# Fin 514: Financial Engineering II

## Lecture 1: Options basics

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Spring, 2018

# Outline

- Puts and calls; payoff and P&L diagrams.
- Put-call parity.
- Simple strategies.
- Combinations.
- Using options to create any payoff profile.

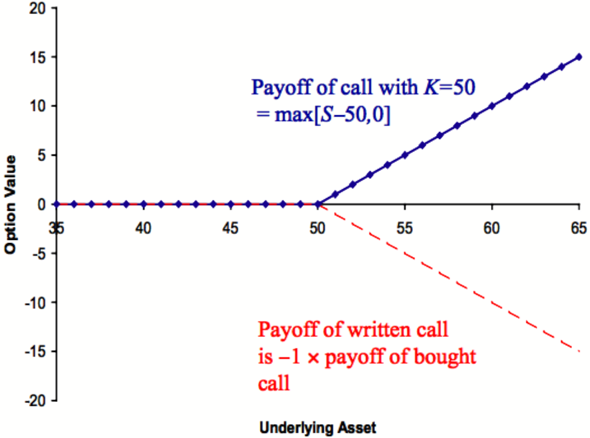
# Calls and puts

- Rights of option owner:
  - Call: right to deliver K, receive S, payoff $= \max[S_T - K, 0]$.
  - Put: right to deliver S, receive K, payoff $= \max[K - S_T, 0]$
- Notation:
  - $t$: current time/date, $T$: expiration date.
  - $\tau = T - t$ the remaining time to amturity
  - $S$, $S_t$, or $S_T$: stock price (or $F$, $F_t$, $F_T$: forward price).
  - $K$: exercise/strike price.
  - $C(t)$ or $C_t$, $P(t)$ or $P_t$: values of call and put options at time $t$.
  - $C(T)$ or $C_T$, $P(T)$ or $P_T$: values of call and put options at expiration T (payoffs)
- In U.S., exchange-traded options are typically on 100 shares
  - payoff of actual call is 100 $\max[S - K, 0]$
  - However, we usually do our analysis on a per-share basis, that is we consider options with payoffs $\max[S - K, 0]$.

# Calls and puts: Exercise style

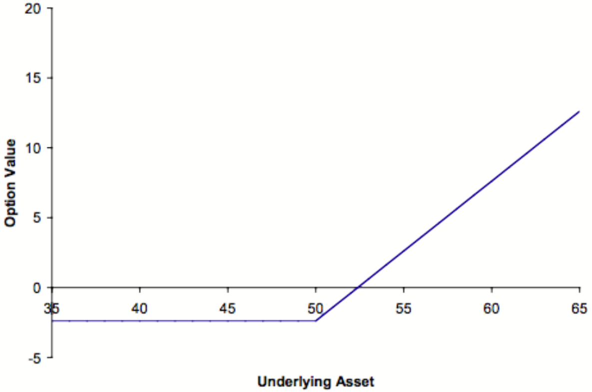
- European: exercisable only at expiration date.
- American: exercisable any time during the life of the option.
- Bermudan/semi-American: exercise at one of a list of dates.
- Where is Bermuda?
- The terminology European and American was chosen by the famous economist (and Nobel laureate) Paul Samuelson, and has nothing to do with where the options are or were traded.

# Call payoffs with $K = 50$



# Call profit/loss

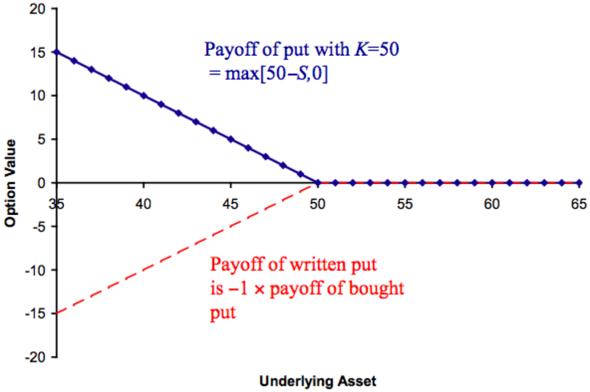
The previous page showed a payoff diagram: where the initial investment in the portfolio is not on the diagram. Practitioners use P/L diagrams where P/L = payoff - investment



# Payoff or P/L

- In this course we often use payoff diagrams.
- We use the diagrams to understand the implications of adding another option/instrument to a portfolio.
- With P/L diagrams it is harder to see the implications of adding another option/instrument the initial investment changes.
- The entire picture shifts up or down, obscuring the impact of the additional option on the portfolio
- However, when drawing payoff diagrams, one must remember initial investment

# Put payoffs with $K = 50$



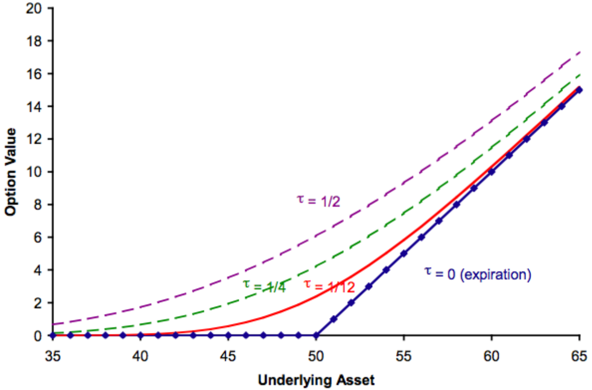
# QUIZ

Are the following statements true or false:

- Buying a call is the same as writing a put.
- Buying a put is the same as writing a call.

