FIN 591: Homework #4

Due on Monday, April 30, 2018

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Problem 1

Assuming $\theta = 1$, V_t is represented as follows.

$$V_t = [C_t^\gamma + \delta E_t[V_{t+1}^\gamma]]^{\frac{1}{\gamma}} \tag{1}$$

Solving the difference equation iteratively, we can solve V_t as follows.

$$\begin{aligned} V_t &= [C_t^\gamma + \delta \mathrm{E}_t[V_{t+1}^\gamma]]^{\frac{1}{\gamma}} \\ &= [C_t^\gamma + \delta \mathrm{E}_t[\{C_{t+1}^\gamma + \delta \mathrm{E}_{t+1}[V_{t+2}^\gamma]^{\gamma \times \frac{1}{\gamma}}\}]]^{\frac{1}{\gamma}} \\ &= [C_t^\gamma + \delta \mathrm{E}_t[C_{t+1}^\gamma + \delta \mathrm{E}_{t+1}[V_{t+2}^\gamma]]^{\frac{1}{\gamma}} \\ &= [C_t^\gamma + \delta \mathrm{E}_t[C_{t+1}^\gamma] + \delta^2 \mathrm{E}_t[V_{t+2}^\gamma]]^{\frac{1}{\gamma}} \end{aligned} \tag{2}$$

Using this procedure iteratively, V_t is derived as follows.

$$\begin{aligned} V_t &= [C_t^\gamma + \delta \mathrm{E}_t[C_{t+1}^\gamma] + \delta^2 \mathrm{E}_t[C_{t+2}^\gamma] + \delta^3 \mathrm{E}_t[C_{t+3}^\gamma] + \delta^4 \mathrm{E}_t[C_{t+4}^\gamma] + \dots]^{\frac{1}{\gamma}} \\ &= \sum_{i=t}^{\infty} \delta^i \mathrm{E}_t[C_i^\gamma]^{\frac{1}{\gamma}} \end{aligned} \tag{3}$$

Considering the discrete-time version of power utility $U(C_t, t) = \delta^t \times C_t^\gamma / \gamma$, γV_t^γ implies a lifetime utility function.

Problem 2