### Problem Set 2

Total 15 points

Due Tuesday, 6 February 2018 before 11:59pm

*Instructions*: This is a group assignment.

Handwritten solutions are acceptable here, please either photograph them to submit electronically or hand them to me in class on Monday. Of course, you can also insert the photos of your handwritten solutions into Excel and upload them electronically.

*(a) The file name should be: PSn.FAMILYNAME1Initial1.doc, for example my solution to Problem set 2 would have the file name PS1.WIDDICKSM.doc.*

*(b) Please make sure that your solutions are well-organized and clear, with appropriate text, explanations, and formatting. If I cannot figure out what you did, then what you did is wrong.*

**Digital Options (Black-Scholes adaptation)**

**Digital options** have payoffs as follows: Calls:



Puts



with exercise price K, cash amount A and time to maturity T.

1. By considering the Black-Scholes equation on a dividend paying stock, write down the value of a Digital puts at t = 0.
2. What portfolio of risk-free assets and digital calls replicates the payoff of a digital put?

JP Morgan are issuing **Dual Directional Trigger jump securities** contracts on the EuroStoxx[[1]](#footnote-1) index documented on Page 4 I also give you interest rate and option data to help you answer. You have determined that the continuously compounded dividend yield is 1%.

1. Find a portfolio of zero-coupon bonds and European options (calls, puts, digitals) that can replicate the payoff of the note. Calculate the estimated value of the Plus (your answer should be between $9.50 and $9.60.
2. Your would like to know the probability that their payoff from this note is less than the face value ($10). What information would you give them?

**Link between binomial model and Black-Scholes PDE (Coming Soon!)**

**2.**  (3 points) Consider a simplified binomial model with time step of *dt* where , (note that this is the CRR model for very small time steps dt). The option value today is *V = V(S,t),* option value in the up-state is *V+ = V(uS, t+dt*), and option value in the down-state is *V- = V(dS, t+dt)* show that in the limit of *dt 🡪 0* we recover the Black-Scholes partial differential equation:



*Hint*. This question is asking you to carry out a small bit of analysis. Expand *V*+ and *V*- in Taylor series (your Taylor series needs to be second order in *S* and first order in *t*), substitute the expansions into the standard binomial pricing formula,

substitute in the definition of q and expand all of the exponential terms using *ex = 1 + x*. With any luck, you should obtain the partial differential equation above.

**Practice with the Binomial spreadsheet**

**3.** (6 points) Adapt the trees from ImpBinomial.xls to value a slightly simplified version of this product:

<https://www.sec.gov/Archives/edgar/data/1114446/000091412118000095/ub40996089-424b2.htm>

Bank of America (BAC) product. I have tried to match the r, , and  as closely as possible!

Face value: $1000

Payoff at maturity: if the stock price is greater than or equal to $31.19 then you receive the face value plus the final coupon payment. If the stock price is below $ you receive a cash amount equivalent to 1000/31.19 = 35.6252 stocks plus the final coupon payment.

Autocall feature: If the stock price is greater than or equal to the initial price on any of the observation dates t = 1/4, 1/2, 3/4 then the notes are immediately called for the face value + coupon.

Coupons: there is a monthly coupon of 6.40% (this is an annual figure) of the face value, payable at t = 1/12, 2/12 etc.

To set up your tree:

Choose N = 50 (i.e the same as in Impbinomial)

S0 = 31.19

T = 1.

r = 1.966%

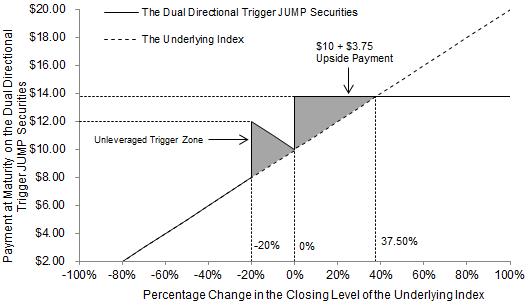
 = 24.650%

Proportional Dividends: 1.75% (annual figure) quarterly at t = 0.15. 0.4, 0.65, 0.9, so assume 0.25 x 1.75% is paid each quarter.

Use the same u, d formulas as given in the Impbinomial spreadsheet.

