Declining Business Dynamism and Worker Mobility*

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Abstract

The firm entry rate in the United States declined precipitously in recent decades, leading to an increase in the share of older, larger businesses. Younger workers tend to sort into younger firms, suggesting that the compositional shift of economic activity towards older firms has harmed the labor market prospects of younger workers. To assess this hypothesis, I develop an equilibrium labor market sorting model featuring both on-the-job search and two-sided, life-cycle heterogeneity. I use the framework to quantify the consequences of the shift in the firm age distribution for workers' careers. I find that changes in the firm age distribution alone account for about 43 percent of the aggregate decline in the employer switching rate and about 23 percent of the aggregate decline in the employment-to-population ratio between 1994 and 2019. Aggregate worker welfare falls by about 0.6 percent along the transition path, with younger workers experiencing larger declines.

Keywords: Business Dynamism, Entrepreneurship, Economic Mobility, Worker Flows
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1 Introduction

Over the past several decades, the United States economy witnessed a dramatic decline in business dynamism. Firm entry and exit rates decreased, increasing the proportion of larger, older businesses in the economy. Moreover, despite large secular changes in economic activity over this time horizon, these trends were pervasive across regions and industries.[1] In this paper, I investigate the implications of declining business dynamism for aggregate labor market outcomes as well as labor market outcomes across older and younger groups of workers.

My analysis begins with the observation that the age composition of employment changes across the firm life-cycle. Using data from the U.S. Census Bureau's Quarterly Workforce Indicators (QWI) database, I show that while more mature firms (11 years and older) employ younger and older workers in proportion with their representation in the labor force, the age composition of employment at younger firms is significantly skewed towards younger workers. This finding is not accounted for by differences in firm size across firm age categories, is not driven by certain sectors or regions, and has remained stable over time.[2] The fact that younger workers differentially sort into younger firms suggests that the decline in the share of young firms in recent decades may have differentially affected the labor market outcomes of younger workers.

To assess this hypothesis and to quantify the labor market effects of the declining share of young firms, I develop an equilibrium model of labor market sorting between heterogeneous workers and heterogeneous firms with on-the-job search (OJS). The model builds on Lise and Robin (2017), who study the business cycle implications of sorting between workers and firms with permanent differences in skill and productivity, respectively. I modify their framework to instead allow both workers and firms to differ by age, which evolves as they progress throughout their life-cycles. I also allow for a time varying firm age distribution, which changes in response to innovations in the rate at which firms enter and exit the economy. These features allow me to study the forces that contribute to the sorting patterns I document in the data and to analyze the consequences of declines in the firm entry rate for labor market outcomes across cohorts of workers.

In the model, both workers and firms differ by the current stage of their life-cycle. Firms enter the economy at the beginning of their life-cycle and post vacancies in order to hire employees. They face random shocks that cause them to close down, the incidence of which depends on how long they have been in operation. Workers enter the labor

[1] See Decker et al. (2014), Hathaway and Litan (2014), and Pugsley and Şahin (2019) for recent evidence. These studies find broad-based declines in business formation across industries and geographic locations.

[2] Ouimet and Zarutskie (2014) also document this pattern using Census microdata for the years 1992–2004.

market at the beginning of their careers and search for jobs. They face random shocks that separate them from their employers back into the unemployment pool. If they remain employed, they may also search on-the-job for a new employer.

Matches between workers and firms at different stages of the life-cycle may differ in the level of output produced, and therefore wages paid. I adopt a simple, reduced form expression for the match-level output function that varies by both worker and firm age. Across the worker life-cycle, this proxies for the fact that human capital accumulation may allow older workers to produce more on average than younger workers. Across the firm life-cycle, this proxies for the fact that older firms are on average larger and more productive than younger firms due to selection or growth effects. Moreover, workers at different stages in the life-cycle may have differential productive capacity when matched with firms at different stages of the life-cycle.

I show how the joint surplus function is sufficient to characterize the distribution of matches between workers and firms at different stages of the life-cycle. Two objects, which I take to be primitives of the model, determine the profile of the joint surplus. First, the incidence of job destruction shocks faced by firms of different ages makes matches at less risky firms more valuable relative to more risky firms. Second, the match-level output function governs the average production level resulting from matches between different workers and firms. I calibrate these objects using data on job destruction rates by firm age and on average wages by worker and firm age.

I choose 1994 as the starting point for my analysis and calibrate the model in steady state to match several features of the worker and firm life-cycle.[3] I target job finding and separation rates by worker age, wages paid by firms of different ages to workers of different ages, and the employment distribution by firm age.[4] The calibrated model reproduces the life-cycle patterns in the data quite well. Both job finding rates and separation decline with worker age and wages display an increasing and hump-shaped trajectory over the worker life-cycle. Moreover, the oldest firms pay wages that are 1.5 times higher than those paid by the youngest firms. Finally, though I do not explicitly target the age composition of employment at young versus old firms, the model captures these life-cycle sorting patterns. This is because young firms, which have higher exit and job destruction rates, sit at the bottom of the job ladder and hire disproportionately from an unemployment pool composed of younger workers.

With the calibrated model in hand, I then explore the implications of the decline in the

[3] I choose this time period as the starting point for my analysis because of data availability reasons; wage data from the QWI are not available before the 1990s. Moreover, previous studies have argued that the negative trend in business dynamism accelerated after 2000 (Decker et al., 2014).

[4] Importantly, I include only *cross-sectional* moments from the initial time period in my calibration strategy.

firm entry rate for labor market outcomes. Starting from the initial steady state, I simulate a decline in business dynamism by allowing the firm age distribution and total number of firms in the economy to evolve in a manner consistent with the data. In the data, firm exit rates conditional on firm age group have remained roughly stable since 1994, but the firm entry rate has dropped precipitously. Therefore, both the share of young firms and the number of firms per worker in the economy have declined over this time period. I calibrate the law of motion for the mass of firms by firm age group to match these patterns. I then feed this law of motion into the model and study the evolution of the economy along the transition path.

I find that through the lens of the model, the change in the firm age distribution results in a decline in labor market mobility for all workers. Along the transition path of the economy, firms post fewer vacancies, leading to an overall decline in labor demand. Therefore, the total number of meetings between workers searching for jobs and firms posting vacancies falls, leading to a drop in the contact rate. The aggregate job finding rate declines by 8 percentage points and the aggregate job-to-job switching rate declines by 0.15 percentage points. In addition, the shift in the firm age distribution towards older firms, which have lower separation rates, leads to a fall in the aggregate job separation rate of about 0.03 percentage points. Because job finding falls more than job separation, the employment rate falls (nonemployment rate rises) by about 1 percentage point.

Additionally, job finding, separation, and switching rates all decline by more for younger cohorts of workers. Job finding falls by more for younger cohorts because in the calibrated model, younger workers have higher search intensity. Therefore, they are more exposed to the decline in business dynamism for a given contact rate. The larger decline in job separation for younger workers is explained by a composition effect. In the initial steady state, younger workers differentially sort into younger firms, which have high separation rates. Along the transition path, as the share of young firms declines, younger workers are reallocated into jobs at older firms, which have lower separation rates. Lastly, job-to-job flows fall by more for younger workers due to the fall in the contact rate combined with their higher average search intensity.

Taking these predictions to the data, I find that the decline in business dynamism accounts for about 43 percent of the aggregate decline in the employer switching rate and about 23 percent of the aggregate decline in the employment-to-population ratio between 1994 and 2019. Notably, the decline in business dynamism also accounts for the fact that worker mobility has fallen by more for younger worker age groups (Bosler and Petrosky-Nadeau, 2016; Mercan, 2017). In the model, as in the data, the aggregate decline in worker mobility is driven by larger declines among younger cohorts of workers.

