NPV, DISCOUNTING, AND THE PRICE OF RISK

NPV

- A decision will increase the value of the firm if:
  Value of benefits > Value of costs
- But how to value/compare benefits and costs…
  - that occur at **different times**?
  - that are more or less **uncertain**?
  - that have different **sensitivity to systematic risks**?
- Main tool: Net Present Value
  NPV = Present Value (Benefits) – Present Value (Costs)
- The **NPV decision rule** says that we should:
  - **Accept** all projects with **NPV>0**
  - **Reject** all projects with **NPV<0**
- We get present values by taking any stream of expected benefits or costs and **discounting** them

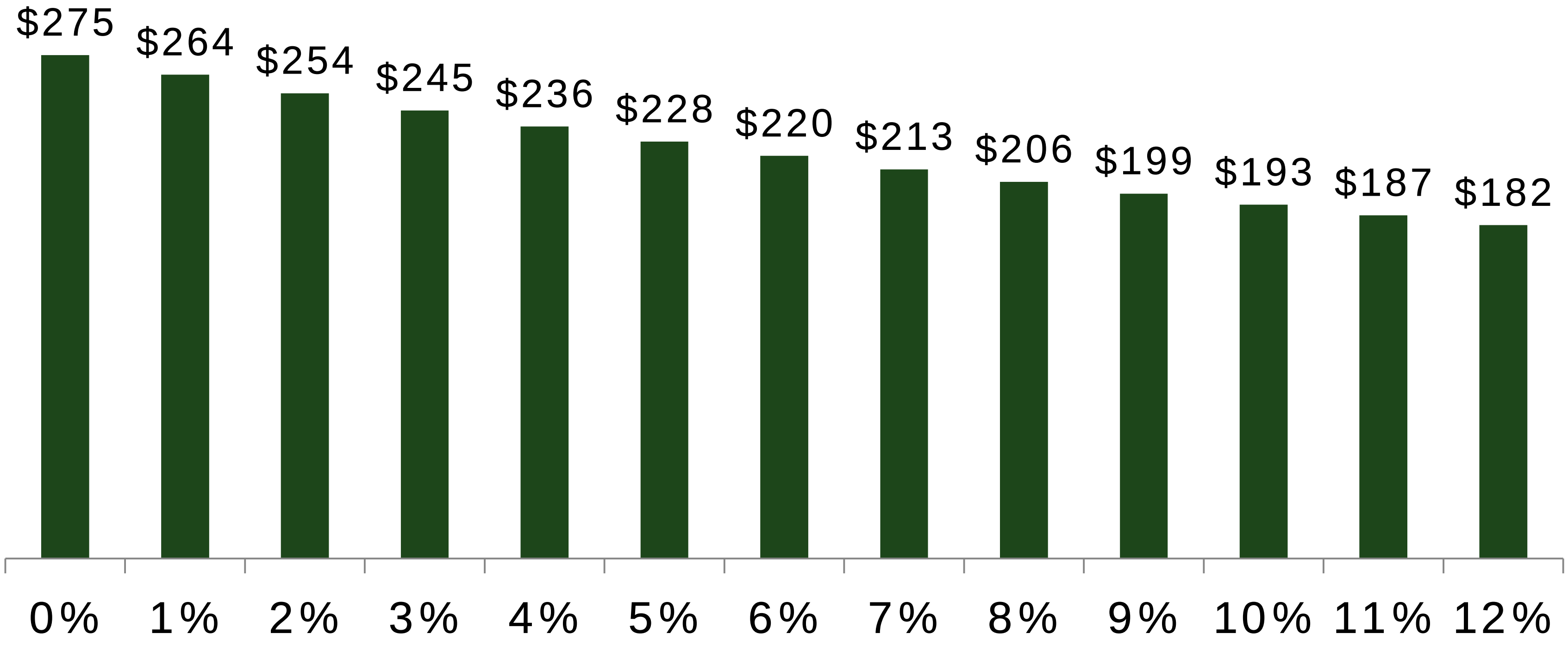
Example: How should we value a stream of cash flows that takes place over time?

