

Machine Learning with Python

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2023 Winter





Day 3: Unsupervised learning

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Agenda

- Day 1: Introduction to machine learning
 - Some key concepts in machine learning
 - Jupyter notebook and some packages usage
- Day 2: Supervised learning
 - Classification
 - Regression
 - Regularization
- Day 3: Unsupervised learning
 - Dimension reduction
 - Clustering













Notation of the slides

Code or Pseudo-Code chunk starts with ">", e.g.
 ▶ print("Hello world!")

Link is underlined

Important terminology is in **bold** font

Overview

Time

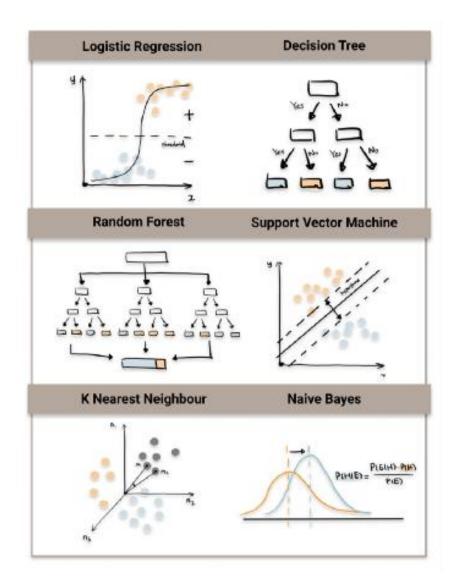
• 3-hour workshop (45min + 45min + 30min + practice/Q&A)

Topics

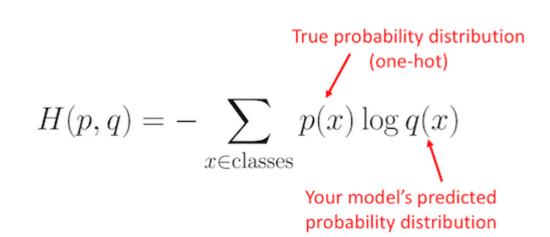
- ☐ Regression
- □ Regularization
- ☐ Dimension reduction
- ☐ Clustering



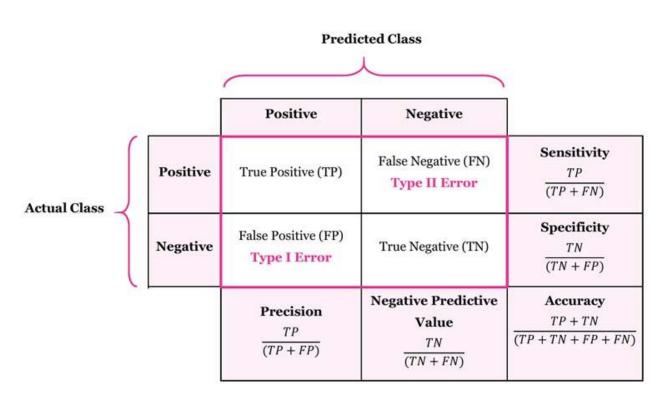
- □ 9 different classification algorithms
 - Logistic regression
 - KNN
 - Naïve bayes
 - Support vector machine
 - Decision tree
 - Random forest
 - Adaboost
 - Gradient boosting
 - Neural network



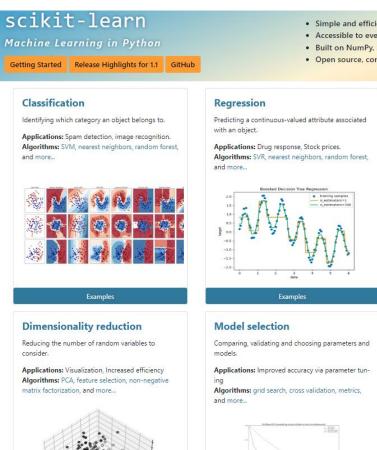
- ☐ 9 different classification algorithms
- ☐ Cross entropy loss function



- □ 9 different classification algorithms
- ☐ Cross entropy loss function
- ☐ Performance measure
 - Accuracy
 - Confusion matrix
 - Precision & Recall
 - ROC, AUC, PR-ROC



- 9 different classification algorithms
- ☐ Cross entropy loss function
- ☐ Performance measure
- □ Practice
 - Training-test-validation set construction
 - Performance measure calculation
 - ROC curve, decision boundary
 - Linearly non-separable example
 - Parameter tuning (overfitting/underfitting)

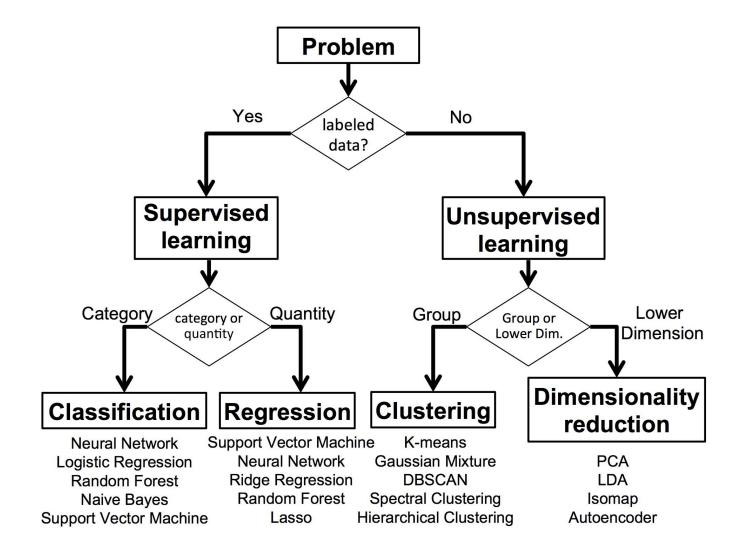


- Simple and efficient tools for predictive data analysis
 Accessible to everybody, and reusable in various contexts
 Built on NumPy, SciPy, and matplotlib
- Open source, commercially usable BSD license



