

FIN 514: Problem Set #3

Due on Sunday, February 18, 2018

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Problem 1

Using the following Black-Scholes formula, the option price was calculated as 10.2479.

$$\begin{aligned} \text{Put option price} &= Ke^{-r(T-t)}N(-d_2) - Se^{-\delta(T-t)}N(-d_1) \\ d_1 &= \frac{\log(\frac{S}{K}) + (r - \delta + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}} \\ d_2 &= d_1 - \sigma\sqrt{T-t} \end{aligned}$$

Then, using Cox, Ross and Rubinstein(CRR), Rendleman and Bartter(RB), Leisen and Reimer(LR) method each, put option value was calculated from $N = 50$ to $N = 1000$. Figure 1 shows the error of each method. The error was calculated using the formula $V_N - V_{EXACT}$. As shown in figure, except LR method, there

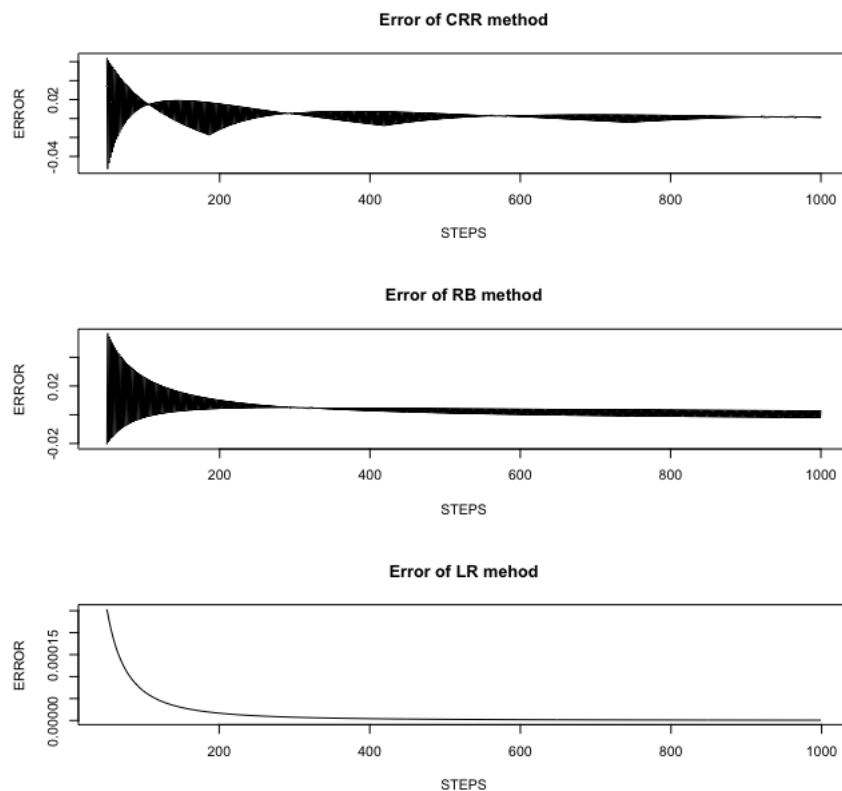


Figure 1: Error of each method

seems to exist some problems. In CRR method, it looks like that error is converging to zero, but it is not monotonic. It means it does not guarantee that applying more steps makes more accurate values. Regarding RB method, it seems better than CRR, but error is increasing from some points (about $N = 300$). The reason for this phenomenon is that option payoff is not linear shape. Since LR method solves this problem when N is odd, the shape of error in LR method seems monotonically decreasing to zero as N goes to some large value. The reason why monotonicity is important is that we can extrapolate values from two

binomial trees to get more accurate values if monotonic error is guaranteed. Figure 2 shows the error after extrapolation ($M = 2N$ is used when using CRR and RB method, $M = 2N - 1$ is used for extrapolation.). Since it is well-known that LR method has $O(1/n^2)$ errors, the extrapolation procedure has changed from

