Response for Project Week 6

Course Code: Fintech545

Course Title: Quantitative Risk Management

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Problem 1

Problem 1

Assume you a call and a put option with the following

- Current Stock Price $165
- Current Date 03/03/2023
- Options Expiration Date 03/17/2023
- Risk Free Rate of 4.25%
- Continuously Compounding Coupon of 0.53%

Calculate the time to maturity using calendar days (not trading days).

For a range of implied volatilities between 10% and 80%, plot the value of the call and the put.

Discuss these graphs. How does the supply and demand affect the implied volatility?

1.1 Calculating the time to maturity

From the problem, the current date is 03/03/2023, and the expiration date is 03/17/2023. Since there are 365 days in a year. Therefore, the time to maturity is below:

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Time to Maturity is 0.0384
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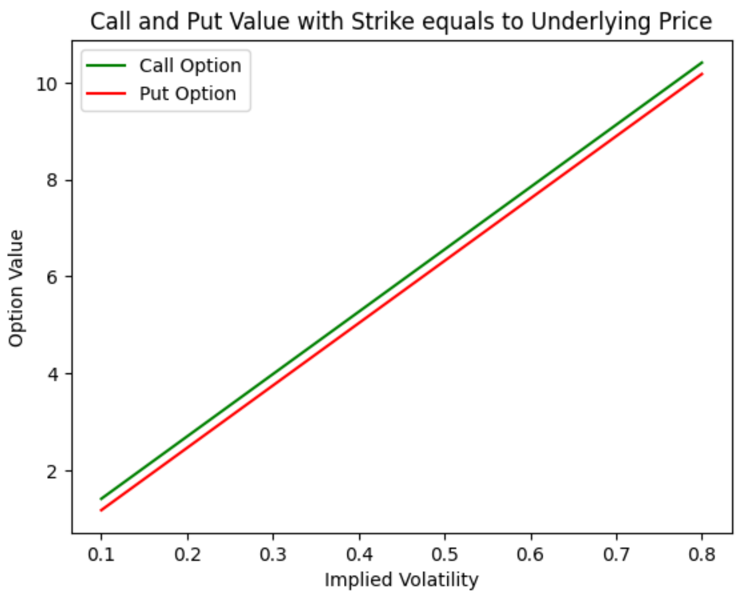
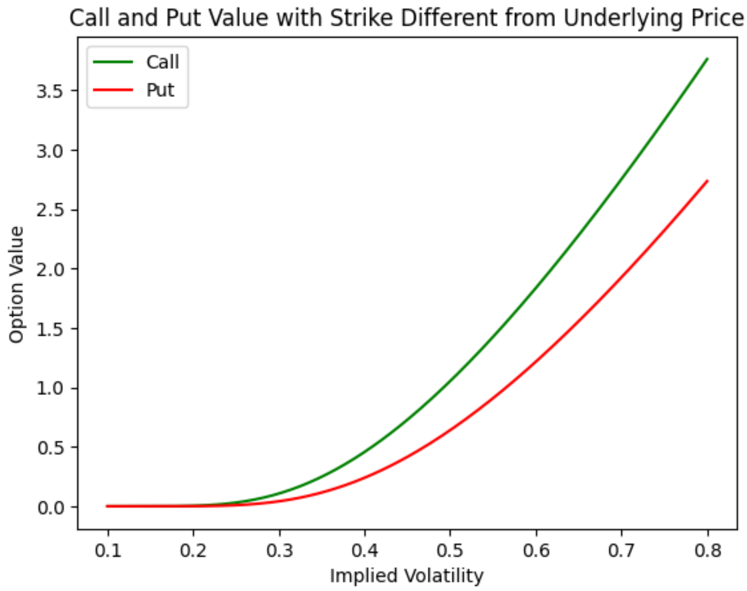
1.2 Plot the value of the call and the put

Before calculating the value of the call and the put, we need to specify the implied volatilities. Since the range is from 10% - 80%, we can generate multiple value of implied volatility from this range. In this homework, I generate 200 evenly spaced volatility from 10% to 80%.

After generating the implied volatility, we can calculate the value of the call and put using the generalized BS model. The figure below is the call and put value with strike price equals to the underlying price.



We can also set the strike price and the underlying price being different. In this homework, I set the strike price is 20 more than underlying price for the call, I also set the strike price is 20 less than underlying price for the put. By using these stats, we can also plot a figure as below:



1.3 Graph Discussion

As we can see the graphs above, the option value increases as the implied volatility increases, which means that the price and the implied volatility are positive correlated. This is quite intuitive since we know that volatility is a measure of risk, and if the risk is higher, then the price should also be higher for taking more risks.

