Recognition and Vulnerability Analysis of Key Nodes in Power Grid Based on Complex Network Centrality

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***Abstract*—The analysis of blackouts, which can inevitably lead to catastrophic damage to power grids, helps to explore the nature of complex power grids but becomes difficult using conventional methods. This brief studies the vulnerability analysis and recognition of key nodes in power grids from a complex network perspective. Based on the ac power flow model and the network topology weighted with admittance, the cascading failure model is established first. The node electrical centrality is further pointed out, using complex network centrality theory, to identify the key nodes in power grids. To effectively analyze the behavior and verify the correctness of node electrical centrality, the net-ability and vulnerability index are introduced to describe the transfer ability and performance under normal operation and assess the vulnerability of the power system under cascading failures, respectively. Simulation results of IEEE 30-bus and IEEE 57-bus test cases show that the key nodes can be effectively identified with high electrical centrality, the resultant cascading failures that eventually lead to a severe decrease in net-ability, verifying the correctness and effectiveness of the analysis.**

***Index Terms*—Cascading failure, complex network, vulnerability, power system, centrality.**

I. Introduction

With the continuous expansion in scale and the increasing integration of components, power grids' structure and characteristics become more and more complicated. Under these circumstances, whenever large blackouts occur, it can cause huge revenue losses and serious damage to the social production and the lives of residents. Therefore, it is of importance to sustain power systems reliability and enhance the robustness. By abstracting buses into network nodes and transmission lines into network links, power grids can be regarded as networks with interaction between units or individuals. The complex network theory provides a new aspect to evaluate the risk of cascading blackouts [1].

Many researchers have applied the complex network theory to study real networks [1]–[4]. In [5], several indices were raised to assess the robustness of power system, i.e., the percentage of unserved nodes (PUN) and the percentage of noncritical links (PNL). Then the influence of network topology and location of generators were explored. Furthermore, it was discovered that the cascading failures are usually triggered by some specific components' malfunction and then cause large blackouts, so that it is of great significance to determine the specific key nodes and branches in view of the stability of power system. In [6], based on the Kirchhoff's law, the definition of electrical betweenness was raised to identify the key branches, which overcame the disadvantage of former methods that power only flows along the shortest paths, reasonably reflecting the actual use of transmission lines by power flowing between "generator-load" node pairs. The effective graph resistance was deployed in [7] as a metric to assess the robustness of power grids against cascading failure, and then four strategies (based on degree product, principle eigenvector, Fiedler vector and effective resistance) were investigated to identify the best pair of nodes to connect in order to optimize the effective graph resistance. Among all these studies, the DC power model was widely applied to calculate the power flow, which cannot provide critical and detailed information about voltage and power quantities, because of the negligence of nonlinearily, which in turn affects the iteration of cascading failures.

In this brief, based on AC power flow model [8], which contains more accurate information compared with DC model, the node electrical centrality is proposed to identify the key nodes in power grids by using the complex network centrality theory. In Section II, a cascading failure model used AC power flow model is introduced at first. Then, by combining the electrical betweenness and eigenvector centrality, an assessment indicator of node importance, i.e., node electrical centrality, is further pointed out in Section III. Section IV introduces the vulnerability indices. Both the node electrical centrality and grid vulnerability indices are applied for the standard test cases of IEEE 30-bus and IEEE 57-bus to identify the key nodes and investigate the effectiveness of identification approach based on node electrical centrality. Simulation results verify that the cascading failures caused by failure of

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nodes with high electrical centrality will eventually lead to a more severe decrease of net-ability. Section V concludes this brief.

II. Model of Cascading Failures in Power Grids

A. Power Flow Model

In order to model cascading failures in a more practical way, this brief applies the AC power flow model to the cascading failure process [8], which has the advantages of rapid iteration and accurate calculation result. The problem of AC power flow calculation is equivalent to solve the non-linear node voltage equations [9]

$$Y_B U_B = \left[\frac{S}{U}\right]_B^*, \tag{1}$$

where Y_B is the node admittance matrix, Y_{ij} is the admittance of the transmission line connecting the node i and j, and $Y_{ii} = -\sum_{j\neq i} Y_{ij}$ represents the self-admittance of the node i. U_B is the bus voltage vector, and S is the injection power vector of each bus.

