Distribution summary

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Distribution	PMF/PDF	Mean	Variance	Note
Bernoulli	$p^k(1-p)^{1-k}$ $k \in \{0,1\}$	p	$p(1-p)$	takes value 1 (success) with probability p, it's a special case of binomial distribution (n=1)
Binomial	$\binom{n}{k}p^k(1-p)^{n-k}$ $k \in N$	np	$np(1-p)$	models the number of successes in a i.i.d Bernoulli trails of size n; can be viewed as drawing n samples **with replacement** from a population of size N
Poisson	$\frac{\lambda^k e^{-\lambda}}{k!}$ $k \in N$	λ	λ	expresses the probability of a given number of events occurring in a fixed interval of time or space if these events occur with a known constant mean rate and independently of the time since the last event
Negative Binomial	$\binom{k+r-1}{r-1}p^k(1-p)^r$ $k \in N$	$\frac{pr}{1-p}$	$\frac{pr}{(1-p)^2}$	models the number of successes in an i.i.d Bernoulli trials before a specified number of failures (denoted as r) occur
Geometric	$p^k(1-p)$ $k \in N$	$\frac{p}{1-p}$	$\frac{p}{(1-p)^2}$	models the number of successes in an i.i.d Bernoulli trails before the first failure occurs, it's a special case of negative binomial (r=1)
Hyper-geometric	$\frac{\binom{K}{k}\binom{N-K}{n-k}}{\binom{N}{n}}$ $k \in \{\max(0, n+K-N), \ldots, \min(n,K)\}$	$n\frac{K}{N}$	$n\frac{K}{N}\frac{N-K}{N}\frac{N-n}{N-1}$	models the probability of k successes in n draws, **without replacement**, from a population of size N that contains exactly K objects with that feature, where each draw is either a success or a failure.
Gaussian	$\frac{1}{\delta\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{\delta^2}}$ $x \in R$	μ	δ^2	Widely used in natural and social science, which is partially due to the central limit theorem. It's the only distribution whose cumulants beyond the first two (i.e., other than the mean and variance) are zero.
Exponential	$\lambda e^{-\lambda x}$ $x \in [0,\infty)$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$	models the lengths of the inter-arrival times in a homogeneous Poisson process. It can be viewed as a continuous counterpart of the geometric distribution
Chi-square	$\frac{1}{2^{k/2}\Gamma(k/2)}x^{k/2-1}e^{-x/2}$ $x \in [0,\infty)$ or $x \in (0,\infty)$ if k=1	k	$2k$	the chi-square distribution with k degrees of freedom is the distribution of a sum of the squares of k independent standard normal random variables
t	$\frac{\Gamma(\frac{\nu+1}{2})}{\sqrt{\nu\pi}\Gamma(\frac{\nu}{2})}(1+\frac{t^2}{\nu})^{-\frac{\nu+1}{2}}$ $x \in R; \nu = dof$	0	$\frac{\nu}{\nu-2}$	t-distribution has heavier tails, meaning that it is more prone to producing values that fall far from its mean.
F	$\frac{\sqrt{\frac{(d_1x)^{d_1}d_2^{d_2}}{(d_1x+d_2)^{(d_1+d_2)}}}}{xB(\frac{d_1}{2},\frac{d_2}{2})}$ $x \in [0,\infty)$ or $x \in (0,\infty)$ if $d_1 = 1$	$\frac{d_2}{d_2-2}$	$\frac{2d_2^2(d_1+d_2-2)}{d_1(d_2-2)^2(d_2-4)}$	a continuous probability distribution that arises frequently as the null distribution of a test statistic, most notably in the analysis of variance (ANOVA) and other F-tests.
Gamma	$\frac{\beta^\alpha x^{\alpha-1}e^{-\beta x}}{\Gamma(\alpha)}$ $x \in (0,\infty)$	$\frac{\alpha}{\beta}$	$\frac{\alpha}{\beta^2}$	usually used to model waiting times (In oncology, the age distribution of cancer incidence often follows the gamma distribution. In genomics, it was applied in Peak calling step)
Beta	$\frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha,\beta)}$ $x \in [0,1]$	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$	models the behavior of random variables limited to intervals of finite length in a wide variety of disciplines.