Thermal Conductivity Experiment

Analysis of Thermal Conductivity Testing for Conductive Materials

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08/03/23

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Dear Mr. Sharma,

The report details the experiment design with the intent of measuring temperatures along a material after being subject to a hot and cold surface in order to determine the thermal conductivity of the desired material.

Thermal conductivity can be calculated using the method defined in the following report. The method details the equations and steps necessary to calculate thermal conductivity through the use of Fourier's Law that relates conductive heat transfer to the heat distribution and Young's Modulus.

In order to properly calculate the thermal conductivity of the desired material, there must be a few assumptions, namely that the system does not experience any convective or radiative heat losses for the duration of the test. The method also assumes one dimensional heat transfer along the testing axis of the material. This method does not require a known reference material and will be examined at steady state.

Limitations to this method of testing primarily include the constraint for materials to be good conductors of heat. Although this method could be a viable option to determine if a material is or is not a good conductor of heat, it would not assist much in determining the appropriate thermal conductivity coefficient.

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Enclosed: Design of Experiment Report

Signed: ______________________________

Summary of Contributions:

Will Buziak (30%):

- Executive Summary
- Data Interrogation
- Data Reduction
- Conclusion
- What I learned

Dishan Desai (40%):

- Introduction
- Apparatus
- Data Reduction
- Conclusion
- What I learned

Bryson Hines (30%):

- Lists of Figures, Tables, and Symbols
- Procedure
- Data Reduction
- Conclusion
- What I learned

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List of Symbols

Symbol	Description	Units
Q_{hot}	The heat supplied to the bar in time Δt	J/s
Q_{cold}	The heat removed from the bar in time Δt	J/s
Δt	Change in time	s
ΔT_{hot}	Temperature Change of the hot end of the Sample	k
ΔT_{water}	Temperature Change of the cold end of the Sample	k
L	Length of the Sample	m
A	Cross-sectional area of the bar	m^2
C_w	Specific heat of water	J/g-k
Δm	Change in mass of the water	kg
k	Thermal Conductivity of the Sample	W/m-k

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Executive Summary

The design of the experiment is centered around utilizing Fourier's law to determine thermal conductivity using the known qualities of heat supplied to the system and the measured temperature change. The testing setup only requires two heat sources, a hot & cold, four thermocouples placed at the two respective ends of the sample & insulation around the testing material. Traditionally, the hot temperature was created using steam, however, for this experiment it will be an electric hot plate with an adjustable temperature.

It is assumed that the testing material is perfectly insulated and therefore has theoretically zero heat flux due to convection & radiation. The testing material is assumed to be uniform & homogeneous. The thermal conductivity will be calculated using temperature measurements after reaching steady state. In other words, the testing material will not be changing temperature any longer & the supplied heat has had a chance to move throughout the testing material.

The experiment will be performed by reading the four temperatures from each thermocouple for varying hot temperatures. The cold temperature will remain constant and the temperature readings must be taken while the sample is in steady state. In order to read the temperature at steady state, first adjust the hot and cold temperatures and wait until all temperature values have settled before collecting data. The time it takes for the material to reach steady state is necessary and therefore the testing sample must be allowed to settle and after each data collection, the system should ideally be allowed to return to room temperature before the next hot temperature data series is collected.

Potential sources of error can be noted from the testing setup. The sample will experience some convective and radiative losses regardless of insulation. Thermocouples must be properly placed & connected to provide appropriate readings. If temperature data is collected without first reaching steady state, the data will be incorrect and the experiment must be restarted, all data must be tossed including time data. Additionally, this testing method is ideal for materials that are good conductors of heat and therefore, if attempting to perform on an insulator would not yield accurate results.

Data validity will be performed by producing temperature distributions for each series of data collection. Graphs and figures are recommended for data visualization by comparing the varying hot temperature data series. The data reduction process will consist of utilizing Fourier's law and manipulating the equation to be placed into terms that can solve for the thermal conductivity. In order to solve for thermal conductivity, the known temperature distribution is necessary along with the known dimensions, supplied heat and relevant time.

Data interrogation will be conducted for both the hot and cold ends of the testing sample for each data series. The hot and cold ends should produce comparable results due to the assumption of no outside energy losses due to the insulation. This assumption allows us to verify the validity of our 1D steady state experiment to calculate the thermal conductivity of a given testing material.

