Declining Business Dynamism and Worker Mobility*

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Abstract

This paper studies the consequences of the recent decline in business dynamism in the United States for the labor market. Firm entry rates declined precipitously in recent decades, leading to an increase in the share of older, larger businesses. Younger workers tend to sort into younger firms, suggesting that the compositional shift of economic activity towards older firms may have harmed the labor market prospects of younger workers. In order to assess this hypothesis, I develop a model of labor market sorting with both on-the-job search and two-sided life-cycle heterogeneity. I calibrate the model to match the life-cycle profiles of worker flows and earnings as well as the employment shares of young versus old firms in the mid-1990s. I then simulate the response of the economy to a decline in the firm entry rate that replicates the shift in the firm age distribution away from younger and towards older firms. I find that the decline in business dynamism accounts for about 43 percent of the decline in employer switching and about 23 percent of the decline in employment rates between 1994 and 2019. Aggregate worker welfare falls by about 0.6 percent along the transition path, with younger workers experiencing larger declines.

Keywords: Business Dynamism, Entrepreneurship, Economic Mobility, Worker Flows

JEL Classification: E24, L26, J62, M13

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1 Introduction

Over the past several decades, the United States economy witnessed a dramatic decline in business dynamism. Firm entry and exit rates decreased, increasing the proportion of larger, older businesses in the economy. Moreover, despite large secular changes in economic activity over this time horizon, these phenomena were pervasive across regions and industries.[1] In this paper, I investigate the implications of declining business dynamism for aggregate labor market outcomes as well as labor market outcomes across older and younger groups of workers.

My analysis starts with the observation that the composition of employment across workers of different ages is quite different within younger and older businesses. Using data from the U.S. Census Bureau's Quarterly Workforce Indicators (QWI) database, I show that while more mature firms employ younger and older workers in roughly equal proportions, the age composition of employment at younger firms is skewed towards younger workers. This finding is not accounted for by differences in firm size across firm age categories, is not driven by certain sectors or regions, and has remained relatively stable over time.[2] The fact that younger workers differentially sort into younger firms suggests that the decline in the share of young firms in recent decades may have differentially affected the labor market outcomes of younger workers.

To assess this hypothesis and to quantify the labor market effects of the declining share of young firms, I develop an equilibrium model of labor market sorting between heterogeneous workers and heterogeneous firms with on-the-job search (OJS). The model builds on Lise and Robin (2017), who study the implications of sorting between workers and firms with permanent differences in skill and productivity, respectively, for U.S. business cycles. I modify their framework to instead allow both workers and firms to differ by age, which evolves over time as they progress throughout their life-cycles. I also allow the distributions of workers and firms by age group to change in response to innovations in the rates at which workers and firms, respectively, enter and exit the economy. This permits me not only to rationalize the sorting patterns between workers and firms at different stages of their life-cycles, but also to analyze the consequences of declining business dynamism for employment outcomes and career mobility across cohorts of workers.

In the model, both workers and firms differ by the current stage of their life-cycle. Firms enter the economy in each period and, if they survive, grow older and hire additional employees. Workers enter the labor market at the beginning of their careers

[1] See Decker et al. (2014a), Hathaway and Litan (2014), and Pugsley and Şahin (2019) for recent evidence on trends in business dynamism across industries and geographic locations.

[2] Ouimet and Zarutskie (2014) and Dinlersoz et al. (2019) also find evidence of this pattern in the cross section.

and search for jobs. They are subject to random shocks that separate them from their employers back into the unemployment pool. If they remain employed, they may also search on-the-job for a new employer. Importantly, the degree to which output differs across matches between workers and firms of different ages controls the life-cycle sorting patterns of employment. Consistent with the labor market sorting literature, the model includes a simple reduced form expression for match-level output, which is allowed to vary by both worker and firm age. Across the worker life-cycle, this proxies for the fact that human capital accumulation may allow older workers to produce more on average than younger workers. Across the firm life-cycle, this proxies for the fact that older firms are on average larger and more productive than younger firms due to selection effects or firm growth. Moreover, this expression captures that workers at different stages in the life-cycle may have differential productive capacity when matched with firms at different stages of the life-cycle. This would occur if, for instance, younger workers possessed more recent vintages of technical skills and young firms were dependent on such skills to create new innovative products (Ouimet and Zarutskie, 2014).

I calibrate the model in steady state to match several features of the U.S. economy in the mid 1990s. I target the life-cycle profiles of worker flows, wages paid by firms of different ages to workers of different ages, and the employment share by firm age in 1994.[3] The wage profile across both the worker and the firm life-cycle helps to discipline the parameters of the match-level output expression. The calibration results capture the empirical patterns of worker mobility over the life-cycle — both job finding rates and job separation rates decline with worker age — as well as the increasing and hump-shaped trajectory of wages over the worker life-cycle. Moreover, the oldest firms in the model pay wages that are 1.5 times as high as those paid by the youngest firms, in line with the data. Finally, though I do not explicitly target the age composition of employment at young versus old firms, the model reproduces these life-cycle sorting patterns quite well.

Starting from an initial steady state in 1994, I then simulate a decline in business dynamism by allowing the firm age distribution and total number of firms in the economy to evolve in a manner consistent with the data. I gather data from the Census Bureau's Business Dynamics Statistics (BDS) database and examine trends in firm dynamics between 1994 and 2019. In the data, firm exit rates conditional on firm age group have remained roughly stable since 1994, but the firm entry rate has dropped precipitously. I calibrate the stochastic process for the mass of firms by firm age group to match these patterns. The

[3] I choose this time period as the starting point for my analysis because of data availability reasons, such as the Current Population Survey (CPS) redesign. Moreover, previous studies have argued that the trend in business dynamism accelerated after 2000 (Decker et al., 2014a).

resulting evolution of the firm age distribution is nearly identical to that in the data. The share of younger firms declines along the transition path as the firm entry rate declines and the firm age distribution shifts towards older firms.

I feed the calibrated process for the firm age distribution into the model and study the consequences for various labor market outcomes on aggregate as well as across worker cohorts. Because workers at different stages of the life-cycle sort into firms of different ages and since mobility rates of younger workers are more tied to aggregate conditions, the decline in dynamism produces changes in labor market outcomes that vary across worker cohorts.

I find that the decline in business dynamism causes a decline in labor market mobility for all workers. Along the transition path of the economy, fewer vacancies are posted by firms, leading to an overall decline in labor demand. Therefore, the total number of meetings between workers searching for jobs and firms posting vacancies fall, leading to a drop in the contact rate. The aggregate job finding rate declines by 8 percentage points and the aggregate job-to-job switching rate declines by 0.15 percentage points. In addition, the shift in the firm age distribution towards older firms, which have lower separation rates, leads to a fall in the aggregate job separation rate of about 0.03 percentage points. Because job finding falls more than job separations, the employment rate falls (nonemployment rate rises) by about 1 percentage point.

In addition to the aggregate decline in worker mobility, job finding, separation, and switching rates all display different declines across worker age groups. In particular, each series declines by more for younger cohorts of workers. The job finding rate falls by more for younger worker cohorts because in my calibrated model, younger workers have higher search intensity. This is consistent with the fact that in the cross section, younger workers on average have higher job finding rates. Therefore, they are more exposed to the decline in business dynamism for a given contact rate. Job separations fall by more for younger workers due to a composition effect. In the initial steady state, younger workers differentially sort into younger firms, which have high separation rates. Along the transition path, as the share of young firms declines, younger workers are reallocated into jobs at older firms, which have lower separation rates. Lastly, job-to-job flows fall by more for younger workers due to the fall in the contact rate combined with their higher average search intensity.

Taking these predictions to the data, I find that the decline in business dynamism explains about 43 percent of the decline in employer switching and about 23 percent of the decline in employment rates between 1994 and 2019. Notably, the decline in business dynamism also accounts for the fact that worker mobility has fallen by more for younger worker age groups (Bosler and Petrosky-Nadeau, 2016; Mercan, 2017). My analysis also shows that declining business dynamism has interesting implications for workers' career trajectories. First, I find that workers on average occupy lower rungs of the job ladder where they receive a smaller share of the surplus. Next, the decline in business dynamism leads to a flattening wage-tenure profile within firms. Finally, wages fall by more for younger workers along the transition path, leading to a widening age-wage gap. Recent empirical evidence finds that this has been the case in several high-income countries, including the U.S., in recent decades (Bianchi and Paradisi, 2022).

Lastly, I quantify the welfare impacts of the decline in business dynamism. I approximate total worker welfare in the economy as the total flow value received by employed workers (in the form of wages) and unemployed workers (in the form of nonemployment benefits). Along the transition path, total welfare changes if either (i) there is a change in the share of workers who are employed or (ii) conditional on being employed, there is a change in average wages paid to workers. I label the former effect the *Extensive Margin* effect and the latter effect the *Intensive Margin* effect. I find that total worker welfare falls, largely due to a decline in the *Intensive Margin*. Though the employment rate falls on aggregate, wages decline severely because workers are more likely to match with large firms and trade off current wages in order to so. Along the transition path, total worker welfare falls by about 0.6 percent.

The welfare results across worker cohorts mirror my findings on worker mobility and wages across the life-cycle. Because employment rates and wages fall by more for younger worker age groups, their welfare measures also display larger declines. Total welfare declines during the period under consideration, but the brunt of the impact is borne by younger workers. Therefore, I argue that the large decline in business dynamism in the U.S. has not only led to a deterioration of labor market prospects for all workers, but also contributed to a widening gap in outcomes between recent and past generations of labor market entrants.

Related literature. My paper contributes to several different strands of the literature studying the causes and consequences of the recent decline in business dynamism. The literature that examines the causes of declining business dynamism is too large to catalog extensively.[4] However, I highlight two recent papers that reflect a growing consensus re-

[4]A large literature documents the pervasiveness of the decline in business dynamism in the aggregate as well as across markets. For instance, Decker et al. (2014b) find a "pronounced declining trend" in the pace of job creation, job destruction, and the firm startup rate and argue that this trend accelerated after 2000. Hathaway and Litan (2014) show that dynamism declined in all fifty U.S. states and the vast majority of U.S. metropolitan areas during the last three decades. More recently, Pugsley and Şahin (2019) argue

garding the explanation for these trends and elaborate on the findings of these papers in the next section. Both Hopenhayn et al. (2022) and Karahan et al. (forthcoming) document that firm dynamics within cohorts of firms have remained stable, implying that observed changes in the firm age distribution have resulted entirely from changes along the entry margin. They show that in models of firm dynamics with linear entry conditions based on Hopenhayn (1992), a decline in labor supply growth produces changes in firm dynamics consistent with these empirical patterns. This implies that trends in average firm size, the firm exit rate, and concentrating on aggregate are due to the changing composition of firms by firm age. In this paper, I additionally examine the implications of trends in business dynamism for labor market outcomes and inequality across worker cohorts.

Next, my paper relates to studies that propose explanations for the declining trend in worker flows.[5] Cairó (2013) documents patterns of changing educational attainment and rising skill demands among employers. The author shows that an increase in job retraining requirements lowers labor market turnover and can explain about one-third of the decline in the job reallocation rate over the past several decades. Mercan (2017) and Pries and Rogerson (2022) document that most of the decline in the employer-to-employer transition rate is accounted for by a decline in short-duration jobs. These paper propose that better ex-ante information about match quality or screening by firms of potential applicants explains the decline in job mobility in recent decades. My paper proposes a new channel for the decline in job mobility though the decline in business dynamism.

The motivation for this study connects to several papers that directly consider the life-cycle determinants of worker mobility, the job ladder, and labor market sorting. First, Topel and Ward (1992) argue that early-career "job shopping" is an important source of life-cycle wage growth. Ouimet and Zarutskie (2014) find that the fortunes of young firms and workers are inextricably linked: young firms disproportionately hire and employ young workers, young workers earn higher wages in young firms, and talented young workers select into young firms that display higher innovation and growth potential. Dinlersoz et al. (2019) also find that young firms tend to employ younger workers and argue that labor market frictions specific to newly created businesses are key for generating these sorting patterns. In my analysis, life-cycle sorting patterns of employment form the basis through which a decline in the share of young firms differentially affects

that the decline in the employment share of startup firms (firms less than 1-year old) is not driven by changes in geographic or industrial composition of economic activity and instead that declines in startup firm employment shares have occurred *within* narrowly defined industries and regions.

[5] Hyatt and Spletzer (2013) find that depending on the data source, hires and separations rates fell between 10 percent and 38 percent between 1998 and 2010. Molloy et al. (2016) document a clear downward trend in the pace of worker flows and of job turnover and discuss competing explanations for these trends.

young versus old workers.

This paper builds on work in Postel-Vinay and Robin (2002), Lise and Robin (2017), and Lentz et al. (2017), who develop models of two-sided heterogeneity and labor market sorting. In these studies, worker types differ by fixed skill or ability and firm types differ by fixed productivity or technology. In contrast to these papers, I allow firms and workers to differ not by skill or productivity, but by age, which evolves over the life-cycle. This allows me not only to capture the life-cycle dimensions of worker mobility and employment sorting, but also to speak to differential changes in labor market outcomes across different cohorts of workers. In order to assess cross-cohort inequality in labor market outcomes, it is crucial to include a life-cycle dimension for both firms and workers.

Finally, my paper is related to recent studies that jointly consider firm dynamics and on-the-job search. Engbom (2019) develops a model of firm and worker dynamics in order to assess the consequences of labor force aging. Importantly, he finds that while the direct effects of labor force aging explain some portion of the decline in worker flows, the majority of the decline results from feedback effects onto the incentives to start new businesses.[6] Recent papers by Bilal et al. (2022) and Elsby and Gottfries (2022) build firm dynamics models with frictional labor markets where workers may search on-the-job. Though models featuring all of these elements have typically been too complex for analysis, both sets of authors show under certain conditions that the problem becomes tractable, allowing them to study firm growth and worker flows in conjunction. I develop a method to include life-cycle dimensions for both worker and firm outcomes while also maintaining analytical tractability under a related, but more restrictive set of assumptions on the production and vacancy posting cost functions.[7] Additionally, both papers focus on the business cycle dimension of worker flows, whereas I study the long-term decline in business dynamism.

Layout. The rest of the paper is structured as follows. In Section 2, I discuss a set of empirical facts that motivate my study of declining business dynamism and its potential impacts across cohorts of workers. In Section 3, I present a model of two-sided labor market sorting between workers and firms at different stages of the life-cycle. Section 4 discusses the calibration strategy of the model. Section 5 explores the effects of a de-

[6] Hopenhayn et al. (2022) and Karahan et al. (forthcoming), who do not consider heterogeneity on the worker side of the labor market, instead study a decline in labor supply growth and find related results. In these studies, most of the declines in aggregate variables are the result of indirect effects of declining labor supply growth on firm entry and exit rates.

[7] See the discussion in Bilal et al. (2022) on the relationship between their model and the model of Lise and Robin (2017), on which my framework is based.

cline in business dynamism on the economy and compares the model's predictions to the data. Section 6 discusses the implications of declining business dynamism for aggregate welfare as well as welfare across worker cohorts. Section 7 concludes.

2 Motivating Evidence

In this section, I review trends in business dynamism and present motivating evidence that the decline in the share of young firms may have had a larger impact on the labor market outcomes of younger workers. First, I discuss recent findings in the literature demonstrating that changes in the distribution of firms by firm age over the past several decades were primarily driven by changes in the firm entry margin. Then, I show empirical evidence that the age composition of employment at younger firms is significantly more skewed towards younger workers. That is, young workers tend to sort into young firms, while the age distribution of employment at older firms is similar across worker age groups. I describe the data sources in Appendix A and the methodology below.

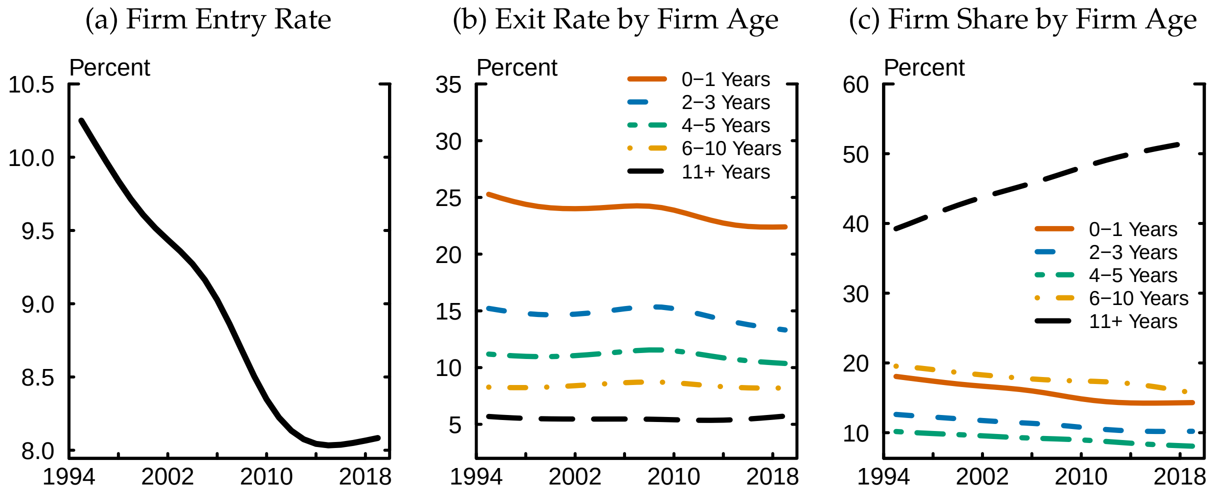
2.1 Decline in Firm Entry and Shift of Firm Age Distribution

Figure 1 displays trends in business dynamism from 1994 to 2019.[8] Over this time period, the entry rate of new businesses declined precipitously, firm exit rates conditional on firm age were roughly constant, and the firm age distribution shifted towards older businesses (11 years and older). As shown previously in the literature, the aggregate decline in firm entry was not the result of compositional shifts of economic activity across sectors or geographic locations (Decker et al., 2014a; Pugsley and Şahin, 2019). In fact, the decline in the firm entry rate was pervasive across markets, occurring even within very narrowly defined industry $\times$ geography cells. Moreover, declining firm entry has not changed other margins of business dynamics conditional on firm age. For instance, average survival and growth rates have remained constant within firm age categories.