Lastly, I quantify the welfare implications of the shift in the firm age distribution in order to assess and unpack the mechanisms through which declining business dynamism affects workers. As the firm entry rate declines, two competing channels affect workers' labor market prospects. First, as there are fewer firms in the economy, the opportunity to match with any given firm declines. Second, as the share of older businesses, which the calibration exercise finds are more productive, increases, the average match in the economy is of higher quality. I refer to the first channel as the "match-level effect" and to the second channel as the "match-distribution effect."

I find that quantitatively, the match-level effect dominates, and workers experience an overall decline in welfare of about 0.6 percent. Though the employment distribution shifts towards more stable jobs at older businesses, overall employment opportunities diminish as the number of firms per worker falls. Similarly, though workers sort into better matches on average than in the initial steady state, average within-match wages fall. The wage setting mechanism in the model implies that with fewer firms competing to poach workers away from other firms, workers experience a decline in their bargaining power (Postel-Vinay and Robin, 2002). Therefore, as the number of firms per worker falls, workers command a lower share of the surplus within matches, on average, and hence are paid lower wages. Moreover, as worker mobility declines, workers are more likely to remain on lower rungs of the job ladder within firms.

The welfare results across worker cohorts mirror my findings on worker mobility and wages across the life-cycle. Because employment rates and wages fall by more for younger worker age groups, their welfare measures also display larger declines. Total welfare declines during the period under consideration, but the brunt of the impact is borne by younger workers. Therefore, I argue that the large decline in business dynamism in the U.S. has not only led to a deterioration of labor market prospects for all workers, but also contributed to a widening gap in outcomes between recent and past generations of labor market entrants.

Related Literature My paper contributes to several different strands of the literature that studies the causes and consequences of the recent decline in business dynamism. The literature that examines the causes of declining business dynamism is too large to catalog extensively.[5] However, I highlight two recent papers that document empirical evidence that motivates my analysis. Both Hopenhayn et al. (2022) and Karahan et al. (forthcoming) find that firm dynamics within cohorts of firms have remained stable in recent decades. Therefore, the changing composition of firms by firm age, in turn driven

[5]See Decker et al. (2016) as well as Ackcigit and Ates (2021, 2023) for an overview of the literature.

by a decline in the firm entry rate, entirely accounts for any observed aggregate trends in firm dynamics such as the firm exit rate, average firm size, and concentration. They show that in firm dynamics models with linear entry conditions based on Hopenhayn (1992), a decline in labor supply growth produces trends consistent with these empirical patterns. In this paper, I additionally examine the implications of trends in business dynamism for labor market outcomes and inequality across worker cohorts.

My study connects to several papers that consider the life-cycle dimension of worker mobility, the job ladder, and labor market sorting. First, Topel and Ward (1992) argue that early-career "job shopping" is an important source of life-cycle wage growth. Next, Ouimet and Zarutskie (2014) document that young firms tend to hire and employ young workers, young workers earn higher wages in young firms, and talented young workers select into young firms that display higher innovation and growth potential. Haltiwanger et al. (2018b) show that business cycles disproportionately affect job ladder dynamics for younger and less educated workers. Last, Dinlersoz et al. (2019) find that labor market frictions specific to newly created businesses are key for generating the observed patterns of sorting between workers and firms at different stages of the life-cycle. In my analysis, life-cycle sorting patterns of employment form the basis through which a decline in the share of young firms differentially affects job mobility rates for young versus old workers.

Next, this paper builds on work in Postel-Vinay and Robin (2002), Lise and Robin (2017), and Lentz et al. (2017), who develop models of two-sided heterogeneity and labor market sorting. In these studies, worker types differ by fixed skill or ability and firm types differ by fixed productivity or technology. In contrast to these papers, I allow firms and workers to differ not by skill or productivity, but by age, which evolves over the life-cycle. This allows me not only to capture the life-cycle dimension of worker mobility and labor market sorting, but also to speak to differential changes in labor market outcomes across different cohorts of workers.

This paper is also related to recent studies that jointly consider firm dynamics and on-the-job search. Engbom (2019) finds that while the direct effects of labor force aging explain some portion of the decline in worker flows in the United States, the majority of the decline results from feedback effects onto the incentives to start new businesses. Bilal et al. (2022) and Elsby and Gottfries (2022) build tractable firm dynamics models with frictional labor markets, on-the-job search, and decreasing returns to scale in production. I develop a method to include life-cycle dimensions for both worker and firm outcomes while also maintaining analytical tractability under a related set of assumptions on the production and vacancy posting cost functions.[6] While both papers focus on the business

[6]See the discussion in Bilal et al. (2022) on the relationship between their model and Lise and Robin (2017).

cycle dimension of worker flows, I study the long-term decline in business dynamism.

Finally, my paper relates to studies that propose explanations for the declining trend in worker mobility.[7] Cairó (2013) shows that an increase in job retraining requirements lowers labor market turnover and can explain about one-third of the decline in the job reallocation rate over the past several decades. Mercan (2017) and Pries and Rogerson (2022) propose that better ex-ante information about match quality or screening by firms of potential applicants can explain the decline in job mobility in recent decades. Relative to these papers, I propose a new channel for the decline in worker mobility through the decline in the firm entry rate. The mechanism at work in my paper is most similar to that in a recent contribution by Bagga (2023), who shows that the decline in the number of firms per worker can explain almost two-thirds of the decline in worker mobility in the U.S. since the 1980s. Relative to her paper, I study the life-cycle dimension of the decline in worker flows and find that declining business dynamism also accounts for the larger decline in employer-to-employer transition rates experienced by younger cohorts.

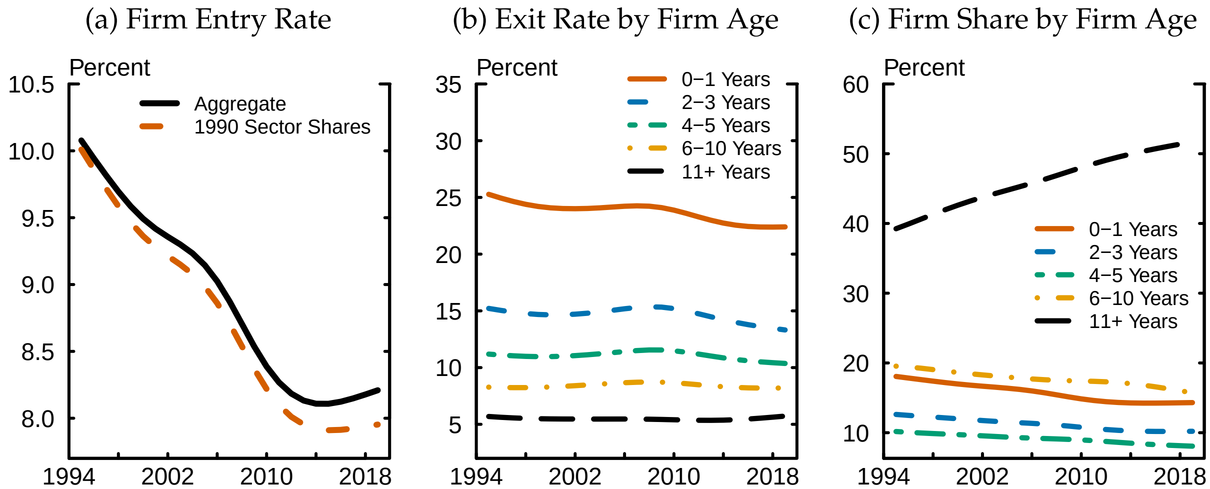
Layout The rest of the paper is structured as follows. In Section 2, I review empirical evidence that motivates my analysis. In Section 3, I present an equilibrium model of labor market sorting between workers and firms at different stages of the life-cycle. Section 4 discusses the numerical implementation and calibration strategy of the model. Section 5 explores the effects of a decline in business dynamism on the economy and compares the model's predictions to the data. Section 6 discusses the welfare implications of declining business dynamism in the aggregate and across worker cohorts. Section 7 concludes.