Introduction

Objectives

The purpose of this experiment is to predict thermal conductivity of a solid material. The materials that are tested are metals due to having high thermal conductivity compared to other materials. In this experiment, the metal is heated on one end by a heat pad on a copper sink, while the other end is being cooled by tap water. The temperature differences for the hot end and cold end are collected by four different thermocouples, which are placed in certain locations of the metal to record accurate temperature differences. The rate of heat transfer for the hot end and cold end can be calculated once the temperature differences are found. Thermal conductivity of the metal is calculated using the rate of heat transfers of both the hot and cold ends. Reference samples will be used to ensure that the prediction of the thermal conductivity is conducted in an appropriate manner.

Background

Thermal conductivity, a measure of a material's ability to conduct heat, is a fundamental property with far-reaching implications in fields as diverse as physics, materials science, engineering, and environmental science. Given the crucial role of heat transfer in numerous physical and engineering systems, precise measurement and characterization of thermal conductivity are of utmost importance. Burger explains the importance of thermal conductivity as functional in many industrial applications as well as everyday electrical devices. It is noted that many uses of thermal conductivity can be both in favor and in avoidance of high thermal conductivity in applications that either seek to retain or dissipate heat in a timely manner [1] . Although various methods to measure thermal conductivity have been developed over the years, each comes with its inherent advantages and limitations.

One such technique that has proven its worth over the decades is the classic method involving a uniformly heated bar and the establishment of a steady-state thermal gradient along its length. This approach is particularly suitable for good conductors, primarily metals, and is characterized by its simplicity, cost-effectiveness, and relatively high accuracy.

The principle of this method lies in one-dimensional steady-state heat transfer: when one end of the bar is heated, and the other end is kept cool, a thermal equilibrium is eventually established such that the heat input at the heated end equals the heat output at the cool end. By carefully measuring the temperatures and heat flow, one can then accurately determine the thermal conductivity of the material.

Although this approach is simple and straightforward, it has often been sidelined in favor of more sophisticated techniques in modern research. It is often the case in engineering problems that the only known values are those that can be measured directly. This produces a desire to calculate the desired material for any given design problem in classical engineering. However, White explains this method to be additionally effective in industries such as the battery industry [2]. Nonetheless, the fundamental elegance of its principle, combined with its provision for direct measurement of thermal conductivity, suggests a substantial untapped potential in this traditional method, especially when integrated with advanced modern sensors and data analysis techniques.

Test Apparatus and Procedure

Apparatus

Experimental procedures measure thermal conductivity of a material where a metal bar is being continuously heated by an electric heat pad element attached to a copper heatsink at one end and cooled continuously at the opposite end by tap water running through a copper coil, where water mass flow rate will be measured using a differential pressure mass flow meter. [3]

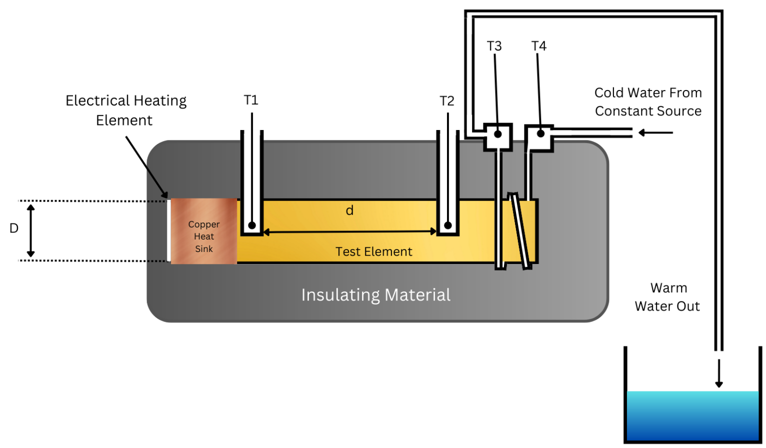

Figure 1: Testing Setup Diagram

The bar is thermally insulated by a Fiberglass composite material that would be used to mitigate heat losses to the cylindrical bar, with a length of 75 cm and a diameter of 7 cm. This length to diameter ratio is to assist in 1D SS heat transfer. Temperature values will be measured by thermocouples on the bar and in the water flow.

The heating pad allows for an adjustable temperature while the cold water source remains constant. Thermocouples must be serviced and tested frequently to verify the validity of the testing setup. A timer will be used to collect the relevant time it takes for the sample to reach steady state.