Given the stability of firm exit rates and the fact that the decline in the aggregate firm entry rate is not the result of changing sectoral or geographic composition, it must be the case that the aging of firms in the economy (Figure 1c) resulted exclusively from changes in the entry margin (Pugsley and Şahin, 2019; Hopenhayn et al., 2022; Karahan et al., forthcoming). Therefore, aggregate trends in firm exit rates, survival rates, growth

[8]The latest statistics on firm entry from the Census Bureau are available in the Business Dynamics Statistics (BDS) database until 2019. Dinlersoz et al. (2021) find that new business applications surged during the COVID-19 pandemic, but this was driven in part by a shift in the composition of applications towards nonemployer businesses. Post-pandemic trends in new business formation remain an open area of research.

Figure 1: Declining Business Dynamism



Notes: Firm Entry Rate is defined as the number of age 0 firms divided by the total number of firms. Exit Rate for each firm age bin is defined as the number of firm deaths in the respective age bin divided by the total number of firms in the respective age bin. Firm Share for each firm age bin is defined as the total number of firms in the respective age bin divided by the total number of firms. Data are from the Census Bureau's Business Dynamics Statistics (BDS) program and are HP-filtered with an annual smoothing parameter. For more details on the BDS, see Appendix A.

rates, average firm size, and concentration are all driven by the shifting age composition of firms induced by a decline in the number of new startup firms created each year. Even more remarkable is that some of these aggregate trends would have reversed had the age composition of businesses in the economy not changed over this time horizon.

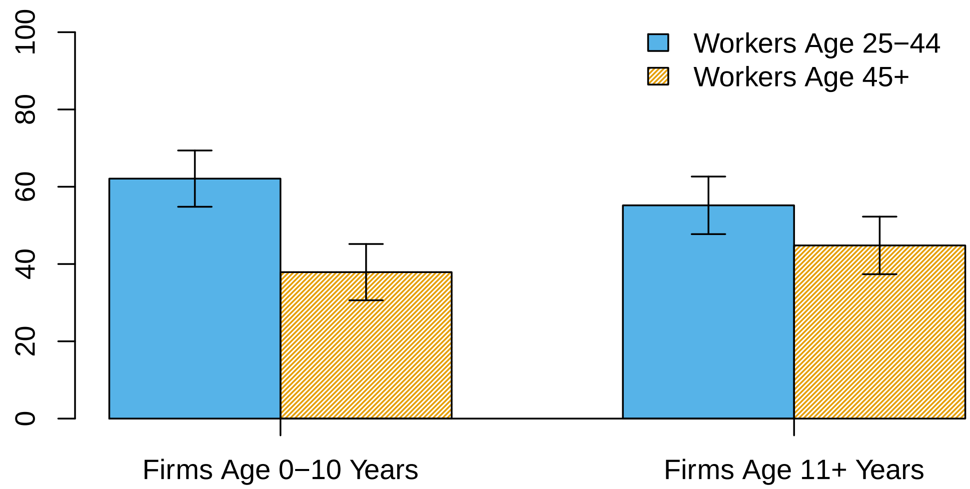
These facts are not new to the literature nor are they unique to my paper. They are taken directly from the aforementioned studies and reproduced here in order to motivate the experiment I conduct in this paper: an exogenous decline in firm entry that shifts the age composition of businesses in the economy towards older firms. I now review evidence on firm and worker sorting patterns that suggests this change in firm demographics may give rise to different labor market outcomes across worker cohorts.

2.2 Worker and Firm Life-Cycle Sorting Patterns

I examine patterns of worker and firm sorting by age in the Census Bureau's Quarterly Workforce Indicators (QWI) database and show that the composition of employment at younger firms is skewed towards younger workers. That is, younger firms tend to employ younger workers. This suggests that a decline in the share of younger firms in the economy may disproportionately affect younger workers.

Figure 2 plots the composition of employment across worker age group by firm age

Figure 2: Age Composition of Employment: Young versus Old Firms



Notes: Figure shows average employment composition, in percentages, by worker age group for firms in different age groups. Data on employment by firm age group and worker age group are from the Census Bureau's Quarterly Workforce Indicators (QWI) database. For more details on the QWI, see Appendix A. For all series, I include only male workers and take averages over 1993-2019. Black bars show standard errors.

group. For instance, the solid blue bar on the left side of the figure shows that roughly 60 percent of employees at firms between 0–10 years old (young firms) are between the ages of 25–44, while the dashed orange bar on the left side of the figure shows that the remaining 40 percent of employees at these firms are 45 years or older.[9] From the figure, a striking pattern emerges. The composition of employment at younger firms is significantly more skewed towards younger workers relative to the employment composition at more mature firms (11+ years old). In fact, the percentage of firm employment composed of workers less than 45 years old is declining in firm age.

This pattern has also been documented by Ouimet and Zarutskie (2014) as well as Dinlersoz et al. (2019) using microdata from the Census Bureau. Ouimet and Zarutskie (2014) also find that young firms disproportionately hire young employees, controlling for firm size, industry, geography and time. Since I do not have firm-level data in my sample, I cannot control for firm-specific factors. However, I use the Quarterly Workforce Indicators (QWI) database to confirm that the pattern I document above holds across NAICS Sectors and when controlling for time fixed effects. I also perform this analysis using firm size (as opposed to firm age) and find that the sorting patterns by firm and

[9] I use a standard classification of young versus old firms as in the literature. The 0-10 year old firm age bin can be further broken down into the following sub-categories: 0–1, 2–3, 4–5, 6–10 years old. Worker age bins may be further broken down into 25–34, 35–44, 45–54, 55+ years old. See Appendix Figure B.1 for sorting patterns using these finer age bins.

worker age are not driven by firm size. This is an important finding as young firms tend to be small and it could instead be the case that young workers sort into smaller firms, with no life-cycle dimension to firm employment policies. Appendix B contains additional figures as well as the results of this regression analysis using data tabulated at the industry level.

3 Model of Firm and Worker Sorting

In this section, I develop an equilibrium model of labor market sorting between ex-ante heterogeneous workers and ex-ante heterogeneous firms that builds on Lise and Robin (2017). While Lise and Robin (2017) study the business cycle implications of two-sided heterogeneity and labor market sorting, I extend their model to capture the employment sorting patterns between workers and firms in different stages of the life-cycle. I also adopt the wage setting protocol developed in a related paper by these authors, Lentz et al. (2017).[10] Below, I elaborate on the model structure and the wage setting protocol.

3.1 Environment

Time, indexed by t, is discrete, and extends forever. Both firms and workers are heterogeneous and differ by type, where worker type is denoted by x and firm type is denoted by y. The density of worker types (ages) at time t is given by $\ell_t(x)$ with mass normalized to 1. Workers enter the economy into the youngest age type at rate η and exit the labor force due to retirement at rate ω_x, which depends on their age type x. The density of firm types (ages) at time t is given by $m_t(y)$ with mass $\mathcal{M}_t$. Firms enter the economy into the youngest age type at rate γ_t and exit at rate $\zeta_t(y)$, which depends on their age type y. Workers and firms age stochastically according the the Markov processes $\Pi(x'|x)$ and $\Pi(y'|y)$, respectively. In the remainder of the paper I use "age" or "type" interchangeably to refer to workers with index x as well as to refer to firms with index y.

Workers can either be employed or unemployed. If a worker of type x matches with a firm of type y, they produce flow match output $p(x, y)$. The worker earns a flow wage of $w_t(x, y)$, which is the equilibrium outcome of a sequential auctions bargaining procedure outlined below. While unemployed, a worker receives a flow nonemployment benefit $b(x)$. Both employed and unemployed workers may search for jobs, so the model features on-the-job search (OJS). Further, worker search intensity $\phi(x, l)$ is set exogenously and depends on both worker type x and employment status $l \in \{e, u\}$. An employed worker

[10] See Lise and Postel-Vinay (2020) for a recent implementation of this wage setting protocol.

of type x contacts a firm at rate $\phi(x,e)\cdot\lambda_t$ and an unemployed worker of type x contacts a firm at rate $\phi(x,u)\cdot\lambda_t$. I assume that $\phi(x,l)=\psi_x\cdot\kappa_l$ so that the worker type component ψ_x and the labor market state component κ_l of search intensity enter multiplicatively. I normalize κ_u to 1. Both firms and workers discount the future at rate $\beta=\frac{1}{1+r}$.

Within each period, there are two stages. At the beginning of the period, matches between workers and firms exist such that some workers are employed and the rest are unemployed. Then, in the first stage ("separation stage"), worker and firm types change according to $\Pi(x'|x)$ and $\Pi(y'|y)$, respectively. Some fraction of workers exit the labor force and are replaced by new entrants to the labor market who have the lowest worker type (youngest age group) and start off unemployed. After firms and workers learn their new types and retirement takes place, some matches dissolve and workers in these matches return to unemployment. Next, in the second stage ("matching stage"), the total stock of unemployed workers (previously unemployed workers plus those newly unemployed) and the total stock of employed workers, each scaled by their search intensity, may form matches with new firms. After new matches form, the economy enters the next period.

3.2 Value Functions

The value function for an unemployed worker of type x is given by

$$\begin{aligned} W^u_t(x) = b(x) + (1-\omega_x)\beta\,\mathrm{E}_{x'}\Big[&(1-\psi_x\lambda_{t+1})W^u_{t+1}(x') \\ &+\psi_x\lambda_{t+1}\int \max\{W^e_{t+1}(x',y'),W^u_{t+1}(x')\}\frac{v_{t+1}(y')}{V_{t+1}}dy'|x\Big] \end{aligned}$$

While unemployed, the worker receives flow benefit $b(x)$. If she does not retire, she may continue on in the labor market and search for jobs in the next period. With probability $(1-\psi_x\lambda_{t+1})$ she fails to contact a firm and remains unemployed, possibly with new type x'. In this expression, λ_{t+1} is the aggregate contact rate, which will be determined according to a matching function specified below, and ψ_x is the search intensity for a worker of type x. With complementary probability $\psi_x\lambda_{t+1}$ she contacts a firm and receives the employed worker value $W^e_{t+1}(x',y')$ provided that the joint surplus of the match is positive such that the firm is willing to hire the worker. Otherwise, she remains in unemployment. Conditional on contacting a firm with a non-negative match surplus, she meets a firm of type y' with probability $\frac{v_{t+1}(y')}{V_{t+1}}$, where $v_t(y)$ is the number of vacancies posted by firms of type y and $V_t=\int v_t(y)\,\mathrm{d}\,y$ is the total number of vacancies in the economy. Therefore, $\frac{v_t(y)}{V_t}$ is the density of vacancies posted by firms of type y. As in Lise and Robin (2017), I make the following assumption about unemployed worker bargaining power:

Assumption 1. *Unemployed workers are assumed to have zero bargaining power so that for workers hired out of unemployment, $W_t^e(x,y) = W_t^u(x)$. Under this assumption, the worker's value function reduces to the following:*

$$W_t^u(x) = b(x) + (1-\omega_x)\beta\, \mathrm{E}_{x'}\left[W_{t+1}^u(x')|x\right] \tag{1}$$

Proof: *See Appendix C.*

This expression states that the value of unemployment is simply the presented discounted value of current and future flow nonemployment benefits $b(x)$, which stands for any per-period utility value a worker receives while unemployed. In particular, it may stand for home production, leisure value, or explicit unemployment benefit payments. It may also vary by worker type. Though as a result of this assumption they are technically indifferent between unemployment and employment, I follow Lise and Robin (2017) and assume that unemployed workers always accept job offers upon contacting a firm.

The value function for employed workers $W_t^e(x,y)$ is not specified because it is not needed for the equilibrium computation. Instead, I proceed to define the *joint* value of an employment relationship. Let $P_t(x,y)$ denote the present discounted value of a match with flow output $p(x,y)$. In other words, $P_t(x,y)$ represents the value of a match between a worker of type x and a firm of type y. The value function for $P_t(x,y)$ is given below.

$$\begin{aligned} P_t(x,y) &= p(x,y) \\ &\quad + (1-\omega_x)\beta\, \mathrm{E}_{x',y'}\Big[\Big(1-(1-\delta_{x,y})\mathbb{1}\{P_{t+1}(x',y') \geq W_{t+1}^u(x')\}\Big)W_{t+1}^u(x') \\ &\qquad + \ (1-\delta_{x,y})\mathbb{1}\{P_{t+1}(x',y') \geq W_{t+1}^u(x')\}\Big((1-\psi_x\kappa_e\lambda_{t+1})P_{t+1}(x',y') \\ &\qquad + \psi_x\kappa_e\lambda_{t+1}\int \max\{P_{t+1}(x',y'), W_{t+1}^e(x',y'',y')\}\frac{v_{t+1}(y'')}{V_{t+1}}\,\mathrm{d}\,y''\Big)|\, x,y\Big] \end{aligned}$$

In the current period, a match between a worker of type x and a firm of type y produces $p(x,y)$. Assuming the worker does not retire, the match dissolves exogenously with probability $\delta_{x,y}$, which may depend on both worker type and firm type. A match dissolves endogenously if, after firms and workers learn their new types, the continuation value of the match drops below the value of the worker's outside option, $P_{t+1}(x',y') < W_{t+1}^u(x')$. Instead, If the match persists, the employed worker has the opportunity to meet a new firm of type y'' with probability $\psi_x\kappa_e\lambda_{t+1}\frac{v_{t+1}(y'')}{V_{t+1}}$. If she fails to meet a new firm, the match persists with the same continuation value. However, if an employed worker successfully meets a new firm, then the current firm ("incumbent firm") and the new firm ("poaching firm") enter into Bertrand competition over the worker's services. This procedure, which follows Postel-Vinay and Robin (2002), is explained in more detail below.

3.3 Sequential Auctions Protocol

Suppose a worker employed at a firm of type y meets a firm of type y'. There are two possible outcomes for the worker's new employer. Either the total match value is higher at the incumbent firm ($P_t(x,y) > P_t(x,y')$) and the worker remains at the incumbent firm, or the total match value is higher at the poaching firm ($P_t(x,y) < P_t(x,y')$) and the worker moves to the poaching firm. In the case where the worker remains at the incumbent, the worker may be able to renegotiate her wage to a higher value. This occurs when the joint match value of the poaching firm $P_t(x,y')$ is higher than the joint match value corresponding to any previous outside offer she has received. If the joint match value of the poaching firm does not exceed the joint match value corresponding to the highest previous outside offer, the worker simply discards the offer from the poaching firm. If the worker is poached, she may negotiate her wage at the poacher such that she receives the entire match value $P_t(x,y)$ from the incumbent firm. In this way, the continuation value of the match turns out to be independent of whether or not the worker is poached and therefore independent of the employed worker value function $W^e_t(x,y)$.

Instead, we may write the joint worker and firm problem in terms of the joint surplus of the match. Let $S_t(x,y) = P_t(x,y) - W^u_t(x)$ be the total surplus at time t from an employment relationship between worker x and firm y. The surplus is the total match value net of the values of the worker's and the firm's outside options.[11] The surplus function determines all allocations in the economy and is given by the expression below.

$$S_t(x,y) = p(x,y) - b(x) + (1-\omega_x)(1-\delta_{x,y})\beta\, \mathrm{E}_{x',y'}\left[\max\{S_{t+1}(x',y'),0\} \mid x,y\right] \qquad (2)$$

This equation states that the joint surplus of a match between worker x and firm y is equal to the flow output of the match net of the workers' flow value of nonemployment, plus any future expected surplus if the match continues. Given flow match output $p(x,y)$ and flow nonemployment value $b(x)$, it is sufficient to solve Equation 2 to determine the surplus value of any possible match in the economy, simplifying the equilibrium computation considerably. Notice that the distribution of employment does not appear in this equation, meaning that the model has the block-recursive property, as shown in Lise and Robin (2017). Block-recursivity stems from the assumption that workers have no bargain-

[11] Due to free entry, an unfilled vacancy has no value and hence the value of a vacancy does not appear in the surplus definition.

ing power out of unemployment (Assumption 1) along with the fact that the sequential auctions protocol renders the match continuation value independent of the employed worker value. For an explicit derivation of Equation 2, see Appendix C.

