# Sectoral Reallocation and the Firm Life Cycle

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## Overview

**Goal**:

- Investigate the interaction between sectoral reallocation and firm dynamics since the 1980s/1990s.
- Conceptually: Sector *i* expands relative to sector *j* if:
  1. Incumbent firms in *i* grow relative to incumbent firms in *j*
  2. The marginal entrepreneur enters in *i* relative to *j*
- Research question: How did each margin contribute to observed changes in employment shares?
  - Current: Quantify 1. using simple structural model
  - Future: Produce new estimates of trends in firm dynamics by sector/cohort

**Methodology**:

1. Document new facts on life cycle firm employment growth
   - Establishment size at entry ↓ in recent cohorts, no change in exit rates
   - Trend stronger in manufacturing relative to service sector
2. Estimate firm dynamics model to uncover structural factors:
   i. Fixed costs of production [De Ridder, 2024]
   ii. Fixed costs of entry [Gutiérrez et al., 2021; Kozeniauskas, 2024]
   iii. Persistence of firm-level productivity [Decker et al., 2020]
   iv. Dispersion of firm-level productivity [Barth et al., 2016; Decker et al., 2020]
3. Decompose structural change using parameter estimates

**Findings**:

1. Sector-level trends defy aggregate trends in firm dynamics
   - Estimated fixed costs decrease in both services and manufacturing
   - Estimated entry costs increase (decrease) in services (manufacturing)
   - Productivity persistence decreases in both services and manufacturing
2. Fixed costs and entry costs explain little of within-sector trends
   → Changes in output mostly driven by changes in productivity process
3. Sector-level firm dynamics work against aggregate reallocation
   → Only within-sector forces ⟹ growth in manufacturing relative to services

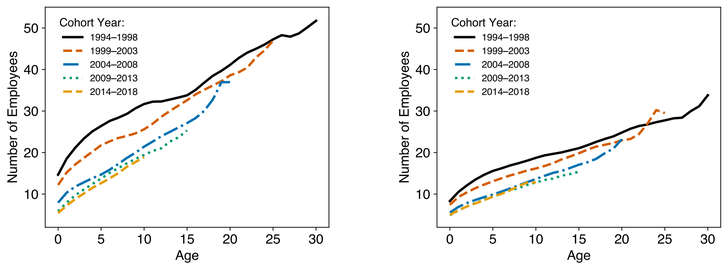
**Literature**:

- Business Dynamism: Decker et al. (2016); Akcigit and Ates (2020, 2023); Hopenhayn et al. (2022); Karahan et al. (2024)
- Structural Change and Labor Reallocation: Hopenhayn and Rogerson (1993); Dent et al. (2016); Ding et al. (2022)

## Motivating Evidence

**Trends in Average Establishment Size by Cohort**

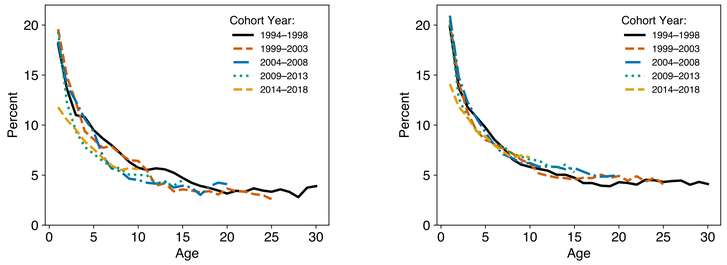
(a) Manufacturing (b) Services




Source: Bureau of Labor Statistics (BLS) Business Employment Dynamics.

**Trends in Exit Rate by Cohort**

(a) Manufacturing (b) Services




Source: Bureau of Labor Statistics (BLS) Business Employment Dynamics.

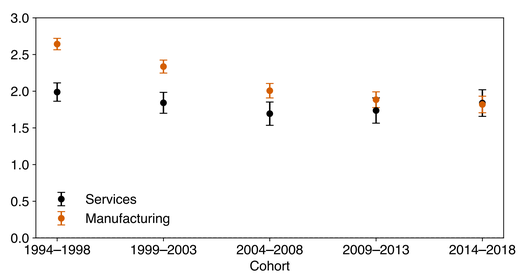
### Regression Evidence

- Estimate sector-specific establishment dynamics by cohort
  - Let $j \equiv$ sector, $k \equiv$ cohort, $t \equiv$ year
  - Group cohorts into 5-year bins
- Parameterize as quadratic in establishment age

(1) $\ln(\text{Average Establishment Size})_{j,k,t} = \alpha_k^j + \beta_k^j \text{Age} + \gamma_k^j \text{Age}^2 + \delta_t + \varepsilon_{j,k,t}$

(2) $\text{Exit Rate}_{j,k,t} = a_k^j + b_k^j \text{Age} + c_k^j \text{Age}^2 + d_t + e_{j,k,t}$

Estimates: Average Firm Size ($\alpha_k$)




## Firm Dynamics Model

### Setup

- Each sector is its own island → Hopenhayn (1992) economy
- Representative household
  - Consumes final output $Y$ and supplies labor $L$ inelastically
- Heterogeneous firms
  - Differ in productivity level $z$ that evolves according to $P(z'|z)$
  - Produce using labor $\ell$ only, no adjustment costs
  - Operate decreasing returns to scale production function $y = z\ell^\alpha$, $\alpha < 1$
  - Incumbent firms pay per period fixed costs $c_f$
  - Potential entrants pay fixed entry cost $c_e$ to enter market, draw $z$
  - Distribution of firms $\mu(z)$ determined in equilibrium

### Firm Problem

- Firm chooses labor input $\ell(z;p)$ and exit $\chi(z;p)$ to maximize

$$V(z;p) = \max_\ell [pz\ell^\alpha - w\ell - c_f] + \beta \max\{\mathrm{E}[V(z';p)], 0\}$$

- Assume productivity $z$ is AR(1) in logs

$$\ln(z') = \rho_z \ln(z) + \varepsilon; \quad \varepsilon \sim N(0, \sigma_z)$$

- Free entry condition

$$\beta \int V(z;p) g(z) \, \mathrm{d}z = c_e$$

where $g(z)$ is stationary distribution of $P(z'|z)$

### Estimation Results

| | Services | | | | Manufacturing | | | |
|---|---|---|---|---|---|---|---|---|
| Year | $c_f$ | $c_e$ | $\rho_z$ | $\sigma_z$ | $c_f$ | $c_e$ | $\rho_z$ | $\sigma_z$ |
| 1994–1998 | 1.237 | 4.298 | 0.968 | 0.180 | 0.837 | 11.346 | 0.970 | 0.215 |
| 1999–2003 | 0.833 | 4.389 | 0.962 | 0.213 | 0.556 | 11.431 | 0.971 | 0.220 |
| 2004–2008 | 0.553 | 4.245 | 0.951 | 0.256 | 0.303 | 9.583 | 0.961 | 0.289 |
| 2009–2013 | 0.582 | 4.877 | 0.924 | 0.308 | 0.297 | 9.100 | 0.942 | 0.366 |
| 2014–2018 | 0.610 | 5.139 | 0.899 | 0.362 | 0.478 | 9.127 | 0.895 | 0.496 |

### Decomposition

(a) Services (b) Manufacturing