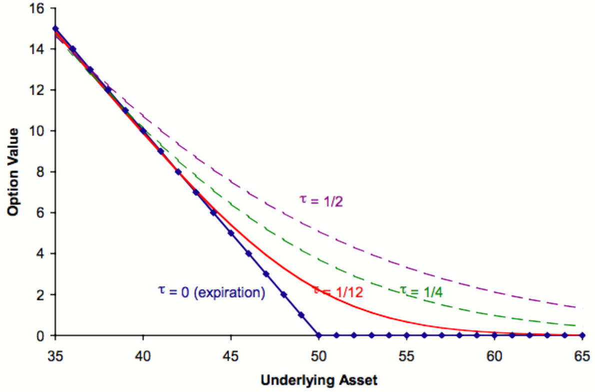
# Call values prior to expiration: using Black-Scholes

Where $\tau = T - t$, time remaining until expiration,
$0 \leq \partial C/\partial S \leq 1, \partial^2 C/\partial S^2 \geq 0, \partial C/\partial \tau \geq 0$



# Put values prior to expiration: using Black-Scholes

Where $\tau = T - t$, time remaining until expiration,
$-1 \leq \partial P/\partial S \leq 0, \partial^2 P/\partial S^2 \geq 0$.



# QUIZ

Which do you think is the riskiest option position?

- Long call
- Long put
- Short call
- Short put

# Put-call parity

- Consider European call and put options on a common stock with strike K expiring at time T
- Payoffs are

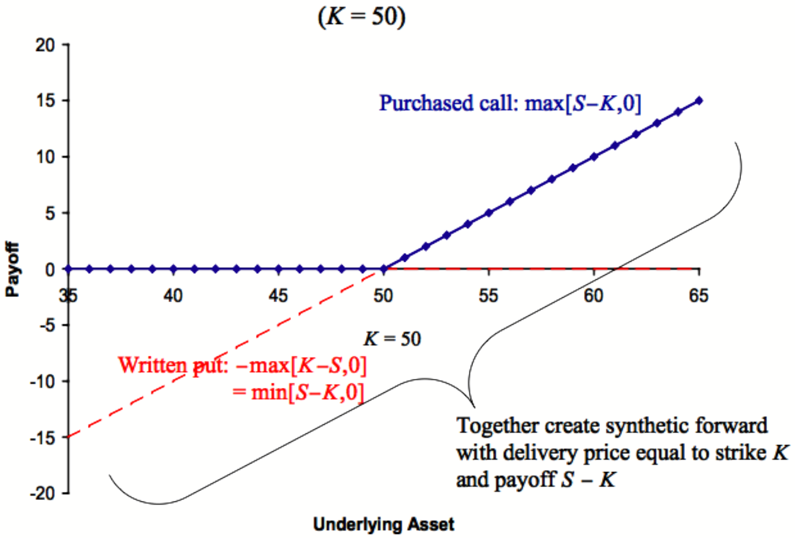
$$\begin{aligned} C(T) &= \max[S_T - K, 0] \\ P(T) &= \max[K - S_T, 0] \end{aligned}$$

- A portfolio of a purchased call and written put expiring at T has payoff $C(T) - P(T)$
- Graph on next slide shows that

$$C(T) - P(T) = S_T - K,$$

that is payoff of portfolio of purchased call and written put is identical to payoff of forward with delivery price equal to the strike $K$.

# Put-Call parity, $K = 50$



# Put-call parity

- At expiration:

$$\begin{aligned} C(T) - P(T) &= max[S_T - K, 0] - max[K - S_T, 0] \\ &= max[S_T - K, 0] + min[S_T - K, 0] \\ &= S_T - K \end{aligned}$$

- At time t: If stock does not pay dividends, then time-$t$ value of $S_T$ is $S_t$ and value of $K$ is $e^{-r(T-t)}K$. Thus at time t,

$$C(t) - P(t) = S_t - e^{-r(T-t)}K$$

If there are dividends,

$$C(t) - P(t) = S_t - PV(divs.) - e^{-r(T-t)}K,$$

where PV(divs.) is the present value of the dividends paid during the term of the option

# Put-call parity

From the relation in terms of the stock, one can obtain a relation in terms of the forward price, $F_t$: without dividends:

$$\begin{aligned} C(t) - P(t) &= S_t - e^{-r(T-t)}K \\ C(t) - P(t) &= e^{-r(T-t)}[S_t e^{r(T-t)} - K] \\ C(t) - P(t) &= e^{-r(T-t)}[F_t - K] \end{aligned}$$

with dividends

$$\begin{aligned} C(t) - P(t) &= S_t - PV(divs.) - e^{-r(T-t)}K \\ C(t) - P(t) &= e^{-r(T-t)}[S_t e^{r(T-t)} - FV(divs.) - K] \\ C(t) - P(t) &= e^{-r(T-t)}[F_t - K] \end{aligned}$$

