CS221 Fall 2018 Homework [car]

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Problem 1

1.a:

Answer:First, we remove all non-ancestor nodes, only C_1, C_2, D_2 remains.
Next, try to convert to factor graph, we then have three factors: $p(c_1), p(c_2|c_1), p(d_2|c_2)$.
Then we apply conditions with $c_2 = 1$ and $d_2 = 0$ and marginalize c_1, we see below formula:

$$\mathbb{P}(D_2 = 0|C_2 = 1) = \sum_{c_1}(p(c_1)p(c_2|c_1))p(d_2 = 0|c_2 = 1)$$

We see $\sum_{c_1}(p(c_1)p(c_2 = 1|c_1)) = 0.5$ and $p(d_2 = 0|c_2 = 1) = \eta$ when $c_2 = 1$ and $1 - \eta$ when $c_2 = 0$.
so we have $\mathbb{P}(D_2 = 0|C_2 = 1) = \frac{0.5*\eta}{0.5*\eta+0.5*(1-\eta)} = \eta$

1.b:

Answer:First we remove D_1 as non-ancestor node, then build factor function with specify conditions and marginalize as follow:

$$\begin{aligned}
f_1(c_2) &= \sum_{c_1} p(c_1)p(c_2|c_1) \\
f_2(c_2) &= p(d_2 = 0|c_2) \\
f_3(c_2) &= \sum_{c_3} p(c_3|c_2)p(d_3 = 1|c_3)
\end{aligned}$$

Then we see

$$\begin{aligned}
\mathbb{P}(C_2 = 1|D_2 = 0, D_3 = 1) &\propto f_1(c_2)f_2(c_2)f_3(c_2) \\
&= \begin{cases} 0.5(1-\eta)((1-\varepsilon)\eta + \varepsilon(1-\eta)) & : c_2 = 0 \\ 0.5\eta(\varepsilon\eta + (1-\varepsilon)(1-\eta)) & : c_2 = 1 \end{cases} \\
&= \frac{(1-\varepsilon)(1-\eta)\eta + \varepsilon\eta^2}{\varepsilon\eta^2 + \varepsilon(1-\eta)^2 + 2(1-\varepsilon)(1-\eta)\eta}
\end{aligned}$$

1.c.1:

Answer:

$$\mathbb{P}(C_2 = 1|D_2 = 0) = 0.2$$
$$\mathbb{P}(C_2 = 1|D_2 = 0, D_3 = 1) \approx 0.4157$$

1.c.2:

Answer: The intuition is $D_3 = 1$ increases possibility that the car was at position 1 at time t_2, even-though $D_2 = 0$ indicates car is at position 0.

1.c.2:

Answer: Calculate $\mathbb{P}(C_2 = 1|D_2 = 0) = \mathbb{P}(C_2 = 1|D_2 = 0, D_3 = 1)$ with $\eta = 0.2$, we have $\varepsilon = 0.5$, the intuition is when $\varepsilon = 0.5$, car's current position wouldn't be influenced by its previous position, so a new condition $D_3 = 1$ doesn't matter in this case.

Problem 5

5.a:

Answer:According to Bayes rule and $e_1 = (e_{11}, e_{12})$, we have:

$$\begin{aligned}\mathbb{P}(C_{11}, C_{12}|E_1 = e_1) &= p(c_{11}, c_{12}|e_{11}, e_{12}) \\ &\propto p(c_{11}, c_{12})p(e_{11}, e_{12}|c_{11}, c_{12})\end{aligned}$$

We know there are two permutation sequences for E_1, which is $(d_{11} = e_{11}, d_{12} = e_{12})$ and $(d_{11} = e_{12}, d_{12} = e_{11})$, we also know c_{11} and c_{12} are independent, then we have:

$$\begin{aligned}\mathbb{P}(C_{11}, C_{12}|E_1 = e_1) &\propto p(c_{11}, c_{12})(p(d_{11} = e_{11}, d_{12} = e_{12}|c_{11}, c_{12}) + p(d_{11} = e_{12}, d_{12} = e_{11}|c_{11}, c_{12})) \\ &\propto p(c_{11})p(c_{12})(p(d_{11} = e_{11}|c_{11})p(d_{12} = e_{12}|c_{12}) + p(d_{11} = e_{12}|c_{11})p(d_{12} = e_{11}|c_{12}))\end{aligned}$$

According the Hint, we know $p(d_{11} = e_{11}|c_{11}) = p\mathcal{N}(e_{11}; \|a_1 - c_{11}\|; \sigma^2)$, after apply this to $p(d_{12} = e_{12}|c_{12})$, $p(d_{11} = e_{12}|c_{11})$ and $p(d_{12} = e_{11}|c_{12})$, we have the final expression below:

$$\begin{aligned}\mathbb{P}(C_{11}, C_{12}|E_1 = e_1) \propto p(c_{11})p(c_{12})(&p\mathcal{N}(e_{11}; \|a_1 - c_{11}\|; \sigma^2)p\mathcal{N}(e_{12}; \|a_1 - c_{12}\|; \sigma^2) \\ &+ p\mathcal{N}(e_{12}; \|a_1 - c_{11}\|; \sigma^2)p\mathcal{N}(e_{11}; \|a_1 - c_{12}\|; \sigma^2))\end{aligned}$$

5.b:

Answer:For any settings of $(c_{11}, ..., c_{1K})$, there are K! permutations to assign the observation value for K cars. Since we know prior $p(c_{1i})$ are same for all i, different permutation of car location assignment sequence will produce same result. So for the setting that could maximize the probability, there are K! permutation assignment sequence, so the maximum value of $\mathbb{P}(C_{11} = c_{11}, ..., C_{1K} = c_{1k}|E_1 = e_1)$ is at least K!.

5.c:

Answer:K-1