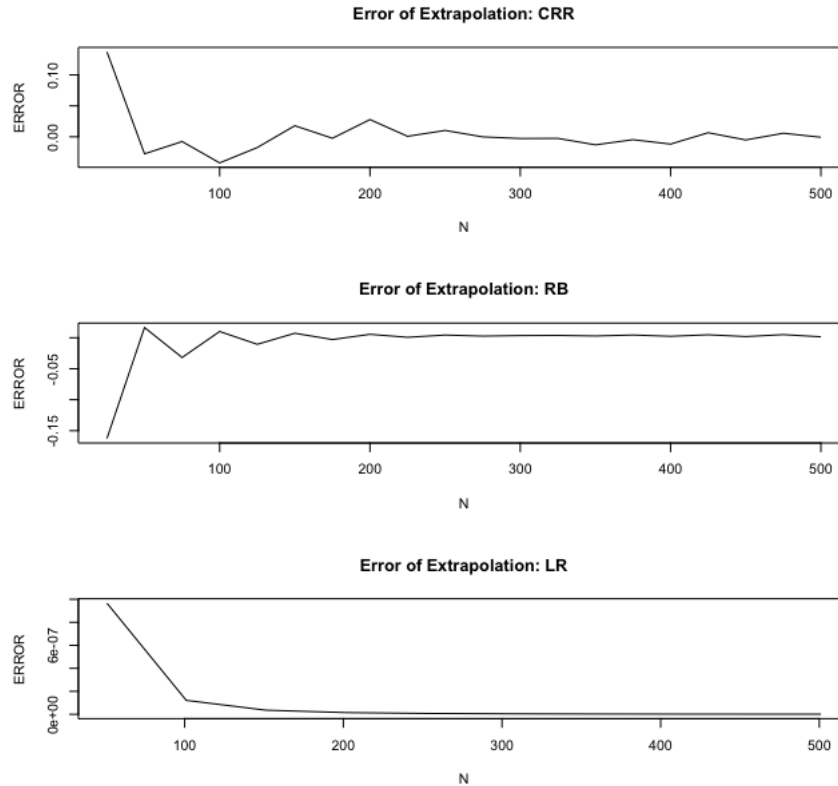


Figure 2: Error of each methods after extrapolation

original one to followings.

$$V_{EXACT} \approx \frac{M^2 V_M - N^2 V_N}{M^2 - N^2} \quad \text{where } M \text{ and } N \text{ are odd numbers.}$$

As shown in Figure 2, the error of CRR and RB methods seems sawtoothing, but error of LR method is converging to zero monotonically. Furthermore, the accuracy of value is even worse at some points for CRR and RB method when extrapolation is applied. Before using extrapolation technique, the maximum error of CRR and RB method is about 0.05, but there are some points where error is about 0.1 after extrapolation. However, in LR method, the error after extrapolation is always smaller than before. That is why monotonicity is important when using extrapolation to get more accurate value.

Problem 2

The value of American put option is calculated as 10.3762 using Broadie and Detemple(BD) method for 10000 steps. All following procedures are performed assuming this number is true value of the option. Using

the same procedure of Problem 1, errors were calculated using CRR, BD, LR method. Figure 3 shows errors of each method. In similar to result of European option, error of CRR method seems to have sawtoothing

