Important Note: There are three (3) problems in this HW set

Problem 1: Solve the following linear system

$$
\begin{aligned}
-x_1 + 2x_2 - 4x_3 &= 13/6 \\
2x_1 + 3x_3 &= -2 \\
3x_1 - 2x_2 &= 17/6
\end{aligned}
$$

using the "Gauss Elimination" technique (show your complete work).

Problem 2: Consider the following linear system

$$
\begin{bmatrix} 0.6536 & 0.0575 & 0.0095 \\ 0.2411 & 0.8725 & 0.1062 \\ 0.0004 & 0.1986 & 0.2331 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0.5691 \\ 0.4624 \\ 0.2146 \end{bmatrix}
$$

1. Use **(1)** Jacobi Iteration, **(2)** Gauss-Seidel, and **(3)** SOR techniques (with $\omega = 1.03$) to solve this system by performing only 2 iterations by hand (show your complete work in all steps for both iterations)
2. Finish up the calculations started in Part (1) by writing a MATLAB code that will continue the iterations until the convergence threshold of $\epsilon = 1 \times 10^{-8}$ has been reached
3. Report the number of iterations required for each method to reach the specified/desired convergence level.
4. Plot (using MATLAB or Excel) the norm of residual, $\|\Delta \mathbf{x}\|$ or $\|\mathbf{R}\|$, versus the iteration number for all three methods.

Problem 3: Consider a randomly-generated linear system of size n = 100 which can be defined in MATLAB using the following pseudo-code

```
% Randomly generated matrix (A)
rng(1234);
A = zeros(n,n);
for i = 1:n
    for j = 1:n
        if (i == j)
            A(i,j) = (n/2)*rand();
        elseif (abs(i-j) < 3)
            A(i,j) = 0.75*rand();
        else
            A(i,j) = 0.001*rand();
        end
    end
end

% Create a randomly generated solution vector
x_exact = rand(n,1);

% Use the solution vector to set the RHS vector
b = A*x_exact;
```

Consider the **Successive-Over-Relaxation, SOR** method (you may use the MATLAB code from Problem 2 to perform SOR iterations).

1. Find the "*optimal*" value of the over-relaxation factor, ω, that will give the least error after the fewest number of iterations. Use $\Delta\omega = 0.05$ to increment your relaxation factor starting from $\omega = 1.0$ (i.e., the Gauss-Seidel method). For each value of over-relaxation, solve the system using the SOR technique and monitor both the number of iterations required as well as the final error (norm of residuals) value.

2. Tabulate the results in terms of over-relaxation factor, number of iterations, and the final error value.

3. Plot the convergence histories for each test (i.e., each ω value) against each other and clearly mark the "optimal value" case.

Note 1: Use $\epsilon = 1 \times 10^{-8}$ as the convergence threshold for all cases.
Note 2: No "hand calculations" are required. Use the MATLAB code exclusively for all calculations.

Due Date: Wednesday, September 15, 2021