Alex Rodriguez “$275 million” contract with the Yankees

Year	Salary	"Signing bonus"	With no discounting	Present value (at 10%)
2008	27	2		29.0
2009	32	1		30.0
2010	32	1		27.3
2011	31	1		24.0
2012	29	1		20.5
2013	28	1		18.0
2014	25	3		15.8
2015	21			10.8
2016	20			9.3
2017	20			8.5
Total	*265*	*10*	*275*	*193.2*

The PV of payments to Rodriguez

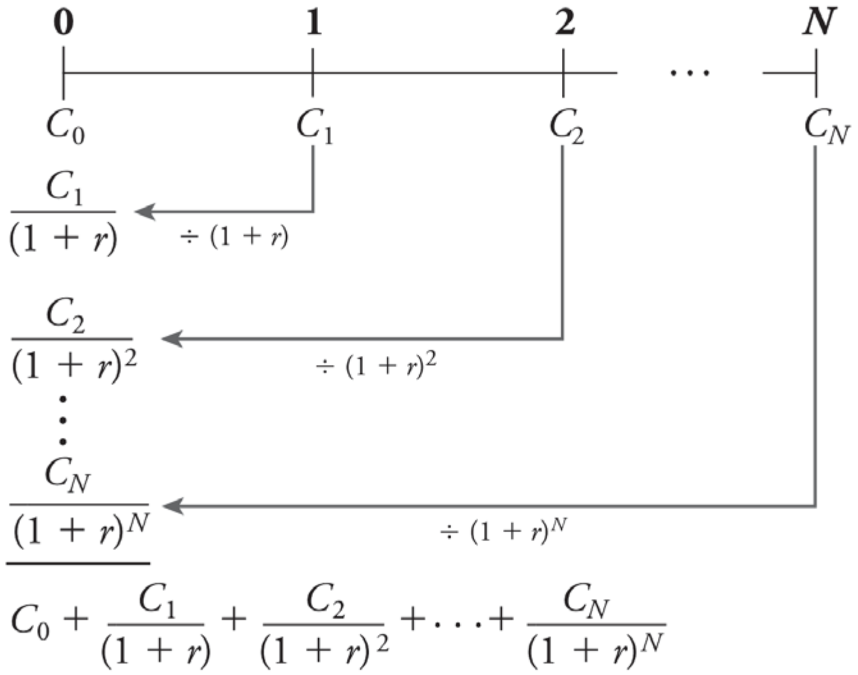
(the calculation is made as if in 2008)



What is the correct discount rate for A-Rod’s contract? Why?

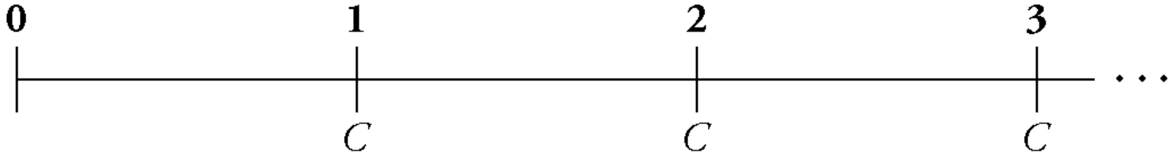
Discounting

- "Discount rates" convert future dollars into dollars today
  - — Idea, a dollar today is worth more than a dollar in the future, but exactly how much more?
- If there are many cash flows in the future, we discount each cash flow separately and then sum up the discounted values
  - — We might ideally want to use a different *r* for every cash flow to reflect the "term structure"



Perpetuities

- A **perpetuity** is a stream of **equal** cash flows that occur at **regular intervals** and **last forever**



Note: The first cash flow does not occur immediately, but at the **first period**

- Present Value of a Perpetuity:

PV(perpetuity)$= \frac{C}{r}$

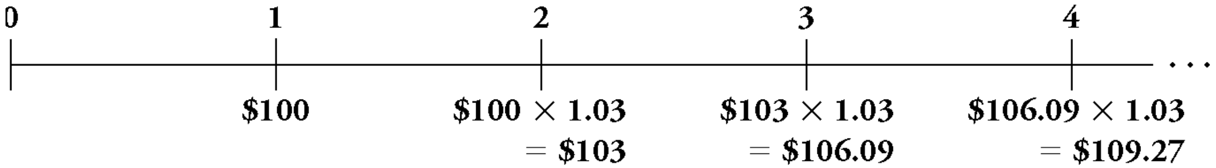
Annuities

- An **annuity** is a stream of ***N*** equal cash flows paid at regular intervals