3.4 Worker Search and Vacancy Posting

Workers search both on and off the job and firms post vacancies to equate the expected benefits and costs of meeting a worker. The worker flow equations, which I specify below, determine the numbers of workers separated to unemployment, hired from unemployment, and poached by firms of different types. Recall that within each period there are two sub-periods: a separation stage where agents realize their new types after which some matches are destroyed and a matching stage where new matches form between searching workers and firms with open vacancies. Let $\tilde{u}_t(x)$ and $\tilde{e}_t(x, y)$ represent the stock of unemployed and employed workers, respectively, after the separation stage. Aggregate search intensity is then given by the expression below.

$$L_t = \int \psi_x \, \tilde{u}_t(x) \, \mathrm{d}\, x \; + \; \int \int \psi_x \, \kappa_e \, \tilde{e}_t(x, y) \, \mathrm{d}\, x \, \mathrm{d}\, y \tag{3}$$

Knowing aggregate search intensity, firms post vacancies in order to hire workers from the pool of total searchers. The value of a filled vacancy is given by

$$\begin{aligned} J_t(y) = \int \frac{\psi_x \tilde{u}_t(x)}{L_t} \max\{S_t(x, y), 0\} \, \mathrm{d}\, x & \\ + \int \int \frac{\psi_x \kappa_e \tilde{e}_t(x, y')}{L_t} \max\{S_t(x, y) - S_t(x, y'), 0\} \, \mathrm{d}\, x \, \mathrm{d}\, y' & \end{aligned} \tag{4}$$

The value of a filled vacancy has two components. Either the worker is hired from unemployment, in which case the firm receives the entire surplus from the match (unemployed workers are assumed to have zero bargaining power) or the worker is hired from employment, in which case the firm receives the surplus of the match net of the surplus of the match at the previous firm. Notice that if $S_t(x, y) < 0$ the match is not formed and that no firm may poach from another firm with a higher surplus.

Firms also face per-unit flow vacancy posting costs $c(n)$ on the number of firm-level vacancies n. I parameterize the vacancy cost function as the iso-elastic function

$$c(n_t(y)) = c_{0,y} \frac{n_t(y)^{1+c_1}}{1 + c_1}$$

where $n_t(y)$ is the number of vacancies posted by each firm of type y at time t. Under this functional form assumption, the parameter $c_{0,y}$ governs the level of vacancy costs, which is allowed to differ by firm type. As I will discuss below, this assumption helps the model match the distribution of employment by firm age in the data. The parameter c_1 governs the curvature of the vacancy cost with respect to the number of firm-level vacancies. I assume that $c_1 > 0$ so that $c(\cdot)$ is a convex function. The free entry condition dictates that firms post vacancies up to the point where the expected value of a filled vacancy is equal to the marginal cost of opening a vacancy. In equilibrium, vacancies are therefore pinned down by the condition

$$c'(n_t(y)) = c_{0,y} n_t(y)^{c_1} = q_t \cdot J_t(y) \tag{5}$$

where q_t is the rate at which firms contact workers. I show how q_t is determined below. Given $J_t(y)$, q_t, and values for $c_{0,y}$ and c_1, the number of firm level vacancies by firm type $n_t(y)$ solves Equation 5. Aggregate vacancies are then given by

$$V_t = \int n_t(y) m_t(y) \, \mathrm{d}\, y = \int v_t(y) \, \mathrm{d}\, y$$

where $v_t(y) = n_t(y) m_t(y)$ is the number of firm-level vacancies $n_t(y)$ multiplied by the mass of firms of type y, $m_t(y)$.

3.5 Matching and Contact Rates

Matches between searching workers L_t and firm vacancies V_t are produced according to a standard constant returns to scale matching function $\Phi(L_t, V_t)$, which is assumed to be Cobb–Douglas with elasticity parameter α.

$$\Phi(L_t, V_t) = L_t^{\alpha} V_t^{1-\alpha}$$

Matching efficiency is normalized to 1. Hence, the contact rate for an unemployed worker of type x is given by $\psi_x \lambda_t = \psi_x \frac{\phi(L_t, V_t)}{L_t} = \psi_x \left(\frac{V_t}{L_t}\right)^{1-\alpha}$ and the contact rate for an employed worker of type x is given by $\psi_x \kappa_e \lambda_t = \psi_x \kappa_e \left(\frac{V_t}{L_t}\right)^{1-\alpha}$. The rate at which firms contact workers is given by $q_t = \frac{\Phi(L_t, V_t)}{V_t} = \left(\frac{L_t}{V_t}\right)^{\alpha}$.

3.6 Worker Flow Equations

Given the surplus function $S_t(x, y)$, total search effort L_t, total vacancies V_t, as well as the masses of employed and unemployed searchers after the separation stage, $\tilde{e}_t(x, y)$ and $\tilde{u}_t(x)$, respectively, the masses of employed and unemployed workers at the end of the period are determined according to the worker flow equations below. The laws of motion for unemployed and employed workers are given by the equations below.

$$u_t(x) = \tilde{u}_t(x)\left[1 - \psi_x \lambda_t \int \frac{v_t(y)}{V_t} \mathbb{1}\{S_t(x,y) \geq 0\} \,\mathrm{d}\, y\right] \tag{6}$$

$$\begin{aligned} e_t(x,y) = \tilde{e}_t(x,y) &+ \underbrace{\psi_x \kappa_e \lambda_t \int \tilde{e}_t(x,y') \frac{v_t(y')}{V_t} \mathbb{1}\{S_t(x,y) > S_t(x,y')\} \,\mathrm{d}\, y'}_{\text{Poaching Hires}} \\ &- \underbrace{\psi_x \kappa_e \lambda_t \int \tilde{e}_t(x,y) \frac{v_t(y')}{V_t} \mathbb{1}\{S_t(x,y') > S_t(x,y)\} \,\mathrm{d}\, y'}_{\text{Poaching Separations}} \\ &+ \underbrace{\psi_x \lambda_t \, \tilde{u}_t(x) \, \frac{v_t(y)}{V_t} \, \mathbb{1}\{S_t(x,y) \geq 0\}}_{\text{Unemployed Hires}} \end{aligned} \tag{7}$$

The law of motion for unemployed workers in Equation 6 makes clear that stock of unemployed workers at the end of the period is made up of workers who fail to find jobs during the matching stage. This can be because they fail to contact a firm or because they contact a firm with negative match surplus. The law of motion for employed workers mirrors the situations that can arise from the sequential auctions bargaining protocol. The stock of type x workers employed at type y firms is equal to previous employment plus any employees poached from other firms net of employees lost to other firms, plus workers hired out of unemployment. Note that all of these expressions are weighted by $\frac{v_t(y)}{V_t}$, which is the share of total vacancies at firms of type y.

Additionally, the objects $\tilde{u}_t(x)$ and $\tilde{e}_t(x,y)$ are determined as follows.

$$\begin{aligned} \tilde{u}_t(x) = (1-\omega_x)\Pi(x'|x)u_t(x) + n_t(x) \\ + \int \big(\mathbb{1}\{S_t(x,y) < 0\} \; + \; \delta_{x,y} \cdot \mathbb{1}\{S_t(x,y) \geq 0\}\big) \cdot e_{t-1}(x,y) \,\mathrm{d}\, y \end{aligned}$$

$$\tilde{e}_t(x,y) = (1-\delta_{x,y}) \cdot \mathbb{1}\{S_t(x,y) \geq 0\} \cdot (1-\omega_x) \cdot \Pi(x'|x) \cdot \Pi(y'|y) \cdot e_{t-1}(x,y)$$

The first expression states that the number of unemployed workers of type x after the separation stage is equal to any previously unemployed workers of type x who do not transition into the next age bin or who do not retire, plus any new labor market entrants, plus any workers who are separated from their jobs either endogenously or exogenously. The number of new entrants $n_t(x)$ is equal to the total number of retiring workers $\int \omega_x u_t(x) \; + \; \int \omega_x e_t(x,y) \, \mathrm{d}\, x \, \mathrm{d}\, y$ for the first age-bin and 0 otherwise. The second expression states that the number of employed workers of type x at firms of type y after the separation stage consists of previously employed workers who survive job destruction, retirement, and who do not transition to another worker or firm age bin. Along with the surplus function, the worker flow equations determine the entire distribution of employment by worker and firm type as well as unemployment by worker type.

3.7 Wage Setting

I adopt the wage setting protocol developed in Lentz et al. (2017). I assume that firms commit to delivering a constant share σ_t of the surplus for the entire duration of a match until and unless the worker receives an outside offer, in which case the surplus share is renegotiated according to the sequential auctions protocol. Therefore, the worker receives a share $\sigma_t(x,y,y')$ of the surplus that depends on her type x, her previous firm (previous outside offer) y', and her current firm y, which is constant until she receives an outside offer. In particular, for $S_t(x,y) \geq S_t(x,y')$,

$$\sigma_t(x,y,y') \equiv \frac{S_t(x,y')}{S_t(x,y)}$$

Since allocations are determined entirely by the surplus function, the wage setting protocol only specifies how the match surplus is splits between firms and workers. For instance, a protocol under which the firm pays a fixed wage until the next renegotiation would result in identical allocations. As shown in Lentz et al. (2017), the convenience of the fixed surplus share rule is that it produces a closed form solution for the wage equation. I present this equation below and leave the full derivation for the appendix. Let $\sigma_t = \sigma_t(x,y,y')$.

$$\begin{aligned} w_t(x,y,\sigma_t) = {} & \sigma_t p(x,y) + (1-\sigma_t) b(x) \\ & - (1-\omega_x)(1-\delta_{x,y}) \beta \, \mathrm{E}_{x',y'} \Big[\\ & \mathbb{1}\{S_{t+1}(x',y') \geq 0\} \cdot \psi_x \kappa_e \lambda_{t+1} \int R_{t+1}(x',y',\sigma_{t+1},y'') \frac{v_{t+1}(y'')}{V_{t+1}} \, \mathrm{d}\, y'' \mid x, y \Big] \end{aligned} \tag{8}$$

where the term $R_t(x,y,\sigma_t,y')$ results from the possible outcomes of the sequential auctions protocol and represents the additional surplus the worker captures due to a renegotiation.

It is given by the piecewise function:

$$R_t(x, y, \sigma_t, y') = \begin{cases} S_t(x, y) - \sigma_t S_t(x, y) & S_t(x, y') > S_t(x, y) \\ S_t(x, y') - \sigma_t S_t(x, y) & \sigma_t S_t(x, y) < S_t(x, y') \leq S_t(x, y) \\ 0 & S_t(x, y') \leq \sigma_t S_t(x, y) \end{cases}$$

The first case corresponds to a situation where the worker is poached. In this case, she is able to capture the entire surplus from her old firm and therefore receives $S_t(x, y)$ net of the previous surplus share $\sigma_t S_t(x, y)$ in her old match. In the second case, the offer is higher than her previous outside offer, but not high enough to trigger a poaching event. The worker is able to renegotiate her surplus share at the incumbent firm in order to extract the full value of the outside offer. She therefore receives $S_t(x, y')$ net of her previous surplus share $\sigma_t S_t(x, y)$. In the third case, the outside offer is not sufficiently high to trigger a renegotiation and the offer is discarded.

Given this assumption on the wage setting protocol, the wage $w_t(x, y, \sigma_t)$ is a weighted average of flow match output $p(x, y)$ and flow nonemployment benefit $b(x)$, net of future expected renegotiation opportunities captured by the final term in Equation 8 containing $R_t(x, y, \sigma_t, y')$. As a result of this term, wages will be lower for lower tenure workers, as these workers expect to have future opportunities to climb the job ladder and renegotiate their wages upward. The distribution of wage contracts across matches (x, y, σ_t) must also be solved as part of the equilibrium in order to compute average wages by (x, y) pair. Details are presented in Appendix C.

3.8 Law of Motion for Mass of Firms

The law of motion for the mass of firms of type y in time period t, which I denote as $m_t(y)$, is as follows:

$$m_{t+1}(y) = \Pi(y'|y)\big(1 - \zeta_t(y)\big)m_t(y) + \gamma_t \cdot \mathbb{1}\{y = \underline{y}\} \tag{9}$$

where $\Pi(y'|y)$ is the transition matrix across firm age bins, $\zeta_t(y)$ is the exit rate for firm age y at time t, and γ_t is the entry rate of firms into the first age bin $\underline{y}$. Given exit rates $\zeta_t(y)$ and entry rate γ_t, the steady state mass of firms by firm age, which I denote $\bar{m}(y)$, is the fixed point of Equation 9.

4 Numerical Implementation and Calibration

This section describes the calibration of the model as well as other details of the numerical implementation. I calibrate the model in steady state in order to match several features of the U.S. economy in the mid-1990s. Below, I describe the specific moments targeted in the calibration procedure and provide an overview of which parameters in the model help to inform certain moments in the model.

4.1 Worker and Firm Age Bins

The Census data used in the empirical section of the paper are defined at the bin-level. I choose the same bins as the units of analysis for the model. There are 4 worker age bins $\{25\text{–}34, 35\text{–}44, 45\text{–}54, 55+\}$ and 5 firm age bins $\{0\text{–}1, 2\text{–}3, 4\text{–}5, 6\text{–}10, 11+\}$, each in years. Worker types evolve stochastically across bins according to the Markov transition matrix $\Pi(x'|x)$ and firm types evolve stochastically across bins according to $\Pi(y'|y)$. The model is set at a monthly frequency, which means that in each time period, $\frac{1}{12\times 10}^{th}$ of 25–34 year-old workers become 35–44 year-old workers, $\frac{1}{12\times 2}^{th}$ of 0–1 year-old firms become 2–3 year-old firms, et cetera.[12] Within bins, however, workers and firms are identical. Hence, the model describes the average worker within a certain age range and the average firm within a certain age range.

4.2 Externally Set Parameters

I externally set a subset of parameters to commonly used values in the literature. Table 1 shows the externally set parameters.

Table 1: Externally Set Parameters

Parameter		Value	Target
β	Discount factor	0.9959	5% annual real interest rate
α	Matching function elasticity	0.8	Lange and Papageorgiou (2020)
κ_u	Unemployed search intensity	1	Normalization
κ_e	Employed search intensity	0.5	Faberman et al. (2022)
c_1	Vacancy cost curvature	1	Normalization

Notes: The frequency is monthly.

[12] Note that workers can only move up age bins, so the transition matrices contain only zeros below the diagonal. Transition matrices $\Pi(x'|x)$ and $\Pi(y'|y)$ are specified explicitly in the appendix.

One period is set to one month in the model, so all rates are monthly. The discount factor is set to correspond to an annual interest rate of 5 percent. The matching function elasticity with respect to searchers α is set to match recent estimates of the elasticity of hires with respect to searchers (Lange and Papageorgiou, 2020). I normalize the search intensity of unemployed workers to 1 and set the search intensity of employed workers to a standard value from the literature.[13] The curvature parameter of the vacancy cost function is set such that the function is exactly quadratic (and the marginal cost of an additional vacancy is exactly linear) in the number of firm-level vacancies posted.

4.3 Directly Estimated Parameters

Next, a subset of parameters is directly informed by the data. I display their values in Table 4 and describe the calibration procedure below.

The retirement rate is set such that workers only face retirement once they enter the oldest age bin (55+). I set the retirement rate for this age bin so as to match the share of workers age 55 and over in the labor force in 1994.

I set the search intensity by age group parameters ψ_x to target the age profile of the job finding rate in 1994. Workers in the youngest age group (25–34) have the highest job finding rates, so I normalize their search intensity to 1. The ψ_x's for all other age groups are set relative to the youngest worker age group (25–34). They are calculated by taking the ratio of the job finding rate for age group x to the job finding rate for age group 25–34. Before setting the ψ_x's, I first HP-filter each job finding rate series to ensure that I extract the trend for each worker age group.

I allow the exogenous separation rate $\delta_{x,y}$ to vary only by firm age such that $\delta_{x,y} = \delta_y$. I then set δ_y directly to the value of the job destruction rate by firm age group from the Census Bureau's Business Dynamics Statistics (BDS) database in 1994. As with the calibration of worker search intensity, I HP-filter each job destruction rate series before extracting the value in 1994. The Census Bureau defines the job destruction rate as the sum of all employment losses from contracting establishments, including establishments shutting down, divided by total employment. It therefore includes employment losses both from employees leaving the firm (continuing firms) and from firm exits (firm deaths). This is the relevant definition of match separation in my model since the boundaries of the firm with a firm age bin are undefined. The separation rate δ_y include both cases: employees leaving a firm that survives as well as employees returning to unemployment

[13] See Holzer (1987) and Faberman et al. (2022) for estimates of the relative time spent searching by employed workers as well as Baley et al. (forthcoming), for a recent discussion.