The thermal insulation is not ideal, but will be considered as such due to negligible heat losses. Temperatures are considered steady and can be collected when the readings have settled and do not change value by .2 degrees within 5 seconds. The sensitivity of the thermocouples and overall conditions of the testing setup may change those standards slightly.

Test Procedure

Figure 1 shows the testing setup for the experiment. Firstly, the testing material was covered in the Fiberglass composite material. The Fiberglass composite acts as thermal insulation for the tasting material, which prevents heat from entering or leaving the system. Fiberglass composite was used as thermal insulation because of its relatively low thermal conductivity, which indicates minimal heat transfer from the environment.

A heating pad adhesive is applied to one end of the metal bar, which is then attached to a copper heat sink to heat up the hot end of the metal. This heat pad cannot exceed 149 degrees Celsius per the physical limitations of the device. On the other end of the bar, a fixed flow rate of tap water is inserted into the copper coil in order to cool the bar. Where tap water from the building should be between 12 and 16 degrees Celsius.

Once the bar is being heated and cooled simultaneously, the thermocouples can be placed onto the testing material. The first thermocouple was placed closest to the end where the end of the bar is being heated, while the second thermocouple was placed in the middle of the bar, where the bar would not be affected by the heating pad. The first two thermocouples will be used in the calculation of the heat transfer rate of the hot end. The fourth thermocouple was placed onto the inlet of the copper coil, where the tap water enters. This thermocouple measures the temperature of the tap water, which is used to cool the end of the bar opposite of the heating pad. The third thermocouple was placed in the copper coil where the water is being pumped out of the metal bar and fiberglass composite. The third and fourth thermocouples' temperatures are used for the calculation of the heat transfer rate of the cool end.

Temperatures of the four thermocouples are recorded once the temperatures reach steady-state, which will be used to calculate the heat transfer rates of the testing sample. Simultaneously, the tap water that is being inserted into the system on the cool end is being pumped out by the copper coil into a bucket. The bucket collects the tap water that was being heated, which is essential to calculate the heat transfer rate of the cool end. This process will be repeated for 5 trials on each testing material. The cold end will not be able to be adjusted due to the water being supplied by the inline tap. It must be recorded at the time of the experiment. The hot end will be controlled by the heating pad and will begin at 40 degrees Celsius and be increased by 5 degrees with each trial.

Following the end of the experiment, calculations of the heat transfer rate of the cool and hot end are found. These equations are used to derive an equation to calculate thermal conductivity of the testing sample. These equations are all forms of Fourier’s Law.

Data Reduction and Validity Check

Using the data that was collected via the data collection software, the values could be reduced using the following methods. Firstly, the measured empirical values were found and input into the following table. These values will be needed in the following calculations.

Table 1: Physical Properties of Setup

Physical Properties		
Item	Value	Unit
L	0.75	m
D	0.07	m
d	0.75	m
c_w	4.184	J/g-k
Mass Flowrate	6.57E-02	Kg/s

$$\left(\frac{Q_{hot}}{\Delta t}\right)_{hot} = -kA\frac{\Delta T_{hot}}{L} \qquad [1]$$

$$\left(\frac{Q_{cold}}{\Delta t}\right)_{cold} = C_w\frac{\Delta m}{\Delta t}\Delta T_{water} \qquad [2]$$

Equations 1 and 2 are the heat transfer rate equations for the cold and hot end of the testing sample. ΔT_{hot} is the temperature difference between the temperature reading of the 1st and 2nd thermocouples, while ΔT_{water} is the temperature difference between the temperature reading of the 3rd and 4th thermocouples. Δm is the mass of the hot water collected pumped out of the system. These equations are classically utilized to find the heat being applied to a given condition, as in early thermodynamics classes the thermal conductivity of a material is almost always given. These equations can be manipulated to solve for the thermal conductivity if all other quantities are known.

$$\left(\frac{Q_{cold}}{\Delta t}\right)_{cold} = \left(\frac{Q_{hot}}{\Delta t}\right)_{hot} \qquad [3]$$

Equation 3 shows that the heat transfer rate of the cold end and hot end are equivalent. This relationship can be made because the assumption system is thermally insulated due to the fiberglass composite, which means that heat is not entering or leaving the system. As a result, the values for equations 1 and 2 should be the same.

$$k = -C_w\frac{L}{A}\frac{\Delta m}{\Delta t}\frac{\Delta T_{water}}{\Delta T_{hot}} \qquad [4]$$

Equation 4 finds the thermal conductivity of the testing sample. This equation is derived from equation 3, where heat transfer rates of the hot and cold end are equal to each other in order to find an equation for thermal conductivity. This process was used for both the unknown sample and the sample with known thermal conductivity. Then using temperature data for the known sample and equation 4 a theoretical value can be found. Then the difference between the actual and the theoretical value could be calculated. An average for this theoretical/actual offset could be found then subtracted from the found value for unknown material. The following value would then be the actual thermal conductivity of the unknown sample.

