FIN 591: Homework #1

Due on Monday, February 5, 2018

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Problem 1

a. *(Short selling restriction)* If an investor cannot sell short risky asset, the amount of investment in risky asset should be nonnegative. Therefore, the maximization problem is stated as the following equation.

$$\begin{aligned} &\max_{A} \mathrm{E}[U(\widetilde{W})] = \max_{A} \mathrm{E}[U(W_0(1+r_f) + A(\widetilde{r} - r_f))] \\ &\text{subject to } A \geq 0 \end{aligned} \tag{1}$$

The maximization problem (1) is equivalent to

$$\max_{A} \mathrm{E}[U(W_0(1+r_f) + A(\widetilde{r} - r_f))] + \lambda A \tag{2}$$

By applying Kuhn-Tucker conditions, the value of A maximizing expected utility must must satisfy the following conditions.

$$\begin{aligned} \mathrm{E}[U'(\widetilde{W})(\widetilde{r} - r_f)] + \lambda = 0 \\ A \geq 0 \\ A\mathrm{E}[U'(\widetilde{W})(\widetilde{r} - r_f)] = 0 \end{aligned} \tag{3}$$

b. A restriction of riskless borrowing implies that the dollar amount of investment in riskless asset should be nonnegative. It leads to the following maximization problem.

$$\begin{aligned} &\max_{A} \mathrm{E}[U(\widetilde{W})] = \max_{A} \mathrm{E}[U(W_0(1+r_f) + A(\widetilde{r} - r_f))] \\ &\text{subject to } W_0 - A \geq 0 \end{aligned} \tag{4}$$

From the analogy of problem 1.a, the value of A which maximizes expected utility must satisfy the following first order conditions.

$$\begin{aligned} \mathrm{E}[U'(\widetilde{W})(\widetilde{r} - r_f)] + \lambda = 0 \\ W_0 - A \geq 0 \\ (W_0 - A)\mathrm{E}[U'(\widetilde{W})(\widetilde{r} - r_f)] = 0 \end{aligned} \tag{5}$$

Problem 2

Problem 3

Problem 4

a.

b.