Design for Julia implementation of the New Harmony algorithm

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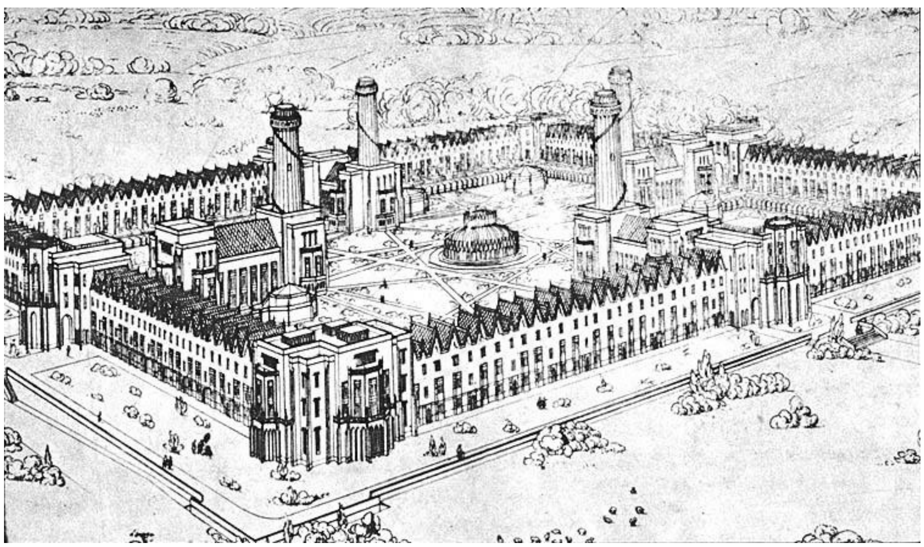

Fig. 1: Robert Owen's design for New Harmony

1 Introduction

The existing java implementation of the harmony algorithm is rather buggy and hard to maintain because

1. There is no good java support for csv files so it is all done by hacked library.
2. There is poor support for matrix abstractions.
3. A the time I wrote it I had not formally thought through the tensor representation of the temporal planning problem.

I intend to try a from scratch new implementation in Julia.

To go from static plans for one time period to dynamic ones we have to replace the matrix formalism of input output analysis to tensors. Vectors and matrices are special limiting case of tensors. Vectors are rank 1 tensors, matrices rank 2 tensors. The rank of a tensor is the number of distinct indices required to identify one of its elements. So the output vector o can be subscripted by one index as in o_i the technology matrix requires two: $a_{i,j}$. We will be describing the temporal planning problem in terms of rank 3 tensors like S which require to be indexed as $s_{i,j,t}$ with the index t representing time period. If you are familiar with older programming languages this is what is called a 3 dimensional array.

1.1 Representing the production functions

An I/O table implicitly defines a set of constraints between economic flows. We need to make the constraints implicit. This can be done using the A matrix: $a_{i,j}$ specifies how much of input i is required to make one unit of output j.

Let U be the rank 3 tensor of planned uses of goods, such that $u_{t,i,j}$ is the use of good i to make output j in time period t.

The matrix O will denote the successive output vectors such that $o_{t,j}$ is the output of j in time period t. Let us further assume that the plan is for a relatively short time interval in which we ignore changes in technology. It follows that

$$o_{t,j} \leq \frac{u_{t,i,j}}{a_{i,j}} \tag{1}$$

If we have an $n \times n$ matrix we will end up with n^2 different inequalities.

1.2 Linking time periods

Let $\mathcal{S}$ be a rank 3 tensor of capital stocks such that $s_{t,i,j}$ denotes the stock of i used to produce j in period t. There are capital constraints analogous to the flow constraints in Equation 1.

$$o_{t,j} \leq \frac{s_{t,i,j}}{c_{i,j}} \tag{2}$$

The stocks of capital in year t are the corresponding stock in year $t-1$, minus depreciation that occurred during year $t-1$ plus output set aside for accumulation in $t-1$.

$$s_{t,i,j} = (1 - d_{i,j})s_{t-1,i,j} + \alpha_{t-1,i,j} \tag{3}$$

where $\alpha_{i,j,t}$ is the accumulation of stocks of type i for use in industry j at time period t. We will denote the accumulation rank 3 tensor as $\mathcal{A}$.