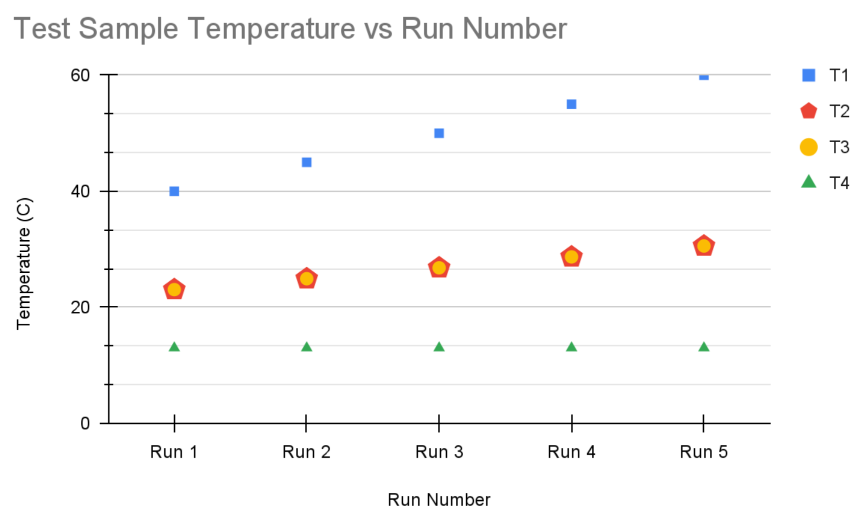

Figure 2: Testing Sample Temperature Plot

Resulting tables and figures prove the validity of the data collected. It is recommended to produce plots that well detail the change in temperature and exhibit expected behavior. The expectation is that there will be a linear increase in temperature as the electric hot plate is incrementally increased. This trend will be less apparent for thermocouples that are placed closer to the cold source. This is because the cold source is constant and therefore the increasing temperature will not have as much of an affect on the temperature reading. Since the temperature is collected at steady state, each respective thermocouple should have settled on its reading, decreasing any stochasticity in the data.

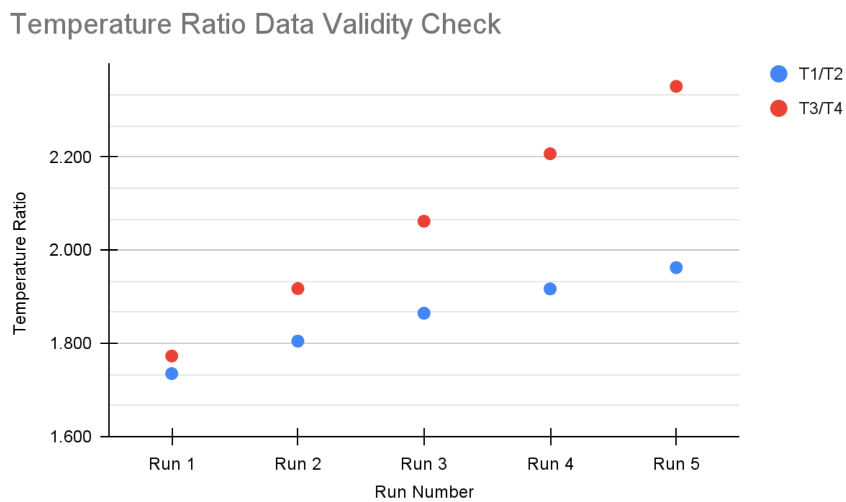

Figure 3: Temperature Ratio Data Validity Check

The final data validity check is the two respective temperature ratios T1/T2 & T3/T4. These ratios should fall within the range of 1.5-1.8 with some allowable error. There is an expected linear growth behavior as the data series increase. This behavior is expected for both ratio sets. Potential expected sources of error can come from the testing setup, specifically with insulation degradation. Allowable ranges of error can be determined by quality of testing setup. Another potential source of error is the testing sample. If the identity of the sample is not a good conductor, this method of measurement is not ideal & therefore can produce skewed data [4].