Table 2: Directly Estimated Parameters

Parameter		Bin	Value	Target
ω_x	Retirement rate	55+	0.0162	Labor force share age 55+
ψ_x	Search intensity by worker age bin	25–34	1.0000	Job finding rate by worker age bin, relative to age bin 25–34
		35–44	0.9063	
		45–54	0.8247	
		55+	0.8030	
δ_y	Separation rate by firm age bin	0–1	0.0300	Job destruction rate by firm age bin
		2–3	0.0242	
		4–5	0.0195	
		6–10	0.0160	
		11+	0.0116	
$\bar{m}(y)$	Mass of firms by firm age bin	0–1	0.0142	Number of firms by firm age bin relative to size of the labor force
		2–3	0.0099	
		4–5	0.0080	
		6–10	0.0152	
		11+	0.0299	

Notes: The frequency is monthly. For all moments, I extract the trend using an HP filter with the appropriate smoothing parameter and take the value in 1994. The labor force share of age 55+ workers is from the Bureau of Labor Statistics' Labor Force Statistics (LFS) database. I include only male workers. The job finding rate for each worker age bin is constructed using the longitudinally linked CPS and is HP-filtered using a monthly smoothing parameter. I use only male workers age 25 years and older. The job destruction rate by firm age bin is from the BDS and is HP-filtered using an annual smoothing parameter. The mass of firms by firm age bin is the ratio of the number of firms in the respective age bin to the total number of workers in the labor force. The number of firms by firm age bin is from the BDS. The total number of workers in the labor force is from the LFS and includes only male workers age 25 years and older. The resulting series are HP-filtered using an annual smoothing parameter.

because their firm has closed down.

I assume that the economy is in steady state in 1994 and set the mass of firms by firm type $\bar{m}(y)$ directly to its empirical value in 1994. I calculate this value for each firm age bin by taking the ratio of the number of firms in that age bin to the total number of workers in the labor force. I then HP-filter each resulting series with an annual smoothing parameter. Data for the number of firms by firm age bin are from the BDS, while data on the number of workers in the labor force is from the Bureau of Labor Statistics' Labor Force Statistics (LFS) database. I include only male workers age 25 years and older.

4.4 Internally Estimated Parameters

I use the remaining parameters related to match-level output $p(x, y)$, flow nonemployment value $b(x)$, and the vacancy cost function $c(\cdot)$ to target several moments in the data. I target worker flows into and out of unemployment by worker age bin, the distribution of employment across firm age bins, and the entire wage grid by firm age bin and worker age bin in 1994. The exact data moments are specified below. I first briefly describe how I parameterize the functions $p(x, y)$ and $b(x)$ before discussing the moment matching procedure and identification.

4.4.1 Functional Form Assumptions

The match-level output grid $p(x, y)$ is a crucial element of the match surplus between firms and workers of different types. The shape of the match surplus function $S(x, y)$ not only determines the sorting patterns between firms and workers but also influences the level of wages for different matches. Moreover, as shown in the previous section, $p(x, y)$ enters directly into the wage equation. Following Lise and Robin (2017), I parameterize the match-level output function $p(x, y)$ as a second order polynomial in worker type x and firm type y.

$$p(x, y) = p_0 + p_1 x + p_2 y + p_3 xy + p_4 x^2 + p_5 y^2$$

This functional form is flexible enough to capture the contours of the wage grid without allowing for too many degrees of freedom.

I also follow Lise and Robin (2017) and set the flow nonemployment benefit $b(x)$ such that it is equal to some fraction b_0 of a worker's maximum attainable match output.

$$b(x) = b_0 \cdot \max_y \{p(x, y)\}$$

In the expression above, $\max_y\{p(x, y)\}$ stands for the match output at worker x's most productive match. The scaling parameter b_0 helps target the overall level of wages. In the model, if unemployment becomes "too costly" – i.e. $b(x)$ is very low relative to $p(x, y)$ – then workers accept wages that are counterfactually too low (even negative) in order to "buy" their way onto the job ladder. This is a well-known feature of the sequential auctions bargaining protocol that I adopt in order to pin down wages in the model. It is especially strong when workers have zero bargaining power out of unemployment, as I assume in order to keep the model tractable. However, setting b_0 sufficiently high helps mitigate this effect such that wages remain positive.

4.4.2 Heuristic Identification Argument

Though the parameters in my moment matching exercise will be jointly identified by the moments in the data, it is useful to consider which moments in particular are informative about specific parameters.

The match-level output function $p(x, y)$ and the flow nonemployment benefit $b(x)$ both enter directly into the wage equation, which I reproduce below.[14] They also affect wages indirectly through the "Expected Renegotiation Benefit" term, as they affect the shape of the surplus function $S(x, y)$. This term captures the amount a worker is willing to have deducted from her wages in order to accept a job on a certain rung of the job ladder. It is higher (wages are lower) when she expects many opportunities to renegotiate her wages upward in the future.

$$\begin{aligned} w(x,y,\sigma) = {} & \sigma p(x,y) + (1-\sigma) b(x) \\ & - (1-\omega_x)(1-\delta_{x,y})\beta \underbrace{\mathrm{E}_{x',y'}\left[\mathbb{1}\{S(x',y') \geq 0\} \cdot \psi_x \kappa_e \lambda \int R(x',y',\sigma,y'') \frac{v(y'')}{V} \,\mathrm{d}\, y'' \,|x,y\right]}_{\text{Expected Renegotiation Benefit}} \end{aligned}$$

Therefore, there is a tension in the effects of $p(x, y)$ and $b(x)$ on wages. First, there is the direct effect that a higher $p(x, y)$ or $b(x)$ increases wages. However, there is an indirect effect through the expected renegotiation benefit: higher $p(x, y)$ or lower $b(x)$ produces higher surplus, meaning that workers will be willing to accept lower wages to gain access to these high surplus matches. Whichever effect dominates is a matter of the specific numerical values from the calibration routine, but it is possible to find a set of parameters such that the shape of the wage profile in the model resembles that of the data. The six parameters of the match-level output function $\{p_0, p_1, p_2, p_3, p_4, p_5\}$ along with the scale parameter b_0 therefore help to determine the shape of this wage profile across worker and firm age bins.

Wages over a worker's life-cycle display a hump shaped pattern: they are low initially for labor market entrants, rise as workers age and gain experience, and then fall slightly as workers near retirement. Intuitively, the parameters p_1 and p_4 determine the average shape of the wage profile across worker age bins. Across firms of different ages, there is a strong correlation between productivity (firm size) and wage. Older firms tend to be larger and more productive, and therefore pay higher wages than younger firms. The parameters p_2 and p_5 control the degree to which older firms pay higher wages on average relative to younger firms. The parameter p_3 is an interaction term that determines the

[14] To conserve on notation, I suppress time subscripts.

degree to which specific firm types may pay higher or lower wages to specific worker types. If $p_3 > 0$, for instance, older firms pay especially high wages to older workers. Finally, the parameters p_0 and b_0 help to target the average level of wages. The parameter b_0 in particular ensures that wages at high surplus firms are not counterfactually too low. See Appendix Figure F.4 for the exact profile of wages across the firm and worker life cycle in the data.

The parameters in the vacancy cost function then help to target the distribution of matches as well as the overall scale of the economy. The level parameters $c_{0,y}$ help to pin down the average job finding and unemployment rates in the economy as parallel shifts in these parameters shift the vacancy cost curve up and down in a parallel manner. They also help match the distribution of employment across firm age bins by differentially scaling up or down the vacancy cost function that each firm type faces. The curvature parameter c_1 directly influences the marginal cost of vacancy posting and therefore helps to pin down the employment distribution across firm age. Larger values of c_1 will mean that is very costly for firms to post more vacancies on the margin, whereas smaller values of c_1 result in firms that are very responsive to changes in the economic environment in their vacancy posting decisions. Note that the only form of decreasing returns in the model enters through the vacancy cost function and therefore c_1 may be interpreted as the firm's span of control.

The equation below shows the solution for the number of vacancies $v_t(y)$ posted by firms in each firm age bin. From this equation, we can see how the level parameters $c_{0,y}$ shift up or down the number of vacancies posted by firms of each age.

$$v_t(y) = m_t(y) \left(\frac{q_t \cdot J_t(y)}{c_{0,y}} \right)^{\frac{1}{c_1}}$$

As discussed above, I normalize $c_1 = 1$ so that the parameters $c_{0,y}$ largely determine the degree to which firms in different bins face different vacancy posting costs. The equation above makes clear that setting the $c_{0,y}$'s appropriately allows me to match the vacancy distribution and therefore the employment distribution across firms.

4.4.3 Method of Moments Estimator

I calibrate 12 parameters in total to match 33 moments in the data. The parameter vector is given by $\theta = \{c_{0,1}, c_{0,2}, c_{0,3}, c_{0,4}, c_{0,5}, p_0, p_1, p_2, p_3, p_4, p_5, b_0\}$. I choose 1994 as the date for the initial steady state and construct the following moments in the data: unemployment-to-employment flow rates by worker age bin from the Current Population Survey (CPS), employment-to-unemployment flow rates by worker age bin from the CPS, employment share by firm age from the BDS, and average earnings per employee by worker and firm age bins from the QWI. This gives me 4 + 4 + 5 + 20 = 33 moments in total, meaning that the model is overidentified. Table 3 summarizes the data moments and their sources.

Table 3: Data Moments

Moment	Bins	Source
Job finding rate	Male workers age {25–34, 35–44, 45–54, 55+}	CPS
Separation rate	Male workers age {25–34, 35–44, 45–54, 55+}	CPS
Employment share	Firms age {0–1, 2–3, 4–5, 6–10, 11+}	BDS
Earnings/employee	Male workers age {25–34, 35–44, 45–54, 55+} $\times$ Firms age {0–1, 2–3, 4–5, 6–10, 11+}	QWI

Notes: Job finding rate is defined as the number of unemployed workers who transition into employment divided by total unemployment for each age bin. Separation rate is defined as the number of employed workers who transition into unemployment divided by total employment for each age bin. Data are from the longitudinally-linked CPS. Employment share is defined as employment in each firm age bin as a percentage of total employment. Data are from the BDS. Earnings/employee is defined as average monthly earnings, in units of 1000s of 1982-1984 dollars. Data are from the QWI and are deflated using the Consumer Price Index for All Urban Consumers. See Appendix A for additional details on moment construction.

Let $m(\theta)$ denote a vector of moments resulting from the solution of the model in steady state under θ. Let $\hat{m}$ denote the vector of data moments. Both $m(\theta)$ and $\hat{m}$ are $N \times 1$ vectors, where $N = 33$. I choose parameter vector $\hat{\theta}$ so as to minimize the following objective function:

$$\hat{\theta} = \arg\min_{\theta} \left\| \frac{1}{\sqrt{N}} \left(\frac{m(\theta) - \hat{m}}{\hat{m}} \right) \right\|$$

where $\|\|$ denotes the norm operator. The objective function may be interpreted in terms of percent differences. An objective function value of 0.10 means that the average deviation between model moments and data moments is 10 percent. This has the advantage of not over-weighting moments that have larger magnitudes (wages) or under-weighting moments with smaller magnitudes (distributions). I use global methods to efficiently and thoroughly search the parameter space. Details are provided in Appendix E.

4.5 Calibration Results

The results of the calibration exercise are presented in Table 4. First, I am able to match the employment share by firm age bin with the vacancy cost level parameters $c_{0,y}$ almost exactly. As in the data, the oldest firms in the model comprise the majority share of employment. Next, the parameters of the match-level output function help to match the shape and level of the wage profile over worker and firm age. The scale parameter of the flow nonemployment benefit b_0 also helps to match the level of wages. Since these 7 parameters are informative about the 20 points on the worker age $\times$ firm age grid, I display their values in Table 4 and plot the wage profile by worker and firm age in Figure 3 below. I slightly underestimate career earnings growth in the model, but the difference between wages paid between old and young firms aligns well with the data. Overall, the model achieves a good fit, with an objective function value of about 15 percent.

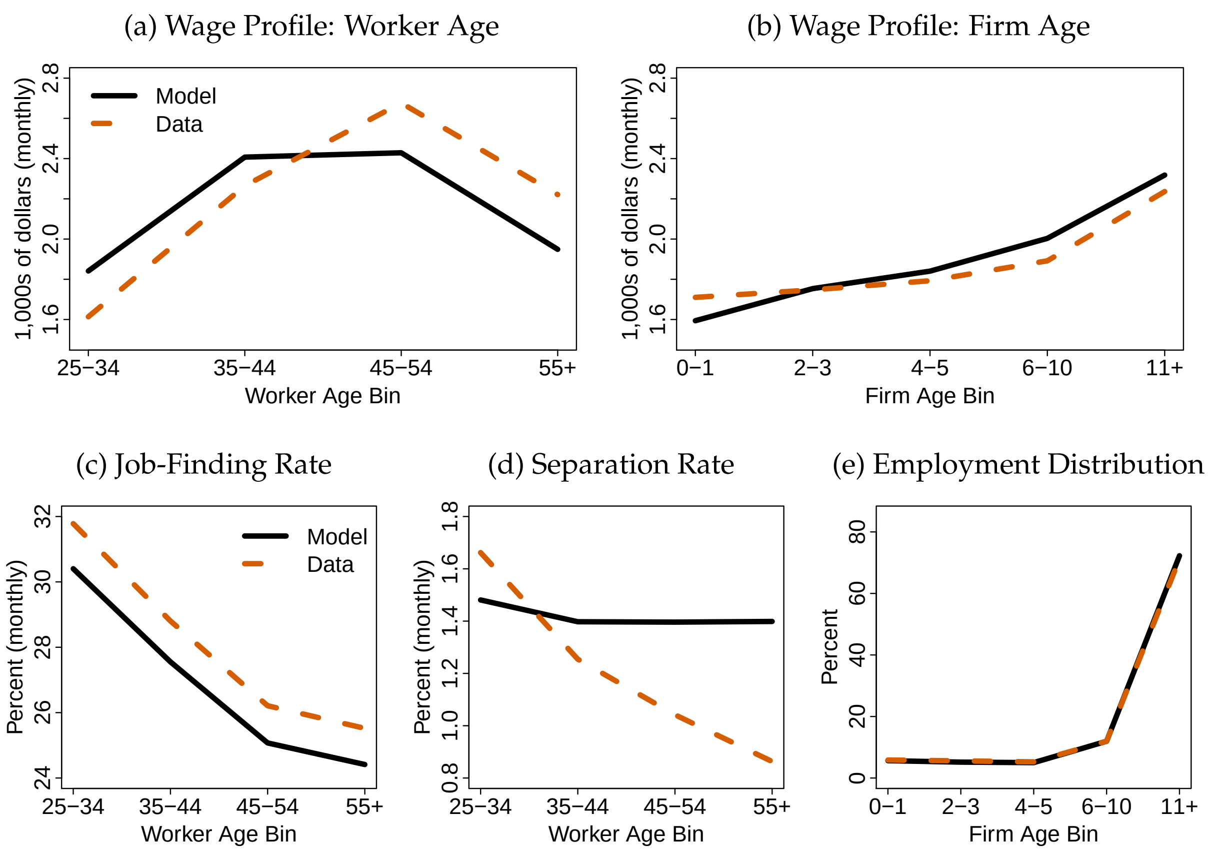
Table 4: Internally Estimated Parameters

Parameter		Bin	Value	Target	Data	Model
$c_{0,1}$	Vacancy cost level by firm age bin	0–1	0.2699	Employment share by firm age bin	5.8467	5.6361
$c_{0,2}$		2–3	0.7452		5.5714	5.1629
$c_{0,3}$		4–5	1.5066		5.2489	5.0019
$c_{0,4}$		6–10	3.2007		11.9643	11.9496
$c_{0,5}$		11+	8.9125		71.1980	72.2496
p_0	Shape of match-level output function	–	2.2001	Wages by worker $\times$ firm age bin	See Figure 3	
p_1		–	2.1622		See Figure 3	
p_2		–	-0.5275		See Figure 3	
p_3		–	0.3609		See Figure 3	
p_4		–	-2.5674		See Figure 3	
p_5		–	0.6122		See Figure 3	
b_0	Scale of $b(x)$	–	0.7851		See Figure 3	

Notes: The employment share by firm age bin is from the BDS. Moments are shown in percentages. Wages by worker $\times$ firm age bin are defined as average monthly earnings-per-employee in units of 1000s of 1982-1984 dollars. Data are from the QWI and are deflated using the Consumer Price Index for All Urban Consumers. See Appendix A for additional details on moment construction.

I also target worker flow rates such as the rate at which workers transition between unemployment-to-employment and employment-to-unemployment by worker age group in the calibration exercise. However, there is not a single set of parameters that help to identify these moments, so I plot the fit to the data in Figure 3 below. As can also be seen in the figure, I match the life-cycle profile of the job finding rate, but slightly underestimate its level. The profile of the job finding rate over the life cycle is captured by the age-specific search intensity parameters ψ_x, which I set outside of the moment matching exercise. Since there is no worker age-specific component to the separation rate, the only differences in separation rates across worker age groups in the model arise from the fact that workers differentially sort into firms of different ages. Though the magnitudes do not match the data exactly, the model captures the fact that separation rates decline over a worker's life cycle. Finally, the figure visually confirms the results shown in Table 4 with respect to the firm employment distribution.

