H.W 9 ME 331

1. $r_c = 13$ $\omega = 15000$ rpm $q_{out} = 2900$ kJ/kg

$r = 80$ mm $h = 50$ mm

$$13 = \frac{\pi(80^2)\cdot 50}{V_{min}} + 1 \Rightarrow V_{min} = 83775 \text{ mm}^3 = V_3$$

$V_{max} = 1.0891e6 = V_1$

$= 1089100$ mm^3

$\Rightarrow 83775 = \pi 80^2 h_{min}$

$h_{min} = 4.1666$ mm

$\Rightarrow \boxed{h_{max} = 54.1666 \text{ mm}}$

$\Delta V = 1005325$ mm^3

$\eta_{TH} = 1 - \frac{1}{r^{k-1}} \Rightarrow 1 - \frac{1}{13^{.4}}$ → assuming perfect gas & air mixture acts as air

$\eta_{TH} = .642 \Rightarrow \boxed{64.2\%}$

$Q_H = C_v(T_3 - T_2)$ → $C_v = \frac{R}{k-1} = \frac{287}{.4}$

$T_1 = 285$ K, $P_1 = 101325$ Pa

$P_2 = P_1$

$T_2 = T_1(13)^{.4} = 285(13)^{.4}$

$2900 = C_v(T_4 - T_1)$ → $\frac{2900 + T_1 C_v}{C_v} = T_4$

$T_3 = T_4(13)^{.4}$

$W = Q_H - 2900$

$P = W \cdot \frac{15000}{60}$

2. $\varepsilon = 1$, $k = 1.4$, $T_C = 300\,K$ & $T_H = 1100\,K$

$T_1 =$ $T_3 =$

$$(\gamma_p)_{max} = \left(\frac{T_H}{T_C}\right)^{\frac{k}{2(k-1)}} = \left(\frac{1100}{300}\right)^{\frac{1.4}{2(.4)}} = 9.716$$

3. $T_1 = 300\,K$

$P_1 = 1\,bar \rightarrow P_2 = 9\,bar$

a) $PV^k = C$

$$w = -\frac{1.4}{.4}(287)(300)\left\{(9/1)^{\frac{.4}{1.4}} - 1\right\} = -263.2$$

$$T_2 = T_1(P_2/P_1)^{\frac{k-1}{k}} \rightarrow 562.03$$

$$.7178(562.03 - 300) + (-263.2) = \boxed{-75.12}$$

b)

$$w = RT_1 \ln(P_1/P_2) = -187.6$$

c) $$w_1 = -287(300)\left(\frac{1.4}{.4}\right)\left\{(3)^{.4/1.4} - 1\right\} = -111.12$$

$$\Rightarrow \boxed{-222.2}$$

4. $T_C = 300\ K$, $T_H = 1500\ K$ $r_p = 9.5$ $\dot{m} = 200\ kg/s$

$\eta_c = .83$ $\eta_t = .87$ $\varepsilon = .8$ $R = 287$ $k = 1.4$

a) $C_p = \frac{k}{k-1} \cdot R = 1004.5$

$T_{2s} = T_1 (r_p)^{\frac{k-1}{k}} = 570.78\ K$

$.83 = \frac{T_{2s} - T_1}{T_2 - T_1} \rightarrow T_2 = T_1 + \frac{(T_{2s} - T_1)}{\eta_c} = 626.24\ K$

$T_{4s} = \frac{T_3}{r_p^{\frac{k-1}{k}}} = 788.39\ K$

$T_4 = T_3 - \eta_T (T_3 - T_{4s}) = 680.9\ K$

$W_t = C_p (T_3 - T_4) = 621.88\ kJ/kg$

$W_c = C_p (T_2 - T_1) = 327.7\ kJ/kg$

$W_{net} = W_t - W_c = 294.7\ kJ/kg$

$\dot{W} = \dot{m} \cdot W_{net} = 200 (294.71) = 5894.2\ kW$

b) $\varepsilon = \frac{T_x - T_2}{T_4 - T_2} \rightarrow T_x = 629.97$

$\dot{Q} = \dot{m} C_p (T_3 - T_x) \rightarrow \dot{Q} = 133405.67$

$\eta = \dot{W}/\dot{Q} = .442$

5. $V_1 = 290$ m/s $T_1 = 230$ K $P_1 = 26.4$ kPa

$P_3/P_2 = 9$ $T_4 = 1500$ K $R = 287$ $k = 1.4$

$\eta_c = .8$ $\eta_T = .85$

$C_p = 1.0045$

$C_v = .7175$

$h_2 - h_1 = V_1^2/2 \rightarrow C_p(T_2 - T_1) = V_1^2/2$

$\Rightarrow T_2 = 271.86$ K

$T_2/T_1 = (P_2/P_1)^{k-1/k} \rightarrow P_2/P_1 = (T_2/T_1)^{k/k-1}$

$\frac{P_2}{26.4} = \frac{271.86}{230}^{1.4/.4}$

$P_2 = 47.4$ kPa

$T_{3s}/T_2 = |P_{3s}/P_2|^{\frac{k-1}{k}} \rightarrow 9^{.4/1.4}(271.86)$

$T_{3s} = 509.31 \Rightarrow .8 = \frac{509.31 - 271.86}{T_3 - 271.86}$

$T_3 = 568.67$

turbine:

$$c_p(T_4 - T_5) = c_p(T_2 - T_1)$$

$$1500 - T_5 = 568.67 - 271.86$$

$$T_5 = 1203.19\,K$$

$$\eta_p = \frac{T_4 - T_5}{T_4 - T_{5s}} \rightarrow T_{5s} = 1150.81$$

$$\frac{T_4}{T_{5s}} = \left(P_4/P_{5s}\right)^{k-1/k}, \quad P_4 = P_3 \Rightarrow P_{5s} = 15$$

$$P_3/P_5 = \left(T_4/T_{5s}\right)^{k/k-1} \rightarrow P_5 = 168.74\,kPa$$

$$P_6 = P_1, \quad T_5/T_6 = \left(P_5/P_6\right)^{k-1/k}$$

$$\Rightarrow T_6 = 708.2\,K$$

$$\Rightarrow V_6 = \sqrt{2c_p(T_5 - T_6)} = 997.21$$

a) $V_4 - V_1 = \boxed{707.2}$

b) $\eta_p = \frac{2V_1}{V_6 + V_1} = \boxed{.45}$