Results and Discussion

Table 2 shows the data collection for iron, which was the known sample that was tested. Table 1 shows the temperatures from the four thermocouples and the comparison of the theoretical thermal conductivity to the actual thermal conductivity of iron.

Table 2: Thermal Conductivity of Known Sample

Known Sample Iron (k= 80.2 W/m-k)								Thermal Conductivity			
	T1	T2	Delta 1	T3	T4	Delta 2	Delta2/ Delta1	k (W/m-k)	Theoretical k	Actual k	Offset
Run 1	40	30.35	9.65	30.35	13	17.35	1.7979		96.32	80.2	16.12
Run 2	45	33.6	11.4	33.6	13	20.6	1.8070		96.80	80.2	16.60
Run 3	50	36.87	13.13	36.87	13	23.87	1.8180		97.39	80.2	17.19
Run 4	55	39.87	15.13	39.87	13	26.87	1.7759		95.14	80.2	14.94
Run 5	60	42.89	17.11	42.89	13	29.89	1.7469		93.59	80.2	13.39

Table 2 shows that the theoretical thermal conductivity that was calculated from the experiment ranged from 91.52 W/m-k to 93.75 W/m-k. The actual thermal conductivity is 80.2 W/m-k [6], which has an offset of 12.09 on average compared to the theoretical thermal conductivity [5]. The offset was accounted for when calculating the actual thermal conductivity of the unknown sample.

Table 3 displays the temperature reading of the four thermocouples of the unknown testing sample. Similar to Table 2, the theoretical thermal conductivity is being compared to the actual thermal conductivity.

Table 3: Thermal Conductivity of Unknown Sample

Unknown Sample k=?								Thermal Conductivity		
	T1	T2	Delta 1	T3	T4	Delta 2	Delta2/ Delta1	k (W/m-k)	Theoretical k	Actual k
Run 1	40	23.05	16.95	23.05	13	10.05	0.5929		31.76	16.12
Run 2	45	24.93	20.07	24.93	13	11.93	0.5944		31.84	16.20
Run 3	50	26.81	23.19	26.81	13	13.81	0.5955		31.90	16.25
Run 4	55	28.69	26.31	28.69	13	15.69	0.5964		31.95	16.30
Run 5	60	30.57	29.43	30.57	13	17.57	0.5970		31.98	16.34
									Average	16.24

The theoretical thermal conductivity that was calculated ranges from 28.27 W/m-k to 28.30 W/m-k, which is a difference of only 0.3 W/m-k. The actual thermal conductivity is calculated by subtracting the theoretical value by the average offset as seen from Table 2. As a result, the average actual thermal conductivity of the unknown sample is 16.19 W/m-k after 5 runs with the offset being considered. The offset is included in the calculation of the actual thermal conductivity in order to adjust for errors that may be caused by the testing apparatus. A thermal conductivity of 16.19 W/m-k is close to that of the actual conductivity of stainless steel. So in a practical sense, it could be assumed that the unknown sample is stainless steel. [6]

The temperature gradient ratio, which is defined as the ratio of Delta 2 and Delta 1, is seen on Table 3. Delta 2 is observed to be around half of Delta 1 for all 5 runs, which indicates that the change in temperature for the cold end is smaller than the hot end.