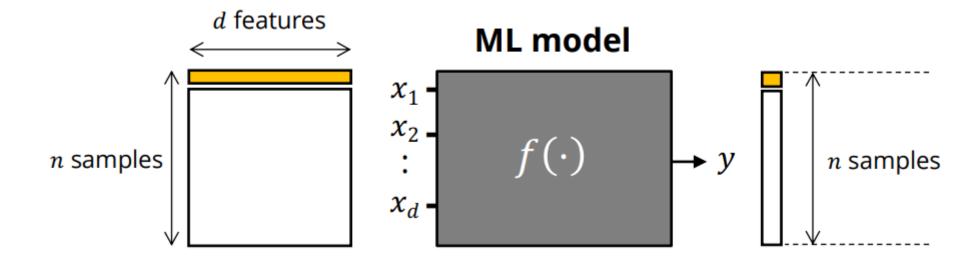


Types of machine learning



Supervised learning

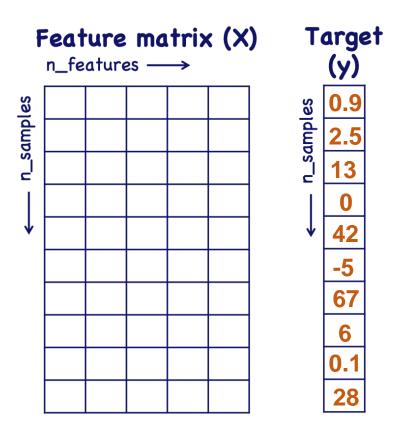
- Training data with n samples of features x and label y
- Learn a function class f(x) to describe y based on x



Different choice of f() for regression tasks

- Linear regression
- Polynomial regression
- SVR
- Tree-based regression
- Boosting
- Neural Network

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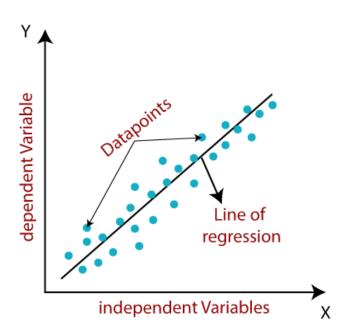
$$\hat{y} = f(x)$$

Linear regression

Make prediction by computing the weighted sum of features

$$\hat{y} = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \dots + \theta_n x_n$$

$$\hat{y} = h_{\boldsymbol{\theta}}(\mathbf{x}) = \boldsymbol{\theta} \cdot \mathbf{x}$$



Linear regression

Make prediction by computing the weighted sum of features

$$\hat{y} = heta_0 + heta_1 x_1 + heta_2 x_2 + \dots + heta_n x_n$$
 $\hat{y} = h_{m{ heta}}(\mathbf{x}) = m{ heta} \cdot \mathbf{x}$

- Train the model: find a parameter set to best fit the data
- Loss function

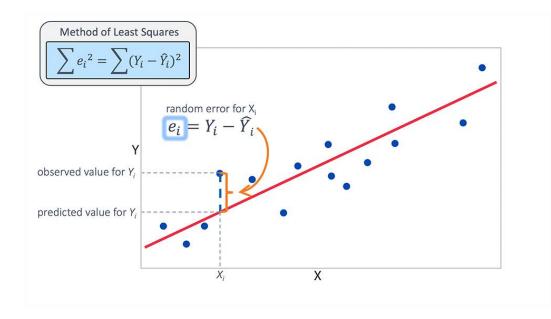
Equation 4-3. MSE cost function for a Linear Regression model

$$ext{MSE}\left(\mathbf{X}, h_{oldsymbol{ heta}}
ight) = rac{1}{m} \sum_{i=1}^{m} \left(oldsymbol{ heta}^\intercal \mathbf{x}^{(i)} - y^{(i)}
ight)^2$$

Sometimes MAE is also used (less sensitive to outliers)

Equation 2-2. Mean absolute error (MAE)

$$ext{MAE}\left(\mathbf{X},h
ight) = rac{1}{m} \sum_{i=1}^{m} \! \left| h\left(\mathbf{x}^{(i)}
ight) - y^{(i)}
ight|$$



Linear regression

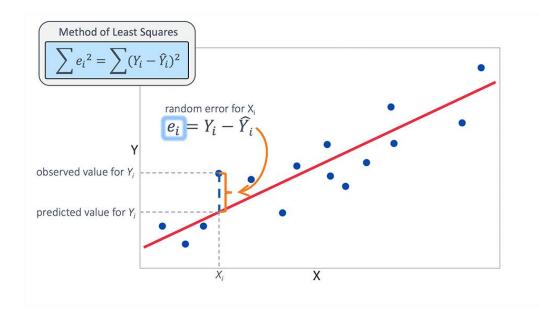
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- Train the model: find a parameter set to best fit the data
- Normal equation

$$\widehat{\boldsymbol{ heta}} = \left(\mathbf{X}^{\intercal} \mathbf{X} \right)^{-1} \mathbf{X}^{\intercal} \mathbf{y}$$

$$\boldsymbol{\theta} = E(\widehat{\boldsymbol{\theta}})$$



Polynomial regression

- When your data is more complex than a straight line
 - Add powers of a feature as new features

$$\hat{y} = \beta_0 + \beta_1 x_1 + \beta_2 x_2$$

$$\hat{y} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1^2 + \beta_4 x_2^2 + \beta_5 x_1 x_2$$

Be aware of the combinatory explosion of features!