Each element of this tensor takes resources of type i in period t and adds it to the capital stocks of all $c_{i,j,k}$ for $k \in t..\omega$ where ω is the end of the plan horizon. We can represent this as a depreciation function $\mathcal{D}$ that takes $\mathcal{A}$ and maps it to a rank 4 tensor such that:

$$\mathcal{D}(\dashv)_{i,j,t,l}$$

where l ranges over time as well.

This allows us to rewrite Equation 3 in terms of a sum over $\mathcal{D}(\dashv)$ such that

$$S = \sum_l \mathcal{D}(\mathcal{A})_{:,:,:,l} \tag{4}$$

Using Julia notation of : indicating the whole of a range.

Productive consumption can be represented as another matrix P where $p_{t,j} = \sum_i u_{t,i,j}$ is the productive consumption of good j in time period t.

Final consumption is represented as matrix F where $f_{t,j}$ is the final consumption of good j in time period t. It is given by output less accumulation and productive consumption.

$$F = O - \mathcal{A} - P \tag{5}$$

2 Define objectives

Let us assume that we have a matrix G where $\mathbf{g}_t$ is the row vector representing the Leontief final demand function for year t. Let the matrix P be the plan where $\mathbf{p}_t$ is the net output vector for year t in plan P. We can thus estimate possible goal fulfillment for each year for any putative plan as

$$\frac{\mathbf{p}_t \cdot \mathbf{g}_t}{|\mathbf{g}_t \cdot \mathbf{g}_t|}$$

that is to say the normalised inner product or cosine metric.

How should we compute the overall objective function for the whole plan period?

In the case of the linear programming we simply summed the plan fulfillment for all years to get the overall objective function.

In this case we will use the Harmony function $\mathcal{H}()$ similar in shape to that suggested in Towards a New Socialism. We want a fnction that mores sharply peanalises falling short of the goal than exceeding it. For example

$$\mathcal{H}(x) = x/(1.1 + x)$$

(6)

The value 1.1 is chosen since it is implausible that $\frac{\mathbf{p}_t \cdot \mathbf{g}_t}{|\mathbf{g}_t \cdot \mathbf{g}_t|}$ will fall below -1. A possible multi-year objective function is then of the form:

$$\text{Maximise} \sum_t \mathcal{H}\left(\frac{\mathbf{p}_t \cdot \mathbf{g}_t}{|\mathbf{g}_t \cdot \mathbf{g}_t|}\right) \tag{7}$$

This may be suitable if you have good reason to think that your planning algorithm will not set some product gross outputs so low that the net output would actually be negative. If we use the cosine metric there is the risk of this happening since we are just optimising for the projection of the multi-dimensional net output onto the plan ray. If we don't want that to happen an alternative is to set the harmony for each year to be the lowest per product harmony achieved that year and use that for maximisation.

$$\mathbf{H}(t, p, g) = \left\lfloor \mathcal{H}\left(\frac{p_{t,i}}{g_{t,i}}\right) \right\rfloor \tag{8}$$