2 Motivating Evidence

In this section, I present motivating evidence on trends in business dynamism and on the age distribution of employment across the firm life-cycle. First, I review recent findings that changes in the firm age distribution over the past several decades were primarily driven by changes in the firm entry margin. Then, I show evidence that the age composition of employment at younger firms is significantly more skewed towards younger workers. I describe the data sources in Appendix A and the methodology below.

[7] Hyatt and Spletzer (2013) find that depending on the data source, hires and separations rates fell between 10 percent and 38 percent between 1998 and 2010. Molloy et al. (2016) document a clear downward trend in the pace of worker flows and of job turnover and discuss competing explanations for these trends.

Figure 1: Trends in Aggregate Business Dynamics



Notes: The left panel shows the firm entry rate (number of age 0 firms divided by total number of firms) on aggregate and for a counterfactual scenario where firm shares by sector are held constant at their 1990 values. The center panel shows the firm exit rate (number of firm deaths divided by total number of firms) by firm age group. The right panel shows the share of firms in each firm age group. Data are from the Census Bureau's Business Dynamics Statistics (BDS) database and are HP-filtered with an annual smoothing parameter. For more details on the BDS, see Appendix A.

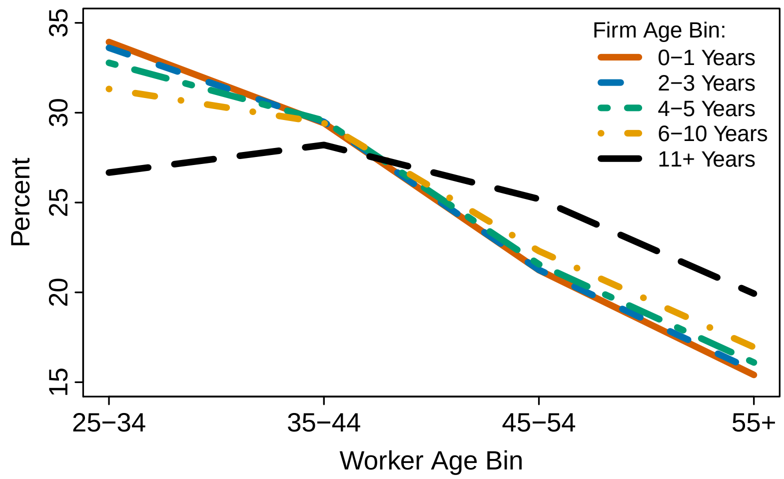
2.1 Decline in Firm Entry and Shift of Firm Age Distribution

Figure 1 displays trends in various measures of business dynamics from 1994 to 2019.[8] Over this time period, the entry rate of new businesses declined precipitously, firm exit rates conditional on firm age were roughly stable, and the firm age distribution shifted towards older businesses. Moreover, the decline in the firm entry rate was a pervasive phenomenon across markets (Decker et al., 2014; Pugsley and Şahin, 2019).[9] It was not a result of the changing industrial composition of economic activity (panel 1a). However, business dynamics conditional on firm age have remained fairly stable over this time horizon (Pugsley and Şahin, 2019; Hopenhayn et al., 2022; Karahan et al., forthcoming). For instance, average survival and growth rates do not display large trends within firm age groups (panel 1b).

Given the stability of firm dynamics conditional on firm age, it must be the case that the changing composition of firms by firm age resulted exclusively from changes in the entry margin (panel 1c). Therefore, any observed *aggregate* trends in firm exit rates, growth rates, average firm size, and concentration are all driven by changes in the firm

[8] I focus on trends that occurred before the onset of the COVID-19 pandemic. Recent research finds that new business applications increased dramatically during the COVID period, but analysis of the specific causes and consequences remains an open area of research (Dinlersoz et al., 2021; Decker and Haltiwanger, 2023).
[9] See Appendix Figure B.1 for sector specific trends in firm entry rates.

Figure 2: Employment Distribution Across Worker Age by Firm Age



Notes: Figure shows average employment composition, in percentages, across worker age group for firms in different age groups. Data on employment by worker and firm age group are from the Census Bureau's Quarterly Workforce Indicators (QWI) database. For all series, I include only male workers and take averages over 1994–2019. For more details on the QWI, see Appendix A.

age distribution, induced by a decline in the number of new startup firms created each year. Even more remarkable is that some of these aggregate trends would have reversed had the age composition of businesses in the economy remained constant during this period (Hopenhayn et al., 2022).

2.2 Worker and Firm Life-Cycle Sorting Patterns

Next, I examine patterns of worker and firm sorting across the life-cycle using the Census Bureau's Quarterly Workforce Indicators (QWI) database and show that the composition of employment at younger firms is significantly more skewed towards younger workers. That is, younger firms tend to employ younger workers in higher proportions.

Figure 2 plots the composition of employment across worker age group by firm age group. For instance, the red, solid line shows that roughly 35 percent of employees at firms between 0–1 years old (startup firms) are between the ages of 25–34, while the dashed, black line shows that only about 27 percent of employees at firms 11 years or older (mature firms) are within this age range. From the figure, a striking pattern emerges. The composition of employment at younger firms is more skewed towards younger workers relative to the employment composition at older firms. In fact, the proportion of employment composed of workers less than 45 years old is declining in firm age.

To test whether factors other than worker and firm age can account for this pattern, I

Table 1: Worker and Firm Sorting Patterns by Firm Age

	(1)	(2)	(3)	(4)
Firm Age 0–10 Years	14.713***	15.068***	14.881***	14.402***
	(0.038)	(0.037)	(0.033)	(0.113)
Frac. Educ. $\leq$ High School				0.161***
				(0.002)
ln(Avg. Firm Size)				1.756***
				(0.049)
Firm Age 0–10 Years $\times$ ln(Avg. Firm Size)				-0.326***
				(0.045)
Year Fixed Effects	X	X	X	X
State Fixed Effects		X	X	X
Industry Fixed Effects			X	X
Observations	464,529	464,529	464,529	367,117
R^2	0.331	0.355	0.528	0.592

Notes: Sample includes only male workers age 25 and over for the years 1994–2019. Frac. Educ. $\leq$ High School is the fraction of a firm's workforce with less than or equal to a high school education. ln(Avg. Firm Size) is the natural logarithm of average firm size. Industry fixed effects are at the 4-digit NAICS level. Standard errors in parentheses. $^{*}p \leq 0.10$;** $p \leq 0.05$;*** $p \leq 0.01$.

estimate the following regression specification

$$\text{Frac. Age 25–44}_{i,j,k,t} = \alpha + \beta \mathbb{1}\{i = \text{Firm Age 0–10 Years}\}_{j,k,t} + \mathbf{X}_{j,k,t} + \varepsilon_{i,j,k,t}$$

where the variable Frac. Age 25–44$_{i,j,k,t}$ denotes the fraction of total employment composed of workers between the ages of 25 and 44, at firms in age group i in state j and industry k during year t. I regress this variable on an indicator for firm age group, so that the coefficient β captures the average difference in employment composition between young (0–10 years) and mature (11+ years) firms within a state $\times$ industry $\times$ year cell. I include fixed effects at the state, 4-digit NAICS industry, and year level to allow for the possibility that the pattern shown in Figure 2 is driven by certain regions, sectors, or time periods. I also include controls for differences in firm size across firm age groups as well as controls for employment composition across different levels of educational attainment. The observed pattern of sorting on age could instead reflect sorting on worker and firm characteristics that also vary across the life-cycle, such as skill and productivity.

Table 1 shows the results of this exercise. The table shows that on average, young firms employ a statistically significantly higher proportion of young workers. Being a firm in the 0–10 year old age category is associated with having an employment composition of age 25–44 year old workers approximately 15 percentage points higher relative to firms in the 11+ age category.[10] Moreover, this pattern does not disappear after controlling for fixed effects at various levels, firm size, or educational composition. Importantly, it is not driven by differences in firm size, as young firms tend to be smaller.[11] If anything, increases in average firm size are associated with having a *higher* proportion of young workers, and this association is weaker for firms in the 0–10 year old age category.