# QUIZ

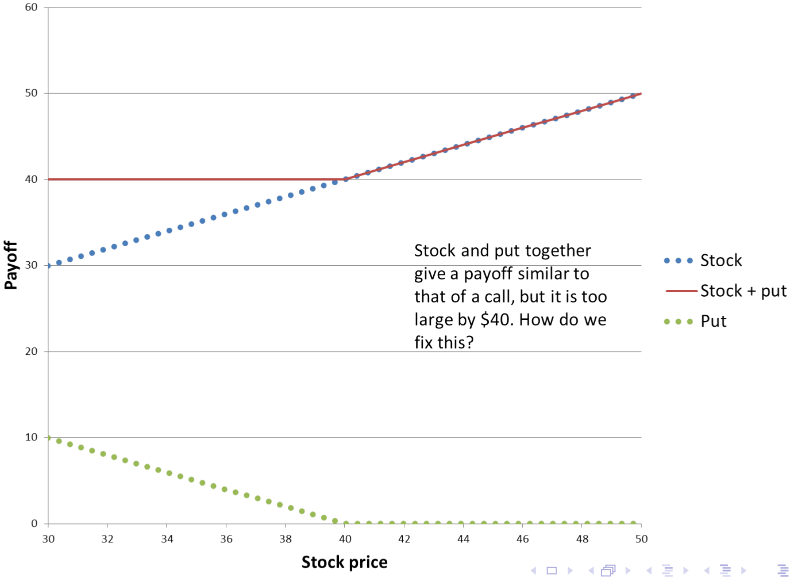
If the risk-free rate is zero, the current price of a put option with an exercise price of \$100, the time to expiry is 1 year, is \$10, the current price of the call option is \$15 and the current underlying asset price is \$104. What is the arbitrage opportunity?

- Buy the call, borrow $K$, sell the put and buy the underlying asset.
- Buy the call, invest $K$, sell the put and sell the underlying asset.
- Sell the call, borrow $K$, buy the put and buy the underlying asset.
- Sell the call, invest $K$, buy the put and sell the underlying asset.

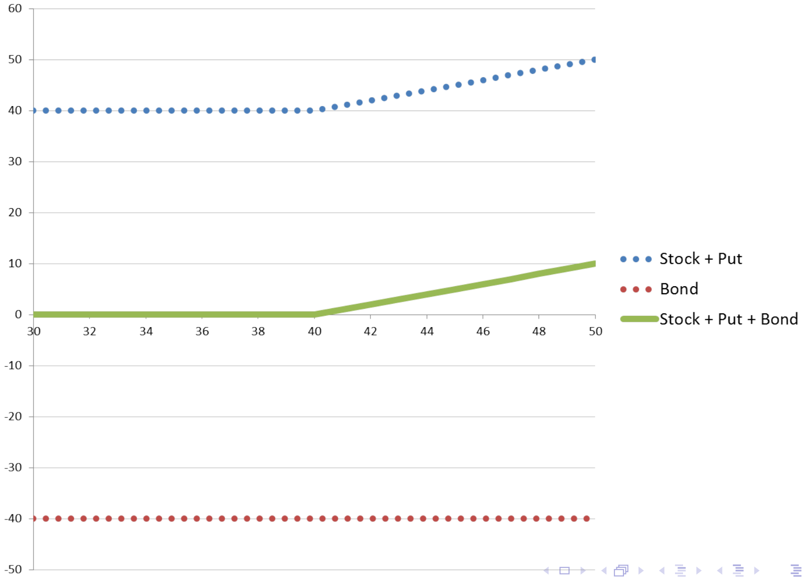
# Put-call parity

- The relation
$$C(T) - P(T) = S_T - K$$
tells us that a bought call and written put create a synthetic foward contract.
- Rearranging,
$$C(T) = S_T - K + P(T)$$
tell us that put plus stock and a loan promising to repay K create a call.
- Moving P, S, or K to the left-hand side will tell us how to create a put, the stock, or a bond/loan promising to pay K.

# Using put, stock, bond to create a call

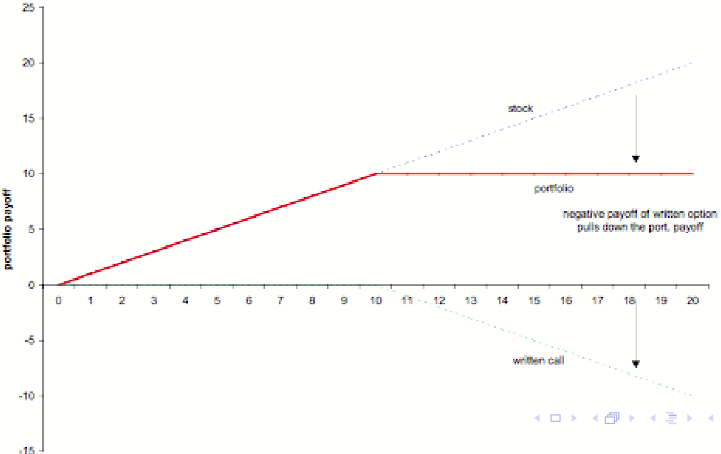


# Using put, stock, bond to create a call



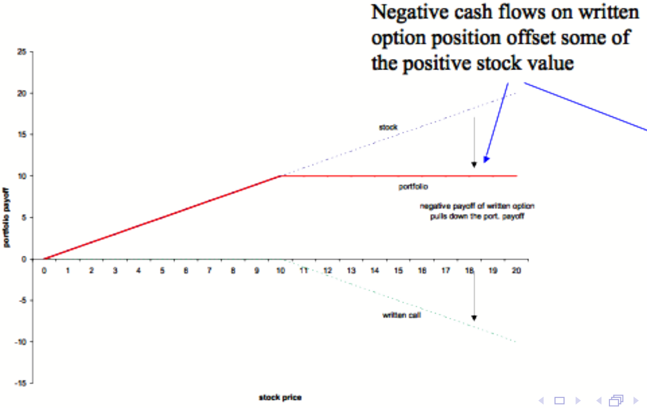
# Hedges: 1:1 hedge

A hedge combines an option with the underlying stock (or other underlying instrument) in such a way that the option protects the stock against loss of the stock protects the option against loss. See article on COMPASS for an analysis of the performance of these strategies.