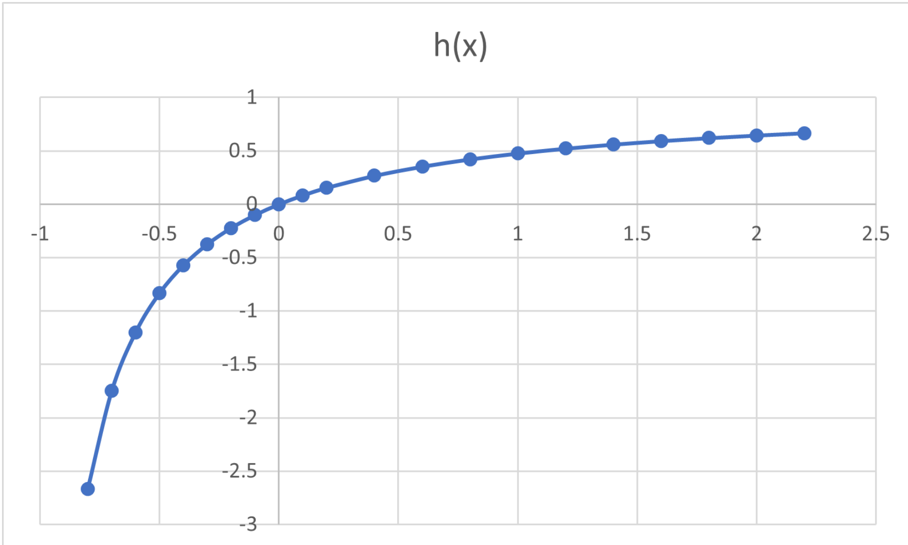

Fig. 2: A possible harmony function $\mathcal{H}(x) = s/(1.1 + x)$.

$$\text{Maximise} \sum_t \mathbf{H}(t, p, g) \tag{9}$$

We will first set out a simple scalar model of the accumulation problem before going on to deal with the full vector form. Suppose we start out with an economy that in year 1 has the following situation

Fixed capital	Labour used	Output	Consumption goal
100	23	50	33

Let us suppose that the fixed capital has a 14 year life expectancy. Clearly if we run a plan that allocates no investment in fixed capital to replace this depreciation then output will fall, and with it the harmony (Figure 3).

3 Investment algorithm

sec:investmentalg Obviously a plan needs to allocate at least enough investment to cover depreciation of fixed capital. So as a first step let us use the following procedure:

1. For the last year of the plan set a net output target such that

   (a) Gross output is such as to ensure full employment of the workforce

   (b) Sufficient investment is being carried out to compensate for depreciation during the year.

   This net output target must be a scaled version of the originally specified target. If the original target would have caused unemployment it will be up-scaled otherwise down-scaled.

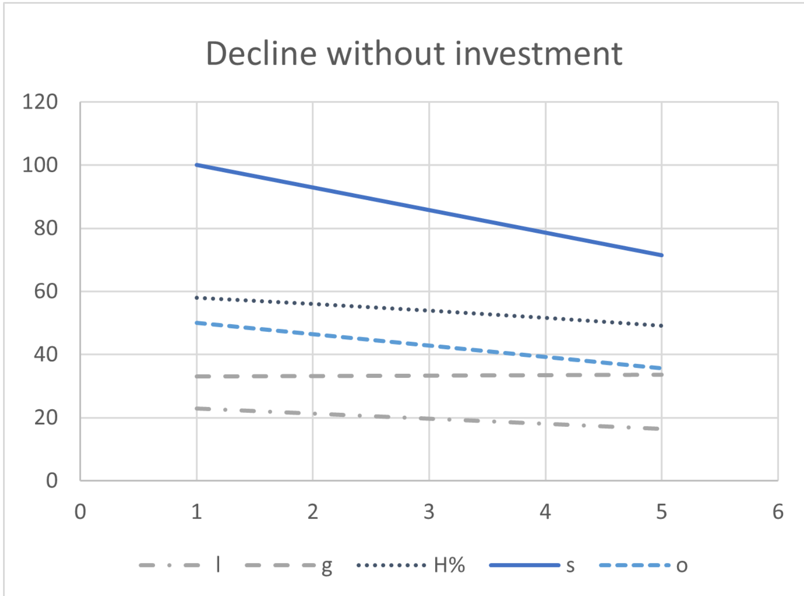

Fig. 3: l= Labour used, g= Consumption goal, H% = Harmony %, s=Stock of fixed capital, o=Total output. Mean harmony over the period is 48%.

2. Assign to each year's fixed capital stock the starting stock depreciated by the appropriate depreciation rate on that type of fixed capital stock.

3. Compute the degree of goal fulfillment that is in principle possible given these fixed capital stocks and the available workforce. From this compute the Harmonies.

4. Compute the mean harmony, and standard deviation of the harmonies over the whole time period.

5. If the coefficient of harmony variation has fallen below some threshold, terminate the plan prepartion, otherwise continue to step 6.