Ouimet and Zarutskie (2014) document a similar pattern using microdata from the Census Bureau for the period 1992 to 2004. My data are at the bin-level, so I cannot control for individual worker and firm-level characteristics. However, I use the QWI to repeat the analysis above using firm size groups and education groups and discuss the results in Appendix C. I do not find that potential sorting along these dimensions masks sorting on worker and firm age, and conclude that there is an important life-cycle component to labor market sorting.

3 Model of Worker and Firm Life-Cycle Sorting

In this section, I develop an equilibrium model of labor market sorting featuring both worker and firm heterogeneity as well as on-the-job search. The model builds on Lise and Robin (2017), who study sorting between workers and firms with fixed types. I instead allow both workers and firms to differ by age in order to capture the sorting patterns between workers and firms at different stages of the life-cycle. I also adopt the wage setting protocol developed in a related paper by these authors, Lentz et al. (2017). Below, I elaborate on the model structure and the wage setting protocol; explicit derivations of key equations are relegated to Appendix D.

3.1 Environment

Time is discrete and extends forever. Both workers and firms are heterogeneous and differ by type, where x denotes worker type and y denotes firm type. For my application, worker type x indexes worker age groups and firm type y indexes firm age groups. Throughout the remainder of the paper I use "age" or "type" interchangeably to refer to workers with index x or to firms with index y. Both workers and firms are risk neutral

[10] Note that these magnitudes are larger than those implied by Figure 2, which uses data aggregated across regions and industries. Comparing Figure 2 with Table 1 reveals that the pattern of worker and firm life-cycle sorting is stronger at finer levels of disaggregation. Appendix Figure B.3 plots the employment distribution across worker age group by firm age group using the state $\times$ industry $\times$ year data.

[11] The literature remains ambivalent about whether firm size is a relevant characteristic for job ladder dynamics and sorting patterns (Moscarini and Postel-Vinay, 2012; Haltiwanger et al., 2018a; Bilal et al., 2022).

and discount the future at rate $\beta = \frac{1}{1+r}$.

The density of worker types is given by $\ell(x)$ with mass normalized to 1. Workers enter the economy into the youngest age group at rate η and exit the labor force due to retirement at rate ω_x, which depends on their age x. The density of firm types at time t is given by $m_t(y)$ with mass $\mathcal{M}_t$. Firms enter the economy into the youngest age group at rate γ_t and exit at rate $\zeta_{y,t}$, which depends on their age y. Workers and firms age stochastically according the the Markov processes $\Pi_{x'|x}$ and $\Pi_{y'|y}$, respectively.

Workers are either employed or unemployed. A worker of type x employed at a firm of type y produces flow match output $p(x, y)$. The worker earns a flow wage of $w_t(x, y)$, which is the equilibrium outcome of a sequential auctions bargaining procedure outlined below. A worker of type x receives flow nonemployment benefit $b(x)$ if they are unemployed.

Both employed and unemployed workers may search for jobs, so the model features on-the-job search (OJS). The contact rate for workers λ_t is determined by a constant returns to scale matching function, defined below. Further, worker search intensity ϕ_x^i is set exogenously and depends on both worker type x and employment status $i \in \{\text{employed } (e), \text{unemployed } (u)\}$. An employed worker of type x contacts a firm at rate $\phi_x^e \lambda_t$ and an unemployed worker of type x contacts a firm at rate $\phi_x^u \lambda_t$.

Timing Within each period, there are two stages. At the beginning of the period, a certain fraction of employed workers are matched to firms and the rest are unemployed. Then, in the first stage ("separation stage"), worker and firm types change according to $\Pi_{x'|x}$ and $\Pi_{y'|y}$, respectively. Some fraction of workers exit the labor force and are replaced by new entrants to the labor market in the youngest age group, who start off unemployed. After firms and workers realize their new types and labor force exit takes place, some matches dissolve and workers in these matches enter unemployment.

Next, in the second stage ("matching stage"), the total stock of unemployed workers (previously unemployed workers plus those newly unemployed) and the total stock of employed workers may form matches with new firms. After new matches form, the economy enters the next period.

3.2 Value Functions

The value function for an unemployed worker of type x is given below.

$$W_t^u(x) = b(x) + (1-\omega_x)\beta\,\mathrm{E}_{x'}\Big[(1-\phi_x^u\lambda_{t+1})W_{t+1}^u(x') \\ + \phi_x^u\lambda_{t+1}\int \max\{W_{t+1}^e(x',y'), W_{t+1}^u(x')\}\frac{v_{t+1}(y')}{V_{t+1}}dy'|x\Big]$$

While unemployed, a worker receives flow nonemployment benefit $b(x)$. If she does not retire, she stays in the labor market and searches for jobs in the next period. With probability $(1-\phi_x^u\lambda_{t+1})$ she fails to contact a firm and remains unemployed, possibly with new type x'. With complementary probability $\phi_x^u\lambda_{t+1}$ she contacts a firm and receives the employed worker value $W_{t+1}^e(x',y')$, provided that it is greater than the continuation value of unemployment. Otherwise, she remains unemployed.

Conditional on contacting a firm, the worker forms a match with a firm of type y' with probability $\frac{v_{t+1}(y')}{V_{t+1}}$, where $v_t(y)$ is the number of vacancies posted by firms of type y and $V_t = \int v_t(y)\,\mathrm{d}\,y$ is the total number of vacancies in the economy. Therefore, $\frac{v_t(y)}{V_t}$ is the density of vacancies posted by firms of type y.

Following Lise and Robin (2017), I assume that unemployed workers have zero bargaining power so that workers hired out of unemployment are offered their reservation value, $W_t^e(x,y) = W_t^u(x)$. Under this assumption, the unemployed worker's value function reduces to the following equation.

$$W_t^u(x) = b(x) + (1-\omega_x)\beta\,\mathrm{E}_{x'}\Big[W_{t+1}^u(x')|x\Big] \tag{1}$$

This expression states that the value of unemployment is simply the present discounted value of current and future flow nonemployment benefits $b(x)$, which represents any per-period utility value a worker receives while unemployed. In particular, it may stand for home production, leisure value, or explicit unemployment benefit payments. It may also vary by worker type. Though the assumption above implies that workers are technically indifferent between unemployment and employment, I follow Lise and Robin (2017) and assume that unemployed workers always accept job offers.

The value function for employed workers $W_t^e(x,y)$ is not specified because it is not needed for the equilibrium computation. Instead, I proceed to define the *joint* value of an employment relationship. Let $P_t(x,y)$ denote the present discounted value of a match with flow output $p(x,y)$. In other words, $P_t(x,y)$ represents the value of a match between a worker of type x and a firm of type y. The value function for $P_t(x,y)$ is given below.

$$\begin{aligned} P_t(x,y) &= p(x,y) \\ &\quad + (1-\omega_x)\beta\,\mathrm{E}_{x',y'}\Big[\big(1-(1-\delta_{x,y})\mathbb{1}\{P_{t+1}(x',y') \geq W^u_{t+1}(x')\}\big)W^u_{t+1}(x') \\ &\qquad + (1-\delta_{x,y})\mathbb{1}\{P_{t+1}(x',y') \geq W^u_{t+1}(x')\}\Big((1-\phi^e_x\lambda_{t+1})P_{t+1}(x',y') \\ &\qquad + \phi^e_x\lambda_{t+1}\int \max\{P_{t+1}(x',y'), W^e_{t+1}(x',y'',y')\}\frac{v_{t+1}(y'')}{V_{t+1}}\,\mathrm{d}\,y''\Big) \mid x,y\Big] \end{aligned}$$

In the current period, a match between a worker of type x and a firm of type y produces $p(x,y)$. Assuming the worker does not retire, the match dissolves exogenously with probability $\delta_{x,y}$, which may depend on both worker type and firm type. A match dissolves endogenously if, after firms and workers learn their new types, the continuation value of the match drops below the value of the worker's outside option, $P_{t+1}(x',y') < W^u_{t+1}(x')$. Instead, if the match persists, the employed worker has the opportunity to meet a new firm of type y'' with probability $\phi^e_x\lambda_{t+1}\frac{v_{t+1}(y'')}{V_{t+1}}$. If she fails to meet a new firm, the match persists with the same continuation value. However, if an employed worker successfully meets a new firm, then the current firm ("incumbent firm") and the new firm ("poaching firm") enter into Bertrand competition over the worker's services. This procedure, which follows Postel-Vinay and Robin (2002), is explained in more detail below.