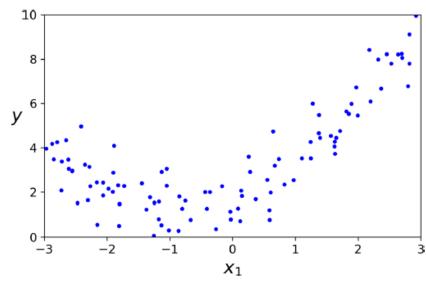
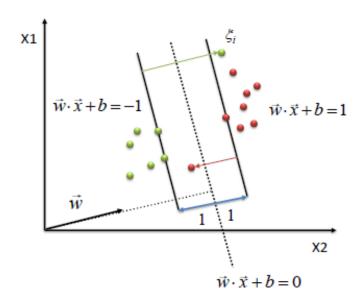


Figure 4-12. Generated nonlinear and noisy dataset

Support vector regression

SVM classification

fit the largest possible street between two classes while limiting margin violations



Constraint becomes:

$$y_i(w \cdot x_i + b) \ge 1 - \xi_i, \ \forall x_i$$

 $\xi_i \ge 0$

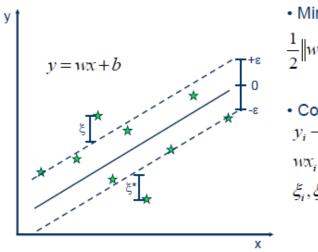
Objective function penalizes for misclassified instances and those within the margin

$$\min \frac{1}{2} \left\| w \right\|^2 + C \sum_{i} \xi_i^{\epsilon}$$

C trades-off margin width and misclassifications

SVM regression

fit as many instances as possible *on* the street while limiting margin violations



Minimize:

$$\frac{1}{2} \| \mathbf{w} \|^2 + C \sum_{i=1}^{N} (\xi_i + \xi_i^*)$$

· Constraints:

$$y_i - wx_i - b \le \varepsilon + \xi_i$$

$$wx_i + b - y_i \le \varepsilon + \xi_i^*$$

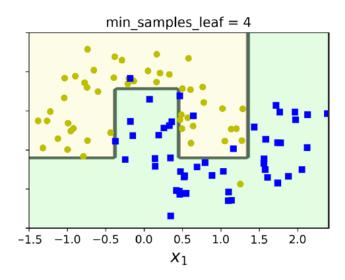
$$\xi_i, \xi_i^* \ge 0$$

The kernel tricks can be used to solve nonlinear regression problem

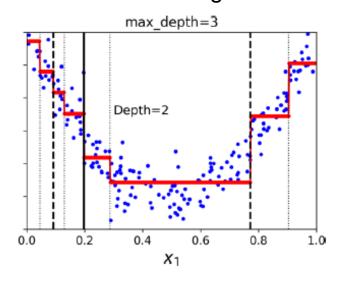
Tree-based regression

 The predicted value for each region is always the average target value of the instances in that region

Decision tree classification



Decision tree regression



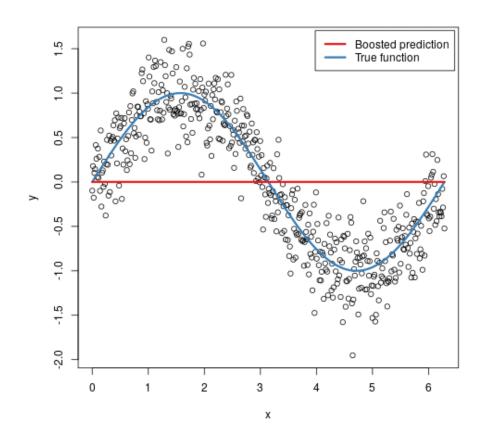
Equation 6-4. CART cost function for regression

$$J\left(k,t_{k}
ight) = rac{m_{ ext{left}}}{m} ext{MSE}_{ ext{left}} + rac{m_{ ext{right}}}{m} ext{MSE}_{ ext{right}} \quad ext{where} \; \left\{ egin{array}{l} ext{MSE}_{ ext{node}} = \sum_{i \in ext{node}} \left(\hat{y}_{ ext{node}} - y^{(i)}
ight)^{2} \ \hat{y}_{ ext{node}} = rac{1}{m_{ ext{node}}} \sum_{i \in ext{node}} y^{(i)} \end{array}
ight.$$

Gradient Boosting Machines

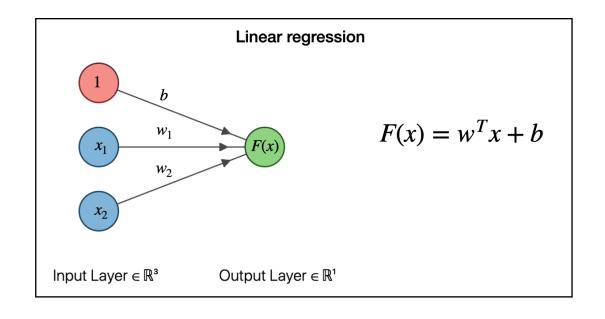
Use decision trees as base-learner, iteratively improves the weak learning model

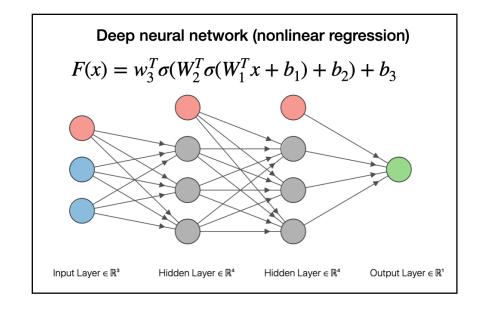
- Fit a decision tree to the data $\hat{y} = F_1(x)$
- Fit the next decision tree to the residuals from previous tree $h_1(x) = y F_1(x)$
- \triangleright Add the new tree to the algorithm $F_2(x) = F_1(x) + h_1(x)$
- > Repeat this process until some mechanism tells us to stop (e.g. cross validation)