- PV(annuity)$= \frac{C}{r}\left(1 - \frac{1}{(1+r)^N}\right)$
  (Hint to remember formula: This is difference between PV of a perpetuity that starts now, minus the PV of a perpetuity that starts in N periods)

Growing Perpetuities

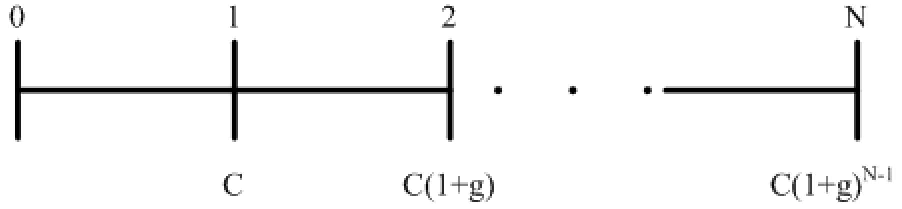
- A growing perpetuity is a stream of perpetual cash flows that occur at regular intervals and **grow at a constant rate forever**
- For example, a growing perpetuity with a first payment of $100 that grows at a rate of 3%:



- PV(growing perpetuity)= $\frac{C}{r-g}$

Growing Annuities

- A growing annuity is a stream of ***N* growing cash flows**, paid at regular intervals



- PV(growing annuity)$= \frac{C}{r-g}\left(1-\left(\frac{1+g}{1+r}\right)^{N}\right)$

Take the lump sum or annuity? (1)

- The Robinson’s of Munford, Tennesse were one of the winner’s in the January 2016 Powerball jackpot
- Choice: Lump sum of $327 million or 30 installments totaling $528.8 million
- Which option should they take?

Photo: AP/Mark Humphrey

Annuity structure

Year	Payout
0	9.43
1	9.81
2	10.20
…	…
27	27.19
28	28.27
29	29.40
Total	**528.8**

Undiscounted total is $528.8 million
Not evenly divided, but a growing annuity (g=4%)

Take the lump sum or annuity? (2)

Considerations:

- Any risk with the annuity?
- What's the correct discount rate?

  For benchmark rates, see http://online.wsj.com/mdc/public/page/2_3022-bondbnchmrk.html
- What if they can invest in the stock market at an expected return of 7%?
- Present value (PV) of the annuity?

Example: Funding an endowed chair

- You want to endow a chair for a famous finance professor at Illinois
  - You want the endowment to add \$100,000 per year to the faculty member's resources (salary, conference travel, purchase of data, etc.)
  - You also want the funding to increase 2% per year to account for inflation
- You expect to earn a rate of return of 4% annually on the endowment
- How much will you need to donate to fund the chair?

Solution:

- The cost of the endowment will start at \$100,000, and increase by 2% each year. This is a growing perpetuity:
- PV(growing perpetuity)$= \frac{C}{r-g} = \frac{100{,}000}{0.04-0.02} = \$5 \text{ million}$

THE PRICE OF RISK

Flashback to Investments: No Arbitrage and the Risk Premium

- "Risk Premium"
  - Additional expected return that investors require to compensate for risk
- The risk premium of a security is determined only by its systematic risk!
- The risk premium for diversifiable/idiosyncratic risk is zero; i.e., investors will not get a premium for taking on such risk
  - Why? Because investors can easily eliminate idiosyncratic risk by diversifying
- Consider a "proof by contradiction":
  - Suppose that idiosyncratic risk of some investment was rewarded with a return premium; then investors could buy these investments, earn the additional premium, but diversify across the investments to eliminate all the risk
  - With this strategy, investors could earn higher returns without taking on any additional risk!
  - Everyone would want to buy these investments, so investors would bid up their prices
  - As they get more expensive, they earn smaller and smaller returns, until they no longer had a return premium and the arbitrage opportunity is eliminated

The Risk Premium

- When an investment is risky, to compute its PV we must discount the *expected cash flow* at the rate:
  r = risk-free interest rate (to account for the time value)
      + risk premium (to account for the systematic risk)
- If an investment is risky but has only idiosyncratic risk, what's the correct discount rate?
  - Where is the idiosyncratic risk captured if not in the discount rate?