# 1:1 hedge

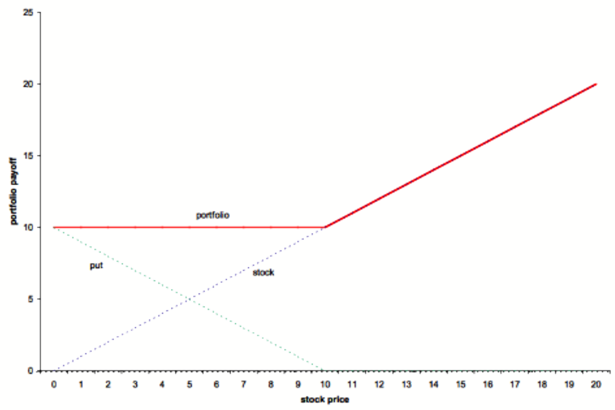
This position has various names: 1:1 hedge, buy-write, write covered call.




| stock | -call | port |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 1 |
| 2 | 0 | 2 |
| 3 | 0 | 3 |
| 4 | 0 | 4 |
| 5 | 0 | 5 |
| 6 | 0 | 6 |
| 7 | 0 | 7 |
| 8 | 0 | 8 |
| 9 | 0 | 9 |
| 10 | 0 | 10 |
| 11 | -1 | 10 |
| 12 | -2 | 10 |
| 13 | -3 | 10 |
| 14 | -4 | 10 |
| 15 | -5 | 10 |
| 16 | -6 | 10 |
| 17 | -7 | 10 |
| 18 | -8 | 10 |
| 19 | -9 | 10 |
| 20 | -10 | 10 |

# 1:1 hedge with put

1:1 hedge with put (also called portfolio insurance)




| stock | put | port |
|---|---|---|
| 0 | 10 | 10 |
| 1 | 9 | 10 |
| 2 | 8 | 10 |
| 3 | 7 | 10 |
| 4 | 6 | 10 |
| 5 | 5 | 10 |
| 6 | 4 | 10 |
| 7 | 3 | 10 |
| 8 | 2 | 10 |
| 9 | 1 | 10 |
| 10 | 0 | 10 |
| 11 | 0 | 11 |
| 12 | 0 | 12 |
| 13 | 0 | 13 |
| 14 | 0 | 14 |
| 15 | 0 | 15 |
| 16 | 0 | 16 |
| 17 | 0 | 17 |
| 18 | 0 | 18 |
| 19 | 0 | 19 |
| 20 | 0 | 20 |

# Spreads

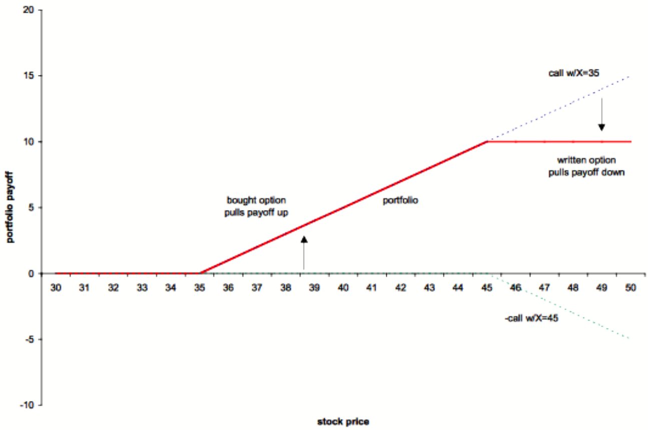
- A spread combines options of same class (call/put) but different series (exercise price or time to expiration)
- Different types
  - Vertical (money, price, perpendicular): 2 options of same class w/same expiration but different strike prices.
  - Horizontal (calendar): 2 options of same class w/same strike but different times to exp.
  - Diagonal: 2 options of same class w/different strikes and different times to expiration
  - Butterfly: 3 options of same class w/different strikes and same time to exp: buy 1 of each of outside strikes write 2 of middle strike.