6. Select the year with the lowest harmony.

7. Estimate by how much the gross production of this year would have to be scaled up in order to be at the mean harmony.

8. Attempt to scale up some fraction ϵ of the way towards the mean harmony by scheduling investment in at least one previous year. This will obviously alter both the consumption in previous years and the fixed capital stocks in one or more years.

9. Go back to step 3.

Comment on step 1 How do we compute the full employment net output which we will designate as N here? We are assumed to know the productivity k of (each kind of) fixed capital, the depreciation rate of fixed capital Δ and the productivity of labour a. We have the relationships for the gross output O as follows

$$O \leq kC$$

$$O \leq aL$$

$$N = O - \Delta C$$

where C is the fixed capital stock and L the labour used. So

$$N \leq kC - \Delta C \tag{10}$$

$$\Delta C \leq kC - N$$

$$N \leq aL - kC - N$$

so

$$kC \leq aL$$

let's assume equality and allow $C = \frac{a}{k}L$ then by (10) we have

$$N = (k - \Delta)\frac{a}{k}L$$

Comment on step 8 When reallocating resources from year *src* to year *dest* there are several complexities to be taken into account. The overriding principle is that no investment we make should reduce the overall harmony across time. If the fall in harmony in the *src* year is less than the rise in the *dest* year the investment is bound to meet this criterion. But determining the change in harmony in the two years is a nice problem.

Remember we are assuming piece-wise Kantorovich demand functions, one for each year. Suppose that there are two types of capital good, ships and vans and that the destination year 'wants' more ships and more vans.

- The first thing to note is that unless the source year immediately precedes the destination one, a ship built in the source year will arrive at the destination year as a depreciated ship. Its productive efficiency will be less than when it was new. We can model this by scaling the amount of capital stock that arrives in the destination year by the appropriate depreciation rate for the good in question

  1 ship in year *src* equals $(1 - \Delta_{ships})^{t-1}$ ships in *dest*

  t years later. If ships depreciate 5% a year then a ship built in 2030, delivered 1st Jan 2031 will count as 95% of a ship on the first of January 2032, 90.25% of a ship at the start of 2033 etc.

- What is the impact on final consumption in the source year if it allocates one ship or one van to investment?

  The van is the easy case since we can assume that vans also enter, to some extent, into final consumption. If in 2030 we allocate one van to capital investment as telephone repair vans in the telecommunication industry, then the output of vans for final consumption in 2030 falls by 1. It is easy to work out the fall along the Kantorovich final consumption ray that this will produce since vans appear directly in the vector of final consumption goods.

  Some people use vans for camping, but nobody buys car ferries for their own personal use. If shipyards in 2030 build a ferry it only affects personal consumption in 2030 to the extent that less labour and other resources will be free to make consumer goods.

  We could use Leontief style input/output analysis to compute the effect that this reallocation will have on final consumption, but that is relatively expensive in computational terms, involving multiplication by what is potentially a rather large matrix. It is computationally preferable to have some sort of valuation vector or price list v that allows you to estimate the cost of the capital investment.

  If we work out the cost of the investment as $i.v$, and know the value $N.v$ of the net output vector that would have been achieved without the investment , we can simply work out the fractional decline in net output as

  $$\frac{N.v - i.v}{N.v}$$

  If F_{old} is the old fulfillment ratio the decline in harmony becomes

  $$\mathcal{H}(F_{old}) - \mathcal{H}(F_{old}\frac{N.v - i.v}{N.v})$$

  If we are dealing with a pre-existing capitalist economy we could just start off by using market prices as the valuations we use. But as planning exercised an increasing influence over the economy it is not clear that market prices would be a reliable measure of cost. For fully planned socialist economies there are two obvious alternatives in the political economy literature : Marxian labour values, Kantorovich objective valuations.

  Kantorovich valuations tend to labour values as the influence of capital stock inadequacies subsides. He held that under long term equilibrium the ODVs would equal labour content ratios.

  The purpose of the iterative adjustment of capital stocks in the investment algorithm is to remove capital stock constraints, leaving the worforce size as the remaining constrain. So at the end of the algorithm, the appropriate valuation would be labour values. For that reason, along with the lower computational cost of labour values we will use them in our valuations.