3.3 Sequential Auctions Protocol

Suppose a worker employed at a firm of type y meets a firm of type y'. There are two possible outcomes for the worker's new employer. Either the total match value is higher at the incumbent firm ($P_t(x,y) > P_t(x,y')$) and the worker remains at the incumbent firm, or the total match value is higher at the poaching firm ($P_t(x,y) < P_t(x,y')$) and the worker moves to the poaching firm.

In the case where the worker remains at the incumbent, the worker may be able to renegotiate her wage to a higher value. This occurs when the joint match value of the poaching firm $P_t(x,y')$ is higher than the joint match value corresponding to any previous outside offer she has received. If the joint match value of the poaching firm does not exceed the joint match value corresponding to the highest previous outside offer, the worker simply discards the offer from the poaching firm.

If the worker is poached, she may negotiate her wage at the poacher such that she receives the entire match value $P_t(x,y)$ from the incumbent firm. In this way, the continuation value of the match turns out to be independent of whether or not the worker is poached and therefore independent of the employed worker value function $W_t^e(x, y, y')$.

Joint Surplus Instead, we may write the joint worker and firm problem in terms of the joint surplus of the match. Let $S_t(x, y) = P_t(x, y) - W_t^u(x)$ be the joint surplus at time t from an employment relationship between worker x and firm y. The surplus function determines all allocations in the economy and is given by the expression below.

$$S_t(x, y) = p(x, y) - b(x) + (1 - \omega_x)(1 - \delta_{x,y})\beta \, \mathrm{E}_{x',y'} \left[\max\{S_{t+1}(x', y'), 0\} \,|\, x, y \right] \tag{2}$$

This equation states that the joint surplus of a match between worker x and firm y is equal to the flow output of the match net of the workers' flow value of nonemployment, plus any expected future surplus if the match continues.

Given flow match output $p(x, y)$ and flow nonemployment value $b(x)$, it is sufficient to solve Equation 2 to determine the surplus value of any possible match in the economy, simplifying the equilibrium computation considerably. Notice that the distribution of employment does not appear in this equation, meaning that the model has the block-recursive property, as shown in Lise and Robin (2017). Block-recursivity stems from the assumption that unemployed workers have no bargaining power along with the fact that the sequential auctions protocol renders the match continuation value independent of the employed worker value.

3.4 Worker Search and Vacancy Posting

Workers search both on- and off-the-job and firms post vacancies to equate the expected benefits and costs of meeting a worker. The worker flow equations, which I specify below, determine the numbers of workers separated to unemployment, hired from unemployment, and poached by firms of different types. Recall that within each period there are two sub-periods: a separation stage where agents realize their new types after which some matches are destroyed and a matching stage where new matches form between searching workers and firms with open vacancies.

Let $\tilde{u}_t(x)$ and $\tilde{e}_t(x, y)$ represent the stock of unemployed and employed workers, respectively, after the separation stage. These objects are determined below. Aggregate search intensity L_t is composed of the stocks of unemployed and employed workers that prevail after the separation stage, scaled by their respective individual search intensities.

$$L_t = \int \phi_x^u \, \tilde{u}_t(x) \, \mathrm{d}\, x \, + \int \int \phi_x^e \, \tilde{e}_t(x, y) \, \mathrm{d}\, x \, \mathrm{d}\, y \tag{3}$$

Knowing aggregate search intensity, firms post vacancies in order to hire workers from the pool of total searchers. The expected value of meeting a worker for a firm of type y is given by the expression below.

$$
\begin{aligned}
J_t(y) = \int \frac{\phi_x^u \tilde{u}_t(x)}{L_t} \max\{S_t(x,y), 0\} \,\mathrm{d}\, x \\
+ \int \int \frac{\phi_x^e \tilde{e}_t(x,y')}{L_t} \max\{S_t(x,y) - S_t(x,y'), 0\} \,\mathrm{d}\, x \,\mathrm{d}\, y'
\end{aligned} \tag{4}
$$

This expression has two components. Either the worker is hired from unemployment, in which case the firm offers the worker her reservation value and extracts the entire match surplus, or the worker is hired from employment, in which case the firm receives any match surplus net of the match surplus at the worker's previous firm. Notice both that if $S_t(x,y) < 0$ the match is not formed and that no firm may poach a worker from another firm with a higher surplus.

Firms face per-unit flow vacancy posting costs on the number of firm-level vacancies $n_t(y)$. Vacancy posting costs are governed by the function $C_y(\cdot)$, which I assume is a convex function, and may depend on firm type y. The free entry condition dictates that firms post vacancies up to the point where the expected value of a filled vacancy is equal to the marginal cost of opening a vacancy. In equilibrium, vacancies are therefore pinned down by the condition

$$
C_y{}'(n_t(y)) = \mu_t \cdot J_t(y) \tag{5}
$$

where μ_t is the rate at which firms contact workers and is the outcome of a meeting process, specified below. Given $J_t(y)$ and μ_t, the number of firm level vacancies by firm type $n_t(y)$ solves Equation 5. Aggregate vacancies are then given by

$$
V_t = \int n_t(y) m_t(y) \,\mathrm{d}\, y = \int v_t(y) \,\mathrm{d}\, y \tag{6}
$$

where $v_t(y) = n_t(y) m_t(y)$ is the total mass of vacancies posted by each firm type y.

Matching and Contact Rates Meetings between the masses of searching workers L_t and firm vacancies V_t are produced according to a constant returns to scale matching function $\Psi(L_t, V_t)$. The rate at which workers contact firms depends on both worker search intensity ϕ_x^i and the aggregate probability of meeting a firm $\lambda_t \equiv \frac{\Psi(L_t,V_t)}{L_t}$. Hence, the contact rate for workers is given by $\phi_x^u \lambda_t = \phi_x^u \frac{\Psi(L_t,V_t)}{L_t}$, which depends on both their labor market status and their age. The rate at which firms contact workers is given by $\mu_t \equiv \frac{\Psi(L_t,V_t)}{V_t}$.

Worker Flow Equations The worker flow equations specify how the distributions of employed and unemployed workers evolve across periods. Given the surplus function $S_t(x, y)$, total search effort L_t and vacancies V_t, as well as the masses of unemployed and employed searchers after the separation stage, $\tilde{u}_t(x)$ and $\tilde{e}_t(x, y)$, respectively, the laws of motion below determine the masses of employed and unemployed workers at the end of the period.

$$u_t(x) = \tilde{u}_t(x) \left[1 - \phi_x^u \lambda_t \int \frac{v_t(y)}{V_t} \mathbb{1}\{S_t(x, y) \geq 0\} \, \mathrm{d}\, y\right] \tag{7}$$

$$\begin{aligned} e_t(x, y) = \tilde{e}_t(x, y) &+ \underbrace{\phi_x^e \lambda_t \int \tilde{e}_t(x, y') \frac{v_t(y')}{V_t} \mathbb{1}\{S_t(x, y) > S_t(x, y')\} \, \mathrm{d}\, y'}_{\text{Poaching Hires}} \\ &- \underbrace{\phi_x^e \lambda_t \int \tilde{e}_t(x, y) \frac{v_t(y')}{V_t} \mathbb{1}\{S_t(x, y') > S_t(x, y)\} \, \mathrm{d}\, y'}_{\text{Poaching Separations}} \\ &+ \underbrace{\phi_x^u \lambda_t \, \tilde{u}_t(x) \, \frac{v_t(y)}{V_t} \, \mathbb{1}\{S_t(x, y) \geq 0\}}_{\text{Unemployed Hires}} \end{aligned} \tag{8}$$

Equation 7 shows that workers who fail to find jobs during the matching stage make up the stock of unemployed workers at the end of the period. This can be because they fail to contact a firm or because they contact a firm with negative match surplus. The terms in Equation 8 mirror the situations that can arise from the sequential auctions bargaining protocol. The stock of type x workers employed at type y firms is equal to previous employment plus any employees poached from other firms net of employees lost to other firms, plus workers hired out of unemployment. Note that each of these components is weighted by $\frac{v_t(y)}{V_t}$, the share of total vacancies at firms of type y.