# Names for spreads

To remember these names, note that a common way (although the WSJ no longer uses this) for displaying options prices is in a table of the form:

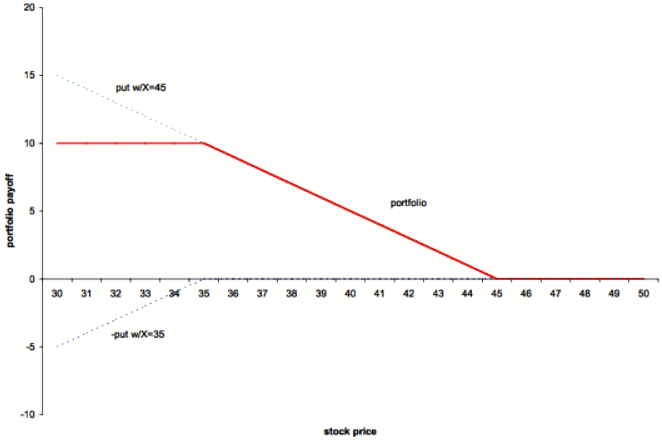
| | | Calls | | | Puts | |
|---|---|---|---|---|---|---|
| Strikes | Mar | Apr | May | Mar | April | May |
| 40 | $\uparrow$ $\nwarrow$ | | | $\leftarrow$ | horizontal | $\rightarrow$ |
| 45 | vertical | diagonal | | | | |
| 50 | $\downarrow$ | | $\searrow$ | | | |

# Bullish vertical spread with calls




| Stk | call | -cal | port |
|---|---|---|---|
| 30 | 0 | 0 | 0 |
| 31 | 0 | 0 | 0 |
| 32 | 0 | 0 | 0 |
| 33 | 0 | 0 | 0 |
| 34 | 0 | 0 | 0 |
| 35 | 0 | 0 | 0 |
| 36 | 1 | 0 | 1 |
| 37 | 2 | 0 | 2 |
| 38 | 3 | 0 | 3 |
| 39 | 4 | 0 | 4 |
| 40 | 5 | 0 | 5 |
| 41 | 6 | 0 | 6 |
| 42 | 7 | 0 | 7 |
| 43 | 8 | 0 | 8 |
| 44 | 9 | 0 | 9 |
| 45 | 10 | 0 | 10 |
| 46 | 11 | -1 | 10 |
| 47 | 12 | -2 | 10 |
| 48 | 13 | -3 | 10 |
| 49 | 14 | -4 | 10 |
| 50 | 15 | -5 | 10 |

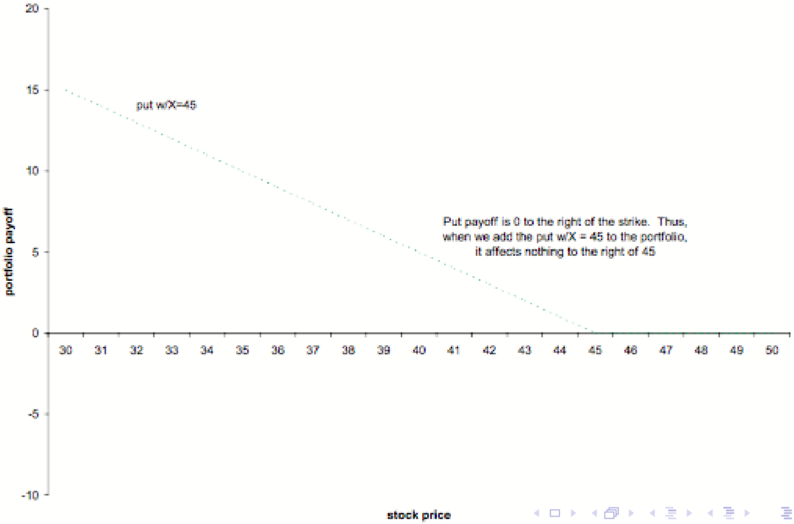
# Bearish vertical spread with puts




| Stk | put | -put | port |
|---|---|---|---|
| 30 | -5 | 15 | 10 |
| 31 | -4 | 14 | 10 |
| 32 | -3 | 13 | 10 |
| 33 | -2 | 12 | 10 |
| 34 | -1 | 11 | 10 |
| 35 | 0 | 10 | 10 |
| 36 | 0 | 9 | 9 |
| 37 | 0 | 8 | 8 |
| 38 | 0 | 7 | 7 |
| 39 | 0 | 6 | 6 |
| 40 | 0 | 5 | 5 |
| 41 | 0 | 4 | 4 |
| 42 | 0 | 3 | 3 |
| 43 | 0 | 2 | 2 |
| 44 | 0 | 1 | 1 |
| 45 | 0 | 0 | 0 |
| 46 | 0 | 0 | 0 |
| 47 | 0 | 0 | 0 |
| 48 | 0 | 0 | 0 |
| 49 | 0 | 0 | 0 |
| 50 | 0 | 0 | 0 |

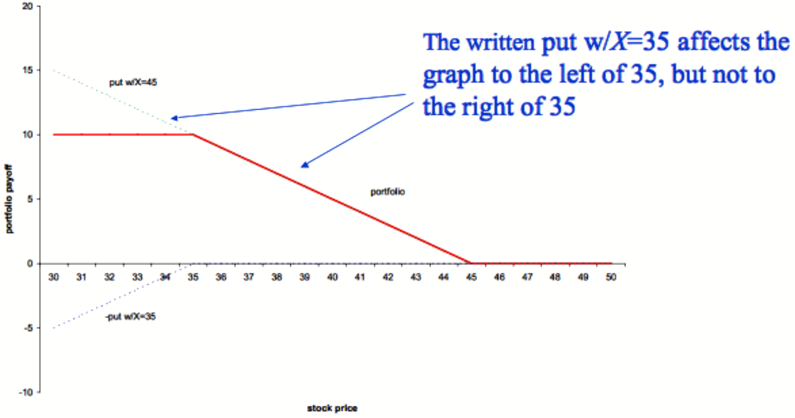
# Drawing the payoff diagrams

Using calls, work from left to right; puts, right to left



# Drawing the payoff diagrams

Add the (written) put w/X=35 this affects nothing to the right of 35 by working from right to left, adding the option doesnt affect anything we did before.