- The next point is that we should not allow any investment to go into the putative plan which would result in net negative products of any goods in the source year.

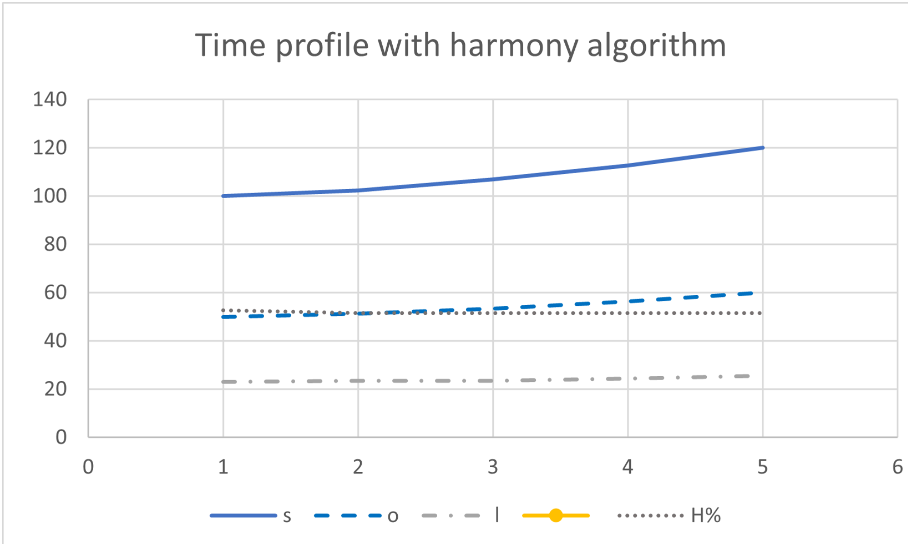

Fig. 4: What happens to output and capital stocks when the harmony algorithm is used. Note that harmony is equalised for all years.

- When selecting the fraction ϵ by which we attempt to move output in year *dest* towards the mean harmony output level care should be taken not to set epsilon so high as to cause oscillations in the plan construction process. As a first suggestion use the depreciation horizon $\frac{1}{\Delta}$ as a guide and set

$$\epsilon = \frac{1}{1 + \frac{1}{\Delta}}$$

When we apply this algorithm to a scalar example we find that the investment level ends up such as to increase output and employment up to full employment in the final year Contrast Figure 4 with 3.

4 Code for scalar example

The Julia code released as `harmony2.jl` was used to produce the results above. You should find it in the same directory as this documentation file.

5 Applying in the full matrix form

The matrix form extension introduces several complexities. The most obvious points are that one has to use a full Leontief formalism for each year. We construct a matrix, conventionally called an A matrix or technology matrix in the Marxian and Input/Output literature. It shows what fraction of a € of each type of input has to be used to produce €1 of output in the corresponding column. The A matrix for the EU is

$$A = \begin{bmatrix} 0.1204 & 0.0252 & 0.0016 & 0.0025 & 0.0086 \\ 0.2020 & 0.3533 & 0.2086 & 0.0749 & 0.6336 \\ 0.0070 & 0.0062 & 0.1968 & 0.0155 & 0.0014 \\ 0.1696 & 0.2117 & 0.1771 & 0.3021 & 0.2861 \\ 0.0277 & 0.0927 & 0.0223 & 0.0210 & 0.0767 \\ 0.1639 & 0.1727 & 0.2249 & 0.3204 & 0.0000 \end{bmatrix} \tag{11}$$

Element $a_{i,j}$ of the matrix specifies how much of the ith input is used to make one unit of the jth output. The 6th row of the matrix encodes labour inputs.

In the case of the example give above the A is highly aggregated and is expressed in money terms. In principle, with sufficient data, one could construct a much larger A matrix in physical not monetary units. But for explanatory purposes this simple matrix will do.

What is the use of A?

Determining resource usage It allows you to determine what resources, in the form of labour and intermediate inputs would be needed to achieve a given scale of gross output.

