1. $y = f(x) = x^3 - 2x^2$

x	y
3	9
3.5	18.375
4	32

a)

$P_0(x)$: $a_0 = 9$

$P_0(x) = 9$

$P_1(x)$: $a_0 + a_1 x$

$$\begin{bmatrix} 1 & 3 \\ 1 & 3.5 \end{bmatrix} = \begin{bmatrix} y_0 \\ y_1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 3 \\ 1 & 3.5 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 9 \\ 18.375 \end{bmatrix}$$

$$R_2 \leftarrow R_2 - R_1 \begin{bmatrix} 1 & 3 \\ 0 & .5 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 9 \\ 9.375 \end{bmatrix} \sim \quad a_0 = -5.0625, \quad a_1 = 4.6875$$

$P_1(x) = -5.0625 + 4.6875x$

$P_2(x)$: $a_0 + a_1 x + a_2 x^2$

$$\begin{bmatrix} 1 & 3 & 9 \\ 1 & 3.5 & 12.25 \\ 1 & 4 & 16 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} y_0 \\ y_1 \\ y_2 \end{bmatrix} \Rightarrow \begin{matrix} R_2 \leftarrow R_2 - R_1 \\ R_3 \leftarrow R_3 - R_2 \end{matrix} \quad \begin{bmatrix} 1 & 3 & 9 \\ 0 & .5 & 3.25 \\ 0 & .5 & 3.75 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 9 \\ 9.375 \\ 13.625 \end{bmatrix}$$

$$R_2 \leftarrow R_2 \quad \begin{bmatrix} 1 & 3 & 9 \\ 0 & 1 & 6.5 \\ 0 & 0 & .5 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 9 \\ 18.75 \\ 4.25 \end{bmatrix}$$

$R_3 \leftarrow R_3 - R_2$

$$\begin{bmatrix} 1 & 3 & 9 \\ 0 & 1 & 6.5 \\ 0 & 0 & .5 \end{bmatrix} \begin{Bmatrix} a_0 \\ a_1 \\ a_2 \end{Bmatrix} = \begin{bmatrix} 9 \\ 18.75 \\ 4.25 \end{bmatrix} \Rightarrow \begin{aligned} a_0 &= -378.75 \\ a_1 &= -36.5 \\ a_2 &= 8.5 \end{aligned}$$

$$P_2(x) = -378.75 + (-36.5)x + 8.5x^2$$

b) $f(3.12) = 10.903$

$P_0(3.12) = \boxed{9}$ error: $|10.903 - 9| = \boxed{1.903}$ $\frac{1.903}{10.903} = \boxed{.1745}$

$P_1(3.12) = -5.0625 + 4.6875(3.12) = \boxed{9.5625}$

error: $|10.903 - 9.5625| = \boxed{1.341}$ $\frac{1.341}{10.903} = \boxed{.1229}$

$P_2(3.12) = -378.75 + (36.5)(3.12) + 8.5(3.12)^2 = \boxed{-409.88}$

- $P_2(x)$ blows up in error, error: $|10.903 + 409.88| = \boxed{420.48}$ $\frac{420.48}{10.903} = \boxed{38.566}$ either there is an error or the convergence blows up.

c) $f(4.5) = 50.625$

$P_0(4.5) = \boxed{9}$ error: $|50.625 - 9| = \boxed{41.625}$

$\frac{41.625}{50.625} = \boxed{.822}$

$P_1(4.5) = -5.0625 + 4.6875(4.5) = \boxed{16.031}$

$P_2(4.5) = -378.75 - 36.5(4.5) + 8.5(4.5^2) = -370.875$ error: $|50.625 - 16.031| = \boxed{34.594}$

$\frac{34.594}{50.625} = \boxed{.677}$

error: $|50.675 + 370.875|$

$= +\boxed{370.875}$

$\frac{+370.875}{50.625} = \boxed{6.332}$

- error blows up at $P_2(x)$ making me believe I made an error.

2)

X	0	2	4	6	8	11	12	15	17	19
y	5	6	7	6	9	8	8	10	12	12

a)

$$\begin{bmatrix} 10 & 95 \\ 95 & 9025 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 83 \\ 923 \end{bmatrix}$$

$$\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 9025 & -95 \\ -95 & 10 \end{bmatrix} \cdot \frac{1}{90{,}250 - 9025} \begin{bmatrix} 83 \\ 923 \end{bmatrix} = \begin{bmatrix} 8.143 \\ .0166 \end{bmatrix}$$

$$A(X) = 8.143 + .0166X$$

b)

y	5	6	7	6	9	8	8	10	12	12
X	0	2	4	6	9	11	12	15	17	19

$$\begin{bmatrix} 10 & 83 \\ 83 & 6889 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 95 \\ 923 \end{bmatrix}$$

$$\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 6889 & -83 \\ -83 & 10 \end{bmatrix} \frac{1}{68{,}890 - 6889} \begin{bmatrix} 95 \\ 923 \end{bmatrix} = \begin{bmatrix} 9.3199 \\ .0217 \end{bmatrix}$$

$$A(X) = 9.3199 + .0217X$$

3) $f''(x_i) \approx \frac{f(x_{i-2}) - 2f(x_{i-1}) + f(x_i)}{h^2}$; $O(h^n)$

a) $f(x) = \cos(x^2 - 7)$ $\quad x = 0$; $h_1 = 2$

$$f''(0) \approx \frac{.1367 - 2(1) + (-.416)}{4} = \boxed{-.5699}$$

error$_1$ = 1.2491

$f''(x) = -4x\sin(x^2-2) \Rightarrow f''(x) = -2\sin(x^2-2) - 4x^2\cos(x^2-2)$

b) $h_2 = 3$ $\quad \boxed{f''(0) = 1.8189}$

$$f''(0) \approx \frac{-.6486 - 2(.7534) + (-.416)}{9} = \boxed{-.308}$$

error = 1.511

$$n = \log_{\left(\frac{-.5699}{-.308}\right)}\left(\frac{1.2491}{1.511}\right)$$

$\boxed{n = 1}$