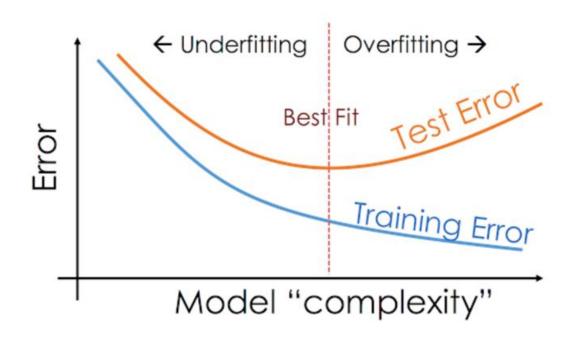
Neural Network

- A universal function approximator
 - Loss function: MSE
 - No sigmoid activation function at the output node

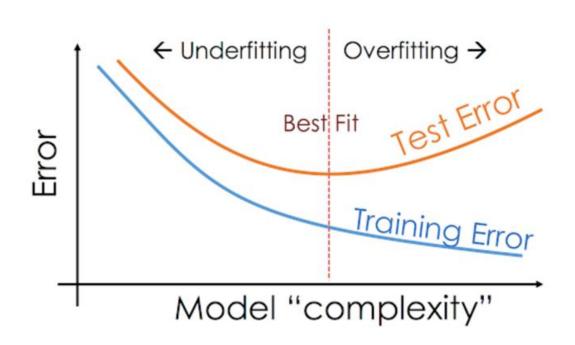


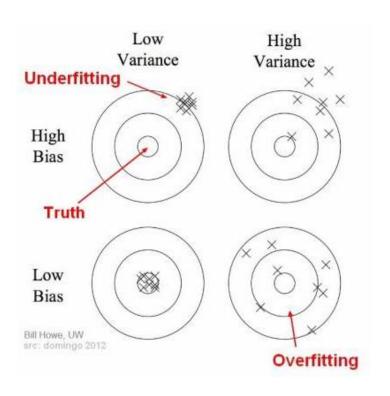


Over/underfit and bias-variance tradeoff



Over/underfit and bias-variance tradeoff

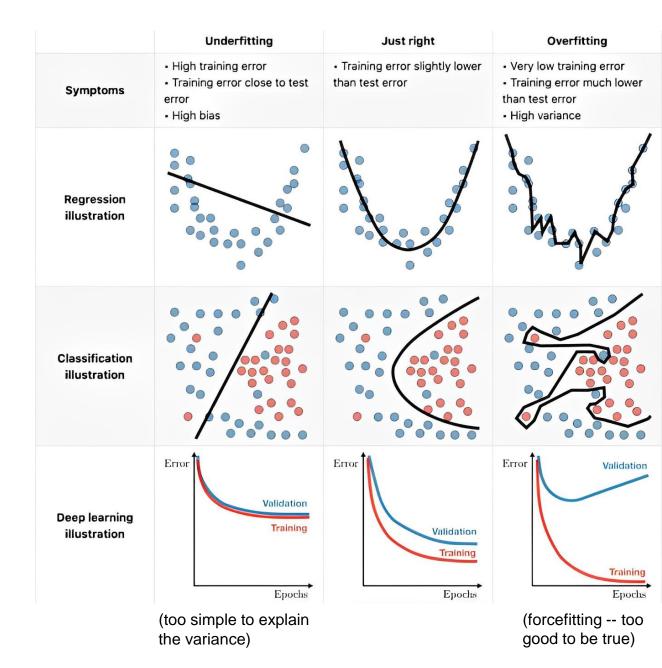




The bias-variance trade off

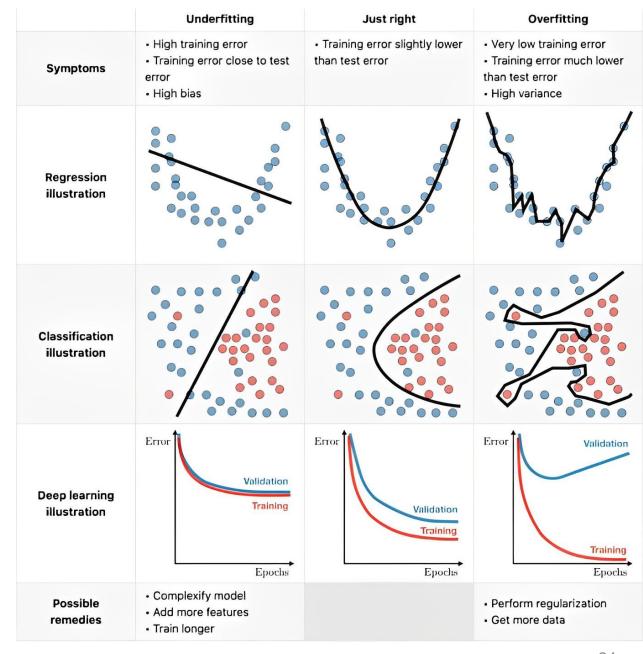
Underfit & Overfit

Symptom and solutions

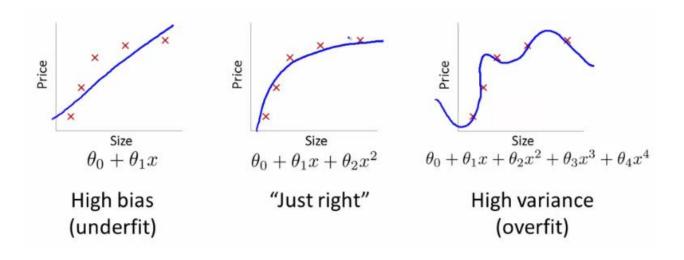


Underfit & Overfit

Symptom and solutions



Regularization techniques in regression



- Regularization: to put constrains on the model
 - The fewer degrees of freedom, the harder it overfits
 - In reality, it's common to have d >> n, regularization is usually needed

3 typical constrains in regression

Lasso (L1 penalty)

$$J(\boldsymbol{\theta}) = ext{MSE}(\boldsymbol{\theta}) + lpha \sum_{i=1}^n | heta_i|$$

Ridge (L2 penalty)

$$J(oldsymbol{ heta}) = ext{MSE}(oldsymbol{ heta}) + lpha rac{1}{2} \sum_{i=1}^n { heta_i}^2$$

Elastic-net

$$J(\boldsymbol{\theta}) = ext{MSE}(\boldsymbol{\theta}) + r \alpha \sum_{i=1}^{n} |\theta_i| + \frac{1-r}{2} \alpha \sum_{i=1}^{n} {\theta_i}^2$$

This forces the learning algorithm to not only fit the data but also keep the model weights as small as possible!