Figure 3: Model Fit: Targeted Moments

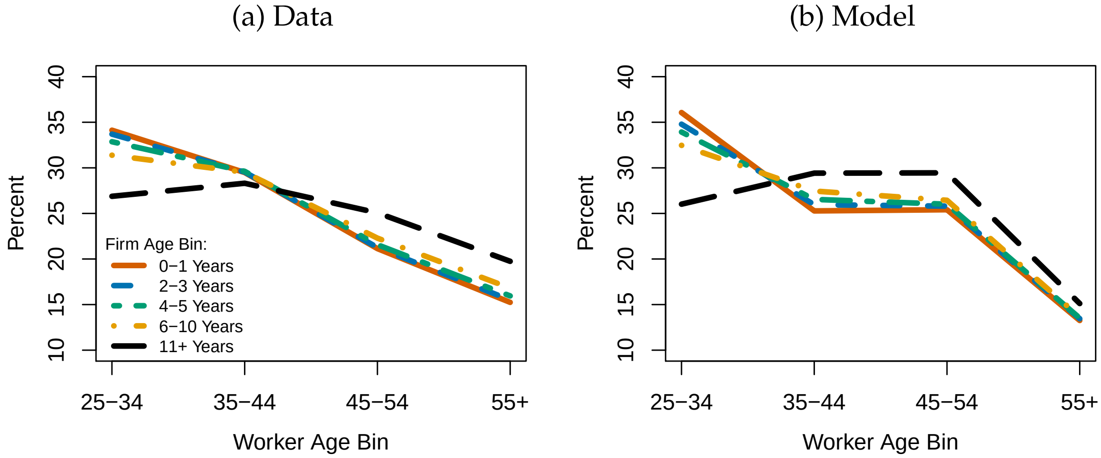


Notes: In each panel, the black solid lines show the model moments and the red dashed lines show the corresponding moments in the data. Job finding rate is defined as the number of unemployed workers who transition into employment divided by total unemployment for each age bin. Separation rate is defined as the number of employed workers who transition into unemployment divided by total employment for each age bin. Data are from the longitudinally-linked CPS (see Appendix A.4). Wage profile is defined as the average wage for each worker age bin, in units of 1000s of 1982-1984 dollars. Data are from the QWI and are deflated using the Consumer Price Index for All Urban Consumers. Firm Employment Distribution is defined as employment in each firm age bin as a percentage of total employment. Data are from the BDS.

4.5.1 Non-Targeted Moments

Though I target the distribution of employment across firm age in aggregate, I do not directly target the distribution of employment across worker age conditional on firm age bin. As discussed in the empirical section of the paper, sorting patterns between firm age and worker age groups are such that young firms play an out-sized role in young worker employment. In particular, the share of employment comprised by younger workers at younger firms is higher than that for older firms.

Figure 4: Distribution of Employment Across Worker Age by Firm Age

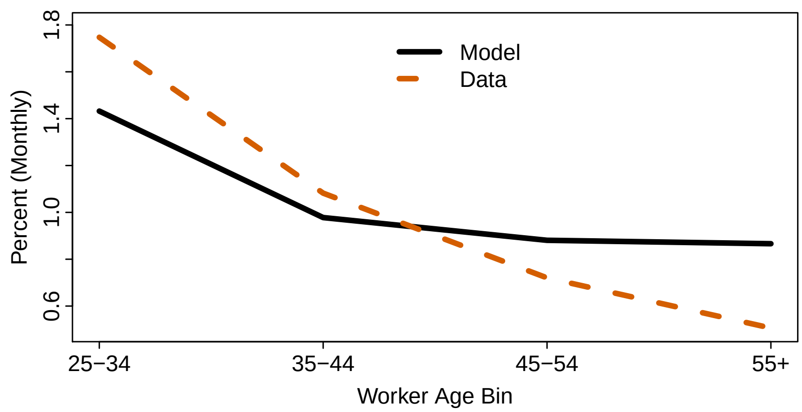


Notes: Left panel shows the distribution of employment across workers age bins for firms of different ages in the data. Data are from the QWI. Right panel shows the distribution of employment across workers age bins for firms of different ages in the model.

As shown in Figure 4, the model matches this feature of the data quite well. Young firms indeed have a higher share of younger workers and a lower share of older workers in the model. Moreover, the distribution of employment for older firms is roungly uniform, in line with the share of old versus young workers in the overall labor force. While the average profile of the employment distribution is informed by the vacancy cost level parameters, the parameters of the match output profile help to inform the age group specific profiles. Therefore, targeting the wage profile across firm and worker age bins directly also helps to match the sorting patterns between workers and firms in the data.

Lastly, the model matches the age profile of the job-to-job flow rate in the data fairly well, though these moments are not targeted directly in the moment matching exercise. Figure 5 shows the model fit the the profile of job-to-job flows over the worker life-cycle. Workers at earlier stages of their careers are more likely to switch jobs as they, on average, have lower human capital (represented by the increasing profile of $p(x, y)$ over the worker life-cycle) and are employed in lower quality matches. However, as workers progress through their careers, they find better matches and therefore do not switch jobs as often. This moment in the model is also informed by the worker-firm sorting patterns displayed in Figure 4 above. As workers have no bargaining power out of unemployment, they will accept any job upon meeting a firm. Therefore, the age profile of hires out of unemployment is the same for all firm types; it aligns exactly with the distribution

Figure 5: Job-to-Job Flows



Notes: The black solid line shows the model moments and the red dashed line shows the corresponding moments in the data. Job-to-job flow rate is defined as the number of workers who directly switch jobs without an intervening spell of unemployment divided by total employment for each age bin. Data are from the J2J.

of unemployment across worker age bins. Hence, the sorting patterns between workers and firms by age are determined by the profile of job-to-job flows: young workers are poached at a higher rate by younger firms and vice versa.

5 Quantifying the Effects of Declining Business Dynamism

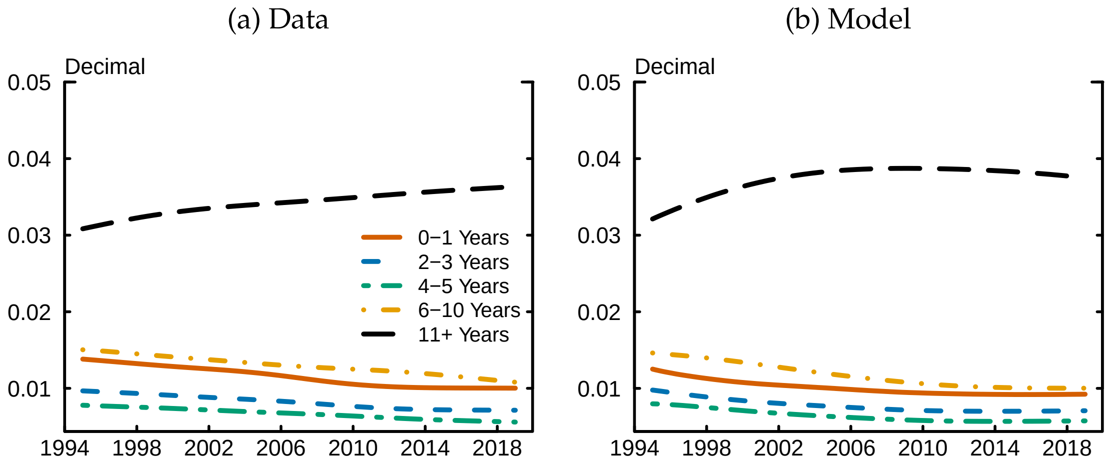
I now simulate a decline in business dynamism in the calibrated model in order to understand how declining dynamism has impacted workers at different stages of the life cycle. Starting from the initial 1994 steady state firm distribution, I decrease the firm entry rate in a manner that aligns with the data and study the effects on labor market outcomes. I also allow exit rates to evolve as in the data. I first describe the calibration of the time path of the entry and exit rates below. Then, I discuss the effects of declining dynamism on labor market outcomes across cohorts of workers and on aggregate.

5.1 Calibrating the Law of Motion for the Mass of Firms

The law of motion for the mass of firms is given in Equation 9. I calibrate the time path of the mass of firms by firm age bin so as to be as close to the data as possible. Then, I feed the resulting evolution of the mass of firms by firm age bin into the model and study the effects on labor market outcomes. This exercise takes *as given* the decline in business dynamism as captured by the firm dynamics inherent in the law of motion for $m_t(y)$.

In the data, I observe the following series: i). exit rates by firm age bin and ii). the

Figure 6: Mass of Firms by Firm Age $m_t(y)$



Notes: The mass of firms by firm age bin is the ratio of the number of firms in the respective age bin to the total number of workers in the labor force. In the data, these series are constructed as follows: the number of firms by firm age bin is from the BDS. The total number of workers in the labor force is from the LFS and includes only male workers age 25 years and older. The resulting series are HP-filtered using an annual smoothing parameter.

ratio of the total number of firms in the economy to the total number of workers in the labor force.[15] To calibrate the law of motion for the mass of firms, I first take exit rates directly from the data. I then back out the time path of the entry rate γ_t so as to match the ratio of firms to the labor force that I see in the data. The resulting process for the mass of firms by firm age is shown in Figure 6. I then feed this process into the model and study the response of the economy along the transition path.

5.2 Effects on the Labor Market

Figures 7 and 8 show the effects of the decline in business dynamism on key labor market variables. As the mass of firms in the economy declines, the total number of vacancies in the economy decreases. This effect can be seen by inspecting the formula for aggregate vacancies in a given time period V_t.

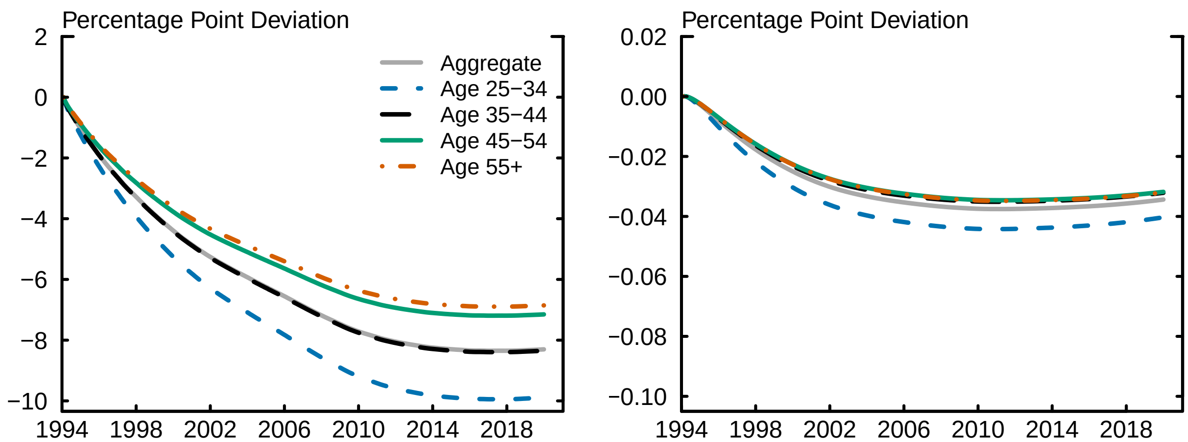
$$V_t = \int n_t(y) m_t(y) \, \mathrm{d}\, y$$

Aggregate vacancies are made up of two components. The first is the number of firm-level vacancies $n_t(y)$, which is pinned down by firms equating the costs and benefits of vacancy

[15] I plot these series in Appendix Figure F.5. I use the HP-filtered versions of each series in order to abstract from business cycle fluctuations.

Figure 7: Effects on Labor Market Flows

(a) Job Finding Rate by Age Group (b) Separation Rate by Age Group



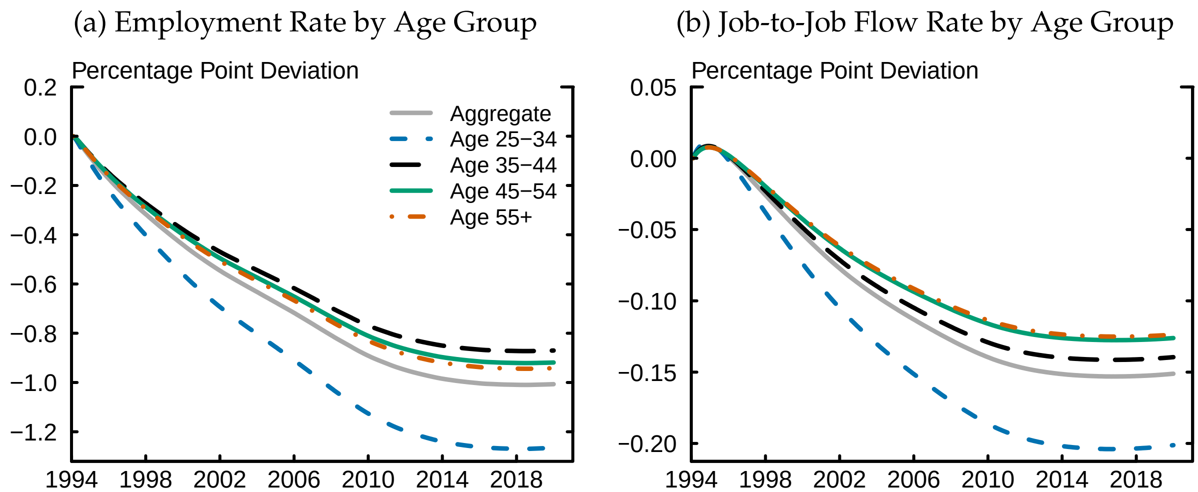
posting. Firm level vacancies are then scaled by the total number of firms in each age bin $m_t(y)$. Due to the decline in business dynamism, the mass of firms $m_t(y)$ declines for all firm types, which directly decreases V_t. The amount that V_t declines along the transition path then depends on the degree to which firm-level vacancies $n_t(y)$ respond to the drop in dynamism. This is determined by the shape of the vacancy cost function, the expected value of a filled vacancy $J_t(y)$, as well as the rate at which firms contact workers q_t.

$$n_t(y) = \left(\frac{q_t \cdot J_t(y)}{c_{0,y}} \right)^{\frac{1}{c_1}}$$

Along the transition path, the expected value of a filled vacancy $J_t(y)$ increases because there are more unemployed workers searching for jobs. Job creation incentives in the model as captured by $J_t(y)$ are quite sensitive to change in the stock of unemployed workers, who search with a higher intensity than employed workers (see Equation 4). In addition, the firm contact rate q_t does not respond along the transition path and stays at a corner solution where a firm posting vacancies will certainly contact a worker. This corner solution arises from the matching function and because the mass of firms is much smaller than the mass of workers in the calibrated model. Therefore, the number of firm level vacancies $n_t(y)$ increases slightly along the transition path due to an increase in the expected value of posting a vacancy $J_t(y)$. However, this positive, indirect effect on V_t is not enough to offset the negative, direct effect of declining dynamism on V_t.

Figure 7 shows that the job finding rate in the economy, which is proportional to the rate at which firms contact workers $\lambda_t = \frac{\Phi(L_t, V_t)}{L_t}$, declines precipitously. With little change in separation probabilities, nonemployment rates increase for all worker age groups, with

Figure 8: Effects on Mobility and Employment



different effects for workers in different stages of their life cycle. Hence, total employment in the economy declines due to lower overall labor demand, shown in Figure 8. Likewise, worker mobility as measured by the job-to-job flow rate also declines, with larger effects for younger worker age groups. The job-to-job flow rate is also proportional to the contact rate λ_t, but it is additionally influenced by the degree to which workers of different age groups are situated on high versus low rungs of the job ladder. Older workers have had more time to search for suitable matches and are on higher rungs of the job ladder.[16] They therefore switch jobs less often on average and are less exposed to the dynamism induced decline in labor demand. Consequently, the largest effects on worker mobility both in terms of movements out of unemployment and in terms of job switching are present for the youngest worker age group: 25–34 year-olds.

5.3 Contribution of Declining Dynamism to Declining Mobility

I now examine the degree to which the model's predictions capture the evolution of certain labor market series in the data during the period under consideration. Table 5 shows the contribution of declining dynamism to changes in labor market outcomes.

Between 1994 and 2019, males between the ages of 25–54 experienced a decline in rates of mobility as well as overall employment. In the data, the rate at which workers switch between jobs at different firms fell by about 0.5 percentage points on a monthly basis. However, this decline was not uniform for all worker age groups. In particular, employer switching fell by more for younger worker age groups, meaning that each suc-

[16]This effect holds to a lesser extent for workers in the 55+ age bin who are nearing retirement, as total match surplus internalizes their higher exit rates.

Table 5: Quantifying the Effects of Declining Dynamism

Change: 1994–to–2019	Data	Model	Explained
Panel A: Employer Switching Rate			
Age 25–34	-0.51 pp	-0.20 pp	39.22%
Age 35–44	-0.25 pp	-0.14 pp	56.00%
Age 45–54	-0.12 pp	-0.13 pp	108.33%
Panel B: Employment–to–Population Ratio			
Age 25–34	-3.24 pp	-1.27 pp	39.20%
Age 35–44	-1.09 pp	-0.87 pp	79.82%
Age 45–54	-2.04 pp	-0.92 pp	45.10%

Notes: Data and Model columns show percentage point (pp) differences. Explained column displays the ratio of the Model column to the Data column, as a percent. Employer switching rate is defined as the percentage of employed workers who switched employers at least once in a year. I use data from the Annual Social and Economic Supplement (ASEC) of the Current Population Survey (CPS) and follow the methodology of Molloy et al. (2016). I HP-filter each series using an annual smoothing parameter and convert to monthly rates in order to be consistent with the model. Employment–to–population ratio is from the Bureau of Labor Statistics Labor Force Statistics (LFS) database. I include only male workers and HP-filter each series using an annual smoothing parameter.

cessive cohort of labor market entrants has faced a lower rate of employment mobility. The counterpart in the model is the job-to-job flow rate, which measures the rate at which workers switch directly between jobs at different firms. As in the data, the model predicts that in response to declining business dynamism, employer switching for younger workers should decline by more. Moreover, the model explains between 35 and 110 percent of the decline in employer switching across worker age groups.[17]

The model also does a good job at capturing the fact that average employment rates declined for workers under the age of 55 between 1994 and 2019. To construct these series in the data, I gather the employment-to-population ratio for males in age groups 25–34, 35–44, and 45–54 from the Bureau of Labor Statistics LFS database and extract the HP-filtered trend of each data series for the years 1994–2019. The table shows that all three age groups experienced declines in their employment-to-population ratio over this time horizon. The model counterpart of these series is the non-employment rate, as the workers in these age groups are highly attached to the labor market and trends in the data are likely not driven by workers dropping out of the labor force for non-economic reasons. The model explains between 35 and 80 of the empirical trends in the employment rate by worker age group.

