financing decisions such as the evaluation of projects, capital structure decisions and executive compensation.
- The second role is to manage risk. Either by assessing the probabilities of losses or by creating a hedging strategy that entirely removes exposures to certain risks. This can include creating products which can be used to hedge existing products or by understanding the relationship between existing products.

Aims: Part I, Binomial model

- We look at option pricing theory, starting from discrete time models where we will revisit the **binomial option pricing formula**, with emphasis on no arbitrage and how this leads to the fundamental theorem of finance.
- This has two aims: to introduce numerical methods and to motivate the more complex continuous time models.
- We will then analyze the binomial pricing model in more detail. We will discuss the accuracy and convergence characteristics of the binomial model and look at practical considerations when using it to value derivatives.
- Within this section we will have a chance to value some real world products as accurately as we can using the binomial model. We will also see some of the limitations of the binomial model.

Aims: Part II, Finite Difference Methods

- We will have an overview of Stochastic (Ito) calculus and continuous time finance which will result in the **Black-Scholes-Merton analysis and the PDE approach**
- This will use results from mathematics to show that derivative pricing can be considered from the point of view of differential equations.
- For some special types of options these partial differential equations can be solved giving us analytic valuations for some standard option types.
- If the derviatives cannot be valued analytically then we can attempt to solve these equations numerically using **Finite Difference Methods**.
- These methods are similar to Binomial trees but we will see the advantages and disadvantages that they have compared to our binomial trees. We will also look at extensions to finite-difference methods and we will use them to value a real world product.

Aims: Part III, Probability model

- We then look at probability theory, martingales in particular, which will lead us to **the probabilistic solution**.
- Here we will look at changing probability measures and how to think of option pricing in terms of calculating expected payoffs and discounting them at appropriate rates.
- The theory here is quite challenging (changing numeraire, Girsanov's theorem etc.) but I will attempt to keep it as intuitive as possible.
- Again, this approach will lead to analytic values but as before many times we will have to use a numerical algorithm. For the probability approach the appropriate numerical scheme is the **Monte Carlo method**.
- As before, we will see the basic idea, discuss its strengths and weaknesses as a practical tool and look at some applications.

Aims: Part IV, Interest rate modeling

- In the remaining time we will look at models for interest rates. We will focus on analytical and binomial tree approaches. In particular, we will look at the **Black model** and the **Black, Derman, Toy** binomial tree. Again, the focus will be practical, but we will have to work harder to calibrate the models to existing data.

Aims: What we will NOT do

- Get too bogged down in mathematical theory, particularly in the stochastic calculus section. The aim is to be rigorous but focus on practical applications.
- Only deal in theory. I will try to use some real world products in your problems sets and projects so that you can see some of the difficulties of derivative valuation.
- Believe that the models we use are flawless. Always bear in mind that financial models are approximations and that they should be used with care, and common sense.

Objectives

Upon successful completion of the course, students should be able to:

- Estimate input parameters into various models.
- Understand the convergence characteristics of different numerical methods.
- How to value complex derivatives via a binomial tree approach.
- Understand the basics of stochastic calculus and use Ito's lemma to derive processes followed by derivative prices.
- Derive the Black-Scholes partial differential equation for various derivative products.
- Solve the Black-Scholes equation analytically for various types of options.
- Value cmplex derivatives using various Finite Difference Methods.
- Derive the Black-Scholes equation using the probabilistic approach and understand what is meant by a numeraire asset.
- Value derivatives using a Monte Carlo method, even when there are early exercise features.
- Understand the Black formula and why it is a valid formula.
- Calibrate and use binomial trees for valuing interest rate derivatives.

Skills you will need

This course links together various skills:

- A lot of the derivations will require knowledge of mathematical and quantitative methods. A knowledge of calculus is required, and some experience with partial differential equations will be useful.
- Most of the work in this class will be mathematicl in nature, you have to enjoy manipulating mathematical equqations and trying to solve them. If you do not - this is not the class for you.
- An understanding of important financial concepts such as arbitrage, fixed income, basic option pricing and portfolio construction will also be essential.
- Finally, to implement the methods will require some programming skills. I will arrange a couple of workshops in VBA for the first few weeks of the semester but you can also use Matlab, C++ etc. if you are familiar with them. At the very least you must be proficient with Excel and be motivated to try out some programming.

Pricing a structured product

- Estimate the market value of the Buffered PLUS (Performance Leveraged Upside Securities) Based on the Level of the S&P 500 Index due November 5, 2018, described in the pricing supplement at: http://www.sec.gov/Archives/edgar/data/83246/000114420416094949/v437259_424b2.htm.
- JP Morgan, a major U.S. investment bank, is selling an issue of Auto Callable Contingent Interest Notes based on the price of Apple. The notes are issued by JP Morgan, but have cash flows based on the price of Apple common stock. Your job is to estimate the fair market value of the notes as of the pricing date, 13 October 2017. The notes are described in detail in the pricing supplement at: https://www.sec.gov/Archives/edgar/data/19617/000089109217007413/e76088_424b2.htm.

Even simple things may be hard to value...

