EE 4302 ADVANCED CONTROL SYSTEM

CA2-Root-Locus Analysis

and Sliding Model Controller

Report

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1. Root-Locus Analysis

The simulink model of a system is shown in figure 1.

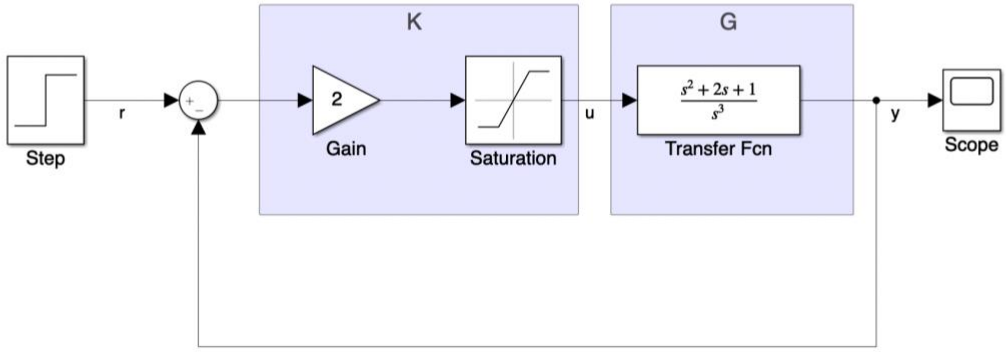


Figure 1 Simulink Model for Root-locus Analysis

1.1 Set block parameters

For Step block as an input single, set its step time as 1 and final value as 1.

For Gain block, set the gain is 2.

For Saturation block, set its upper limit and lower limit as +1 and -1 respectively.

For Transfer Fcn block, we assume the original transfer function of the system is $\frac{s^3+2s+1}{s^3}$, and set its numerator coefficients as [1, 2, 1] and denominator coefficients as [1, 0, 0, 0].

1.2 Run and watch the trace on scope.

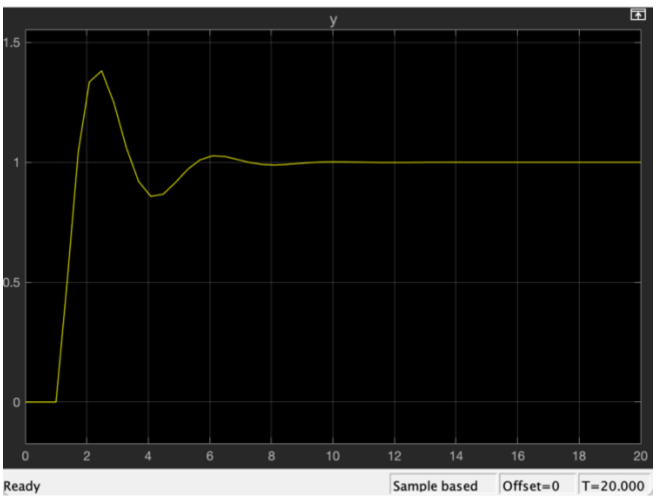


Figure 2 Final value (e) = 1

1.3 Determine the Final Value that gives sustained oscillation

Set the input step signal with final value = 1,2,3,4 and get their output figures on scope.

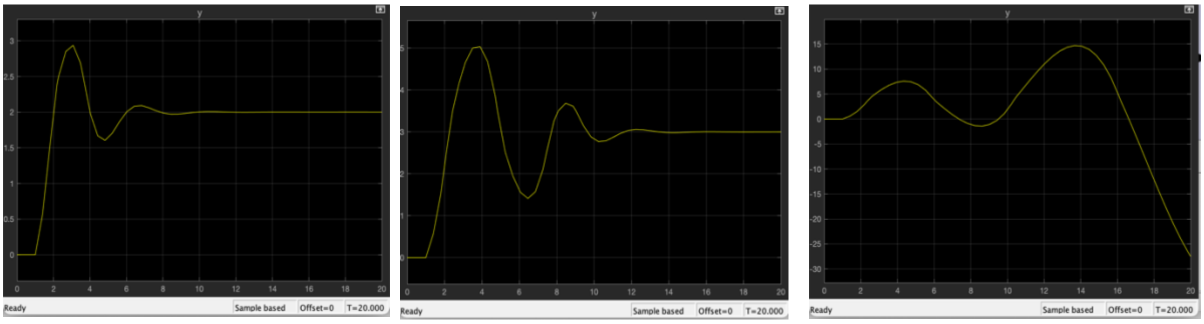
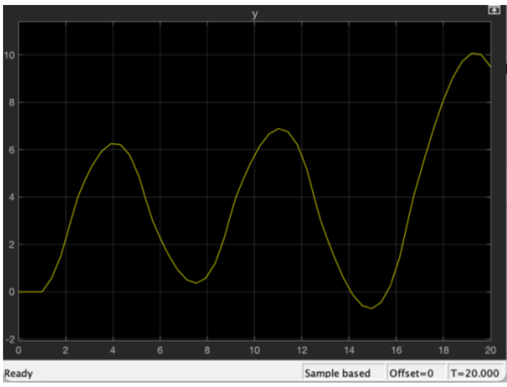


Figure 3 Final value (e) = 2, 3, 4

From the figures above, we can get that when input signal is 1,2 and 3, the system is stable and from value = 4, the system begins to be unstable.