This brief applies the Newton-Raphson power calculation method to solve the AC power flow (1), which is a commonly used iterative method for solving nonlinear equations.

B. Overload Cascading Model

In a power system, the links or nodes, whose load exceeds their capacities, will be tripped by the circuit breakers to prevent them from permanent damage by the overloading. It is assumed that the capacity of a node or link is proportional to its initial load, which is obtained by the Newton-Raphson flow calculation with the initial parameter setting. The capacity of node i and link $(i-j)$ is defined as [5]

$$\begin{aligned} S_i &= (1+\alpha)L_i \\ S_{ij} &= (1+\beta)F_{ij}, \end{aligned} \tag{2}$$

where L_i is the initial load of the node i, and F_{ij} is the initial flow of the link $i-j$, α and β denote the safety margins of lines and nodes in the power system, respectively. Practically, the safety margins are limited because of the economic consideration. In this brief, the safety margins are set as $\alpha = 0.4$, $\beta = 0.2$.

The cascading failure process can be described as follows:

1) Initialization setting: set the power flow parameters, such as power load demand, generation power, voltage magnitude and phase, and the admittance of transmission lines, etc. Then, the initial power flow information can be obtained by Newton-Raphson flow calculation, and the capacities of each nodes and links are fixed.
2) Initial attack: select a node or link as the first failed component.
3) Calculation of power flow redistribution: according to the first failure in the system, the node admittance matrix will be altered, and then the Newton-Raphson flow calculation will be applied to obtain the new power flow distribution in the network.
4) Cascading process: identify every connected subgraphs and the overloaded nodes and links, and then remove the overloaded components. This procedure is duplicated until there is no nodes or links, whose load exceeds its capacity. The cascading iteration ended and the balanced condition of the system can be obtained.

III. Assessment Indicator of Node Centrality

A. Electrical Betweenness Centrality

A power grid can be abstracted into a complex network as an weighted undirected graph, where nodes represent generators, loads and distribution buses, and links are transmission lines. All links are weighted by the admittance values of the transmission lines, which can be used to express the power transfer capability of the line. The weighted adjacency matrix can be represented by $W = (W_{ij})$, which means that W_{ij} equals to the admittance of link $i-j$ if the node i is linked to the node j, so that $W_{ii} = 0$.

In power systems, because the power essentially doesn't only flow along the shortest paths, it is unreasonable to evaluate the importance of nodes and links in the network by the betweenness as in graph theory, which simply presents the ratio of the number of shortest paths passing through nodes or links to the number of all shortest paths between node i and j. Considering the capacity of nodes, the impedance of transmission lines and weighted adjacency matrix, the electrical betweenness can truly reflect the occupancy of a node in power transfer between "generator-load" nodes, and the influence of generation power and load level of different nodes can be taken into account.

The electrical betweenness [10] of node n is defined as

$$B_e(n) = \sum_{i\in G, j\in L} \sqrt{W_i W_j} B_{e,ij}(n), \tag{3}$$

where, G is the set of generation nodes, L is the set of load nodes. (i, j) represents the "generator-load" node pair. The weight of generator node, W_i, is defined as the rated generation active power of generator i and the weight of load nodes, W_j, is defined as the actual or peak load. $B_{e,ij}(n)$ is the electrical betweenness of node n by the unit injection current to the node pair (i, j),

$$B_{e,ij}(n) = \begin{cases} \frac{1}{2}\sum_m \left|I_{ij}(m,n)\right|, & n \neq i, j \\ 1, & n = i, j, \end{cases} \tag{4}$$

where m represents all the nodes directly connected to the node n; $I_{ij}(m,n)$ is the current of the link $(m-n)$ caused by the unit injection current to the "generator-load" node pair (i, j).