[17]For the oldest worker age group, the business dynamism induced decline in employer switching is larger than the decline in employer switching in the data. See Table 5.

6 Welfare Implications of Declining Business Dynamism

I now examine the consequences of the decline in business dynamism for total welfare in the economy as well as welfare for workers at different stages of their life-cycle. The most natural measure of welfare in the model would be the value function for unemployed workers $W^u(x)$. However, this is exogenously pinned down by the sequential auctions protocol, so I instead use a flow value concept of welfare. Let $\bar{w}_t$ denote the flow value of employed workers, $\bar{b}_t$ denote the flow value of unemployed workers, $\bar{f}_t$ denote the flow value of filled vacancies, and $\bar{c}_t$ denote the flow value of unfilled vacancies at time t.

These objects are defined as follows:

$$\bar{w}_t = \int\int e_t(x,y) w_t(x,y)\,\mathrm{d}\,x\,\mathrm{d}\,y$$

$$\bar{b}_t = \int u_t(x) b(x)\,\mathrm{d}\,x$$

$$\bar{f}_t = \int m_t(y)(p_t(y) - w_t(y))\,\mathrm{d}\,y$$

$$\bar{c}_t = \int m_t(y)\tilde{c}_t(y)\,\mathrm{d}\,y$$

where $p_t(y) = \int p(x,y)\,e(x,y)\,\mathrm{d}\,x$ denotes total match output by firm age bin, $w_t(y) = \int w_t(x,y)\,e(x,y)\,\mathrm{d}\,x$ denotes total wages by firm age bin, $\tilde{c}_t(y) = c\big(\pi_v^u(y)\,n_t(y)\big)$ denotes flow vacancy posting costs by firm age bin, and $\pi_v^u(y)$ is the share of unfilled vacancies by firm age bin. Total welfare in the economy at time t is then given by $\bar{U}_t = \bar{w}_t + \bar{b}_t + \bar{f}_t - \bar{c}_t$.

6.1 Decomposition of Total Welfare

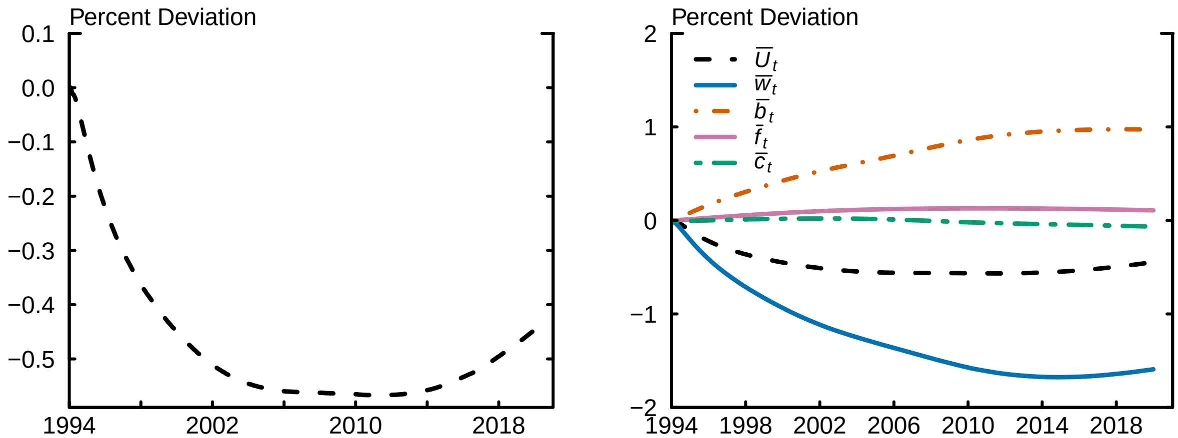
Using the definition of total welfare, we can decompose the percentage change in welfare in the economy into each of its components. Let $dX_t = X_t - X_0$ denote a deviation of the variable X_t from its steady state level X_0. Also, let $\hat{X}_t = \frac{dX_t}{X_0}$ denote a percentage deviation of the variable X_t from its steady state level X_0. Changes in total welfare may be decomposed as follows.

$$\hat{\bar{U}}_t = \frac{d\bar{w}_t}{\bar{U}_0} + \frac{d\bar{b}_t}{\bar{U}_0} + \frac{d\bar{f}_t}{\bar{U}_0} - \frac{d\bar{c}_t}{\bar{U}_0} \tag{10}$$

The results of this decomposition exercise are plotted in Figure 9. The largest components of the total change in welfare are the change employed workers welfare $\bar{w}_t$ and the change unemployed workers welfare $\bar{b}_t$. Along the transition path, these measures impact overall welfare in opposite directions. A decline in $\bar{w}_t$ decreases $\bar{U}_t$, while an increase in $\bar{b}_t$ has a positive effect on $\bar{U}_t$. The former effect dominates for the entirety of the transition path, meaning that total welfare falls over this time horizon.

Figure 9: Welfare Decomposition

(a) Total Welfare　　(b) Components of Total Welfare



The changes in employed and unemployed worker welfare follow from the results presented in the previous section. As business dynamism falls, there is a large decline in employment and a corresponding increase in the unemployment rate for all age groups. Therefore, summing across a smaller (larger) number of employed (unemployed) workers results in lower (higher) welfare for these workers collectively, notwithstanding changes in the flow benefits that each group receives. For unemployed workers, these are constant along the transition path because $b(x)$ does not change over time (there are no changes in match-level output $p(x, y)$, though there are changes in how workers are distributed across this grid). Hence, the effects on $\bar{b}_t$ are straightforward to understand: a larger number of unemployed workers receiving the same flow benefit $b(x)$ results in overall larger $\bar{b}_t$. For employed workers, wages $w_t(x, y)$ change along the transition path, so the effect on $\bar{w}_t$ is not as clear. I further decompose the different margins that affect $\bar{w}_t$ below.

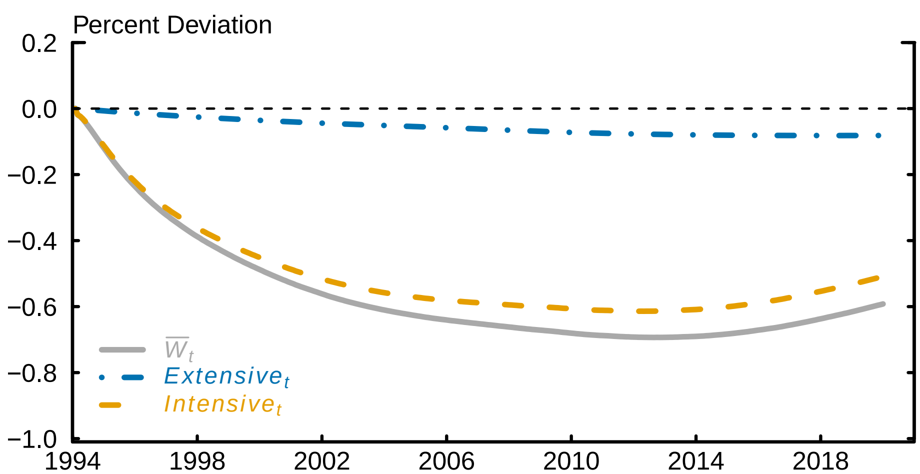
6.2 Employed Worker Welfare: Intensive versus Extensive Margin

I now decompose changes in employed workers' welfare $\bar{w}_t$ into an extensive margin stemming from changes in employment rates and an intensive margin stemming from changes in workers' wages. Let $e_0(x, y)$ denote match-level employment in steady state and let $w_0(x, y)$ denote match-level wages in steady state for matches between workers of type x and firms of type y. The percent deviation of employed workers' welfare from steady state may be written as:

$$\hat{\bar{w}}_t = \frac{1}{\bar{w}_0}\left(\underbrace{\left(\int\int e_0(x,y)w_t(x,y)\,\mathrm{d}\,x\,\mathrm{d}\,y - 1\right)}_{Intensive\ Margin} + \underbrace{\left(\int\int e_t(x,y)w_0(x,y)\,\mathrm{d}\,x\,\mathrm{d}\,y - 1\right)}_{Extensive\ Margin}\right) \quad (11)$$

Intuitively, overall employed worker welfare may change due to changes in the number of workers that are employed or to changes in the wages workers earn when they are employed. In the above expression, the $Intensive\ Margin$ term captures the degree to which employed worker welfare $\bar{w}_t$ changes due to changes in match-level wages, holding employment by worker age and firm age constant. The $Extensive\ Margin$ term captures the degree to which $\bar{w}_t$ changes due to changes in employment in the economy, holding match-level wages constant. Figure 10 plots the evolution of employed worker welfare $\bar{w}_t$ as well as the intensive and extensive margin components of this measure along the transition path.

Figure 10: Worker Welfare: Intensive versus Extensive Margin



This decomposition sheds light on whether overall welfare falls because the probability of being employed falls along the transition path or because workers simply earn less for a given match. It is clear from the figure that the former effect dominates. As discussed above, employment rates fall along the transition path both on aggregate and across worker age groups. In addition, all worker age groups experience declines in wages. Therefore, the intensive margin accounts for the largest portion of the drop in employed worker welfare, which is also the largest component of overall welfare. This exercise shows that a fall in business dynamism generates a large drop in aggregate labor demand, decreasing employment and wages and depressing total welfare in the economy.

6.3 Welfare Changes by Worker Cohort

Lastly, I explore the welfare implications of declining business dynamism across worker age groups. Let $\bar{w}_t(x)$ denote employed workers' welfare for worker age group x. Cohort-specific worker welfare is given by the expression below.

$$\bar{w}_t(x) = \int e_t(x, y)\, w_t(x, y)\, \mathrm{d}\, y$$

Similarly to overall worker welfare, this expression is composed of the employment rate and the wages earned by a given age group of workers. As is clear from the section above, younger age groups experience larger declines in labor market mobility as well as larger declines in their employment rates. Therefore, declines in business dynamism produce unequal changes in welfare for workers at different stages of the life-cycle.

Figure 11: Worker Welfare by Age Group

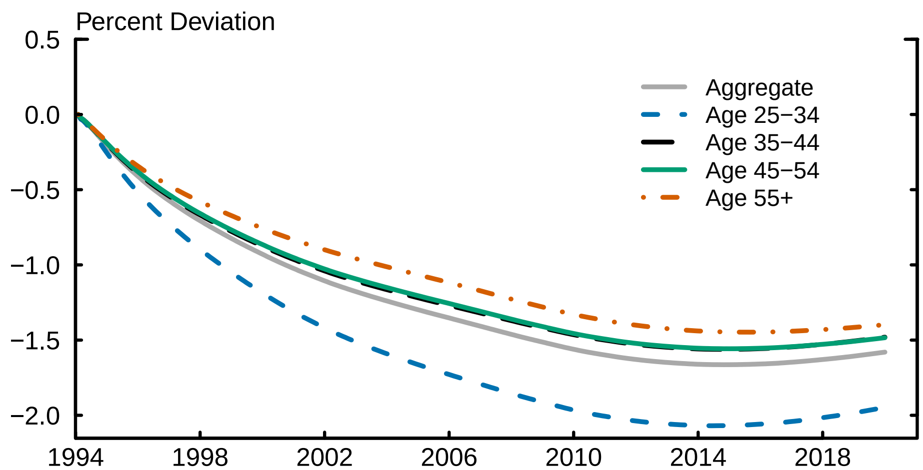


Figure 11 quantifies the degree to which different worker cohorts experience different declines in welfare (conditional on being employed) in response to a decline in business dynamism. While all age groups experience a decline in welfare, younger age groups are hit harder. Aggregate worker welfare $\bar{w}_t$ falls by about 1.5 percent along the transition path, while the youngest age group of workers (25–34) experiences more than a 2 percent decline in welfare. This is because younger workers are more likely to sort into young firms, so they experience larger declines in employment rates as well as mobility when young firms begin to disappear from the economy. Moreover, older cohorts of workers are higher up the job ladder owing to the fact that they have had more time to find stable matches and to the fact that they are more likely to match with firms that have lower separation rates.

7 Conclusion

In this paper, I assess the consequences of the recent decline in business dynamism in the United States for labor market outcomes and total welfare in the economy across different cohorts of workers. I first review several empirical patterns that suggest a link between the rate of business dynamism – the share of young relative to old firms in the economy – and labor market mobility along a worker's life cycle. I show that in the data, there has been a decline in the rate at which new firms enter the economy, resulting in a shift of the firm age distribution towards older firms that tend to also be larger. Moreover, I show that young firms are more likely to employ younger workers in that the employment distribution of young firms is on average skewed towards young workers. This suggests that the decline in business dynamism may have affected the labor market outcomes of more recent cohorts of workers.

Then, I set up a model of labor market sorting between heterogeneous firms and heterogeneous workers subject to search frictions in order to test this hypothesis. In the model, workers differ by the length of time since they entered the labor market and firms differ by the length of time since they entered the economy. I calibrate the model to match several features of the labor market in 1994 and then simulate a decline in business dynamism in line with the data. I find that aggregate employment declines along the transition path, leading to a decline in total welfare in the economy. However, these effects are not felt equally by all workers. Younger workers are more sensitive to changes in business dynamism as they have a larger share of employment at younger firms. Mobility and employment rates decline by more for younger cohort, leading to a decline in welfare that is more severe for these groups of workers.

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A Data Description

A.1 Quarterly Workforce Indicators (QWI)

The Longitudinal Employer Household Dynamics (LEHD database is a linked employer-employee dataset constructed from state administrative records and maintained by the Census Bureau. Access to the underlying microdata in the LEHD is restricted, but the Census Bureau publishes tabulations of the data at different levels of aggregation such as industry, geography, firm size and age, as well as worker demographics. In particular, the Census maintains the Quarterly Workforce Indicators (QWI), which contain information on hires, separations, turnover, employment growth, and wages by industry, worker demographics, and firm age and size. The data can be downloaded from the webpage: https://lehd.ces.census.gov/data/#qwi.

A.2 Job-to-Job Flows (J2J)

To complement the QWI, the Census Bureau publishes additional detail on worker flows in the Job-to-Job Flows (J2J) database. The tabulations are similar to those in the QWI and statistics are available by firm characteristics (industry, age, and size) and by worker demographics (sex by age, sex by education, and race by ethnicity). These data contain measures of *direct* job-to-job transitions across employers and also allow to distinguish hires from other firms (poaching) from hires from the unemployment pool. They also allow to distinguish separations to another firm (job-to-job separations) from separations to nonemployment. The data can be downloaded from the webpage: https://lehd.ces.census.gov/data/#j2j.

A.3 Business Dynamics Statistics (BDS)

The Business Dynamics Statistics (BDS) datasets are maintained by the U.S. Census Bureau and contain annual measures of business dynamics such as job creation, job destruction, establishment births and deaths, and firm startups and exits. The data are available for the overall economy as well as by different establishment and firm characteristics. The BDS is derived from the Census Bureau's Longitudinal Business Database (LBD), a confidential census of business establishments and firms in the U.S. with paid employees comprised of survey and administrative records. Data may be downloaded from https://www.census.gov/data/datasets/time-series/econ/bds/bds-datasets.html.

A.4 Current Population Survey (CPS)

A.4.1 Annual Social and Economic Supplement (ASEC)

To construct the measure of employer switching, I use data from the Annual Social and Economic Supplement (ASEC) of the Current Population Survey (CPS). The ASEC is based on a survey of more than 75,000 U.S. households and contains detailed questions on the social and economic characteristics of each person who is a household member as of the interview date. Questions in the survey pertain to the previous calendar year.

Specifically, I use a variable that records the responses to the following survey question: "For how many employers did (name/you) work in [year]? If more than one at the same time, only count it as one employer." Since the question asks respondents to count simultaneous employment at multiple firms as only one employer, any respondent who answers that she had more than one employer in a given year must have switched jobs between firms at some point during that year. Therefore, this allows me to estimate the percentage of employed workers who switched employers in each year in my sample by simply summing up the number of respondents who had more than one employer and dividing this number by total employment.[18]

I follow the approach of Molloy et al. (2016), which is the first paper to construct this specific measure of job switching to my knowledge. I download the variable NUMEMPS, which contains information on the number of employers from the IPUMS CPS website (`https://cps.ipums.org/cps-action/variables/NUMEMPS`). I select wage and salary workers in the private sector who reported that they were employed or had a job during the previous calendar year.