Example: Risky Cash Flows

- Suppose there is a 50/50 probability of either "State A" or "State B" happening next year, and the risk-free interest rate is 4%
- Below are the Cash Flows (in $) of:
  - X: A risk-free security (always pays the same, regardless of state)
  - Y: A risky security (pays more in state A than in state B)
- What is the value of these securities?

	Cash flow in one year	
	State A	**State B**
X: Risk-free security	1100	1100
Y: Risky security	1400	800

Example: Risky Cash Flows (cont.)

- The Risk-free Security always pays \$1,100
- We can value it by discounting using the one-year risk-free rate:
  Price(risk-free security) = \$1,100/1.04=\$1,056

- For the Risky Security, we first calculate the *expected cash flow:*
  Expected cash flow(risky security)=½ (\$1400) + ½ (\$800) = \$1,100
- To get the current value of this security, we need to discount the expected cash flow
- But what is the appropriate discount rate?

- Recall that the appropriate discount rate is:
  r = Risk-free rate + Risk premium,
  so if we know the risk-free rate (e.g. from Treasuries with a similar one-year maturity), we just need to figure out the risk premium

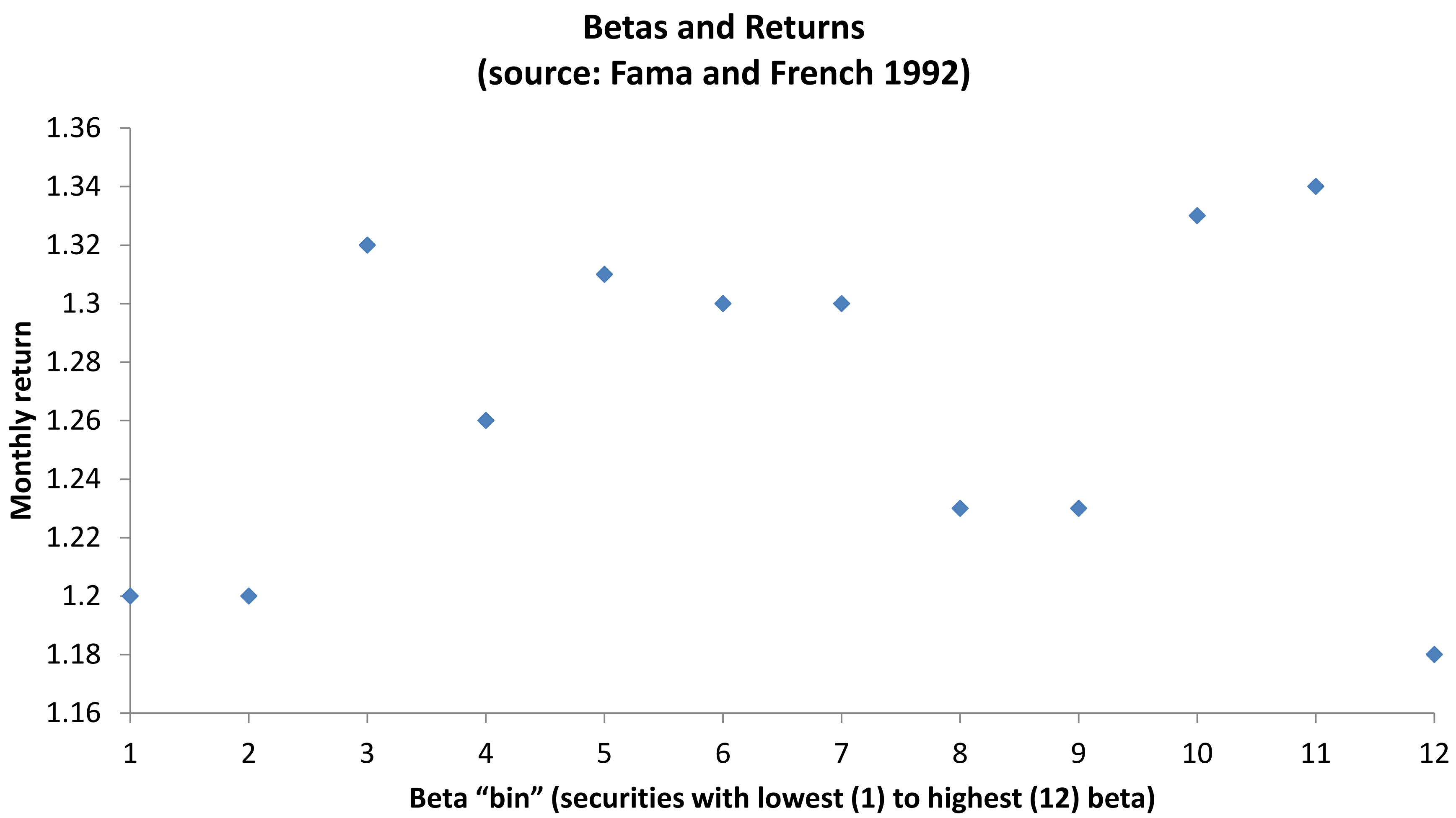
Example: Risky Cash Flows (cont.)

