CS221 Fall 2018 Homework [scheduling]

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By turning in this assignment, I agree by the Stanford honor code and declare that all of this is my own work.

Problem 1

1.a:

Variables:
X = $(X_1, X_2, ..., X_m)$
$Domain_i$ = { 0, 1 }
Factors:
$F_i(x) = \sum_{j=1}^{m} K_j$ mod 2 where $K_j = x_j$ when $i \in T_j$ otherwise 0, $i = 1, 2, .., n$

Further explanation:
We have m variables, each one (X_j) indicate button j is turned on or off. Domain of X_j is {0, 1}, where 0 means button j has been pressed 2n times (unpressed state), 1 means button j has been pressed 2n+1 times (pressed state).
We have n factor functions, each one represents the constrain to one light bulb, x is an assignment $x = (x_1, x_2, ..., x_m)$, the scope of the factor function is $(x_1, x_2, ..., x_m)$, the constraint should be:

$$Weight(x) = \prod_{i=1}^{n} f_i(X) = 1$$

1.b:

i:

There are two consistent assignments: {0, 1, 0} and {1, 0, 1} .

ii:

Call stacks: (number of '==>' indicates layers of call stack)

CSP without using any heuristics: (9 times backtrack calls)
backtrack(∅, 1, (X_1={0, 1}, X_2={0, 1}, X_3={0, 1}))
==> backtrack({$X_1 : 0$}, 1, (X_2={0, 1}, X_3={0, 1}))
==>==> backtrack({$X_1 : 0$, $X_3 : 0$}, 1, (X_2={0, 1}))

==>==>==> backtrack($\{X_1:0,\ X_2:1,\ X_3:0\}$, 1, ()) – success complete assignment
==>==> backtrack($\{X_1:0,\ X_3:1\}$, 1, (X_2={0, 1})) – failed with X_2
==> backtrack($\{X_1:1\}$, 1, (X_2={0, 1}, X_3={0, 1}))
==>==> backtrack($\{X_1:1,\ X_3:0\}$, 1, (X_2={0, 1})) – failed with X_2
==>==> backtrack($\{X_1:1,\ X_3:1\}$, 1, (X_2={0, 1}))
==>==>==> backtrack($\{X_1:1,\ X_2:0,\ X_3:1\}$, 1, ()) – success complete assignment

iii:

Call stacks: (number of '==>' indicates layers of call stack)

CSP with AC-3: (7 times backtrack calls)
backtrack(∅, 1, (X_1={0, 1}, X_2={0, 1}, X_3={0, 1}))
==> backtrack($\{X_1:0\}$, 1, (X_2={1}, X_3={0}))
==>==> backtrack($\{X_1:0,\ X_3:0\}$, 1, (X_2={1}))
==>==>==> backtrack($\{X_1:0,\ X_2:1,\ X_3:0\ \}$, 1, ()) – success complete assignment
==> backtrack($\{X_1:1\}$, 1, (X_2={0}, X_3={1}))
==>==> backtrack($\{X_1:1,\ X_3:1\}$, 1, (X_2={0}))
==>==>==> backtrack($\{X_1:1,\ X_2:0,\ X_3:1\}$, 1, ()) – success complete assignment

Problem 2

2.a:

Answer:
The straightforward method is to set a single auxiliary variable A, and construct binary factors A[1] = X_1, A[2] = $X_2 + A[1]$, A[3] = $X_3 + A[2]$ and a final unary constraint that A[3] has to meet $A[3] \leq K$.
Let's we can use the adder-circuit- like structure given in class to define this in a formal way:
We have three auxiliary variables: B_1, B_2, B_3, B_i represents $\sum_{j=1}^{i} X_j$.
$Domain(B_i) = \{(m,n)|0 \leq m \leq 6, 0 \leq n \leq 6\}$
The factors are:

Initialization (Unary factor)	$[B_1[1] = 0]$
Processing (Binary factor)	$[B_i[2] = X_i + B_i[1]]$
Consistency (Binary factor)	$[B_{i+1}[1] = B_i[2]]$
Constraint on final output (Unary factor)	$[B_3[2] \leq K]$

Problem 3

3.c:

profile.txt:

```
# Unit limit per quarter.
minUnits 3
maxUnits 5

# These are the quarters that I need to fill. It is assumed that
# the quarters are sorted in chronological order.
register Win2018
register Spr2019

# Courses I've already taken
taken CS221
taken PHYSICS41

# Courses that I'm requesting
request EARTHSCI400 weight 5 # Win
request EARTHSYS164 # Win
request CS224N    # Aut
request CS224S    # Spr
```

Here's the best schedule:

Quarter	Units	Course
Win2018	4	EARTHSYS164
Spr2019	3	EARTHSCI400

Problem 4

4.a:

Answer: Let's consider a worst case that we have at least one $p \in P$ which contains all n variables, we need n-ary factor. As all these variables are all independent, when we eliminate one variable, we got a n-1 factor. According to Markov blanket definition, the treewidth for this worst case is $n - 1$.