So, to find the final value that gives sustained oscillation, we trial and error the values between 3 and 4.



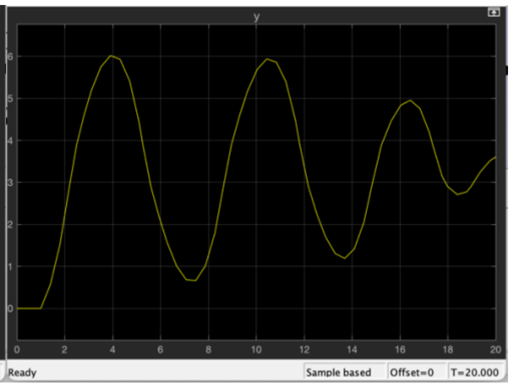


Figure 4 Final value (e) = 3.5 **Figure 5 Final value (e) = 3.4**

Finally, we determine the final value that gives sustained oscillation should be value between 3.4 and 3.5. And when final value is 3.43, we can get a good result.

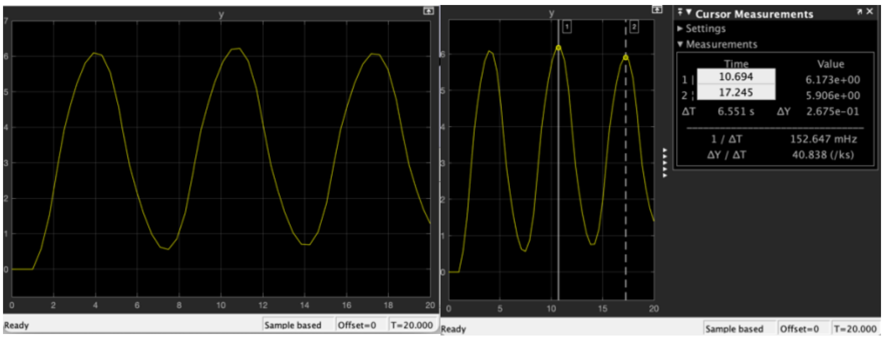


Figure 6 Final value (e) = 3.43

When $e = 3.43, u = 1, K = \frac{u}{e} = 0.2915.$

The period of the sustained oscillation is 6.551 and the corresponding gain is 0.2915.

1.4 The locus of the closed-loop poles Analysis.

We can get a table of values of e, u and K when the final value changes from 1 to 4.

e	u	K
1	1	1
2	1	0.5
3	1	0.3333
4	1	0.25

Table 1

Analyze the locus of the closed-loop poles through the figures. When e = 1,2 and 3, the closed-loop system is stable, so the roots are on the left half plane. Besides, when e increases, the value of K increases, so the roots will move towards the imaginary axis, and the oscillation time of the system increases.

When e is bigger than 4, the closed-loop system is unstable and the roots are on the right half plane. After that, with e increases, value of K begins to decrease, he oscillation phenomenon of the closed loop system increases, and the roots will move further away from the imaginary axis.

Between them, when e is about 3.43, the system is margined stable and sustained oscillation, so the roots of the closed loop system in the imaginary axis.

2. Sliding Mode Control Design

(a) Variable-structure controller design.

The unstable system is

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u \tag{2.1}$$

And the transfer function is

$$\frac{Y(s)}{U(s)} = \frac{1}{s(s-1)} \tag{2.2}$$

Design a variable-structure controller. Choose the switching line as

$$\sigma = x_1 + x_2 \tag{2.3}$$

The control law is

$$u(t) = -\frac{p^T f}{p^T g} - \frac{k}{p^T g} sing\big(\sigma(x)\big) = -[2,0]x - 0.5sign\big(\sigma(x)\big)$$

$$k = 0.5$$

$$p^T = [1 \quad 1]$$

$$f = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$g = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{2.4}$$

(b) Simulate the system using simulink.

Use a inherent block to control the sample time of the loop system as there is no input signal.

Use a gain block (K*u vector), i.e. $[2,0][x1, x2]^T$ to simulate the $"-[2,0]x(t)"$ part of the controller.