- The risk premium depends on how risky security *correlates* with other risk factors that investors are concerned about
  - Risk premium has the form ($Beta_{R[security],R[risk\ factor]}$)*(Premium per unit of beta)
  - For example, in CAPM: how the payoff from an investment correlates with the stock market
  - There are also alternative risk models for systematic risk, and we can have more than factor (with a beta and premium)
    - e.g., the Fama-French 3-factor model
- Suppose security Y, the risky security, is positively correlated with the stock market (*i.e.*, beta>0), then CAPM predicts a positive risk premium
  - Suppose this risk premium is 6% (e.g, a beta of 1.2 and a market risk premium of 5%), then the correct discount rate is: 4%+6%=10%
  - Then the value today of the risky security is: Value(risky security) = Expected cash flow/(1+r) = $1,100/1.10=$1,000
  - Thus, the risky security is worth less than the risk-free security even though the expected cash flow from both are the same!
- What if the payoff from Y is uncorrelated with the stock market (beta=0)?
- Or what if it is negatively correlated with the stock market (beta<0)?

CAPM

- We will mostly use CAPM in the course, but, CAPM has a big problem!
- What is the main empirical prediction in CAPM?

$$E[R_i] = R_f + \beta_{i,market} * (E[R_{market}] - R_f)$$


Betas and Returns
(source: Fama and French 1992)
Monthly return
1.36
1.34
1.32
1.3
1.28
1.26
1.24
1.22
1.2
1.18
1.16
1
2
3
4
5
6
7
8
9
10
11
12
Beta “bin” (securities with lowest (1) to highest (12) beta)

Fama-French 3-factor model

- So if CAPM doesn't work, are there any alternatives?
- Most academics and many practitioners use the **Fama-French 3-factor model** as a benchmark:

$$E[R_i] = R_f + \beta_{i,market} * \left(E[R_{market}] - R_f\right) + \beta_{i,SMB} * (E[R_{SMB}]) + \beta_{i,HML} * (E[R_{HML}])$$

- Equation starts out as CAPM but adds two new "risk factors":
  - How security *i* correlates with a portfolio called "small minus big" that is long "small stocks" (low market cap) and short "big stocks" (high market cap)
  - How security *i* correlates with a portfolio called "high minus low" that is long "value stocks" (high B/M) and short "growth stocks" (low B/M)
  - $E[R_{SMB}]$ and $E[R_{HML}]$ are positive and quite large (especially the premium on HML)
  - Empirically, this model works much better than CAPM!
- Many don't stop here, but add a "momentum factor", a "liquidity factor", etc…
  - All these alternative models work the same, just add more terms to the expected return of *i*!