Suppose we specify the the gross output as a column vector o, then the Matrix vector product $A.o$ evaluates to the vector of intermediate products needed to make output mix o. We will define this vector of resources as r such that

$$r = A.o \tag{12}$$

Let us initially define o to be the actual vector of gross outputs for the EU in 2006 in millions of €, so that

$$o = \begin{bmatrix} 381876 \\ 6851944 \\ 1720275 \\ 13286792 \\ 1527213 \end{bmatrix}$$

Evaluating the product using Octave[1] we find that

$$A.o = \begin{bmatrix} 267750.31100 \\ 4819603.00980 \\ 591788.67900 \\ 6270858.91940 \\ 1080275.17560 \\ 5889898.20950 \end{bmatrix} = r$$

which, to within rounding errors is effectively the same as the actual EU intermediate consumption in 2006. The rounding errors arise from our having printed A in this book to only 4 decimal digit precision.

[1] Octave is an open source matrix language similar to the proprietary Matlab package. You may find it useful to install and use it to verify the examples.

The following equation holds for an A matrix:

$$o = r + f = A.o + f \tag{13}$$

where f is the final demand of the economy. The equation holds because the total output of the economy is divided between that part which is productively consumed and that part which is available for final use. Using Equation 13 we can also work out how much will be available for final consumption for given a particular vector of gross output:

$$f = o - A.o$$

For the Eu in 2006 the final consumption vector f was

$$f = \begin{bmatrix} 113575 \\ 2031855 \\ 1128416 \\ 7015443 \\ 447070 \\ 0 \end{bmatrix}$$

The final zero is the purchases of labour power by the household sector, we set it zero in the sense that no such transaction occurs.

With a little algebra we can use the A matrix to work back from a desired change in final consumption to the structure of gross output that this would involve. Given that $o = A.o + f$ thus $o - A.o = f$ and then takin o as a common factor

$$(I - A).o = f$$

where I is the identity matrix. Dividing through by $(I - A)$ we get

$$o = (I - A)^{-1}.f \tag{14}$$

This allows us to derive the gross output vector o as a function of A and the final demand f.

If you are going to calculate this you have to expand the A matrix to a 6×6 square matrix since you can not take the inverse of a non square matrix. To get round the fact that the A matrix we have used so far has 5 columns and 6 rows, this simply append a new zero filled the last column to indicate that there is no capitalist sector producing labour power. Some economists treat the consumption of the personal sector as producing labour power and would put in non zero values here, but we think that is economically invalid.

1. Labour power is not produced in the same way as other outputs.
2. The final consumption vector for the EU consists of consumption by all classes plus capital investment. Since it contains consumption out of property income, you cant use it as a proxy for workers' consumption.

Tab. 1: Old and new final demand mixes for the EU in €Millions.

	New f	Old f
Agriculture +hunting + fishing	113575	113575
Industry incl.Energ	2131855	2031855
Construction	1128416	1128416
Services	6915443	7015443
Foreign trade	447070	447070
Labour Power	0	0

3. The final consumption vector determines living standards, but the same amount of labour could still be performed at lower living standards, so unless one is dealing with a subsistence economy, having a column for the production of labour power is unrealistic.

Following this procedure we get:

$$(I - A) = \begin{bmatrix} 0.8796 & -0.0252 & -0.0016 & -0.0025 & -0.0086 & 0 \\ -0.202 & 0.6467 & -0.2086 & -0.0749 & -0.6336 & 0 \\ -0.007 & -0.0062 & 0.8032 & -0.0155 & -0.0014 & 0 \\ -0.1696 & -0.2117 & -0.1771 & 0.6979 & -0.2861 & 0 \\ -0.0277 & -0.0927 & -0.0223 & -0.021 & 0.9233 & 0 \\ -0.1639 & -0.1727 & -0.2249 & -0.3204 & 0 & 1 \end{bmatrix}$$

If you use Octave or a similar a Matrix calculator you will find that, to within rounding errors, $(I - A)^{-1}.f$ now[2] yields the same gross outputs as that shown in Table **??**.

A change in final demand Assuming that you have verified that you can derive the gross outputs of the EU from its final consumption, lets look at what happens if we want to change the mix of final consumption. Suppose we want to reduce consumption of services by €100Billion and increase consumption of industrial products by the same amount, as shown in Table 1.