Use a sign block and a gain block to simulate the $"-0.5sign\big(\sigma(x)\big)"$ part of the controller.

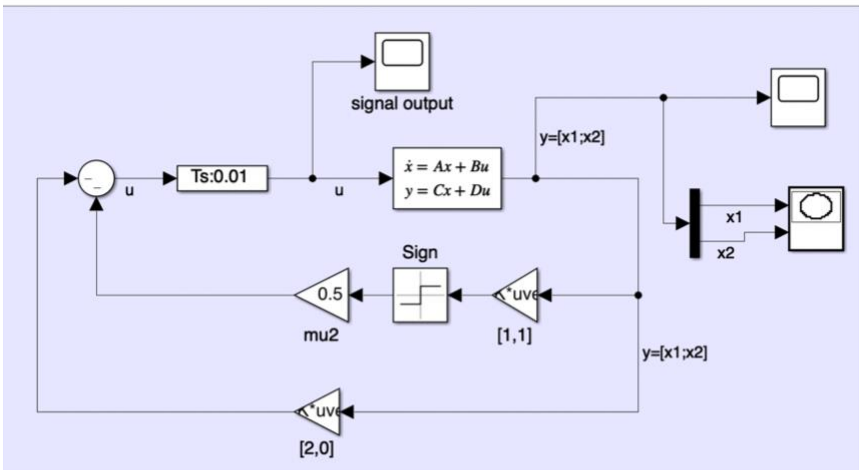


Figure 7 Sliding Mode Control Simulink

And the states, control signal and phase plane trajectory plots are as follow.

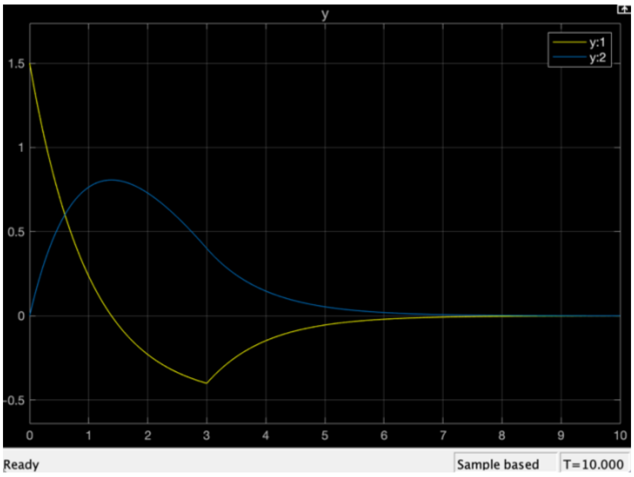


Figure 8 State Variables Plot

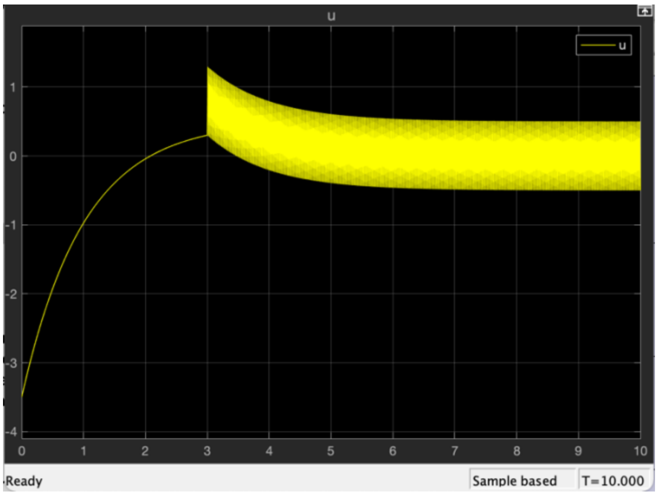


Figure 9 Control Signal Plot

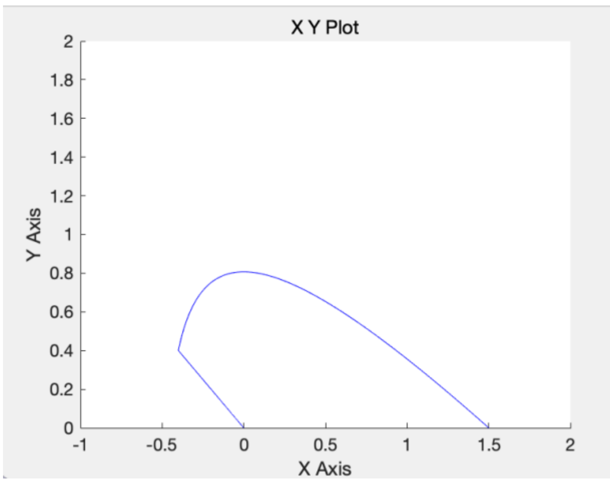


Figure 10 Phase Plane Trajectory plot

(c) Smooth controller Simulation

Modify the variable-structure controller such that chattering is eliminated and simulate the system.