The equation (4) reflects the occupancy of node n in power transfer between node pair (i, j), which sums up $B_{e,ij}(n)$ of all the "generator-load" node pairs according to the corresponding weight in the network. Therefore, the electrical betweenness can quantify the significance of each node in whole network power transfer.

By normalizing the indicator, the electrical betweenness centrality is defined as the solution of

$$C_{be}(n) = \frac{B_e(n)}{\sum_{i\in G, j\in L} \sqrt{W_i W_j}}. \tag{5}$$

B. Eigenvector Centrality

The eigenvector centrality is a topological graph metric, quantifying the centrality of a node in a network, according to the idea that a node is important if it is linked by other important nodes. The eigenvector centrality E_i of node i is therefore defined as [11]

$$E_i = e_i = \frac{1}{\lambda} \sum_{j \in M(i)} e_j = \frac{1}{\lambda} \sum_{j=1}^{n} W_{ij} e_j, \tag{6}$$

where $M(i)$ is the set of nodes directly connected to the node i, n is the number of nodes in the whole network, and λ is a constant. With this rearrangement, it can be rewritten in the vector notation as the eigenvector equation, i.e.,

$$Wx = \lambda x. \tag{7}$$

C. Node Electrical Centrality

By combining the electrical betweenness centrality and eigenvector centrality, the electrical centrality of a node can be obtained, which not only reflects the electrical characteristic by the occupancy of a node in the whole network power transfer, but also indicates the significance of a node from the topological aspect. The electrical centrality of the node i is defined as the solution of

$$NEC(i) = \mu C_{be}(i) + (1 - \mu) E_i. \tag{8}$$

Here, μ is the distribution coefficient indicating the weight of two indicators (i.e., electrical betweenness centrality and eigenvector centrality) in electrical centrality. It further represents the weight of electrical characteristics and topological properties in nodal significance analysis. μ is obtained based on the statistical characteristics of two indicators as follows,

$$\mu = \frac{\text{avg}(C_{be})/\text{var}(C_{be})}{\text{avg}(C_{be})/\text{var}(C_{be}) + \text{avg}(E)/\text{var}(E)}, \tag{9}$$

where avg$(\cdot)$ represents the average function, var$(\cdot)$ represents the variance function. Although μ contains the information about both electrical betweenness centrality and eigenvector centrality, which consist of the representation of μ, either one with smaller variance weight will occupy higher weight. Such combination is built based on the simulation results about electrical betweenness centrality and eigenvector centrality, the latter of which always only has several dominant results so that most of results cannot provide significant contribution.

IV. Case Study

In this section, the effectiveness of key nodes identification approach based on node electrical centrality is verified. Firstly, a vulnerability index proposed in [12] is introduced to evaluate the damage of cascading failures. The proposed identification approach of key nodes is further applied to standard IEEE 30-bus case and 57-bus case, and the vulnerability of systems is calculated under cascading failures caused by every nodes. If the key nodes identified with high node electrical centrality have high vulnerability, which means that the removal of these nodes will cause huge damage to power system, the effectiveness of node electrical centrality can be effectively verified.

A. Vulnerability Index

1) Electrical Distance: The electrical distance is defined as the equivalent impedance Z_{ij} between node i and node j, which can be acquired by injecting a unit current at the node i and extracting at the node j. Thus, the equivalent impedance is equal to the voltage between two nodes U_{ij},

$$Z_{ij} = \frac{U_{ij}}{I_i} = U_{ij}. \tag{10}$$

2) Grid Vulnerability Index: The net-ability of power system is to evaluate the transfer ability and performance under normal operating conditions [12], which is affected by network structure, impedance of each transmission line, rated generators power and load node demand. By taking all the factors above into account, the net-ability of a power transmission grid is explained as

$$NA = \frac{1}{N_G N_L} \sum_{i \in G} \sum_{j \in L} \frac{P_i}{L_j \left| e^{Z_{ij}} \right|}, \tag{11}$$

where N_G and N_L are the number of generation nodes and load nodes, respectively. P_i represents the generation power, and L_j represents the maximum load of buses. If the load bus is directly supplied with generation units, Z_{ij} will become zero. For other combinations of generation and load nodes, the exponential item is used to indicate that the contribution of the generation node i to the load node j is negative correlated to the value of Z_{ij}.