A.4.2 Longitudinally Linked CPS

In order to construct measures of the job finding rate and job separation rate by worker age group, I follow the procedure described in Shimer (2012) to link respondents in the CPS Basic Monthly Survey (BMS) across months. I download data from IPUMS CPS and use the unique identifier CPSIDP constructed by IPUMS to link individuals across surveys (`https://cps.ipums.org/cps-action/variables/cpsidp`). I also implement additional matching criteria to ensure that individuals match on age, sex, and race characteristics.

After linking individuals, information on their labor market status – unemployed (U), employed (E), or not in the labor force (N) – allows me to construct job finding and

[18] In practice, I weight each observation using the weighting variable ASECWT provided by IPUMS CPS.

job separation probabilities. The monthly job finding probability $P(UE)_t$ is defined as the number of unemployed individuals in month $t-1$ who are employed in month t. The monthly job separation probability $P(EU)_t$ is defined as the number of employed individuals in month $t-1$ who are unemployed in month t. Formulas are given below.

$$P(UE)_t = \frac{\#(\text{Unemployed in month } t-1 \text{ who are Employed in month } t)}{\#(\text{Unemployed in month } t-1)}$$

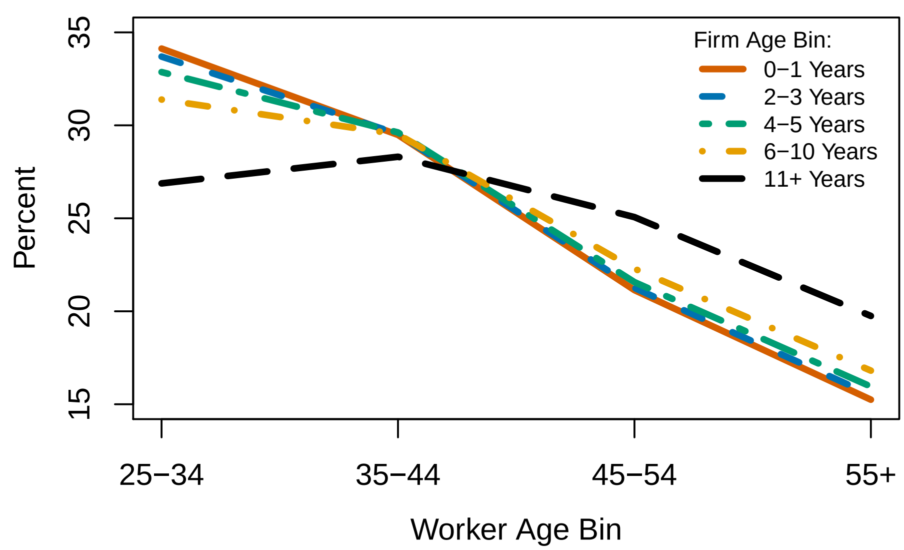
$$P(EU)_t = \frac{\#(\text{Employed in month } t-1 \text{ who are Unemployed in month } t)}{\#(\text{Employed in month } t-1)}$$

The job finding and job separation rates by age group are simply constructed by applying the above formulas for the relevant age sub-sample.

B Additional Empirical Results

B.1 Employment Sorting by Firm Age and Worker Age

Figure B.1: Employment Distribution Across Worker Age by Firm Age



I formally test the sorting patterns by firm age and worker age (plotted here in Figure **??**) using data from the QWI. I gather quarterly data from 1993Q1 to 2018 Q4 across 19 broad NAICS sectors and test the following regression specification:

$$Employment\ Share^g_{i,j,t} = \alpha_i + \alpha_t + \sum_j \mathbb{1}\{Firm\ Age = j\} + \varepsilon_{i,j,t}$$

The variable $Employment\ Share^{g}_{i,j,t}$ records the fraction of total employment of workers in a given age group g, at firms in age group j in sector i at time t. I regress this variable on sector and time dummy variables as well as an indicator variable for firm age category. The base category is mature firms – those age 11 and older. Therefore, the regression coefficient on each firm age category may be interpreted as the percentage point difference in the employment share of a particular worker age group within firms of a certain age group, relative to 11+ year old firms. This regression essentially captures the same information as the figure above, but it allows to control for sectoral differences in the employment distribution as well as time trends. Therefore, it can be thought of as capturing the "steady state" distribution of employment across worker age and firm age, which is exactly the concept of this employment distribution in the model. Results are shown in Table B.1 below.

Table B.1: Young Workers Sort Into Young Firms

	(1) Frac. Age 25-34	(2) Frac. Age 35-44	(3) Frac. Age 45-54	(4) Frac. Age 55+
0-1 Years	6.897***	2.460***	-2.104***	-3.549***
	(0.113)	(0.0880)	(0.0791)	(0.0747)
2-3 Years	6.793***	2.439***	-2.149***	-3.379***
	(0.113)	(0.0880)	(0.0791)	(0.0747)
4-5 Years	6.105***	2.456***	-1.950***	-2.908***
	(0.113)	(0.0880)	(0.0791)	(0.0747)
6-10 Years	4.823***	2.338***	-1.423***	-2.034***
	(0.113)	(0.0880)	(0.0791)	(0.0747)
Time FE	Yes	Yes	Yes	Yes
Sector FE	Yes	Yes	Yes	Yes
Observations	10735	10735	10735	10735

Standard errors in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

We can see that the regression results confirm the patterns show in the figures on employment sorting by firm age and worker age. Younger firms have an employment distribution that is more heavily skewed towards younger workers. For instance, we can see from row 1 of the table that the employment share of age 25-34 year old workers at startup firms (age 0-1 years) is almost 7 percentage points higher than at mature firms (11+ years). This is of comparable magnitude to the figure above. Likewise, startup firms have an employment share of age 45-54 year old workers that is 4 percentage points below that of mature firms. These results shows that the sorting patterns documented in the main text hold even when controlling for sectoral variation as well as time trends.

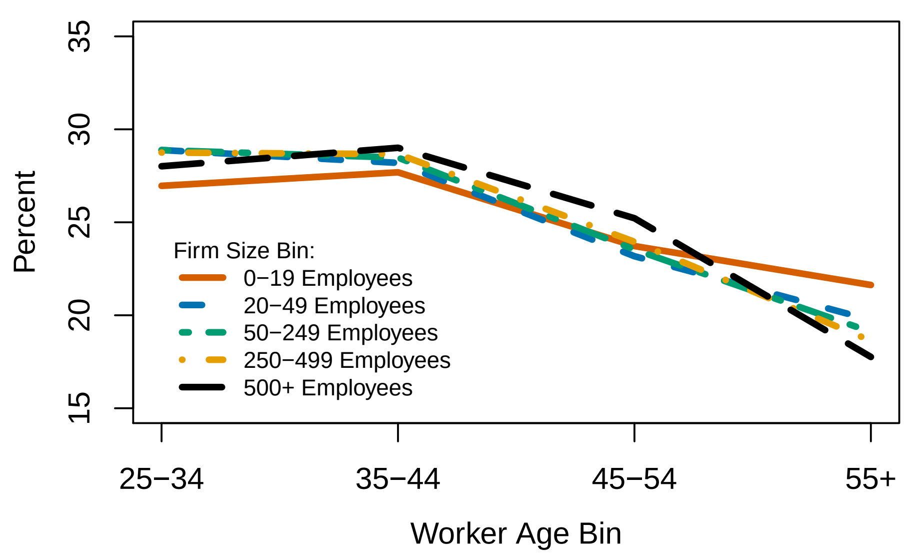
Next, as young firms tend to be small, I test whether similar patterns hold by firm size. If they do, it could indicate that the sorting patterns by firm and worker age are instead driven by differential employment policies with respect to worker age by firm size. This would imply that there is no life-cycle dimension to a firm's employment policies for workers of different ages and that the patterns I document in the data are instead due to firm size. To this end, I test the regression specification:

$$Employment\ Share^{g}_{i,k,t} = \alpha_i + \alpha_t + \sum_k \mathbb{1}\{Firm\ Size = k\} + \varepsilon_{i,j,t}$$

The firm size categories are as follows: 0-19 Employees, 20-49 Employees, 50-249 Employees, 250-499 Employees, 500+ Employees.

The results of this exercise are shown in Figure B.2 and Table B.2. In the regression table, I use large firms (500+ employees) as the omitted category. As opposed to firm age, there are no clear sorting patterns by firm size. First, the magnitudes of the coefficients are much smaller than in the case of firm age. If there are differences in the distribution across worker age by firm size, they are about half as large in magnitude as the differences with respect to firm age. Second, the patterns in the regression table are not consistent with the patterns in the figure, suggesting that any differences may be attributable to cross-sector differences. For instance, the coefficient on 0-19 Employees in the first column of the table is positive, suggesting that small firms have a larger share of young workers than large firms (500+ Employees). However, in the figure, the red solid line corresponding to firms with 0-19 employees is clearly below that black dashed line corresponding to firms with

Figure B.2: Employment Distribution Across Worker Age by Firm Size



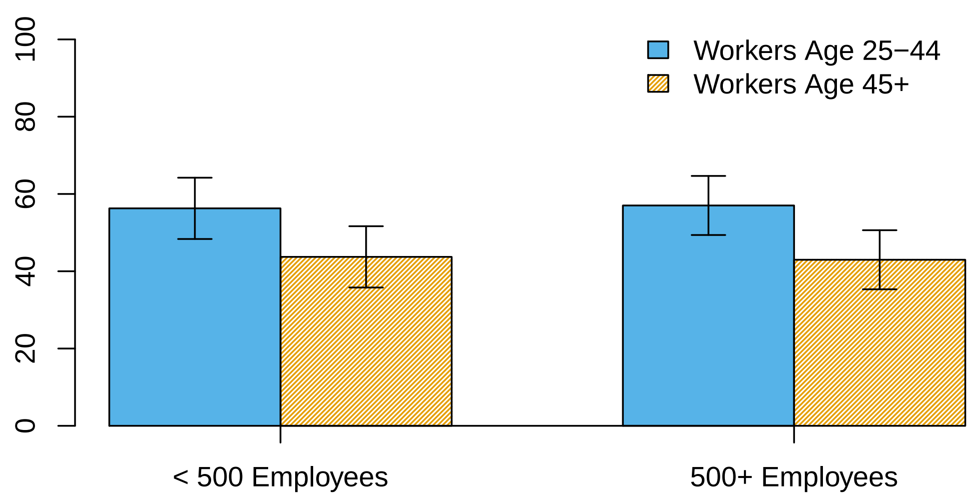
500 or more employees. Lastly, there is no clear ordering to the line in the figure or the coefficients in the regression table by firm size. Therefore, the patterns of employment sorting by firm age and worker age are likely not due to differences in firm size and there does appear to be a life-cycle dimension to a firm's employment composition.

Table B.2: No Strong Sorting Patterns by Firm Size

	(1) Frac. Age 25-34	(2) Frac. Age 35-44	(3) Frac. Age 45-54	(4) Frac. Age 55+
0-19 Employees	-1.251***	-0.671***	-0.132	5.758***
	(0.122)	(0.0939)	(0.0806)	(0.0732)
20-49 Employees	0.868***	-0.0398	-0.598***	3.474***
	(0.122)	(0.0939)	(0.0806)	(0.0732)
50-249 Employees	1.669***	0.438***	-0.575***	2.172***
	(0.122)	(0.0939)	(0.0806)	(0.0732)
250-499 Employees	1.709***	0.625***	-0.220**	1.590***
	(0.122)	(0.0939)	(0.0806)	(0.0732)
Time FE	Yes	Yes	Yes	Yes
Sector FE	Yes	Yes	Yes	Yes
Observations	10735	10735	10735	10735

Standard errors in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Figure B.3: Age Composition of Employment: Small versus Large Firms



Notes: Figure shows average employment composition, in percentages, by worker age group for firms in different size groups. Data on employment by firm age group and worker age group are from the Census Bureau's Quarterly Workforce Indicators (QWI) database. For all series, I include only male workers and take averages over 1993-2019. Black bars show standard errors.

C Derivations and Proofs

To keep the notation simple, I present the following derivations with worker search intensity normalized to 1 such that $\phi(x,l) = \psi_x \cdot \kappa_l = 1$ and with no retirement $\omega_x = 0 \; \forall x$. This is without loss of generality and the same derivations hold in the case with differences in search intensity as well as retirement rates.

C.1 Unemployed Worker Value Function

Under Assumption 1, $W_t^u(x) = b(x) + \beta \, \mathrm{E}_{x'} \left[W_{t+1}^u(x') \mid x \right]$.

Proof. Start with the equation for the worker's value of unemployment.

$$W_t^u(x) = b(x) + \beta \, \mathrm{E}_{x'} \left[(1-\lambda_{t+1}) W_{t+1}^u(x') + \lambda_{t+1} \int \max\{W_{t+1}^e(x',y'), W_{t+1}^u(x')\} \frac{v_{t+1}(y')}{V_{t+1}} dy' \mid x \right]$$

Under Assumption 1, workers hired out of unemployment have zero bargaining power and therefore receive zero surplus share. In other words, firms are able to extract the entire match surplus. Therefore, workers hired out of unemployment simply receive the value of unemployment as their continuation value when matching with a firm. This implies that $W_t^e(x,y,y') = W_t^u(x)$ (we can also see this using the definition of worker value in terms of surplus share $W_t^e(x,y,\sigma) = W_t^u(x) + \sigma_t S_t(x,y)$ and setting $\sigma_t = 0$, see below). Substituting this into the equation above and reducing yields the desired expression.

$$\begin{aligned}
W_t^u(x) &= b(x) + \beta \, \mathrm{E}_{x'} \left[(1-\lambda_{t+1}) W_{t+1}^u(x') + \lambda_{t+1} \int \max\{W_{t+1}^e(x',y'), W_{t+1}^u(x')\} \frac{v_{t+1}(y')}{V_{t+1}} dy' \mid x \right] \\
&= b(x) + \beta \, \mathrm{E}_{x'} \left[(1-\lambda_{t+1}) W_{t+1}^u(x') + \lambda_{t+1} \int \max\{W_{t+1}^u(x'), W_{t+1}^u(x')\} \frac{v_{t+1}(y')}{V_{t+1}} dy' \mid x \right] \\
&= b(x) + \beta \, \mathrm{E}_{x'} \left[(1-\lambda_{t+1}) W_{t+1}^u(x') + \lambda_{t+1} \int W_{t+1}^u(x') \frac{v_{t+1}(y')}{V_{t+1}} dy' \mid x \right] \\
&= b(x) + \beta \, \mathrm{E}_{x'} \left[(1-\lambda_{t+1}) W_{t+1}^u(x') + \lambda_{t+1} W_{t+1}^u(x') \mid x \right] \\
&= b(x) + \beta \, \mathrm{E}_{x'} \left[W_{t+1}^u(x') \mid x \right]
\end{aligned}$$

□

C.2 Surplus Function

The surplus function is defined as the match value $P_t(x, y)$ net of the worker's value of unemployment $W_t^u(x)$. Therefore, $S_t(x, y) = P_t(x, y) - W_t^u(x)$. To derive the value function for the surplus, first, we start with match value $P_t(x, y)$.

$$\begin{aligned} P_t(x, y) = p(x, y) & \\ & + \beta \operatorname{E}\Big[\big(1 - (1-\delta_{x,y})\mathbb{1}\{P_{t+1}(x', y') \geq W_{t+1}^u(x')\}\big)W_{t+1}^u(x') \\ & + (1-\delta_{x,y})\mathbb{1}\{P_{t+1}(x', y') \geq W_{t+1}^u(x')\}\Big((1-\lambda_{t+1})P_{t+1}(x', y') \\ & + \lambda_{t+1} \int \max\{P_{t+1}(x', y'), W_{t+1}^e(x', y'', y')\} \frac{v_{t+1}(y'')}{V_{t+1}} \operatorname{d} y''\Big)\Big] \end{aligned}$$

Due to the sequential auctions framework, the continuation value in the case that an employed worker contacts another firm is independent of the worker value $W_t^e(x, y, y')$. Therefore, this equation reduces to the following:

$$\begin{aligned} P_t(x, y) = p(x, y) & \\ & + \beta \operatorname{E}\Big[\big(1 - (1-\delta_{x,y})\mathbb{1}\{P_{t+1}(x', y') \geq W_{t+1}^u(x')\}\big)W_{t+1}^u(x') \\ & + (1-\delta_{x,y})\mathbb{1}\{P_{t+1}(x', y') \geq W_{t+1}^u(x')\}P_{t+1}(x', y')\Big] \end{aligned}$$

Subtracting $W_t^u(x)$ from both sides yields

$$\begin{aligned} P_t(x, y) - W_t^u(x) = p(x, y) & \\ & + \beta \operatorname{E}\Big[\big(1 - (1-\delta_{x,y})\mathbb{1}\{P_{t+1}(x', y') \geq W_{t+1}^u(x')\}\big)W_{t+1}^u(x') \\ & + (1-\delta_{x,y})\mathbb{1}\{P_{t+1}(x', y') \geq W_{t+1}^u(x')\}P_{t+1}(x', y')\Big] \\ & - b(x) - \beta \operatorname{E}\Big[W_{t+1}^u(x')\Big] \end{aligned}$$

since under Assumption 1, $W_t^u(x) = b(x) + \beta \operatorname{E}\left[W_{t+1}^u(x')\right]$. Finally, rearranging and using the definition of surplus yields the desired result.