Additionally, the objects $\tilde{u}_t(x)$ and $\tilde{e}_t(x, y)$ are determined as follows.

$$\begin{aligned} \tilde{u}_t(x') = &\Pi_{x'|x} \cdot (1 - \omega_x) \left[u_t(x) + \int \big(\mathbb{1}\{S_t(x, y) < 0\} + \delta_{x,y} \cdot \mathbb{1}\{S_t(x, y) \geq 0\}\big) \cdot e_{t-1}(x, y) \, \mathrm{d}\, y\right] \\ &+ \eta_t \cdot \mathbb{1}\{x' = \underline{x}\} \end{aligned}$$

This expression states that the number of unemployed workers of type x' after the separation stage is equal to any previously unemployed workers of type x plus any workers who are separated from their jobs either endogenously or exogenously, who do not retire and who transition into that type, plus new labor market entrants into the lowest worker type $\underline{x}$. The number of new labor market entrants η_t is equal to the total number of retiring workers $\int \omega_x u_t(x) \ + \ \int \omega_x e_t(x,y) \, \mathrm{d}\, x \, \mathrm{d}\, y$. Lastly,

$$\tilde{e}_t(x',y') = \Pi_{x'|x} \cdot \Pi_{y'|y} \cdot (1-\omega_x)(1-\delta_{x,y}) \cdot \mathbb{1}\{S_t(x,y) \geq 0\} \cdot e_{t-1}(x,y)$$

This expression states that the number of employed workers of type x' at firms of type y' after the separation stage consists of workers already employed at these firms who survive job destruction and retirement.

3.5 Wage Setting

I adopt the wage setting protocol developed in Lentz et al. (2017). I assume that firms commit to delivering a constant share σ_t of the surplus for the entire duration of a match until and unless the worker receives an outside offer, in which case the surplus share is renegotiated according to the sequential auctions protocol. Therefore, the worker receives a share $\sigma_t(x,y,y')$ of the surplus that depends on her type x, her current firm y, and her previous firm (previous outside offer) y'; this surplus share is constant until she receives an outside offer. In particular, for $S_t(x,y) \geq S_t(x,y')$,

$$\sigma_t \equiv \sigma_t(x,y,y') = \frac{S_t(x,y')}{S_t(x,y)} \tag{9}$$

Since allocations are determined entirely by the surplus function, wages only specify how workers and firms split the match surplus. Assuming that the worker's surplus share is fixed between renegotiations is convenient because it produces a closed form solution for the wage equation. Wages evolve according to the equation:

$$\begin{aligned} w_t(x,y,\sigma_t) = \; & \sigma_t p(x,y) + (1-\sigma_t) b(x) \\ & - (1-\omega_x)(1-\delta_{x,y})\beta \, \mathrm{E}_{x',y'} \Big[\mathbb{1}\{S_{t+1}(x',y') \geq 0\} \\ & \cdot \phi_x^e \lambda_{t+1} \int R_{t+1}(x',y',\sigma_{t+1},y'') \frac{v_{t+1}(y'')}{V_{t+1}} \, \mathrm{d}\, y'' \mid x, y \Big] \end{aligned} \tag{10}$$

where the term $R_t(x,y,\sigma_t,y')$ results from the possible outcomes of the sequential auctions protocol and represents the additional surplus the worker captures due to a renegotiation. It is given by the piecewise function:

$$R_t(x,y,\sigma_t,y') = \begin{cases} S_t(x,y) - \sigma_t S_t(x,y) & S_t(x,y') > S_t(x,y) \\ S_t(x,y') - \sigma_t S_t(x,y) & \sigma_t S_t(x,y) < S_t(x,y') \leq S_t(x,y) \\ 0 & S_t(x,y') \leq \sigma_t S_t(x,y) \end{cases}$$

The first case corresponds to a situation where the worker is poached. In this case, she is able to capture the entire surplus from her old firm and therefore receives $S_t(x, y)$ net of the previous surplus share $\sigma_t S_t(x, y)$ in her old match. In the second case, the offer is higher than her previous outside offer, but not high enough to trigger a poaching event. The worker is able to renegotiate her surplus share at the incumbent firm in order to extract the full value of the outside offer. She therefore receives $S_t(x, y')$ net of her previous surplus share $\sigma_t S_t(x, y)$. In the third case, the outside offer is not sufficiently high to trigger a renegotiation and the offer is discarded.

Given this assumption on the wage setting protocol, the wage $w_t(x, y, \sigma_t)$ is a weighted average of flow match output $p(x, y)$ and flow nonemployment benefit $b(x)$, net of future expected renegotiation opportunities captured by the final term in Equation 10 containing $R_t(x, y, \sigma_t, y')$. As a result of this term, wages will be lower for lower tenure workers, as these workers expect to have future opportunities to climb the job ladder and renegotiate their wages upward.

3.6 Law of Motion for Mass of Firms

The law of motion for the mass of firms of type y' in time period t is as follows:

$$m_{t+1}(y') = \Pi_{y'|y} \cdot \big(1 - \zeta_{y,t}\big) m_t(y) + \gamma_t \cdot \mathbb{1}\{y' = \underline{y}\} \tag{11}$$

where $\Pi_{y'|y}$ is the transition matrix across firm age bins, $\zeta_{y,t}$ is the exit rate for firm age y at time t, and γ_t is the entry rate of firms into the lowest firm type $\underline{y}$. Given exit rates $\zeta_{y,t}$ and entry rate γ_t, the steady state mass of firms by firm age, which I denote $\bar{m}(y)$, is the fixed point of Equation 11.

4 Numerical Implementation and Calibration

In this section, I describe the details of the numerical implementation and calibration of the model. I calibrate the model in steady state in order to match several features of the U.S. economy in the mid-1990s. Below, I outline the specific moments targeted in the calibration procedure and provide an overview of which moments in the data help to inform certain parameters.

4.1 Worker and Firm Age Bins

The Census data used in the empirical section of the paper are defined at the bin-level. I choose the same bins as the units of analysis for the model. There are 4 worker age bins $\{25\text{–}34, 35\text{–}44, 45\text{–}54, 55+\}$ and 5 firm age bins $\{0\text{–}1, 2\text{–}3, 4\text{–}5, 6\text{–}10, 11+\}$, each in years. Worker types evolve stochastically across bins according to the Markov transition matrix $\Pi_{x'|x}$ and firm types evolve stochastically across bins according to $\Pi_{y'|y}$. The model is set at a monthly frequency, which means that in each time period, $\frac{1}{12\times10}^{th}$ of 25–34 year-old workers become 35–44 year-old workers, $\frac{1}{12\times2}^{th}$ of 0–1 year-old firms become 2–3 year-old firms, et cetera.[12] Within bins, however, workers and firms are identical. Hence, the model describes the average worker within a certain age range and the average firm within a certain age range.

4.2 Functional Form Assumptions

I assume the matching function $\Psi(L_t, V_t)$ is Cobb–Douglas with elasticity parameter α.

$$\Psi(L_t, V_t) = L_t^{\alpha} V_t^{1-\alpha}$$

I normalize aggregate matching efficiency to 1. Hence, the contact rate for an unemployed worker of type x is given by $\phi_x^u \lambda_t = \phi_x^u \frac{\Psi(L_t,V_t)}{L_t} = \phi_x^u \left(\frac{V_t}{L_t}\right)^{1-\alpha}$ and the contact rate for an employed worker of type x is given by $\phi_x^e \lambda_t = \phi_x^e \frac{\Psi(L_t,V_t)}{L_t} = \phi_x^e \left(\frac{V_t}{L_t}\right)^{1-\alpha}$.