The vulnerability of the power system can be thus assessed according to the decrease of the net-ability. The vulnerability of a node i is defined as the net-ability drop when the node i is removed from the network.

$$V_{NA}(i) = \frac{NA - NA_i}{NA}, \tag{12}$$

where NA is the net-ability of the initial network, NA_i is the net-ability after the cascading failure caused by removing the node i.

B. Numerical Analysis

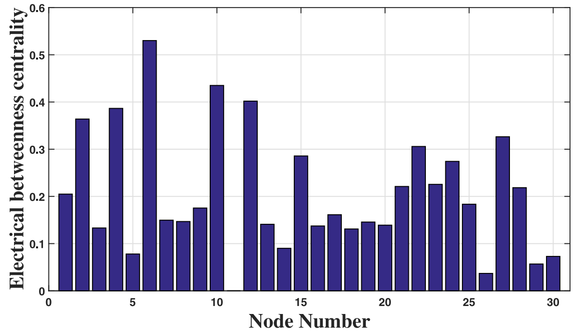
Simulations are conducted using MATLAB in this brief, where the toolbox library MATPOWER [13] is used to solve the AC power flow equations.

Firstly, the node electrical centrality is calculated based on the network topology and initial power flow information, in order to recognize the key nodes in the system. And then the vulnerability of each node is obtained through the cascading failure simulation caused by removal of nodes such that the effectiveness of node electrical centrality is verified.

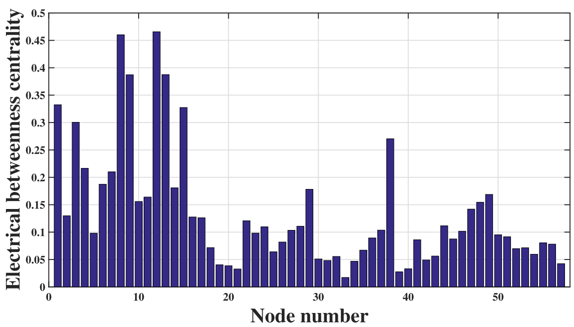
In order to derive the electrical centrality of nodes, the electrical betweenness of each node is calculated and obtained by setting the injection current to generator node i as 1, and the injection current to load node j as -1. Then the electrical betweenness is normalized to get the electrical betweenness centrality according to (5), as shown in Fig. 1. The weighted adjacency matrix W is further used to obtain the eigenvector centrality of each node, as shown in Fig. 2.

According to electrical betweenness centrality and eigenvector centrality obtained above, by using (9), μ can be obtained, i.e., $\mu = 0.741$ for IEEE 30-bus system and $\mu = 0.663$ for IEEE 57-bus system. As a result, the electrical centrality of nodes are obtained for IEEE 30-bus and IEEE 57-bus systems, respectively, and shown in Fig. 3.

From Fig. 3, it can be observed that there is wide variation on the electrical centrality of each node, indicating that