$$\begin{aligned}
P_t(x,y) - W_t^u(x) &= p(x,y) - b(x) \\
&\quad + \beta \operatorname{E}\Big[\big(1 - (1-\delta_{x,y})\mathbb{1}\{P_{t+1}(x',y') \geq W_{t+1}^u(x')\}\big) W_{t+1}^u(x') \\
&\quad + (1-\delta_{x,y})\mathbb{1}\{P_{t+1}(x',y') \geq W_{t+1}^u(x')\} P_{t+1}(x',y') - W_{t+1}^u(x')\Big] \\
&= p(x,y) - b(x) \\
&\quad + (1-\delta_{x,y})\beta \operatorname{E}\Big[\mathbb{1}\{P_{t+1}(x',y') \geq W_{t+1}^u(x')\}\big)\Big(P_{t+1}(x',y') - W_{t+1}^u(x')\Big)\Big] \\
\implies S_t(x,y) &= p(x,y) - b(x) \\
&\quad + (1-\delta_{x,y})\beta \operatorname{E}\Big[\mathbb{1}\{S_{t+1}(x',y') \geq 0\}\big)\Big(S_{t+1}(x',y')\Big)\Big] \\
&= p(x,y) - b(x) \\
&\quad + (1-\delta_{x,y})\beta \operatorname{E}\Big[\max\{S_{t+1}(x',y'), 0\}\Big]
\end{aligned}$$

$\square$

C.3 Wage Equation

The wage setting protocol is taken from Lentz et al. (2017). Recall that a worker receives a share σ_t of the surplus that depends on her type x, current firm y, and previous outside offer y', which stays constant until she receives another outside offer. The surplus share, for $S_t(x,y) \geq S_t(x,y')$, is given by

$$\sigma_t \equiv \sigma_t(x,y,y') = \frac{S_t(x,y')}{S_t(x,y)}$$

Therefore, we may represent the worker's value of employment as

$$W_t^e(x,y,y') = W_t^u(x) + \sigma_t(x,y,y') \cdot S_t(x,y)$$

or more conveniently as

$$W_t^e(x,y,\sigma) = W_t^u(x) + \sigma_t S_t(x,y)$$

From this equation, we can see that hiring from unemployment entails setting $\sigma_t = 0$, which is a just a restatement of Assumption 1. Then, if a worker employed at some firm y meets another firm y', the result of Bertrand competition framework is such that the surplus share σ_t is updated in the following manner.

$$\sigma_t' = \begin{cases} \frac{S_t(x,y)}{S_t(x,y')} & S_t(x,y') > S_t(x,y) \\ \frac{S_t(x,y')}{S_t(x,y)} & \sigma S_t(x,y) < S_t(x,y') \leq S_t(x,y) \\ \sigma_t & S_t(x,y') \leq \sigma S_t(x,y) \end{cases}$$

In the first case, the worker is poached and moves to firm y', extracting the entire surplus $S_t(x,y)$ of her previous match at firm y. In the second case, the worker stays at firm y, but is able to renegotiate her surplus share to match the full amount of the surplus $S_t(x,y')$ at firm y'. In the third case, the offer is below her previous surplus share and is therefore too low to trigger a renegotiation; the worker simply discards the offer and stays at firm y.

Now, starting from the definition of the employed worker value $W_t^e(x,y,\sigma) = W_t^u(x) + \sigma S_t(x,y)$, we can solve for a wage $w_t(x,y,\sigma)$ that implements this contract.

$$\begin{aligned} W_t^e(x,y,\sigma) &= W_t^u(x) + \sigma_t S_t(x,y) \\ &= w_t(x,y,\sigma_t) + \beta \, \mathrm{E}\left[W_{t+1}^u(x')\right] \\ &\quad - (1-\delta_{x,y})\beta \, \mathrm{E}\left[\mathbb{1}\{S_{t+1}(x',y') \geq 0\}\left(\lambda_{t+1} \int Q_{t+1}(x',y',\sigma_{t+1},y'') \frac{v_{t+1}(y'')}{V_{t+1}} \, \mathrm{d}\, y'' \right.\right. \\ &\quad \left.\left. + (1-\lambda_{t+1})\sigma_{t+1} S_{t+1}(x',y')\right)\right] \end{aligned}$$

where $Q_t(x,y,\sigma_t,y')$ is defined similarly to σ_t' above and represents the surplus the worker captures due to a renegotiation. In other words, it is the second best of the three values $\sigma_t S_t(x,y)$, $S_t(x,y')$, and $S_t(x,y)$.

$$Q_t(x,y,\sigma_t,y') = \begin{cases} S_t(x,y) & S_t(x,y') > S_t(x,y) \\ S_t(x,y') & \sigma_t S_t(x,y) < S_t(x,y') \leq S_t(x,y) \\ \sigma_t S_t(x,y) & S_t(x,y') \leq \sigma_t S_t(x,y) \end{cases}$$

Next, notice that from expression for the unemployed worker's value function, we have that $\beta \, \mathrm{E}\left[W_{t+1}^u(x')\right] = W_t^u(x) - b(x)$, so we can use this to eliminate $\beta \, \mathrm{E}\left[W_{t+1}^u(x')\right]$ and

$W_t^u(x)$ from the above equation. We then have

$$\sigma_t S_t(x,y) = w_t(x,y,\sigma_t) - b(x)$$
$$- (1-\delta_{x,y})\beta\,\mathrm{E}\Big[\mathbb{1}\{S_{t+1}(x',y') \geq 0\}\Big(\lambda_{t+1}\int Q(x',y',\sigma_{t+1},y'')\frac{v_{t+1}(y'')}{V_{t+1}}\,\mathrm{d}\,y''$$
$$+ (1-\lambda_{t+1})\sigma_{t+1}S_{t+1}(x',y')\Big)\Big]$$

Lastly, we substitute the definition of the surplus equation into this equation and solve for $w_t(x,y,\sigma_t)$, which yields the desired result.

$$w_t(x,y,\sigma_t) = \sigma_t p(x,y) + (1-\sigma_t)b(x)$$
$$- (1-\delta_{x,y})\beta\,\mathrm{E}\left[\mathbb{1}\{S_{t+1}(x',y') \geq 0\}\cdot\lambda_{t+1}\int R_{t+1}(x',y',\sigma_{t+1},y'')\frac{v_{t+1}(y'')}{V_{t+1}}\,\mathrm{d}\,y''\right]$$

where $R_t(x,y,\sigma_t,y') \equiv Q_t(x,y,\sigma_t,y') - \sigma_t S_t(x,y)$ is defined as in the main text and represents the additional surplus the worker captures due to a renegotiation.

$$R_t(x,y,\sigma_t,y') = \begin{cases} S_t(x,y) - \sigma_t S_t(x,y) & S_t(x,y') > S_t(x,y) \\ S_t(x,y') - \sigma_t S_t(x,y) & \sigma S_t(x,y) < S_t(x,y') \leq S_t(x,y) \\ 0 & S_t(x,y') \leq \sigma_t S_t(x,y) \end{cases}$$

C.4 Contract Distribution

Average wages by (x,y) pair are given by

$$w_t(x,y) = \int w_t(x,y,\sigma_t)G_t(x,y,\sigma_t)\,\mathrm{d}\,\sigma$$

where $w_t(x,y,\sigma_t)$ is the wage for a worker of type x employed at firm y with surplus share σ_t and $G_t(x,y,\sigma_t)$ is the distribution of σ's within all (x,y) matches. The contract distribution is defined similarly to the worker flow equations by the law of motion

$$G_t(x,y,\sigma_t) = \tilde{G}_t(x,y,\sigma_t)\Big[1+\lambda_t - \lambda_t\int \frac{v_t(y')}{V_t}\mathbb{1}\{S_t(x,y') > \sigma_t S_t(x,y)\}\,\mathrm{d}\,y'\Big]$$
$$+ \lambda_t \int \tilde{e}_t(x,y')\frac{v_t(y')}{V_t}\mathbb{1}\{\sigma_t S_t(x,y) > S_t(x,y')\}\,\mathrm{d}\,y'$$
$$+ \lambda_t\,\tilde{u}_t(x)\,\frac{v_t(y)}{V_t}\,\mathbb{1}\{S_t(x,y) \geq 0\}$$

where $\tilde{G}_t(x,y,\sigma_t) = (1-\delta_{x,y})\mathbb{1}\{S_t(x,y) \geq 0\}G_{t-1}(x,y,\sigma_{t-1})$.

D Additional Model Details

The transition matrices for worker type (worker age) and firm type (firm age) are given by the following expressions. Note that the model is set to monthly frequency.

$$\Pi(x'|x) = \begin{bmatrix} 1-\frac{1}{120} & \frac{1}{120} & 0 & 0 \\ 0 & 1-\frac{1}{120} & \frac{1}{120} & 0 \\ 0 & 0 & 1-\frac{1}{120} & \frac{1}{120} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Pi(y'|y) = \begin{bmatrix} 1-\frac{1}{24} & \frac{1}{24} & 0 & 0 & 0 \\ 0 & 1-\frac{1}{24} & \frac{1}{24} & 0 & 0 \\ 0 & 0 & 1-\frac{1}{24} & \frac{1}{24} & 0 \\ 0 & 0 & 0 & 1-\frac{1}{60} & \frac{1}{60} \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

D.1 Model Solution

To solve the model, I use standard numerical techniques to solve the value functions that determine match surplus as well as the equations that govern the steady state distribution of employment across firm and worker types. Given values for $p(x,y)$, $b(x)$, and $\delta_{x,y}$, I first solve for the surplus function in Equation 2 by value function iteration. Then, I iterate on the worker flow Equations 6 and 7 in order to solve for the steady state employment distribution, starting from an initial guess where all workers are unemployed. Each step of the iteration requires solving for the value of a filled vacancy using Equation 4 as well as aggregate search intensity and aggregate vacancies in order to pin down the contact rates λ_t and q_t. This also determines the vacancy distribution across firm types $\frac{v_t(y)}{V_t}$. Next, I solve for wages at the match level by first using Equation 8 to obtain the wage $w_t(x,y,\sigma_t)$ for any pair (x,y) and any possible surplus share $\sigma_t = \sigma_t(x,y,y')$; then, I iterate on the law of motion for the distribution of contracts across σ_t within an (x,y) pair. This allows me to compute average wages by (x,y) pair. Appendix C.4 contains more details on the law of motion for the distribution of wage contracts. With few worker and firm types, the entire solution algorithm converges very quickly.

E Additional Calibration Details

E.1 Global Optimization Algorithm

Since the parameter space is fairly large and the objective function is not well behaved, I use global methods to find the parameters that minimize the distance between the model and data moments. I use a multiple restart procedure in order to select a set of candidate solutions as starting points and then run a local optimization routine from each of these starting values. The algorithm proceeds as follows:

1. Select a set of $S = 250,000$ candidate starting points using Sobol sequences.

2. Evaluate the objective function at each of these points and store the results in a vector.

3. Keep the best (i.e. lowest function value) $S^* = 1,000$ of these points.

4. Run a local optimization routine (Nelder-Mead algorithm) starting from each of these S^* points and store the resulting function values and parameter vectors.

   (a) Let f^* denote the $1 \times S^*$ vector of objective function values at the local optima corresponding to the S^* starting points.

   (b) Let θ^* denote the $N \times S^*$ matrix of parameter values at the local optima corresponding to the S^* starting points.

5. Find the lowest function value among f^* and call this $\hat{f}$; find the parameter vector in θ^* that corresponds to $\hat{f}$.

6. Let $\hat{\theta}$ denote the parameter vector that corresponds to $\hat{f}$. $\hat{\theta}$ is the global minimum.

E.2 Construction of Targeted Moments

E.2.1 Earnings-per-Employee

In the model, there is no intensive margin of labor supply. Therefore, the concept of wages is akin to earnings. To calibrate the wage profile in the model, I target the profile of average earnings-per-employee by worker age group and firm age group in the data.

The data are from the QWI. I use the variable `earns` from the QWI database, which corresponds to average monthly earnings of workers who worked for their respective firms for the entire quarter. Documentation on the variables in the QWI database can be found at the following link: https://lehd.ces.census.gov/doc/QWI_101.pdf. I (mean) collapse the series into the worker age and firm age bins in my model using the appropriate employment weights. I also average across quarters to obtain a yearly series for each worker age bin $\times$ firm age bin cell. I then deflate each resulting yearly series by the Consumer Price Index for All Urban Consumers: All Items in U.S. City Average (FRED code: CPIAUCSL). This price index measure uses the years 1982-1984 as the base years. Lastly, I HP-filter each deflated series using an annual smoothing parameter and normalize the units to thousands of dollars. Therefore, the units of my resulting average earnings measures are: thousands of 1982-1984 dollars earned per month per worker. See Figure F.4 below for a plot of the wage (earnings) profile across worker and firm age bins.

F Additional Figures

Figure F.4: Wage Profile by Worker Age and Firm Age

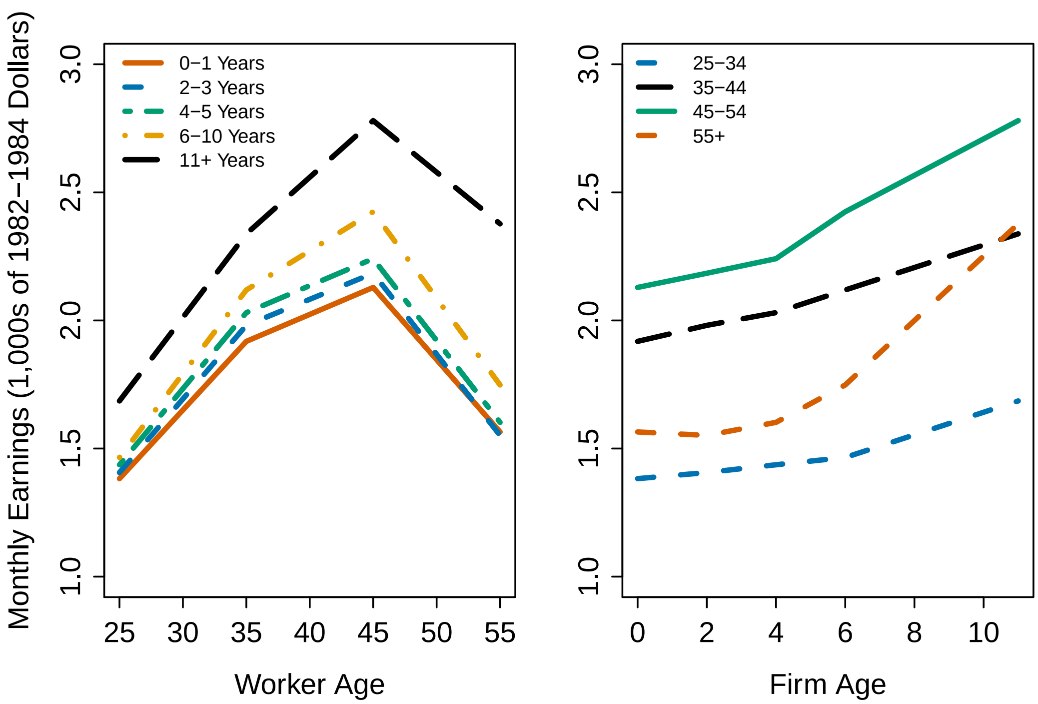
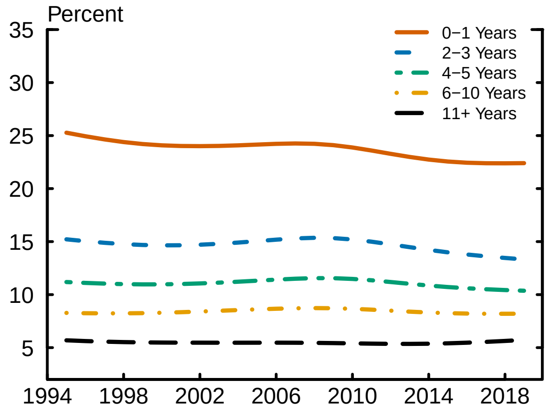
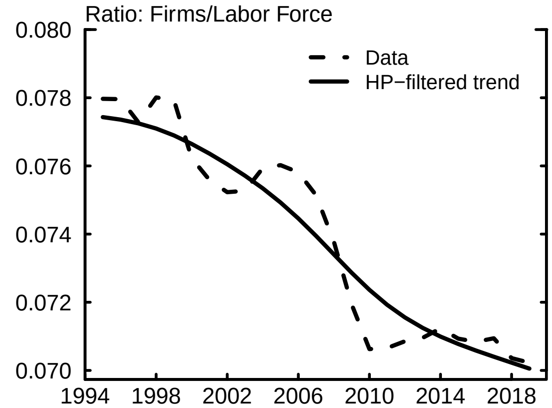


Figure F.5: Calibrating the Law of Motion for the Mass of Firms

(a) Firm Exit Rate by Firm Age



(b) Number of Firms-per-Worker



Notes: Firm exit rate for each firm age bin is defined as the number of firm deaths in the respective age bin divided by the total number of firms in the respective age bin. Data are from the BDS. The ratio of firms/labor force is defined in the same way as the model: the total number of firms in the economy divided by the total number of male workers over the age of 25 in the labor force. Data on the total number of firms in the economy is from the BDS. Data on the labor force is from the Bureau of Labor Statistics (BLS) Labor Force Statistics database. Series are HP-filtered with an annual smoothing parameter.