**Information for Question 1**

|  |  |
| --- | --- |
| **FINAL TERMS** | |
| **Issuer:** | JPMorgan Chase Financial Company LLC |
| **Underlying index:** | EURO STOXX Banks® |
| **Payment at maturity:** | If the final index value is *greater than or equal to*the initial index value, for each $10 stated principal amount security: |
|  | $10 + upside payment |
|  | If the final index value is *less than*the initial index value but is *greater than or equal to* the trigger level, for each $10 stated principal amount security: |
|  | $10 + ($10 × absolute index return) |
|  | *In this scenario, you will receive a 1% positive return on the securities for each 1% negative return on the underlying index.  In no event will this amount exceed the stated principal amount plus $2.00.* |
|  | If the final index value is *less than*the trigger level, for each $10 stated principal amount security: |
|  | $10 × index performance factor |
|  | *This amount will be less than the stated principal amount of $10 per security and will represent a loss of more than 20%, and possibly all, of your investment.* |
| **Upside payment:** | $3.75 per security (37.50% of the stated principal amount). |
| **Index percent change:** | (final index value – initial index value) / initial index value |
| **Absolute index return:** | The absolute value of the index percent change.  For example, a -5% index percent change will result in a +5% absolute index return. |
| **Initial index value:** | The closing level of the underlying index on the pricing date, which was 150 |
| **Final index value:** | The closing level of the underlying index on the valuation date |
| **Trigger level:** | 120, which is 80% of the initial index value |
| **Index performance factor:** | final index value / initial index value |
| **Stated principal amount:** | $10 per security |
| **Issue price:** | $10 per security (see “Commissions and issue price” below) |
| **Pricing date:** | November 30, 2017 |
| **Original issue date (settlement date):** | November 30, 2017 |
| **Valuation date:** | May 30, 2020 |
| **Maturity date:** | May 30, 2020 |
|  |  |
|  |  |



**Interest rate data**

|  |  |  |  |
| --- | --- | --- | --- |
| date quoted | days | maturity date | **USD LIBOR rate (Simple interest, Actual/360 day count)** |
| 20171130 | 7 | 7-Dec-17 | **1.0000%** |
| 20171130 | 31 | 31-Dec-17 | **1.0135%** |
| 20171130 | 33 | 2-Jan-18 | **1.0146%** |
| 20171130 | 68 | 6-Feb-18 | **1.0342%** |
| 20171130 | 96 | 6-Mar-18 | **1.0499%** |
| 20171130 | 124 | 3-Apr-18 | **1.0656%** |
| 20171130 | 159 | 8-May-18 | **1.0852%** |
| 20171130 | 187 | 5-Jun-18 | **1.1009%** |
| 20171130 | 278 | 4-Sep-18 | **1.1520%** |
| 20171130 | 369 | 4-Dec-18 | **1.2030%** |
| 20171130 | 460 | 5-Mar-19 | **1.2540%** |
| 20171130 | 566 | 19-Jun-19 | **1.3135%** |
| 20171130 | 642 | 3-Sep-19 | **1.3561%** |
| 20171130 | 733 | 3-Dec-19 | **1.4071%** |
| 20171130 | 824 | 3-Mar-20 | **1.4582%** |
| 20171130 | 912 | 30-May-20 | **1.5075%** |
| 20171130 | 1013 | 8-Sep-20 | **1.5642%** |

**Option Price Data**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| K | Call | Put | N(d1) | N(d2) |
| *120* | 31.85 | 11.98 | 0.72294 | 0.57166 |
| *127.5* | 27.94 | 15.29 | 0.67151 | 0.51320 |
| *135* | 24.43 | 19.00 | 0.61982 | 0.45780 |
| *150* | 18.55 | 27.57 | 0.51940 | 0.35855 |
| *157.5* | 16.12 | 32.37 | 0.47207 | 0.31524 |
| *165* | 13.99 | 37.46 | 0.42729 | 0.27617 |
| *172.5* | 12.12 | 42.82 | 0.38536 | 0.24121 |
| *179.25* | 10.65 | 47.84 | 0.35018 | 0.21307 |
| *187.5* | 9.08 | 54.22 | 0.31054 | 0.18265 |
| *195* | 7.85 | 60.21 | 0.27768 | 0.15848 |
| *208.5* | 6.03 | 71.40 | 0.22584 | 0.12229 |

1. Technically the Eurostoxx index is denominated in Euros but the payoff of the security is in USD. This makes this product a “quanto” which makes the valuation a little more challenging. For now just treat the Eurostoxx product as if it were a regular US index like the S&P 500. We will see quantos later in the course. [↑](#footnote-ref-1)