Important Note: For all four problems in this HW, show your **complete work** for the first two iterations (for each root of the given non-linear equation). Next, write a **MATLAB code** that will continue the iterations until the desired level of accuracy (convergence threshold) has been reached. Report the number of iterations and the final error value at the end of the iterations.

Problem 1 (20 Points): The function $f(x) = 2\sin(x) - \frac{e^x}{4} - 1$ is zero for two values near $x = -5$.

1. **(15 Points)** Use the **Bisection** method to find both roots using the following intervals as the initial guesses:

   $[a, b] = [-7, -5]$ and $[a, b] = [-5, -3]$

2. **(5 Points)** How many iterations are required to get results that agree to **six** significant figure?

Problem 2 (25 Points): Consider the same non-linear equation from Problem 1, i.e.,

$$f(x) = 2\sin(x) - \frac{e^x}{4} - 1$$

1. **(15 Points)** Use the **Secant** method to find both roots using the following starting points as the initial guesses:

   $[x_0, x_1] = [-7, -5]$ and $[x_0, x_1] = [-5, -3]$

2. **(5 Points)** How many iterations are required to get results that agree to **six** significant figures?

3. **(5 Points)** Compare your final results (in terms of number of iterations and the final error values) to those obtained using the **Bisection** method in Problem 1 (**required:** [1] plot "function value" vs. "iteration number" (one single plot with both curves on top of each other and use **semi-log-y**), [2] a table to report number of iterations + final error value.

Problem 3 (25 Points): The function $f(x) = 4x^3 - 1 - exp(x^2/2)$ has values of zero near $x = 1.0$ and $x = 3.0$.

1. **(5 Points)** What is the "analytical" derivative of the function, i.e., $f'(x)$?
2. **(8 Points)** If you begin Newton's method at $x = 2$, which root is reached? How many iterations are required to achieve an error less than 10^{-6}? (you must show your complete work for the first two iterations)
3. **(8 Points)** Begin Newton's method at another starting point to find the other root (show your complete work for the first two iterations).
4. **(4 Points)** For both parts (2) and (3), tabulate the number of correct digits at each iteration.

Problem 4 (30 Points): Determine the solution of the following system of non-linear equations

$$y = -x^2 + x + 0.5$$
$$y + 5xy = x^2$$

1. **(20 Points)** Using Newton's method and employ initial guesses of $\left[x^{(0)}, y^{(0)}\right] = [1.2, 1.2]$ and $\left[x^{(0)}, y^{(0)}\right] = [-0.5, -1.0]$.
   Continue iterations until the norm of the residuals falls below $\epsilon = 10^{-7}$ (perform **two iterations** by hand and show your complete work, then write a MATLAB code to continue the iterations).
2. **(10 Points)** Using the graphical approach by plotting the two functions and identifying all roots for this non-linear system (**Hint:** you may use "desmos.com/calculator" for visualization. Make sure all roots are marked/highlighted).

Due Date: Wednesday, September 29, 2021