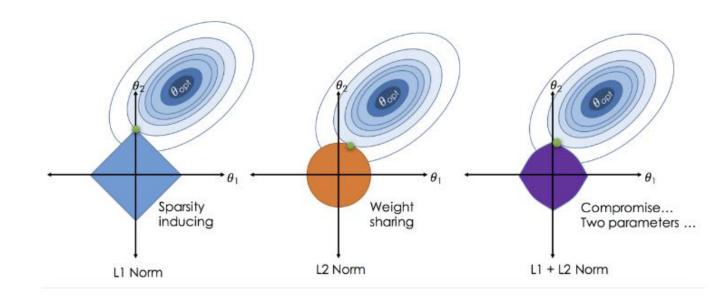
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Summary

- 6 regression algorithms
- MSE/MAE loss function
- Over/Underfit and Bias-variance tradeoff
- Regularization
 - LASSO
 - Ridge
 - Elastic net

Unsupervised learning

- Dimension reduction
 - PCA
 - t-SNE
 - Autoencoder

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- Clustering
 - K-means
 - Hierarchical
 - DBSCAN

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Dimension reduction

High dimensional: a blessing and a curse

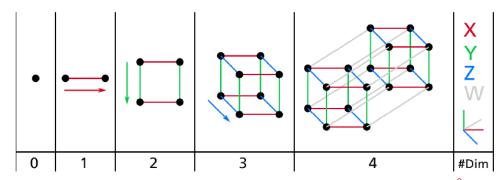
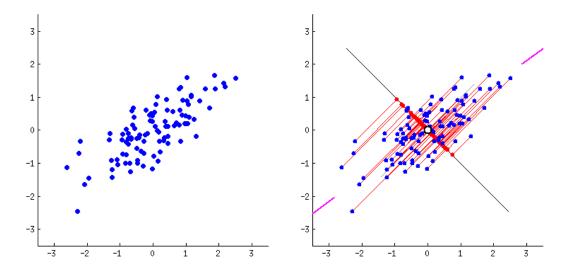


Figure 8-1. Point, segment, square, cube, and tesseract (0D to 4D hypercubes)²

- Curse of dimensionality:
 - Many machine learning algorithm have hard time to find good solutions in high dimensional setting
 - The training can be extremely slow when dimension (number of features) is high

Preserving the maximum variance

- Data compression
- Noise removal
- Data visualization



Preserving the maximum variance

- Data compression
- Noise removal
- Data visualization

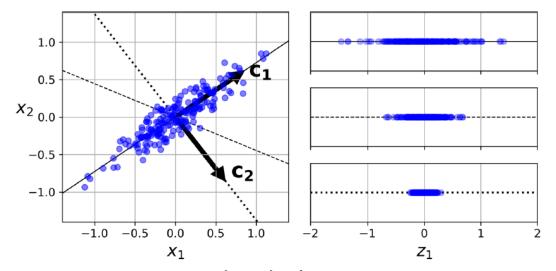


Figure 8-7. Selecting the subspace to project on

Preserving the maximum variance

- Data compression
- Noise removal
- Data visualization

Goal:

• find an orthogonal set of r linear basis vectors $w_j \in R^d$ and the corresponding score $z_i \in R^r$, such that we minimize the average **reconstruction error**

$$J(\mathbf{W}, \mathbf{Z}) = \frac{1}{N} \sum_{i=1}^{N} ||\mathbf{x}_i - \hat{\mathbf{x}}_i||^2$$

where
$$\hat{\mathbf{x}}_i = \mathbf{W}\mathbf{z}_i$$

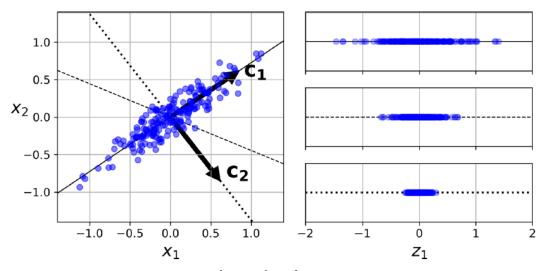
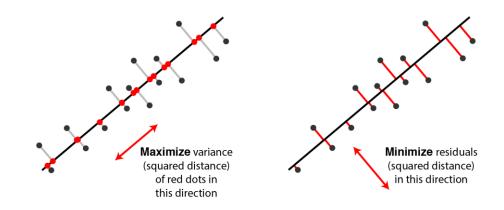
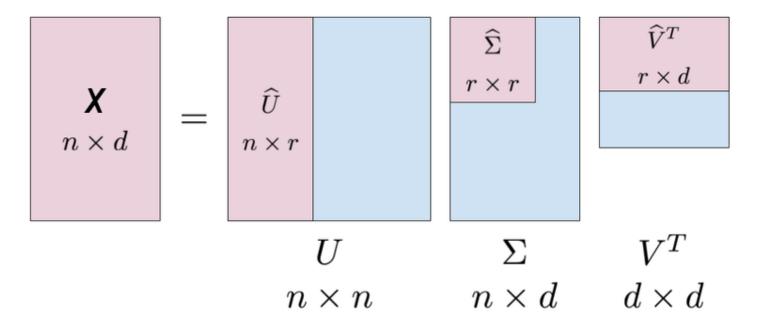