- Consider a 25-year loan of \$100,000, perhaps a student loan, where you must pay fixed interest (5%) every quarter (assume that the principal is repaid at the end of the 25 years).
- As part of the loan contract this loan can be prepaid/refinanced after 5 years (of course, in reality it can typically be repaid at many different times).
- On each payment date, you are scheduled to pay $0.05 \times 0.25 \times \$100,000 = \$1250$.
- At the prepayment date, $t = 5$, the borrower has a choice between prepaying (or refinancing the \$100,000 at the $t = 5$ interest rates) or continuing:

Even simple things may be hard to value...

- Thus the value to the borrower is:

$$\begin{aligned}\text{Value} &= \max(-\$100,000, -\text{PV}(\text{remaining payments})) \\ &= -\text{PV}(\text{rem. pymnts}) + \max(\text{PV}(\text{rem. pymnts}) - \$100,000, 0) \\ &= \text{Current loan} + \text{`Option'}\end{aligned}$$

- What is this 'option' here? Well, what is varying is the present value of the remaining payments and the strike price is the face value of the loan.
- The Present value of the remaining payments will depend upon the interest rates at $t = 5$ and so this is essentially an interest rate option.
- In fact, it is called a **European Swaption** which we will consider at the end of the course.

Even simple things may be hard to value...

- One remaining question: what happens if the loan can be prepaid/refinanced at any time? This becomes an American (or Bermudan) swaption...

Stochastic calculus

- What is a good model for the uncertainty of asset prices?
- Can we perform calculus on functions of these random asset prices?
- Is it possible to derive formulas for the price of a derivative on a random asset price?

Derivative pricing

- What is the expected return for a derivative product?
- Can we replicate this product using simpler traded assets?
- Can we write down a formula for a European style derivative?
- Can we write down a formula for an American style derivative?
- Can we think of option prices in terms of discounted expectations? What assumptions need to be made in order to do this?

Numerical algorithms

- How do we develop algorithms to price options with no analytic solutions?
- How do we know that these prices are accurate?
- Are some numerical methods better than others? How do we distinguish between the competing methods?
- Can we use Binomial trees to value derivatives on large numbers of assets?
- Can we adapt Monte Carlo methods to value options with early exercise features?

Course structure

- There are twenty nine classes in the spring semester:
    - Mondays: 9:30-11:00
    - Wednesdays: 9:30-11:00

  Office hours will take place in 330 Wohlers. The times are as follows:
    - Mondays: 11:00 - 12:00
    - Tuesdays: 2:00 - 3:00
    - Wednesdays: 11:00 - 12:00
- My email is widdicks@illinois.edu, please put FIN514 in the subject heading and make efforts to use your name@illinois.edu account.

Assessment

The breakdown of your grade is as follows:

- Problem Sets: 20%
- Valuation Projects: 30%
- Midterm: 20%
- Final Exam (cumulative): 30%

There will be **seven problem sets** throughout the course.
The midterm will take place on Wednesday 14th March with full details to be announced.
The projects will be performed in small groups based upon the valuations of a complex, real-world products. More details will be announced in class.

Notes on course

- The course will follow the syllabus, please read the appropriate chapter(s) (as indicated on the syllabus) before class. Be aware that we may run faster or slower than the syllabus so keep up to date with where we are at.
- The slides will be posted on COMPASS before class, you should endeavour to read the slides and, if possible, print out the slides and bring them to class.
- Additionally, I will bring hardcopies to class.
- I will often use examples in lectures which are not on the slides, so be prepared to take additional notes.
- There will be occasional quiz questions littered throughout each lecture I expect you to participate.
- I will occasionally post extra articles on COMPASS which you may find of interest. This is recommended rather than required reading.

QUIZ

Will you be willing to put your hand up to participate in quiz questions?

- Yes, of course.
- No, absolutely not.
- Maybe

QUIZ

Will you be willing to put your hand up to participate in quiz questions?

- Yes, of course. CORRECT
- No, absolutely not.
- Maybe

Reading

All required materials will be presented in the lecture notes but I strongly recommend that you purchase the following book:

- **Joshi, M., The Concepts and Practice of Mathematical Finance, Cambridge University Press: Cambridge, UK. ISBN: 0521514088**

The following text books also cover some important but none is a comprehensive as BM

- Bjork, T., 2004, Arbitrage Theory in Continuous Time, Oxford University Press: Oxford, UK. ISBN: 0199271267
- Paul Wilmott Introduces Quantitative Finance, 2001, Wiley: NY. ISBN 0-471-49862-9 (Paperback).

Reading

- Hull, J., 2005, Options, Futures and other Derivatives (6th edition), Prentice-Hall ISBN: 0131499084.
- Baxter, M, Rennie, A, Financial Calculus: An Introduction to Derivative Pricing, Cambridge University Press. ISBN: 0521552893.
- Joshi, M., More Mathematical Finance, Pilot Whale Press. ISBN: 0987122803.
- Rendleman, Richard J., 2002, Applied Derivatives, Blackwell: Oxford. ISBN: 0-631-21590-5.
- Paul Wilmott on Quantitative Finance, 2000, Wiley: NY. ISBN: 0471874388.