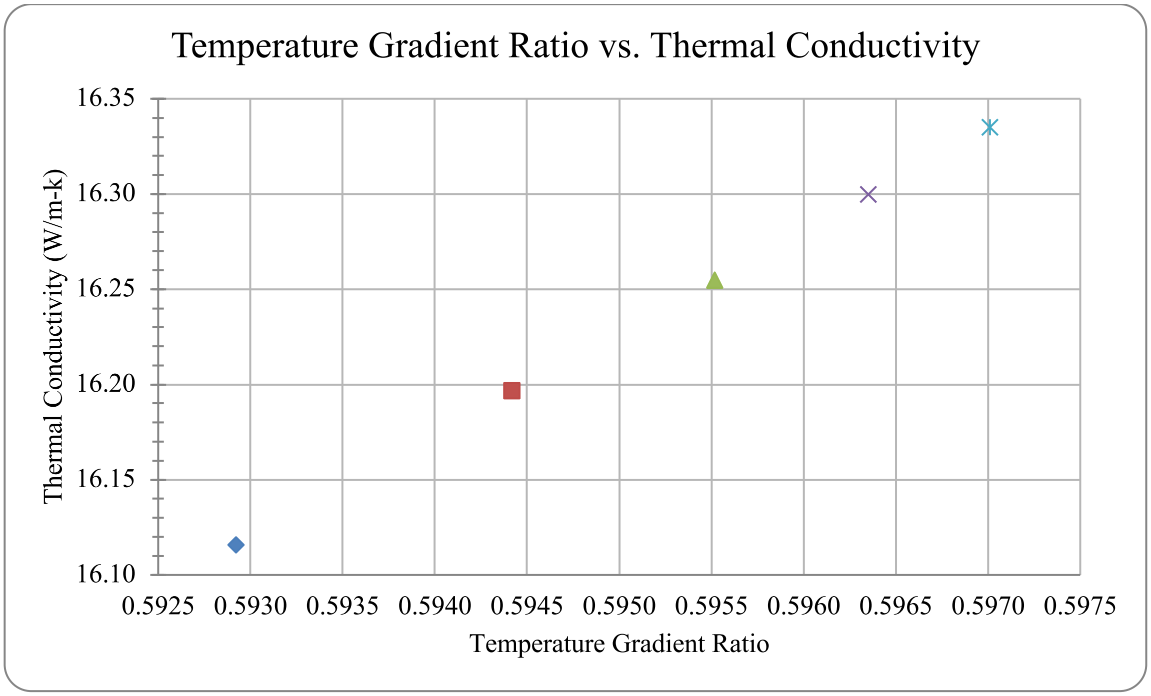

Figure 4: Temperature Gradient Ratio vs. Thermal Conductivity

Figure 4 shows that there is a positive linear relationship between the temperature gradient ratio and thermal conductivity of the unknown sample. The same relationship is also present in iron as Table 2 shows that theoretical thermal conductivity increases as temperature gradient ratio increases. The significance of Figure 4 indicates that in order for a material to have higher thermal conductivity is to have a higher temperature gradient ratio. In order to achieve a higher temperature gradient ratio, the change in temperature of the cool end has to be significantly larger than the change in temperature of the hot end. This phenomenon is present in the data collection for iron. Another method to increase temperature gradient ratio is to have the change in temperature of the hot end to be significantly lower than the change in temperature of the cool end.

Conclusions and Recommendations

From the analysis of the compiled data, the research revealed that the thermal conductivity measurements using this detailed approach could be widely applicable to a real world application of finding a metallic sample of unknown metal's thermal conductivity. It was found that the easiest method to obtain legitimate results in regards to the thermal conductivity of a metallic sample was to set the experimental setup in such a way that the system would become a one dimensional steady state heat flow. So therefore, this would mean that the setup would have to be well thermally insulated so that heat escaping in the radial direction would be negligible. Since it was assumed that the one dimensional steady state is true, then heat transfer using fourier's equation would be applicable in this case. This method does not require a reference material for anything other than verifying the validity of the testing setup. The reference material can also serve as a validity check for the method itself. This method is ideal for materials that are good conductors of heat and although still valid for many other materials, the behavior is more apparent in the data reduction section when performed on a material with a higher thermal conductivity.

Errors that were present in this experiment can be seen in the calculation of the thermal conductivity of iron as the theoretical value was not equivalent to the actual value. As a result, offset calculations were used when finding the thermal conductivity of the unknown sample. Possible errors include that perfect thermal contact may not be achieved between the thermocouples and the metal bar, and that would affect the temperature gradient. Another limitation to the experiment may be that the metal bar may not be of uniform material. The experiment assumes the bar to be of uniform material, so there may be variation within the testing sample that affects heat conduction.

Recommendation for the experiment in the future is to ensure that the testing sample is of uniform material in order to make sure that finding thermal conductivity would be more accurate. Another recommendation that could be made is to use another method to cool down the testing sample other than tap water. This change would allow for more accurate readings on the thermocouple, and this would allow the calculations of the temperature gradient to be more accurate. Finally, it is recommended that additional thermocouples be added to the system in order to get a more accurate temperature gradient reading.