Figure 8-7. Selecting the subspace to project on



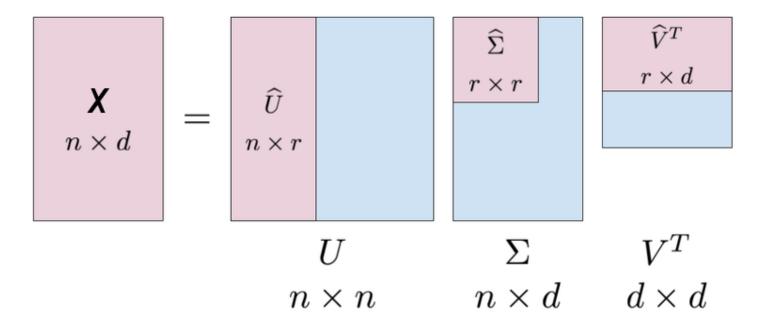
PCA achieved by Singular value decomposition (SVD)



 \widehat{U} $\widehat{\Sigma}$: Principal component scores

 \hat{V} : Principal directions

PCA achieved by Singular value decomposition (SVD)



Project data on to the reduced dimension space

$$X_{d-proj} = X\widehat{V}$$

t-Distributed Stochastic Neighbor Embedding (t-SNE)

- Nonlinear dimensionality reduction
 - PCA uses the global covariance matrix
 - t-SNE focus more on the local structure
- Core algorithm
 - In high dimensional space

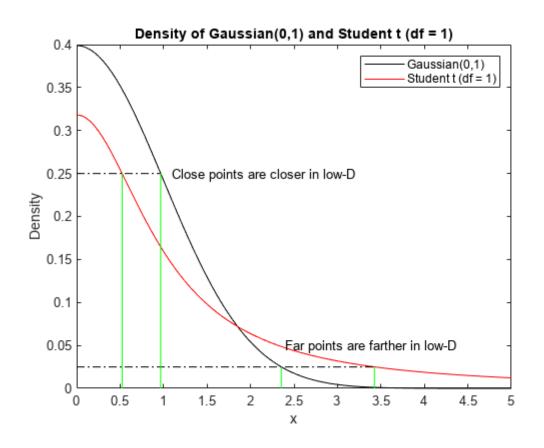
$$p_{ij} = rac{\exp(-\left\|x_{i} - x_{j}
ight\|^{2}/2\sigma_{i}^{2})}{\sum_{k
eq l} \exp(-\left\|x_{k} - x_{l}
ight\|^{2}/2\sigma_{i}^{2})}$$

In lower dimensional space

$$q_{ij} = rac{(1 + \|y_i - y_j\|^2)^{-1}}{\sum_{k
eq l} (1 + \|y_k - y_l\|^2)^{-1}}$$

• Try to minimize the difference of 2 distributions

$$C = D_{\mathrm{KL}}(P \parallel Q) = \sum_{x \in \mathcal{X}} P(x) \log \left(\frac{P(x)}{Q(x)} \right)$$



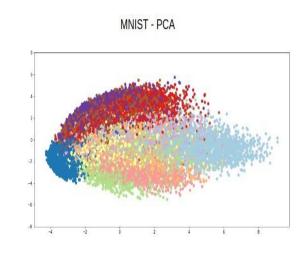
t-Distributed Stochastic Neighbor Embedding (t-SNE)

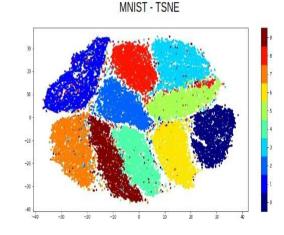
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In lower dimensional space

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Try to minimize the difference of 2 distributions

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Autoencoder

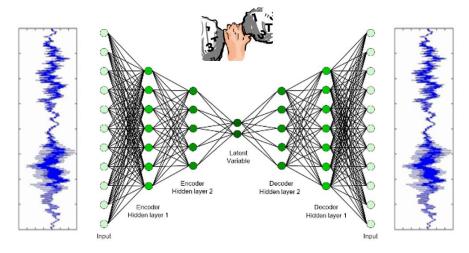
Nonlinear dimension reduction with NN

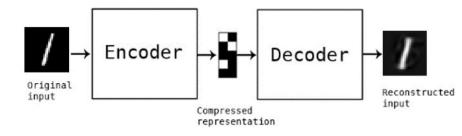


Autoencoder

Nonlinear dimension reduction with NN

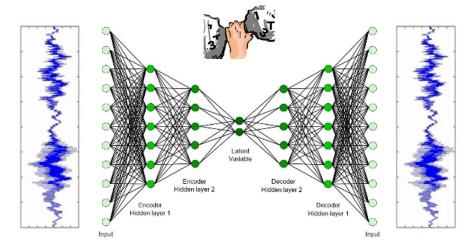
- Key idea:
 - An encoder function z = f(x)
 - A Decoder function x = g(z)
 - Learn to set $g(f(x)) \cong x$

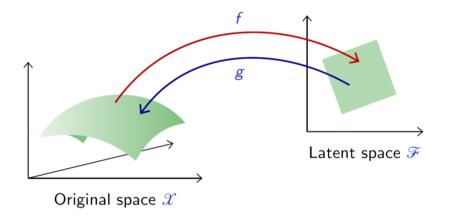




Autoencoder

- Nonlinear dimension reduction with NN
- Key idea:
 - An encoder function z = f(x)
 - A Decoder function x = g(z)
 - Learn to set $g(f(x)) \cong x$
- Loss function: reconstruction error
 - Minimize $||X g \circ f(X)||^2$





Clustering

The vast majority of data is unlabeled. The clustering algorithm tries to identify similar instances and assigning them to *clusters*

- K-means
- Hierarchical clustering
- DBSCAN

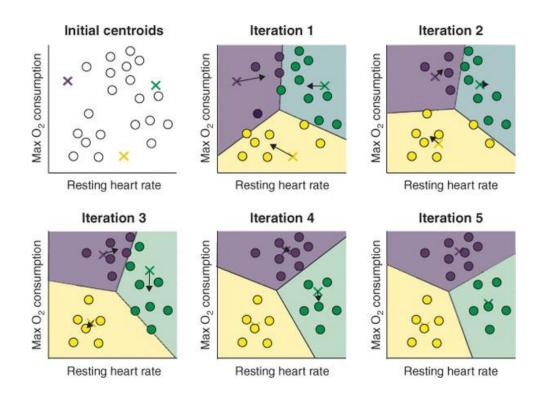


Look for instances centered around a particular point (centroid)

Core algorithm

Algorithm 1 k-means algorithm

- 1: Specify the number k of clusters to assign.
- 2: Randomly initialize k centroids.
- 3: repeat
- 4: **expectation:** Assign each point to its closest centroid.
- 5: **maximization:** Compute the new centroid (mean) of each cluster.
- 6: until The centroid positions do not change.



