Problem 1: Use the **Taylor Series Expansion** to *approximate* function $f(x) = x\sin(x)$ with point $x = \pi$ as the base point.

1. Perform (a) zeroth-order, (b) first-order, (c) second-order, and (d) third-order approximations by including more and more terms in the series.
2. Evaluate function $f(x)$ at points $x_1 = \pi + 0.1$ and $x_2 = \pi + 0.05$ using each of the four approximations obtained in part **(1)** and verify the "order of accuracy" using **error analysis** for each approximation.
3. Finally, write a MATLAB code to visualize these function approximations (i.e., zeroth-order, first-order, . . .) against the actual function itself.

Problem 2: A typical finite-difference formula, commonly used in Computational Fluid Dynamics (CFD), is

$$f'(x) \approx \frac{-3f(x) + 4f(x+h) - f(x+2h)}{2h} \quad ; \quad \mathcal{O}\left(h^n\right)$$

1. Use the **Taylor Series** expansion of an arbitrary function to **re-derive** this formula (show your complete work) and determine the **"order of accuracy"**, i.e., find n in $\mathcal{O}\left(h^n\right)$

Problem 3: Use the finite-difference formula from **Problem 2** to approximate the first derivative ($f'(x)$) of function $f(x) = \cos(x^2 - 1)$ at point $x = 1.0$:

1. Perform this approximation twice: once using $h_1 = 0.1$ and another time using $h_2 = 0.05$.
2. Prove the order of accuracy for the finite-difference formula that was determined in the previous problem by performing an error analysis for the approximations using h_1 and h_2.

Due Date: Friday, August 27, 2021