If we feed the new consumption vector through equation 14 we can derive the gross levels of production in the different sectors needed to support the modified consumption pattern - Table 2. Given a new gross output it is easy to use the A matrix to reconstruct the complete Input/Output table for the new configuration of the economy.

Recall that $a_{i,j}$ encodes the fraction of a € worth of product i that must flow to sector j in order to produce €1 of output in sector j. Given the a gross output vector o' we go through A and multiply each $a_{i,j}$ by o'_j to get the new Input/Output table. This is essential just the reverse of what we did to compute A from a raw Input/Output table.

This however does not deal with investment over time as the plan is still just for one year. For simplicity let us assume that the technology matrix A does not change for the period of the plan. It is a relatively simple extension to take the A matrix and replace it by a rank 3 tensor indexed by time and products, but lets ignore that for now.

In the standard input/output formalism the only constraint on production is assumed to be the flows between sectors captured in A. In fact production is also constrained by

[2] The equivalent Octave expression is `inv(I-A)*f` .

Tab. 2: Gross output needed to support the modified European consumption pattern shown in Table 1

	New gross output	Old gross output	New/Old
Agriculture +hunting + fishing	385670	381212	101%
Industry incl.Energ	7010310	6850772	102%
Construction	1719742	1720148	100%
Services	13197413	13285530	99%
Foreign trade	1541324	1527186	101%
Labour Power	5889113 1	5889154	100%

capital stocks available in each industry. You can, if you have the right data, also construct an analogous C matrix such that $c_{i,j}$ is the amount of capital stock produced in industry i used in industry j to produce €1. Suppose we have a rank 3 tensor $\mathcal{S}$ which represents the stock of investments that have accumulated as fixed capital, then given C and and a vector λ of labour available per year, we can evaluate the maximum scale with which a given target gross output can be achieved each year. Suppose that our gross output target for year y and product p is $o_{y,p}$ then if we go up column p of the C matrix we get the required capital of each type to produce $o_{y,p}$. If we do this for all industries we get a capital requirement matrix for year y which we can compare to the stock of capital available, the matrix $\mathcal{S}_y$. The cell for which the ratio $\frac{\mathcal{S}_{y,i,j}}{c_{i,j}}$ is smallest then constitutes the tightest capital constraint on producing outputs in the ratios specified by o_y. You can then use the A matrix to see if the total labour required imposes a tighter constraint.

6 Matrix version code

The approach above is used in the full matrix version code which is in the file `csvplan.jl`. It takes its inputs in the form of CSV (comma separated value) which are readily handled by Julia. It must be provided with four such files:

```
solvePlanProblem("jeuflows.csv", "jeucap.csv", "jeudep.csv", "jeulabtargs.csv")
```

In all of the files an io table format is used but only the column headers of the io table are included. Assume that the row headers go in the same order as the column ones when preparing your tables. The first is a more or less standard input output table, with the last row being gross output and the second last row being labour inputs (compensation of employees is a typical row heading for this in orginal tables). The next give capital stocks at the start of the plan, and the depreciation rates of the capital stocks. The last, gives the goals and labour supply laid out like table 3

We follow Kantorovich, defining for each year a plan ray - a specified mix of outputs: Table 3.

Unlike the scalar example which used straight line depreciation, `csvplan.jl` uses exponential depreciation.

€ Million	Agriculture	Industry	Construc-tion	Services	Foreign trade	Labour
year1	101765	1469520	48082	6559004	283372	5890041
year2	101998	1470520	49082	6559004	283372	5990336
year3	102231	1471520	50082	6559004	283372	6047519
year4	102696	1472520	51082	6559004	283372	6075071
year5	102463	1473520	52082	6559004	283372	6061295

Tab. 3: For each year we have a plan ray, also called a Leontief demand function which specifies the desired mix of final output. Year 1 is based on actual figures for 2006. The last column specifies the labour resources available. These are based on the actual growth of the EU labour force up to 2009 (year4) after which it is held constant. In the unplanned reality, employment fell after 2009 due to the financial crisis. Food demand is assumed to grow linearly with the workforce, demand for industry and construction also rises relative to services.