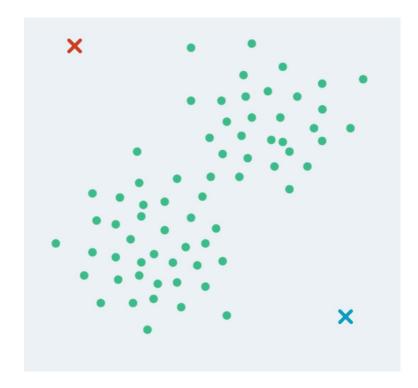
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The algorithm is guaranteed to converge in a finite number of steps (usually quite small).



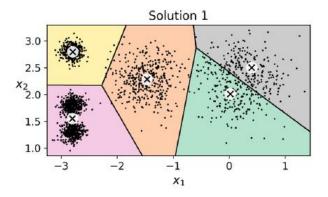
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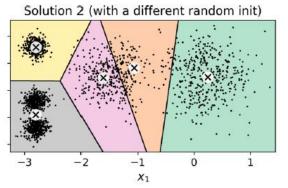
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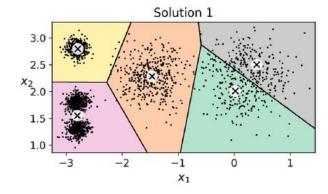
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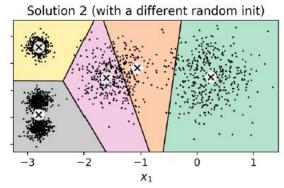
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Solution:

- If you know the approximate position of centroids, manually set them during initialization
- Run the algorithm multiple times with different random initialization, keep the best solution

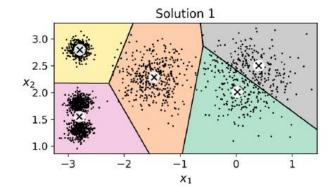
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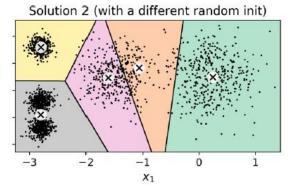
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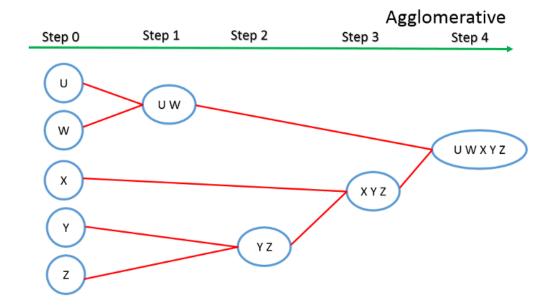
Note:

- K-means favor cluster with similar size, it doesn't perform well with varying sizes
- Need to specify the number of clusters

Hierarchical (Agglomerative) clustering

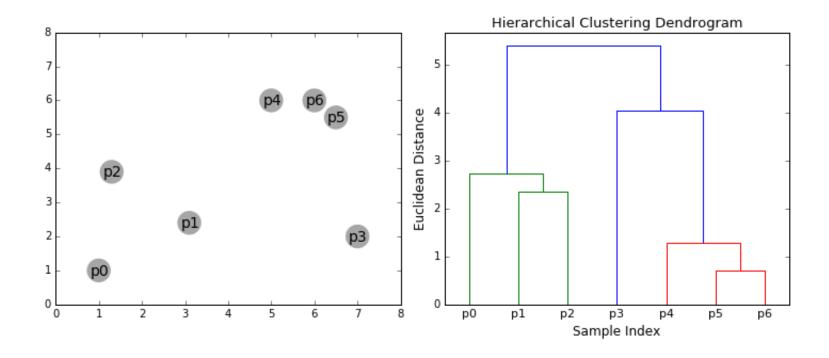
A bottom-Up approach to connect the nearest pair of clusters

```
\begin{split} t &= 0 \\ Choose \ R_0 &= [C_i = x_i, i = 1, ..., N] \ as \ initial \ clustering \\ Repeat \\ t &= t + 1 \\ Find \ the \ closest \ clusters \ C_i, C_j \ in \ the \ existing \ clustering \ R_{t-1} \ such \ that \\ g(C_i, C_j) &= \max_{r,s} (C_r, C_s) \ if \ g \ is \ similarity \ function \\ g(C_i, C_j) &= \min_{r,s} (C_r, C_s) \ if \ g \ is \ dissimilarity \ function \\ Define \ C_q &= C_i \cup C_j \ and \ produce \ the \ new \ clustering \ R_t &= [R_{t-1} - C_i - C_j] \cup C_q \\ Until \ only \ one \ cluster \ is \ left \end{split}
```



Hierarchical (Agglomerative) clustering

A bottom-Up approach to connect the nearest pair of clusters



Scale up well to large number of instance or clusters

DBSCAN

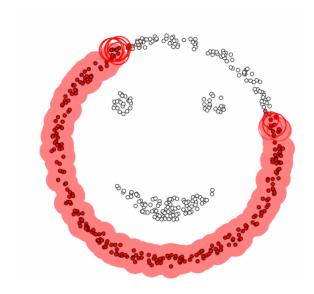
Defines clusters as continuous regions of high density