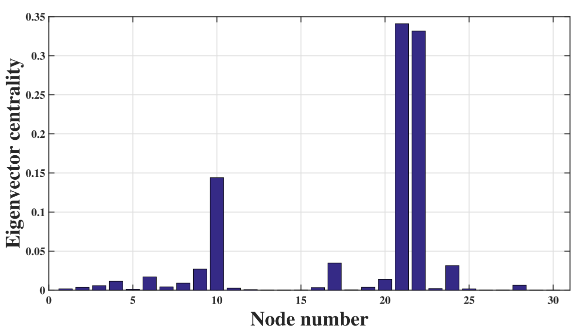


(a) IEEE 30-bus

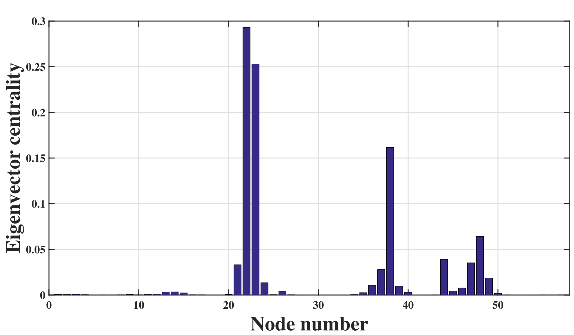


(b) IEEE 57-bus

Fig. 1. Electrical betweenness centrality of all nodes in IEEE 30-bus and IEEE-57 bus.



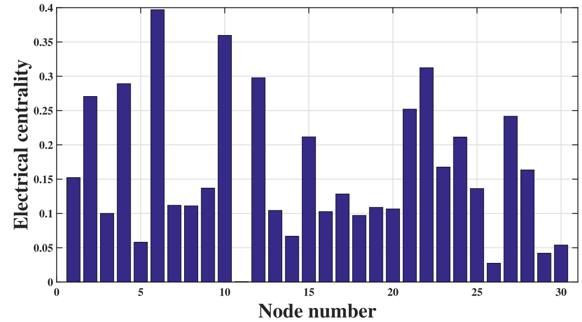
(a) IEEE 30-bus



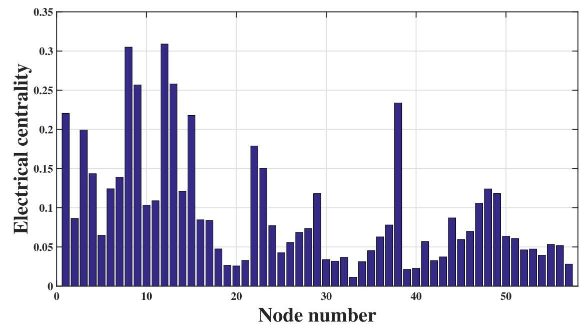
(b) IEEE 57-bus

Fig. 2. Eigenvector centrality of all nodes in IEEE 30-bus and IEEE 57-bus.

different nodes own various significance in the power transfer of grids. The difference between electrical centrality of each node is the result of the interaction of the node location in the network topology and its transmission capacity. For instance, in IEEE 30-bus system, the maximum electrical



(a) IEEE 30-bus



(b) IEEE 57-bus

Fig. 3. Electrical centrality of all nodes in IEEE 30-bus and IEEE 57-bus.

TABLE I
TOP 5 NODES OF ELECTRICAL BETWEENNESS CENTRALITY, NODE ELECTRICAL CENTRALITY, AND VULNERABILITY IN IEEE 30-BUS SYSTEM

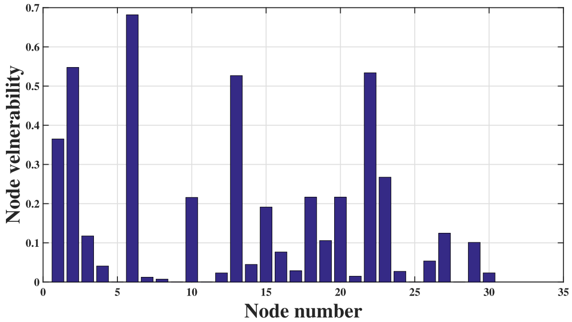
Nodes	$C_{be}(n)$	Nodes	$NEC(n)$	Nodes	$V_{NA}(n)$
6	0.530	6	0.397	6	0.682
10	0.435	10	0.360	2	0.547
12	0.402	22	0.313	22	0.534
4	0.387	12	0.298	13	0.527
2	0.364	4	0.289	1	0.365

centrality among all nodes is 0.318 for the node 6, and the minimum electrical centrality is 0.0006 for the node 11.