Worker search intensity ϕ_x^i is set exogenously and depends on both worker type x and employment status $i \in \{\text{employed } (e), \text{unemployed } (u)\}$. I assume that the labor market status component of search intensity, which I denote by κ_i, and the worker type component, which I denote by ψ_x, enter multiplicatively, so that $\phi_x^i = \kappa_i \cdot \psi_x$.

I parameterize the vacancy posting cost function as the iso-elastic function

$$C_y(n_t(y)) = c_y \frac{n_t(y)}{2}^2$$

where $n_t(y)$ is the number of vacancies posted by each firm of type y at time t. Under this functional form assumption, c_y governs the level of vacancy costs, which is allowed to differ by firm type. As I will discuss below, the c_y parameters are pinned down by the distribution of employment by firm age in the data. The curvature parameter of the vacancy cost function is set such that the function is exactly quadratic (and the marginal cost

[12]Note that workers and firms can only move up age bins, so the transition matrices contain only zeros below the diagonal. Transition matrices $\Pi_{x'|x}$ and $\Pi_{y'|y}$ are specified explicitly in Appendix E.

of an additional vacancy is exactly linear) in the number of firm-level vacancies posted.

Worker Flow Values I parameterize the match-level output function $p(x, y)$ as a second order polynomial in worker type x and firm type y.

$$p(x, y) = p_0 + p_1 x + p_2 y + p_3 xy + p_4 x^2 + p_5 y^2$$

The match-level output function $p(x, y)$ is a crucial element of the match surplus between workers and firms of different types. The shape of the joint surplus function $S(x, y)$ not only determines the sorting patterns between firms and workers but also influences the level of wages for different matches. Moreover, as shown in the previous section, $p(x, y)$ enters directly into the wage equation. The assumed functional form is flexible enough to capture the contours of the wage grid without allowing for too many degrees of freedom.

I follow Lise and Robin (2017) and set the flow nonemployment benefit $b(x)$ such that it is equal to some fraction b_0 of a worker's maximum attainable match output.

$$b(x) = b_0 \cdot \max_y \{p(x, y)\}$$

In the expression above, $\max_y\{p(x, y)\}$ stands for the match output at worker x's most productive match. The scaling parameter b_0 helps control the overall level of wages. In the model, if unemployment becomes "too costly" – i.e. $b(x)$ is very low relative to $p(x, y)$ – then workers accept wages that are counterfactually too low (even negative) in order to "buy" their way onto the job ladder. This is a well-known feature of the sequential auctions bargaining protocol. It is especially strong when workers have zero bargaining power out of unemployment, as I assume in order to keep the model tractable. However, setting b_0 sufficiently high helps mitigate this effect such that wages remain positive.

4.3 Calibration

Table 2 shows the calibrated parameters. I assume that the economy is in steady state in 1994 and calibrate the model in three steps. First, I externally set a subset of parameters to commonly used values in the literature (Panel A). Next, a subset of parameters is directly informed by the data (Panel B). Last, I perform a moment matching exercise designed to target different features of the worker and firm life-cycle (Panel C).

Externally Set Parameters One period is set to one month in the model, so all rates are monthly. The discount factor is set to correspond to an annual interest rate of 5 percent.

Table 2: Model Calibration

Parameter		Bin	Value	Target	Data	Model
Panel A: Externally Set						
β	Discount factor	–	0.996	5% annual real interest rate		
α	Matching function elasticity	–	0.8	Lange and Papageorgiou (2020)		
κ_e	Employed search intensity	–	0.5	Faberman et al. (2022)		
κ_u	Unemployed search intensity	–	1	Normalization		
Panel B: Directly Estimated						
ω_x	Retirement rate	55+	0.016	Labor force share age 55+	0.147	0.147
ψ_x	Search intensity by worker age bin	25–34	1.000	Job finding rate by worker age bin, relative to age bin 25–34	0.318	0.304
		35–44	0.906		0.288	0.276
		45–54	0.825		0.262	0.251
		55+	0.803		0.255	0.244
δ_y	Separation rate by firm age bin	0–1	0.030	Job destruction rate by firm age bin	0.030	0.030
		2–3	0.024		0.024	0.024
		4–5	0.020		0.020	0.020
		6–10	0.016		0.016	0.016
		11+	0.012		0.012	0.012
$\bar{m}(y)$	Steady-state mass of firms by firm age bin	0–1	0.014	Number of firms by firm age bin, relative to labor force	0.014	0.014
		2–3	0.010		0.010	0.010
		4–5	0.008		0.008	0.008
		6–10	0.015		0.015	0.015
		11+	0.030		0.030	0.030
Panel C: Internally Estimated						
c_y	Vacancy cost level by firm age bin	0–1	0.270	Employment share by firm age bin	5.847	5.636
		2–3	0.745		5.571	5.163
		4–5	1.507		5.249	5.002
		6–10	3.201		11.964	11.950
		11+	8.913		71.198	72.250
p_0	Shape of match-level output function	–	2.200	Average earnings by worker $\times$ firm age bin	See Figure 3	
p_1		–	2.162		See Figure 3	
p_2		–	-0.528		See Figure 3	
p_3		–	0.361		See Figure 3	
p_4		–	-2.567		See Figure 3	
p_5		–	0.612		See Figure 3	
b_0	Scale of $b(x)$	–	0.785		See Figure 3	

Notes: The frequency is monthly. For details on moment construction, see Appendix F.

The matching function elasticity α is set to match recent estimates of the elasticity of hires with respect to searchers. I normalize the search intensity of unemployed workers to 1 and set the search intensity of employed workers to a standard value from the literature.[13]

Directly Estimated Parameters The retirement rate is set such that workers only face retirement once they enter the oldest age bin (55+). I set the retirement rate for this age bin so as to match the share of workers age 55 and over in the labor force in 1994.

I set the search intensity by age group parameters ψ_x to target the age profile of the job finding rate in 1994. Workers in the youngest age group (25–34) have the highest job finding rates, so I normalize their search intensity to 1. The ψ_x's for all other age groups are set relative to the youngest worker age group (25–34). They are calculated by taking the ratio of the job finding rate for age group x to the job finding rate for age group 25–34.

I allow the exogenous separation rate $\delta_{x,y}$ to vary only by firm age such that $\delta_{x,y} = \delta_y$. I then set δ_y directly to the value of the job destruction rate by firm age group from the BDS in 1994. The Census Bureau defines the job destruction rate as the sum of all employment losses from contracting establishments, including establishments shutting down, divided by total employment. It therefore includes employment losses both from employees leaving the firm (continuing firms) and from firm exits (firm deaths). This is the relevant definition of match separation in my model since the boundaries of the firm with a firm age bin are undefined. The separation rate δ_y include both cases: employees leaving a firm that survives as well as employees returning to unemployment because their firm has closed down.

I set the steady-state mass of firms by firm age bin $\bar{m}(y)$ directly to its empirical value in 1994. I calculate this value for each firm age bin by taking the ratio of the number of firms in that age bin to the total number of workers in the labor force.