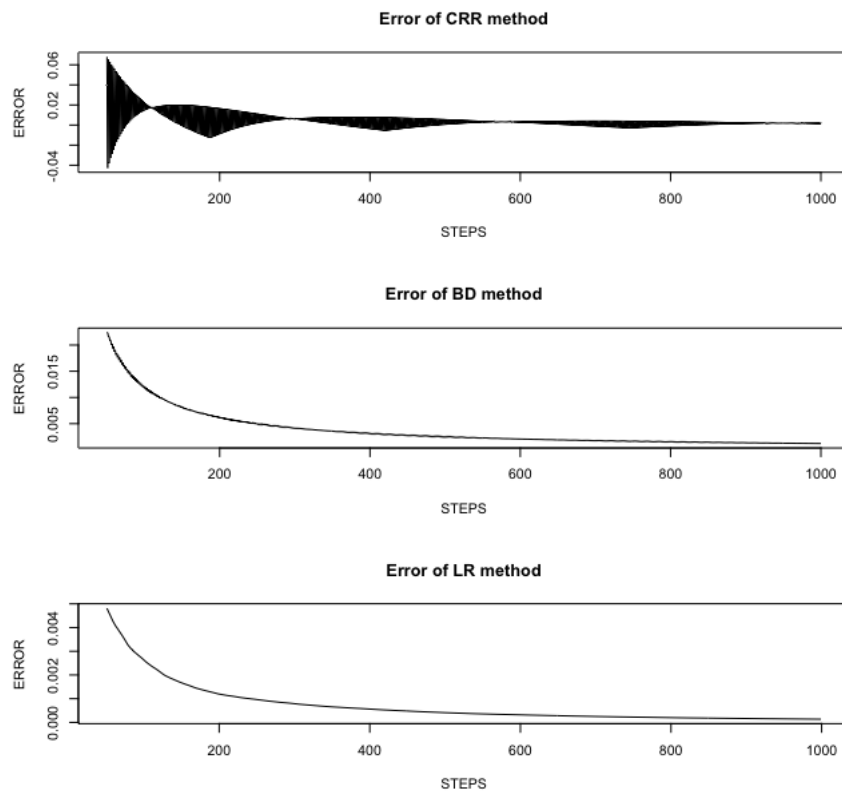


Figure 3: Error of each method

shape and periodic humps when valuing American options. Moreover, the amount of error is even larger than errors of case in European options. In contrast, the other two methods seem to have errors converging to zero monotonically. Since it is already known that sawtoothing and periodic humps of error come from non-linearity of payoff, and in European or American option case, the non-linearity occurs only at the maturity of option, error of BD method has monotonicity and convergence because the method avoids pricing option at maturity. LR method also avoids non-linearity using other parameters to construct binomial tree, errors of LR method also have good features. In other words, CRR method has problems because it has no efforts to handle non-linearity problems. The problem of CRR method also occurs when calculating exercise bound of options. Figure 4 shows the exercise boundary using CRR method. (the number 0 of plot means early exercise is not optimal at the step.) Theoretically, the exercise boundary of a put option should be a smooth increasing function. However, as shown in figure, the shape of boundary is sawtoothing, which does not coincide with theory. Since the optimal exercise boundary is distorted under CRR method, exercise-or-not decision at each node will be distorted consequently, and finally it causes distortion of option prices.

Similar to Problem 1, it is possible to reduce error using extrapolation technique, if monotonically decreasing