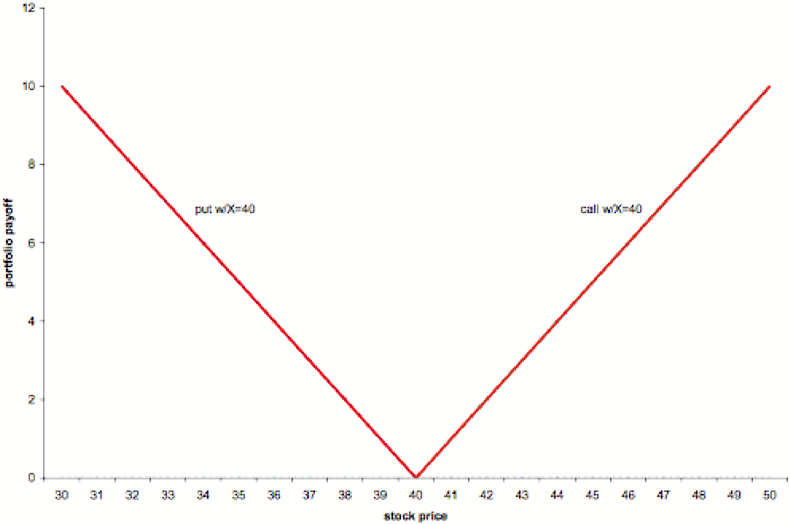
# Slopes and Combinations

- Note that **the slope** of the line is the size of the (in-the-money) option positions for that stock. In the above example the slopes were -1 and zero. So for S between 35 and 45 you are (net) short one call ($K = 35$) or long one put ($K = 45$).

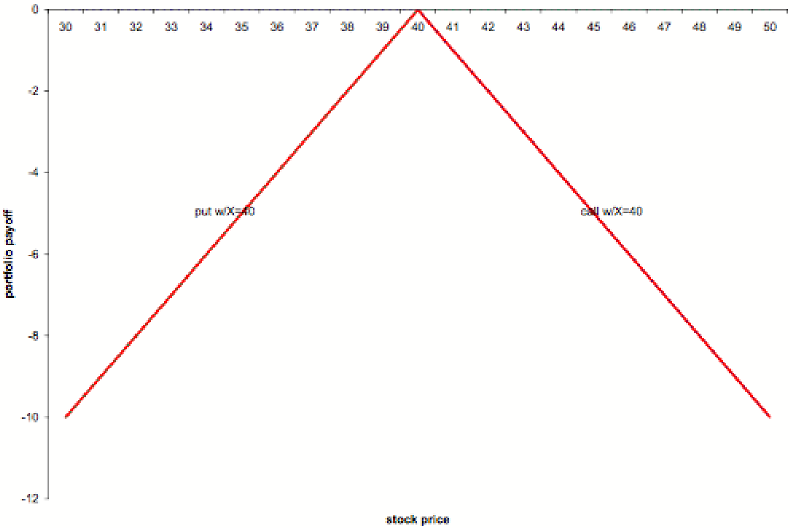
Combinations:

- Combination: options of different types (i.e., put and call) on same stock, both options are either bought or written
- Straddle: bought put and call on w/same strike (written straddle is written put and call)
- Strangle: bought put and call w/different strikes.

# Bought straddle



# Written straddle



# QUIZ

Who would be interested writing in a straddle? Someone who thinks that...

- The stock price will go up a lot.
- The stock price will go up a little.
- The stock price will go down a lot.
- The stock price will go down a little.
- Volatility will be high.
- Volatility will be low.

# Using options to create specific payoffs

- We looked at the 'named' positions for two reasons:
  - You should be somewhat familiar w/these standard positions
  - They provide some practice in working with payoff diagrams
- But, we are interested in a more important skill - to understand how options can be used to create (almost) any payoff profile.

# Using options to create specific payoffs

- Suppose your customer/client wants to make a particular bet, where a 'bet' corresponds to a payoff profile.
- You want to know what options portfolio corresponds to the payoff profile
- This is obviously useful if you/the client will trade in the options.
- Alternatively, if the bet will take the form of a customized derivative contract (e.g., a swap), the interpretation of the transaction as a portfolio of options is useful in the pricing and hedging of the transaction.

