FIN 591: Homework #3

Due on Wednesday, April 11, 2018

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Problem 1

a. Since the final payoff of P is 1, using continuous-time version stochastic discount factor, $P_t(\tau)$ is derived as follows.

$$
\begin{aligned}
P_t(\tau) &= \mathrm{E}_t\left[\frac{U_c(C_{t+\tau,t+\tau})}{U_c(C_t,t)} \times 1\right] \\
&= \mathrm{E}_t\left[\frac{e^{-\phi(t+\tau)}C_{t+\tau}^{\gamma-1}}{e^{\phi t}C_t^{\gamma-1}}\right] \\
&= \mathrm{E}_t\left[e^{-\phi\tau}\frac{C_{t+\tau}^{\gamma-1}}{C_t^{\gamma-1}}\right]
\end{aligned}
\tag{1}
$$

b. From $P_t(\tau) = \mathrm{E}_t\left[\frac{e^{-\phi(t+\tau)}C_{t+\tau}^{\gamma-1}}{e^{\phi t}C_t^{\gamma-1}}\right]$, we can find that process M_t is equal to $e^{-\phi t}C_t^{\gamma-1}$. Therefore, using Ito's lemma, dynamics of M_t can be derived as equation (2).

$$
\begin{aligned}
dM_t &= -\phi e^{-\phi t}C^{\gamma-1}dt + e^{-\phi t}(\gamma-1)C^{\gamma-2}C[(\mu_c - \lambda k)dt + \sigma_c dZ_c] \\
&\quad + \frac{1}{2}e^{-\phi t}(\gamma-1)(\gamma-2)C^2C^{\gamma-3}\sigma_c^2 dt + [e^{-\phi t}(YC)^{\gamma-1} - e^{-\phi t}C^{\gamma-1}]dq \\
&= [-\phi + (\gamma-1)(\mu_c - \lambda k) + \frac{1}{2}(\gamma-1)(\gamma-2)\sigma_c^2]Mdt + (\gamma-1)\sigma_c MdZ_c + (Y^{\gamma-1} - 1)Mdq
\end{aligned}
\tag{2}
$$

c.

d.