*Quiz 6 - Homogeneous ODEs*

Name: Will Buziak At the end of the quiz, scan a pdf of you paper and turn it in at the back. Then upload your scan to Canvas by 5pm.

1. a. What is the time constant $\tau$ of the following system? (1 pt)

$$\dot{x} + 2x = 0$$

$\ddot{y} + 2\zeta\omega_n\dot{y} + \omega_n^2 y = 0$

b. What is the general solution form for the system in 1 a.? (1 pt)

$y(t) = C_1 e^{P_1 t} + C_2 e^{P_2 t}$

$\zeta = 0 \quad \omega_n = \sqrt{2}$

2. Consider the following system:

$$\ddot{x} + \dot{x} + x = 0$$

a. What is the natural frequency $\omega_n$? (1 pt)

$\omega_n = 1$

$\frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm \sqrt{-3}}{2}$

b. What is the damping ratio $\zeta$? (1 pt)

$\zeta = 1/2$

c. What is the damped natural frequency $\omega_d$? (1 pt)

$\omega_d = \omega_n\sqrt{\zeta^2 - 1} = \sqrt{1/2^2 - 1} = \sqrt{3/4}$

c. What are the poles of the system? (1 pt)

$P_{1,2} = -1/2 \pm i\sqrt{3}/2$

d. According to your previous answers, is the system overdamped, critically damped, underdamped, or undamped? (1 pt) overdamped

3. Solve the following ODE subject to initial conditions $x(0) = 1$, $\dot{x}(0) = 0$. (3 pts)

$$\ddot{x} + x = 0$$

$0 = \sqrt{1}\left(\frac{t}{\sqrt{1}} - C_2\right)e^{\sqrt{1}t} - \sqrt{1}\,C_2 e^{-\sqrt{1}t}$

$0 = \ln(t - \sqrt{1}C_2)\sqrt{1}t - \ln(\sqrt{1}C_2)(-\sqrt{1}t)$

$= \ln(t - \sqrt{1}C_2)\cancel{\sqrt{1}t} + \ln(\sqrt{1}C_2)\cancel{(-\sqrt{1}t)}$

$C_2 = 0$

$\boxed{y(t) = \frac{t}{\sqrt{1}}e^{\sqrt{1}t} + 0}$

$\cancel{x}\ \ r^2 + 1$

$r = \pm\sqrt{1}$

$x(t) = C_1 e^{\sqrt{1}t} + C_2 e^{-\sqrt{1}t}$

$x(0) = 1 = C_1 e^{\sqrt{1}t} + C_2 e^{-\sqrt{1}t}$

$x'(0) = 0 = \sqrt{1}C_1 e^{\sqrt{1}t} - \sqrt{1}C_2 e^{-\sqrt{1}t}$

$\ln(1) = \ln(C_1)\cdot\ln e^{\sqrt{1}t} + \ln(C_2)\ln e^{-\sqrt{1}t}$

$\ln(1) = \ln(C_1)\sqrt{1}t - \ln(C_2)\sqrt{1}t$

$\frac{t}{\sqrt{1}} - C_2 = C_1$