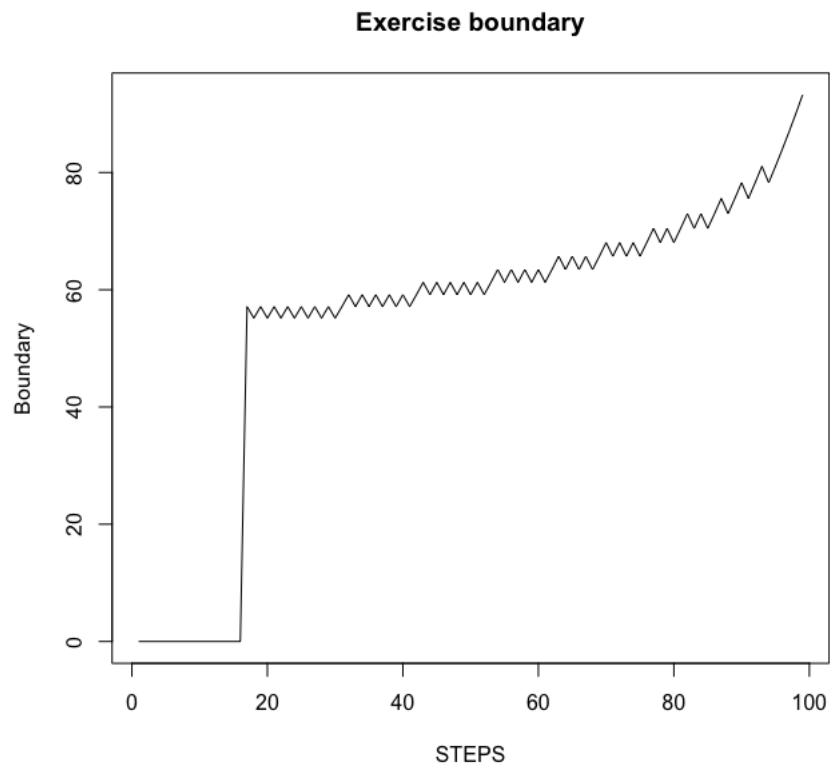


Figure 4: Exercise boundary: CRR method

error is guaranteed. For each method, we calculated errors of option price using extrapolation technique. Figure 5 shows the error of each method using extrapolation technique. errors of all three methods seems to converge to zero. However, because of non-monotonicity, errors in CRR methods using extrapolation is larger than errors not using extrapolation technique in some points. The other methods seem to have smaller errors than not using extrapolation because they have monotonic errors. The interesting thing is errors of LR method. In graph of LR method of Figure 5, the red line is the value of errors using extrapolation of error of order 2, and the green line is which of order 1. The horizontal black line is $Error = 0$ line. It is easy to expect that extrapolating with order 2 is better than with order 1. Of course, extrapolating with order 2 has faster convergence speed, however, at small N , extrapolating with order 1 is more accurate than extrapolating with order 2.

Problem 3

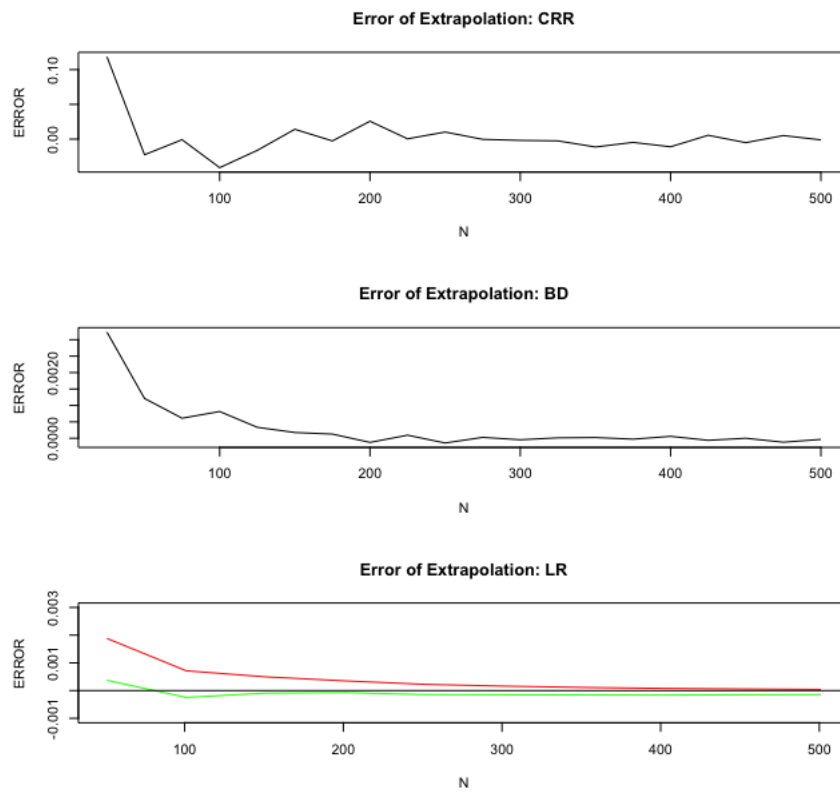


Figure 5: Error of each methods after extrapolation