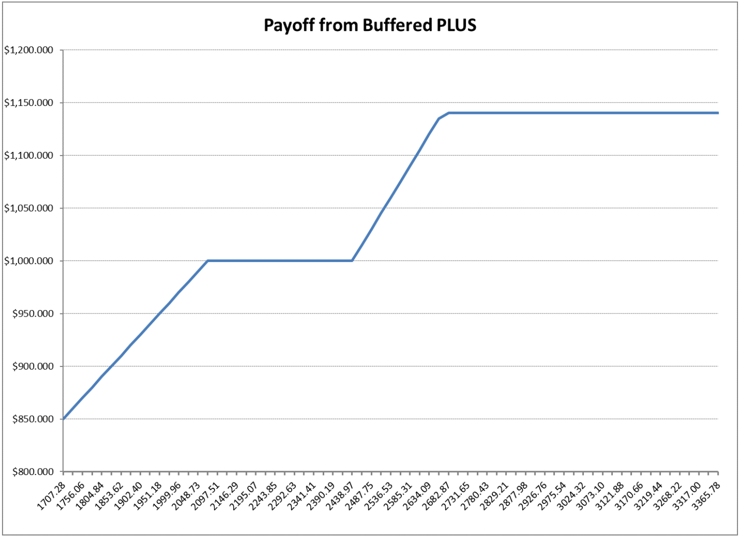
# Example

- Morgan Stanley issued Buffered PLUS Contracts denominated in USD (full details are on COMPASS) that matured on August 23 , 2019 and had payoffs based on the value of the S & P 500 Index on August 23, 2019. These notes were marketed and priced on August 24, 2017, on which date the S & P 500 index closed at 2438.97.
- The payoff on August 23, 2019 is as follows:

| Scenario | Payoff |
|---|---|
| The final index value (Aug 23) is greater than the initial index value. | \$1000 + Leveraged Upside Payment, up to a maximum of \$1140.40. |
| The final index value (Aug 23) is less than or equal to the initial index value but has decreased from the initial index value by an amount less than or equal to 15%: | \$1000 |
| If the final index value (Aug 23) is less than the initial index value and has decreased from the initial index value by an amount greater than 15%: | (\$1000 x Index performance factor) + \$150 |

Leveraged Upside payment = \$1000 $\times$ 1.5 $\times$ Index return
The index return = (final index value - initial index value) / initial index value
Index performance factor = (final index value) / initial index value

# Draw the payoff diagram



# Example

- Now we can recreate this diagram by analyzing the gradients of the lines. One way of recreating it is:
  1. a zero-coupon bond with a face value of \$1000 maturing on 23 Aug 2019
  2. (1000/2438.97) written SPX puts expiring on 23 Aug 2019 with strike price of $0.85 \times 2438.97 = 2037.1245$
  3. $1.5 \times (1000/2438.97)$ purchased SPX calls expiring on 23 Aug 2019 with strike price of 2438.97.
  4. $1.5 \times (1000/2438.97)$ written SPX calls expiring on 23 Aug 2019 with strike price of $1.0936 \times 2438.97 = 2667.257592$
- Other portfolios can also work, based on put call parity.
- **Next, to value the product we need a way of valuing the options . . .**

# QUIZ

Make the payoff using only bonds and put options:

- .
- .
- .
- .

# Black-Scholes-Merton formula

- For a given current stock price, $S_t$ at time $t$ the Black-Scholes-Merton formula gives us the value of call and put options

$$C(S_t, t) = S_t N(d_1) - e^{-r(T-t)} K N(d_2)$$

where

$$\begin{aligned} d_1 &= \frac{\ln(S_t/K) + (r + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}} \\ d_2 &= \frac{\ln(S_t/K) + (r - \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}} = d_1 - \sigma\sqrt{T-t} \end{aligned}$$

where $\sigma$ is the volatility of the stock price, $r$ is the continuously compounded risk-free rate, $T$ is the maturity time measured in years.

- For a put option we have

$$P(S_t, t) = -S_t N(-d_1) + e^{-r(T-t)} K N(-d_2)$$

where $d_1$ and $d_2$ are defined as above.

# Black-Scholes-Merton formula with dividends

- If the underlying asset (perhaps a stock index) pays a flow of dividends at an annualized constant rate $\delta$, then the call option value is:

$$C(S_t, t) = Se^{-\delta(T-t)}N(d_1) - e^{-r(T-t)}KN(d_2)$$

where

$$\begin{aligned} d_1 &= \frac{\ln(S_t/K) + (r - \delta + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{(T-t)}} \\ d_2 &= \frac{\ln(S)t/K) + (r - \delta - \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{(T-t)}} = d_1 - \sigma\sqrt{T-t}. \end{aligned}$$

- For a European put option,

$$P(S_t, t) = -S_t e^{-\delta(T-t)}N(-d_1) + e^{-r(T-t)}KN(-d_2)$$

where $d_1$ and $d_2$ are defined as above.

# Interpretation

As it must, Black-Scholes formula has a probabilistic interpretation. With no dividends:

$$
\begin{aligned}
C(S_t, t) &= S_t N(d_1) - e^{-r(T-t)} K N(d_2) \\
C(S_t, t) &= e^{-r(T-t)} \left[ S_t e^{r(T-t)} N(d_1) - K N(d_2) \right]
\end{aligned}
$$

- $e^{-r(T-t)}$ is the discount.
- $S_t e^{r(T-t)} N(d_1)$ is the risk-neutral expected value of $S_T$ conditional on $S_T > K$ at maturity.
- $-K$ is the amount that you pay.
- $N(d_2)$ is the risk-neutral probability that $S_T > K$.

# Interpretation

And with dividends,

$$
\begin{aligned}
C(S_t, t) &= S_t e^{-\delta(T-t)} N(d_1) - e^{-r(T-t)} K N(d_2) \\
C(S_t, t) &= e^{-r(T-t)} \left[ S_t e^{(r-\delta)(T-t)} N(d_1) - K N(d_2) \right]
\end{aligned}
$$

where now

- $S_t e^{(r-\delta)(T-t)} N(d_1)$ is the risk-neutral expected value of $S_T$ conditional on $S_T > K$ at maturity.