As for the demand and supply, we know that demand and supply will affect price. According to the basic law of demand and supply, when demand increases or the supply decreases, the price increases. When demand decreases or supply increases, the price drops. Since the implied volatility is positively correlated with the price. In the case of demand increases or supply decreases, implied volatility will increase; when demand decreases or supply increases, implied volatility will decrease.

Problem 2

Problem 2
Use the options found in AAPL_Options.csv

- Current AAPL price is 151.03
- Current Date, Risk Free Rate and Dividend Rate are the same as problem #1.

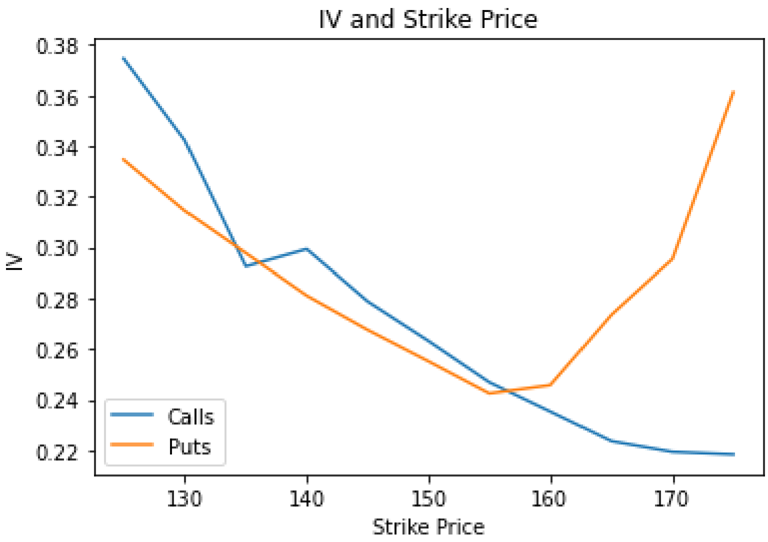
Calculate the implied volatility for each option.

Plot the implied volatility vs the strike price for Puts and Calls. Discuss the shape of these graphs. What market dynamics could make these graphs?

There are bonus points available on this question based on your discussion. Take some time to research if needed.

2.1 Calculating the implied volatility for each option

First, we need to build functions to calculate the implied volatility. Then, we can calculate the implied volatility for each option. After calculation, we can plot the graph.



2.2 Discussion

Strangely, the volatility smile is not displayed for the call options in the graph. The term "volatility smile" refers to the U-shaped pattern formed by the implied volatility of options with the same expiration date but differing strike prices. The reason it seems like a "smile" is due of its appearance.

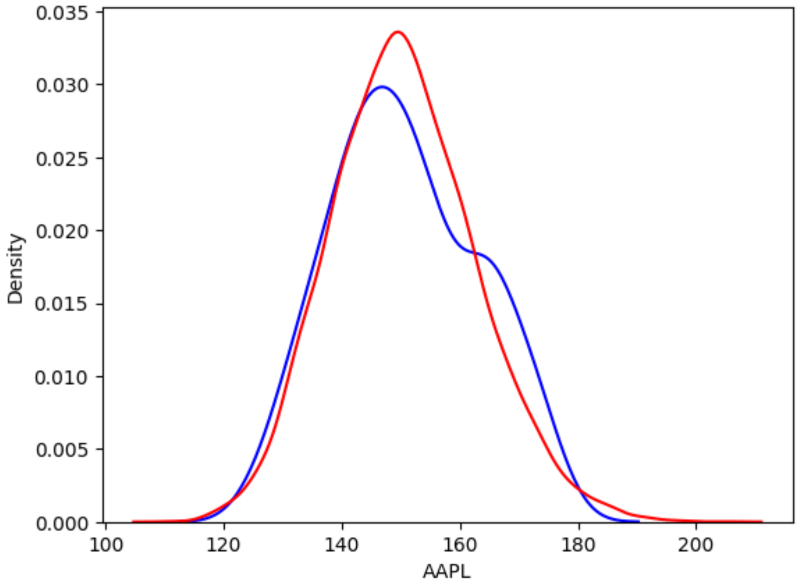
Assuming they have the same expiration date, options with higher strike prices often have lower implied volatility than options with lower strike prices in a normal market. The implied volatility for at-the-money options, however, may occasionally be lower than for out-of-the-money or in-the-money options. This results in the volatility grin, a U-shaped curve.

The market's expectation of more price volatility for the underlying asset in the near future is shown by the volatility smile, which raises the implied volatility for some option contracts. For puts options, a stock may have a volatility smile, but not for call options. This can happen when market participants' estimates for future price changes for the underlying stock at various strike prices deviate.