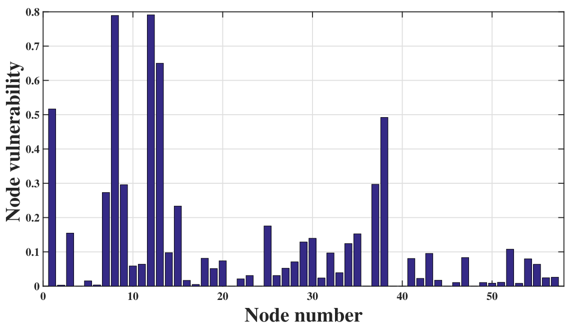
To evaluate the influence by nodes to the grids vulnerability and verify the correctness of the electrical centrality, the net-ability of two tests is firstly calculated according to the equation (11). The net-ability results regarding two systems are 4.792 and 20.616, respectively.

Furthermore, in order to obtain the net-ability after cascading failures triggered by every node, the failure node is picked up in sequence and removed at a time to simulate a cascading failure process. The cascading failures will continue until all the remaining components operate normally. When the system is under stable operation condition, the net-ability of the system will be re-calculated, and the grid vulnerability index can be obtained according to (12). The vulnerability of each node is shown in Fig. 4. List of top 5 nodes of the electrical betweenness centrality, electrical centrality and vulnerability is shown in Table I and Table II for IEEE 30-bus and IEEE 57-bus, respectively.

As can be observed, the node with the maximum electrical centrality always has the maximum vulnerability, indicating that the removal of this node has the most severe damage to the network power transfer capability.



(a) IEEE 30-bus



(b) IEEE 57-bus

Fig. 4. Vulnerability of all nodes in IEEE 30-bus and IEEE 57-bus.

TABLE II
TOP 5 NODES OF ELECTRICAL BETWEENNESS CENTRALITY, NODE ELECTRICAL CENTRALITY, AND VULNERABILITY IN IEEE 57-BUS SYSTEM

Nodes	$C_{be}(n)$	Nodes	$NEC(n)$	Nodes	$V_{NA}(n)$
12	0.466	12	0.309	12	0.791
8	0.460	8	0.305	8	0.789
13	0.388	13	0.258	13	0.650
9	0.387	9	0.257	1	0.517
1	0.332	38	0.234	38	0.492

As shown in Fig. 2, the eigenvector centrality can only reflect the importance of nodes in view of network topology, and the result shows only several nodes have significant eigenvector centrality. Based on the comparison between electrical betweenness centrality and vulnerability index simulation results from Table I to II, although the nodal ordinal regarding the maximum of these two indicators are identical, the following nodal ordinal regarding these two indicators is not exactly the same. Therefore, either the electrical betweenness centrality or eigenvector centrality cannot fully reflect the significance of all nodes so that their combination becomes necessary.

By means of the combination of these two indicators, the electrical centrality contains the information about both electrical characteristics and topological properties of each node. According to the results in Table I, the effectiveness of two indices is compared, i.e., the index from [10] and electrical centrality proposed in Section III. As can be seen, the key nodes identified by electrical centrality is more consistent with the vulnerability result because the eigenvector centrality supplements the topological properties of each node to electrical centrality, which enhances the importance of some nodes with significant topological position, such as node 22 in IEEE 57-bus system and node 38 in IEEE 57-bus system. Therefore, the accuracy and effectiveness of key node recognition method based on electrical centrality is superior to the method using either electrical betweenness centrality or eigenvector centrality.

V. CONCLUSION

Considering the practical characteristic of power flow in power system, this brief proposes a novel key nodes recognition method in power grids based on electrical centrality. By using the AC power flow model, the weighted topology with admittance and complex network centrality theory are fully applied to acquire the node electrical centrality in order to identify the key nodes. Simulation results on IEEE 30-bus and IEEE 57-bus test cases verify that the key nodes can be identified by high electrical centrality and the further cascading failures caused by these identified nodes will lead to a more severe decrease of net-ability. Compared with electrical betweenness index, the identification result of electrical centrality can offer superior accuracy and reflect not only the electrical characteristics but also network topology features. Therefore, the proposed electrical betweenness centrality is more suitable to reveal the characteristic of real power systems.

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