What I Learned

I learned that, in order to find an unknown value. It is essential to isolate that value from known relations. This requires lots of follow up by the experimenters to be able create an experiment that does not allow outside impacts to contaminate the test results. For this reason, we had to create processes that isolate the values that we wanted to find and those that we wanted to control via thermodynamic properties. In addition, I learned about methods for testing thermal conductivity. How to take an observable phenomenon and use it to calculate an unmeasurable quality.

Editing Statement

"I Testify that I have read and edited this report before submitting it."

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[1] Burger, Nicolas, "Review of thermal conductivity in composites: Mechanisms, parameters and theory", Progress in Polymer Science (2016): 1-28

[2] White, Gavin, "Novel methods for measuring the thermal diffusivity and the thermal conductivity of a lithium-ion battery", Applied Thermal Engineering (2022)

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[5] R. K. Agrawal, *Physics Practicals: Part-II*. Krishna Prakashan, 2008.

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APPENDIX A: Original Test Data

DOE Thermal Conductivity Test

June 27 2023

Will Buziak, Bryson Hines, Dishan Desai

Table 4: Thermal Conductivity of Iron

Themal Conductivity			
k (W/m-k)	Theoretical k	Actual k	Offset
	96.32	80.2	16.12
	96.80	80.2	16.60
	97.39	80.2	17.19
	95.14	80.2	14.94
	93.59	80.2	13.39
		Average	15.65

Table 5: Thermal Conductivity of Testing Material

Thermal Conductivity		
k (W/m-k)	Theoretical k	Actual k
	31.76	16.12
	31.84	16.20
	31.90	16.25
	31.95	16.30
	31.98	16.34
	Average	16.24

Table 6: Data Validity Check

Data Validity Check		
	T1/T2	T3/T4
Run 1	1.735	1.773
Run 2	1.805	1.918
Run 3	1.865	2.062
Run 4	1.917	2.207
Run 5	1.963	2.352

APPENDIX B: Equipment List and Specifications

DOE Test Equipment List

Name of Equipment	Model #	Serial #	Manufacturer	Purpose	Resolution	Accuracy
Thermocouple	5SC-TT-K-36-72-ROHS	N/A	OMEGA	Temperature data collection	0-260 °C	N/A
Heating pad with thermostat	KHLVA-0504/2-P	N/A	OMEGA	Electric heater for heat source	-40°C - 149°C	N/A
DAQ	Digital I/O DAQ	N/A	ioTech	Acquire temperature readings from software program	N/A	N/A
Water Supply	N/A	N/A	N/A	Tap water from Lab Building	12°C - 16°C	N/A
Fiberglass Composite Insulation	N/A	N/A	N/A	Housing to inhibit the heat transfer in the radial direction	N/A	N/A

APPENDIX C: Test Procedure

DOE Thermal Conductivity Test

Procedure

Stage 1: Testing Sample Setup
In this initial stage, the testing sample is covered in the fiberglass composite. The adhesive heating pad should be placed on one end connected to a copper heat sink, and the copper coil should be wrapped around the other end of the other bar. The bucket for the water collection needs to be placed directly under the outlet of the copper coil.

1. Place the four thermocouples on the bar in the following order: above the adhesive heating pad, middle of the bar, inlet of the copper coil, and outlet of the copper coil
2. Verify that the copper heat sink is in contact with adhesive heating pad
3. Verify that the water supply is connected to the inlet of the copper coil
4. Confirm that the DAQ is connected to the thermocouples in order to get temperature readings.

Stage 2: Thermal Conductivity Test
In this stage, the copper heat sink begins to heat up the hot end of the testing sample, and a fixed flow rate of tap water enters the copper coil to begin cooling down the cool end of the bar. The DAQ will provide temperature readings to find the change in temperature of the hot end and cool end at steady state. This test will be down for 5 runs per testing sample

1. Every 5 minutes, record the temperatures of the four thermocouples until the temperatures reach steady-state. Record the time it takes for temperature to reach steady state
2. Once the temperatures reaches steady state, collect the water that is being pumped out of the copper coil with a bucket
3. Weigh the mass of the water that is collected by the bucket

Stage 3: Clean Up
In the final stage, the copper heat sink stops heating up the adhesive heating pad and the water stops entering the inlet of the copper coil.

1. Carefully detach the copper heating sink from the testing sample
2. Turn off the water supply to the copper coil
3. Ensure that the temperature readings are inserted into the excel file
4. Shut down the DAQ once data collection is complete and saved