# QUIZ

What is $N(-d_2)$

- The probability that $S_T > S_t$.
- The probability that $S_T < S_t$.
- The probability that $S_T > K$.
- The probability that $S_T < K$.

# Input parameters

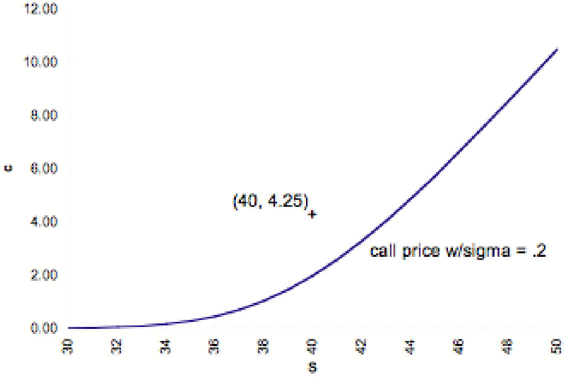
- The Black-Scholes formulae require six input parameters: $S_0, K, T, r, d$ and $\sigma$.
- The first three are very easy to find simply being the current value of the underlying asset, the exercise price of the option and the remaining time to expiry (this is usually denoted as $(T - t)$ where the option is issued at $t = 0$ and the current time is $t$).
- r is the annual continuously compounded risk-free rate of interest, typically the LIBOR rate for the appropriate period, either quoted or from swap data. This is then converted to a continuously compounded rate.
- $d$ is the continuous dividend yield. This can be estimated from historical data, or more usually from futures prices, where we know that $F_{t,T} = S_t e^{(r-d)(T-t)}$. Thus, knowing the futures price gives us an estimate for $d$.
- $\sigma$ is the annual standard deviation of the underlying asset's continuously compounded return. How to estimate this?

# Implied volatility

- Historical estimates of volatility are inherently backward-looking.
- Is it possible to develop a forward-looking volatility estimate?
- Suppose
  - $S_t = 40$; $K = 40$; $r = 0.04$; $T - t = 0.3$; $C(S_t, t) = 4.25$
  - The next page graphs the point $(S_t = 40, C(S_t, t) = 4.25)$, along with a function showing the theoretical value of a call option when $\sigma = 0.20$.

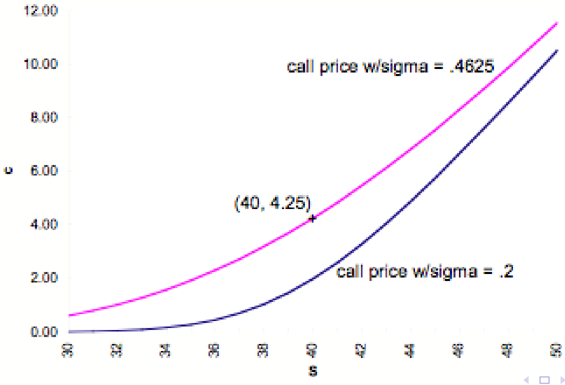
# Implied volatility

- Actual option price plus theoretical prices for $\sigma = 0.2$.
- We can see from the graph that 0.20 or 20% is not the market's estimate of volatility over the life of the option.



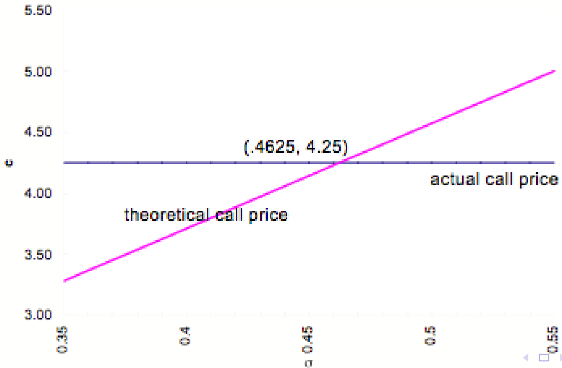
# Implied volatility

- Actual option price, along with theoretical prices for $\sigma = 0.2$ and $\sigma = 0.4625$.
- We see from this graph that a volatility of 0.4625 or 46.25% is implicit in the price of the option.



# Implied volatility

- Graph shows the actual and theoretical option prices as a function of the volatility input into a pricing model (The $\sigma$ on the horizontal axis is not the actual volatility, but the input you use with the theoretical pricing model. Thus, the theoretical price depends on $\sigma$, but the actual market price does not.)



# Implied volatility calculation

- Let $\sigma$ denote the volatility input.
- $c(\sigma)$ denote the theoretical option price, as a function of the volatility input.
- $c_{actual}$ denote the actual market option price, which does not vary with $\sigma$.
- The implied volatility is the $\sigma$ that satisfies equation:

$$c(\sigma) - c_{actual} = 0$$

# Outline

- We have seen an introduction to the payoffs from call and put options.
- We have seen the put-call parity relationship and how this relates call and put prices, options to futures/forwards and how to synthetically create a European option payoff.
- Then we looked at combinations of options, using them for insurance for hedging or for more complex trading strategies.
- Finally, we had a brief introduction to the Black-Scholes formula and implied volatility.