Core algorithm

Algorithm 3: DBSCAN Clustering

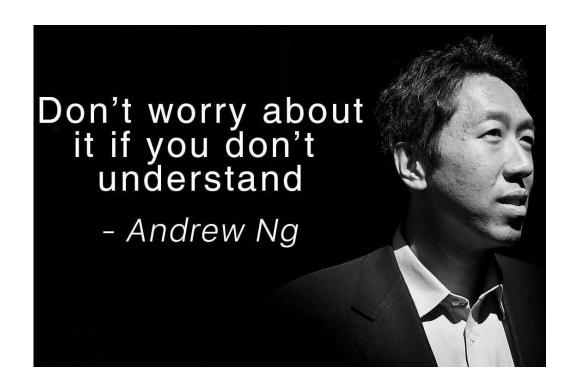
Input: $2D_Data$ obtained by Algorithm1 as the input data, |Data| objects to be clustered, the neighborhood radius (ϵ) and minimum points (μ)

- 1: Randomly select a point P
- 2: Retrieve all points density-reachable from P based on ε and μ and Similarity Metric (*Algorithm2*)
- 3: If *P* is a core point, a cluster is formed.
- 4: If *P* is a border point, no points are density-reachable from P and DBSCAN selects the next no-visited point randomly.
- 5: Continue the procedure until all points have been processed.



- Works pretty well if clusters are dense enough and separated well by low-density regions
- Robust to outliers
- Computation complexity is $O(m \log m)$

Let's do some practice!





Machine learning be like

> git clone https://github.com/wbvguo/qcbio-ML_w_Python.git

Summary

We have knowledge about ☐ Machine learning's definition and categories ☐ The workflow for train a machine learning model ☐ Rationale of several major machine learning algorithms! ☐ Performance measure for evaluating models ☐ Challenges in machine learning and potential solutions And we have experience in ☐ Jupyter notebook ☐ NumPy, matplotlib, scikit-learn, keras ☐ Build, train, evaluate a classifier or regressor ☐ Tune hyperparameters ☐ Unsupervised learning (PCA, K-Means)

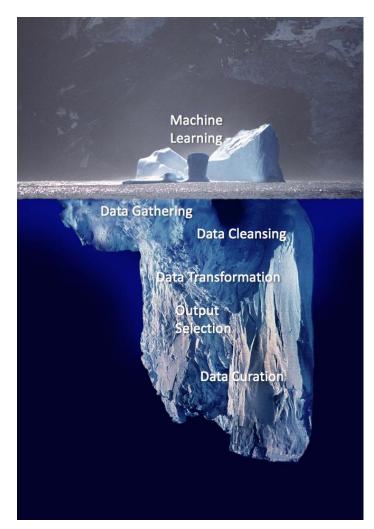
Summary of algorithms

Supervised learning		Unsupervised learning	
Classification	Regression	Dimension reduction	Clustering
Logistic regression	Linear regression	PCA	K-means
KNN	Polynomial regression	t-SNE	Hierarchical
Naïve bayes	SVR	Autoencoder	DBSCAN
SVM	Tree-based		
Decision tree	GBM		
Random forest	Neural Network		
Adaboost			
Gradient boosting			
Neural network			

Beyond machine learning algorithms

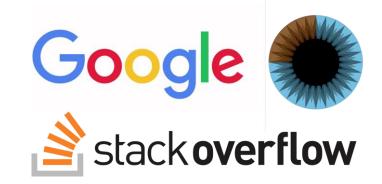
- Problem formulation
- Data cleansing
- Feature engineering
 - Feature encoding
 - Imputation: Missing data handling
 - Transformation/Normalization/Standardization

. . .



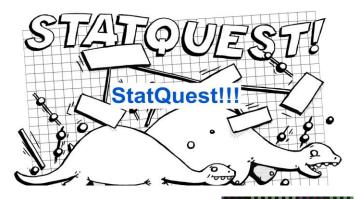
Where to get help?

- https://www.google.com
- https://stackoverflow.com
- https://stats.stackexchange.com/
- https://towardsdatascience.com/
- https://www.3blue1brown.com
- https://statquest.org/
- https://openai.com/blog/chatgpt/





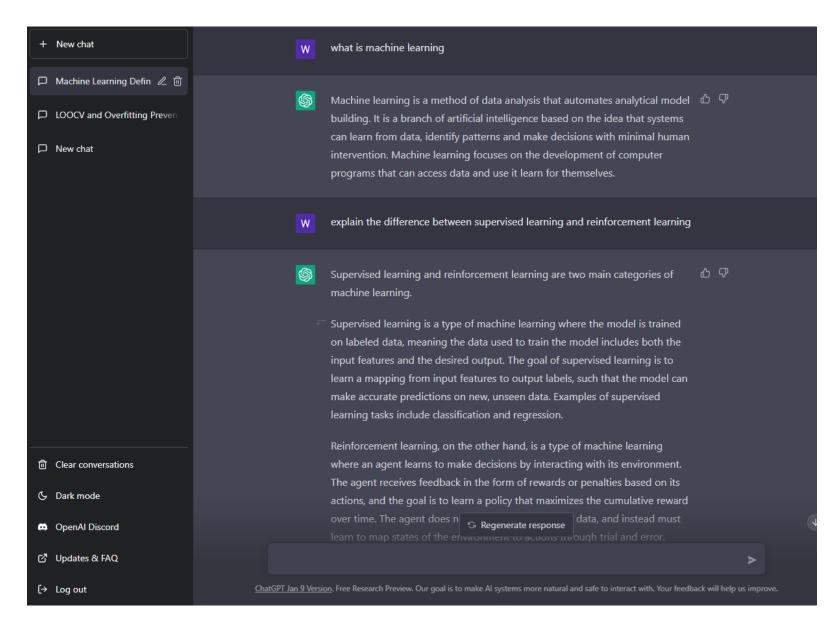
towards data science







ChatGPT



Lastly, GLHF!



"good luck, have fun with machine learning"



Q&A

Google docs