The smooth control law is

$$u(t) = -\frac{p^T f}{p^T g} - \frac{k}{p^T g} sat(\sigma(x), \varepsilon) = -2x_1 - 0.5 sat(\sigma(x), \varepsilon) \qquad (2.5)$$

Replace the sign block with a saturation block, we can get the system in figure 11.

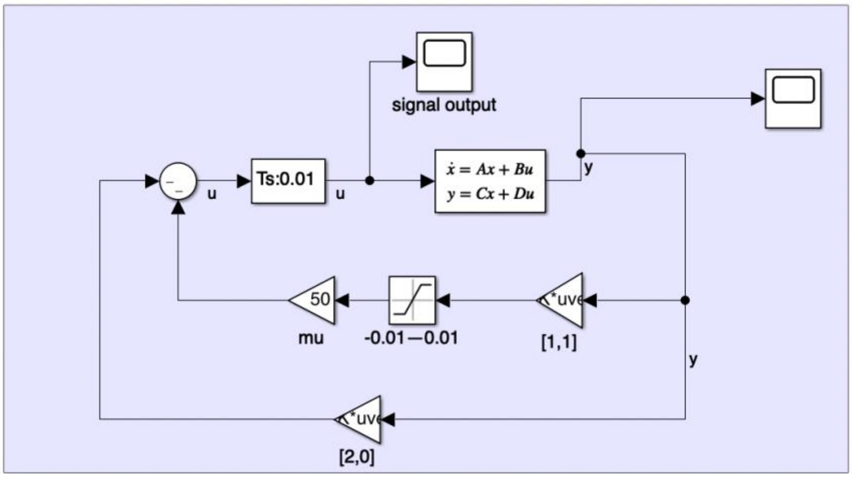


Figure 11 Smooth Control System Simulation

Plot the states and control signal in following figures.

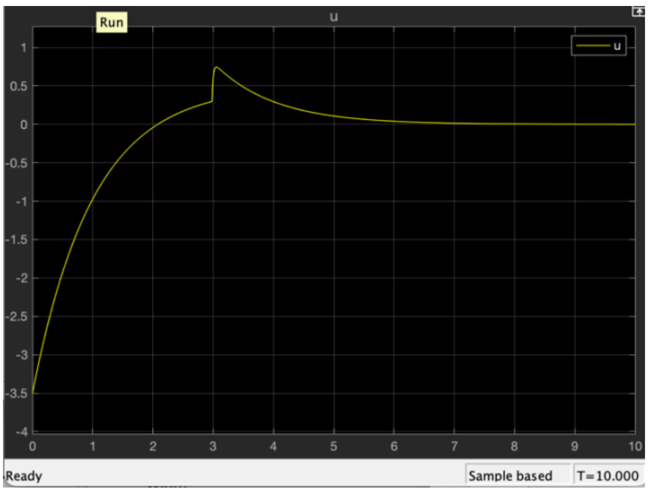


Figure 11 State Variables Plot

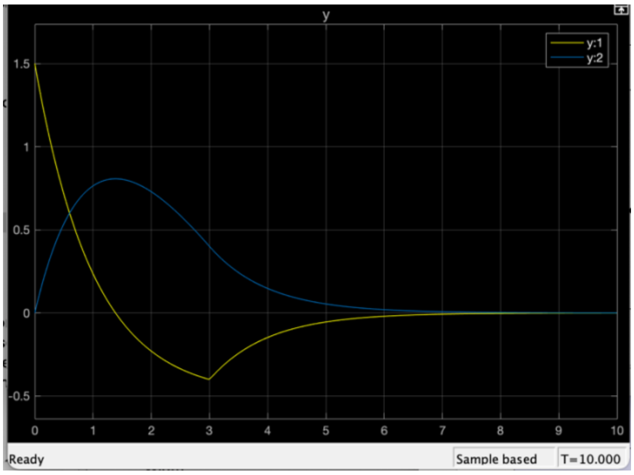


Figure 12 Control Signal Plot

(d) Results Analysis

Through sliding controller, the system can finally achieve a steady state which can be observed in figure 8, and will stay on the switching surface (σ) once it reached. The reason is the derivative of the Lyapunov function ($V = \sigma^2$) is negative definite, so the system is asymptotically stable, that is, σ tends to zero.

However, form figure 9 Control signal Plot in 2(b), we can observe that the control signal has a drawback that rely chatters. To avoid it, we then try the saturation function instead of the sign function and get a smooth controller. And the corresponding plots are shown in figure 11 and figure 12.