One explanation is that market participants expect greater safety for out-of-the-money puts than out-of-the-money calls because they are more concerned about downside risks. Because of this, put options with lower strike prices have higher implied volatility, creating a volatility smile.

On the other hand, because market players could have various expectations for the potential upside of the stock, the implied volatility for call options might not follow the same trend. They might not require as much protection for out-of-the-money calls as they do for out-of-the-money puts because they are less concerned about upside risks.

This anomaly implies that the BS model assumption is deficient. We can plot the actual price and compare it to the lognormal assumption.



As we can see the graph, the actual distribution is different from the assumed distribution. That's probably another reason of volatility smile.

Problem 3

Problem 3

Use the portfolios found in problem3.csv

- Current AAPL price is 151.03
- Current Date, Risk Free Rate and Dividend Rate are the same as problem #1.

For each of the portfolios, graph the portfolio value over a range of underlying values. Plot the portfolio values and discuss the shapes. Bonus points available for tying these graphs to other topics discussed in the lecture.

Using DailyPrices.csv. Calculate the log returns of AAPL. Demean the series so there is 0 mean. Fit an AR(1) model to AAPL returns. Simulate AAPL returns 10 days ahead and apply those returns to the current AAPL price (above). Calculate Mean, VaR and ES. Discuss.

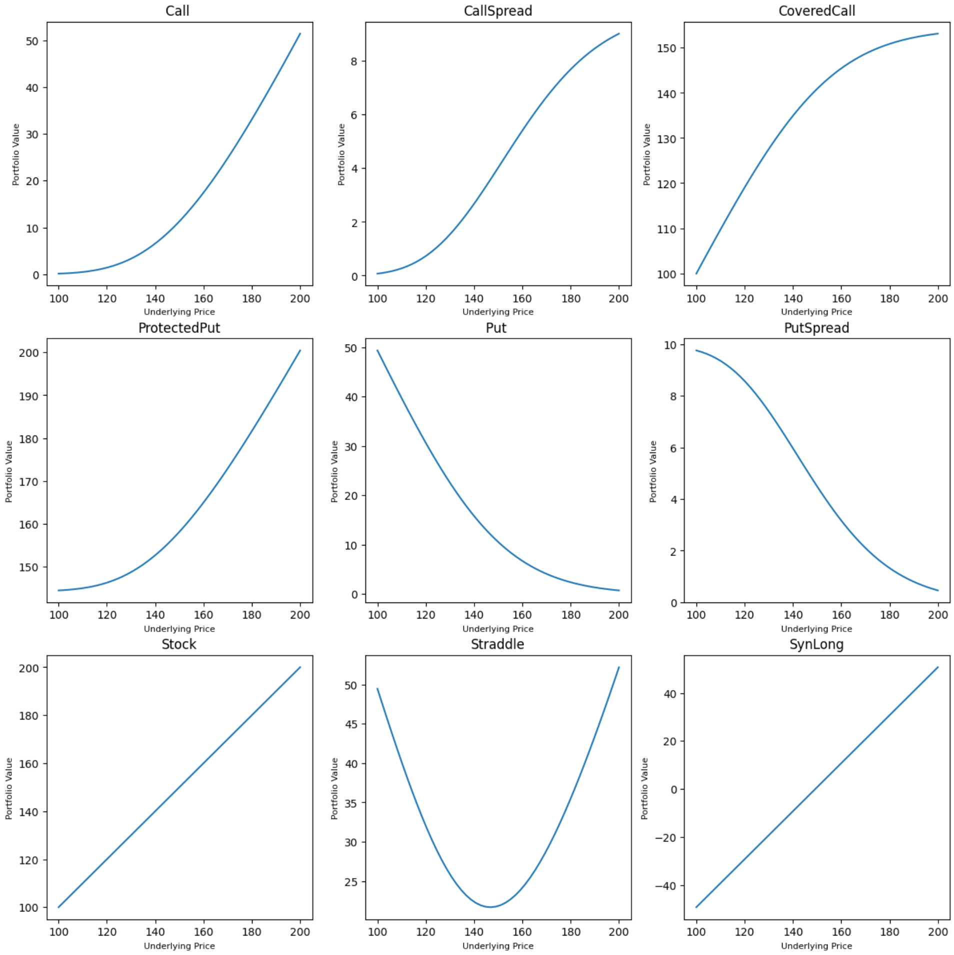
Hints:

- you will need to calculate the implied volatility - might not be the same as #2
- you need to take into account the change in dates for option valuations. You are simulating forward in time and options valuations are a function of time
- Calculate the PL from the current portfolio value using Current Date

3.1 Graph the portfolio

We can plot the graph of the portfolio, and the graph is shown below:

Discussion in 3.3 section



3.2 Calculate Mean, VaR and ES

After calculating the mean, VaR and ES, we can get a graph as shown below:

	Mean of Portfolio Value($)	Mean of Losses/Gains($)	VaR($)	ES($)	VaR(%)	ES(%)	Current Value (on 2023/3/3)
Call	11.604734	-4.804734	3.025212	3.942142	44.488412	57.972675	6.80
CallSpread	4.194345	0.395655	2.748020	3.141400	59.869719	68.440082	9.01
CoveredCall	142.023566	4.956434	14.715407	17.735371	10.011843	12.066520	155.08
ProtectedPut	158.680324	-4.640324	4.478038	5.654142	2.907062	3.670567	154.04
Put	9.656584	-4.806584	1.428960	2.265502	29.463086	46.711376	4.85
PutSpread	4.281372	-1.271372	1.086802	1.509383	36.106374	50.145608	6.69
Stock	151.354218	-0.324218	16.042058	19.762249	10.621770	13.084982	151.03
Straddle	21.261319	-9.611319	-7.742564	-7.738140	-66.459776	-66.421807	11.65
SynLong	1.948150	0.001850	16.358861	20.076945	838.915928	1029.586898	11.65

3.3 Discussion

The strategies can be divided into four kinds:

1. Stock and SynLong:

"Synthetic Long" is a trading strategy in which a position is created that behaves similarly to owning a long call option on an underlying asset, but with a different combination of other options and/or the underlying asset. An investor would typically combine a long position in a stock or futures contract with a long position in a put option on the same underlying asset to create a synthetic long position. Because the put option provides downside protection (similar to the long call), and the stock or futures position provides upside potential, this combination effectively creates a position that mimics the behavior of owning a long call option on the underlying asset.

An investor may be able to achieve the same exposure and potential profit by constructing a synthetic long position, while potentially lowering the cost of the position or adjusting the risk profile. However, depending on the specific options and underlying assets used in the strategy, synthetic long positions may have different risks and costs than owning the actual long call option.

These two has the two largest VaR and ES.

2. Call, Put, Covered Call, Protected Put.

These four are very similar because put and call options have put call parity. A covered call strategy entails holding a long position in an asset while writing (selling) call options on the same asset. A protective put, also known as a married put, is a strategy that entails purchasing a put option on an asset while simultaneously holding a long position in that asset. As the asset is held, these two are not naked. Because they involve holding a long position in the underlying asset in addition to the option position, CoveredCall and ProtectPut typically have higher VaR than buying a Call or Put option.

In a CoveredCall strategy, the investor owns the underlying asset and sells call options on that asset. This means that if the underlying asset's price falls, the value of the long position will also fall, potentially resulting in larger losses than simply holding the call option. As a result, the VaR of a CoveredCall strategy will be higher than that of a single call option.

Similarly, in a ProtectPut strategy, the investor holds a long position in the underlying asset while also purchasing a put option on that asset. While the put option can protect against losses if the underlying asset's price falls, the cost of the put option (the premium) is an additional expense that adds to the strategy's overall risk. This means that the VaR of a ProtectPut strategy will be greater than that of a single put option.

In contrast, buying a Call or Put option alone involves only the premium paid for the option, which is the maximum loss that can be incurred. As a result, the VaR of a simple option position is typically lower than that of a strategy involving a long position in the underlying asset.

3. Callspread and Putspread

A call spread is the simultaneous purchase of a call option on an asset with a specific strike price and the sale of a call option on the same asset with a higher strike price. When an investor expects the underlying asset's price to rise but wants to limit the potential downside risk, this strategy is used. With this strategy, the maximum profit is the difference between the two strike prices minus the cost of the options, while the maximum loss is the premium paid for the options. A put spread involves purchasing a put option with a specific strike price on an asset while simultaneously selling a put option with a lower strike price on the same asset.

When an investor expects the underlying asset's price to fall but wants to limit the potential downside risk, he or she employs this strategy. With this strategy, the maximum profit is the difference between the two strike prices minus the cost of the options, while the maximum loss is the premium paid for the options. As a result, the VaR and ES for these two strategies are low.

4. Straddle:

A straddle is an options trading strategy in which you buy both a call option and a put option on the same underlying asset with the same strike price and expiration date. The VaR and ES for this strategy are the lowest. In fact, it is interesting that the VaR and the ES is negative here. Since we use the AR(1) model, the price is more volatile. Therefore, the estimated price will be either lower or higher. In each case, the payoff of this strategy is high. Therefore, the VaR and the ES is negative here.

A straddle's goal is to profit from significant price movements in the underlying asset, regardless of whether the price rises or falls. If the asset price rises significantly, the call option becomes profitable, while the put option becomes worthless. If the asset price falls significantly, the put option will be profitable, while the call option will be worthless.