Internally Estimated Parameters I calibrate the parameters related to match-level output $p(x, y)$, flow nonemployment value $b(x)$, and the vacancy cost function $C_y(\cdot)$ by targeting worker flows into and out of unemployment by worker age bin, the distribution of employment across firm age bins, and the entire wage grid by worker age bin and firm age bin in the data in 1994. I calibrate 12 parameters in total to match 33 bin-level moments in the data, meaning that the model is overidentified. The parameter vector is

$$\theta = \{c_{0\text{–}1}, c_{2\text{–}3}, c_{4\text{–}5}, c_{6\text{–}10}, c_{11+}, p_0, p_1, p_2, p_3, p_4, p_5, b_0\}$$

Let $m(\theta)$ denote a vector of moments resulting from the solution of the model in

[13] See Holzer (1987) and Faberman et al. (2022) for estimates of the relative time spent searching by employed workers as well as Baley et al. (2022) for a recent implementation of this calibration strategy.

steady state under θ. Let $\hat{m}$ denote the vector of data moments. Both $m(\theta)$ and $\hat{m}$ are $N \times 1$ vectors, where $N = 33$. I choose parameter vector $\hat{\theta}$ so as to minimize the following objective function:

$$\hat{\theta} = \arg\min_{\theta} \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\frac{m_i(\theta) - \hat{m}_i}{\hat{m}_i}\right)^2}$$

where i indexes moments. The objective function may be interpreted in terms of percent differences. An objective function value of 0.10 means that the average deviation between model moments and data moments is 10 percent. This has the advantage of not over-weighting moments that have larger magnitudes (wages) or under-weighting moments with smaller magnitudes (distributions). I use global methods to efficiently and thoroughly search the parameter space. Details are provided in Appendix F.

Though the parameters in my moment matching exercise are jointly identified by the moments in the data, it is useful to consider which moments in particular are informative about specific parameters. The match-level output function $p(x, y)$ and the flow nonemployment benefit $b(x)$ both enter directly into the wage equation, which I reproduce below.[14] They also affect wages indirectly through the "Expected Renegotiation Benefit" term, as they affect the shape of the surplus function $S(x, y)$. This term captures the amount a worker is willing to have deducted from her wages in order to accept a job on a certain rung of the job ladder. It is higher (wages are lower) when she expects many opportunities to renegotiate her wages upward in the future.

$$\begin{aligned} w(x, y, \sigma) = \;& \sigma p(x, y) + (1 - \sigma) b(x) \\ & - (1 - \omega_x)(1 - \delta_{x,y})\beta \underbrace{\mathrm{E}_{x',y'}\left[\mathbb{1}\{S(x', y') \geq 0\} \cdot \kappa_e \psi_x \lambda \int R(x', y', \sigma, y'') \frac{v(y'')}{V}\, \mathrm{d}\, y'' \,|x, y\right]}_{\text{Expected Renegotiation Benefit}} \end{aligned}$$

Therefore, the six parameters of the match-level output function $\{p_0, p_1, p_2, p_3, p_4, p_5\}$ along with the scale parameter b_0 primarily help to determine the shape of the wage profile across worker and firm age bins.

Next, the parameters in the vacancy cost function pin down both the employment distribution across firms and the overall scale of the economy. The equation below shows the solution for the number of vacancies $v_t(y)$ posted by firms in each firm age bin. It is found by combining the definition of $v_t(y)$, the expression for the vacancy cost function

[14] To conserve on notation, I suppress time subscripts.

$C_y(\cdot)$, and Equation 5, and solving for $v_t(y)$.

$$v_t(y) = m_t(y) \, \frac{\mu_t \cdot J_t(y)}{c_y}$$

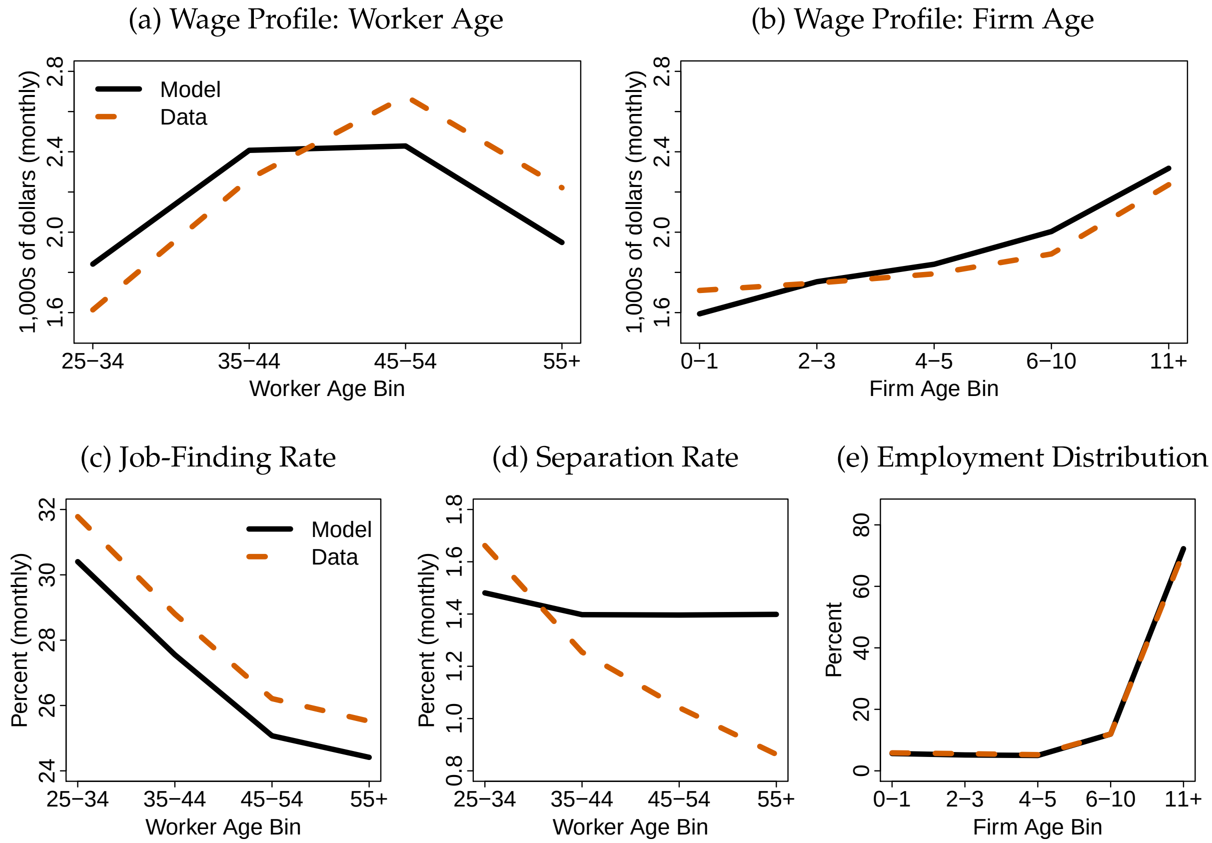
From this equation, we can see how the vacancy cost parameters c_y shift up or down the number of vacancies posted by firms of each age. Therefore, appropriately setting the c_y's pins down the vacancy distribution as well as the employment distribution across firms. Moreover, they control the *total* number of vacancies in the economy, which influences the contact rate λ_t.

Model Fit The last two columns of Table 2 compare the model implied moments to their empirical counterparts in the data. First, I am able to match employment shares by firm age bin almost exactly. As in the data, the oldest firms in the model comprise the majority share of employment. Next, the parameters of the match-level output function help to match the shape and level of the wage profile over worker and firm age. The scale parameter of the flow nonemployment benefit b_0 also helps to match the level of wages. Since these 7 parameters are informative about the 20 points on the worker age $\times$ firm age grid, I display their values in Table 2 and plot the wage profile by worker and firm age in Figure 3. I slightly underestimate career earnings growth in the model, but the difference between wages paid between old and young firms aligns well with the data.

Next, the figure shows that I match the life-cycle profile of the job finding rate, but slightly underestimate its level. The profile of the job finding rate over the life-cycle is captured by the age-specific search intensity parameters ψ_x, which I set outside of the moment matching exercise. Since there is no worker age-specific component to the separation rate, the only differences in separation rates across worker age groups in the model arise from the fact that workers differentially sort into firms of different ages. Though the magnitudes do not match the data exactly, the model captures the fact that separation rates decline over a worker's life-cycle. Overall, the model achieves a good fit, with an objective function value of about 15 percent.

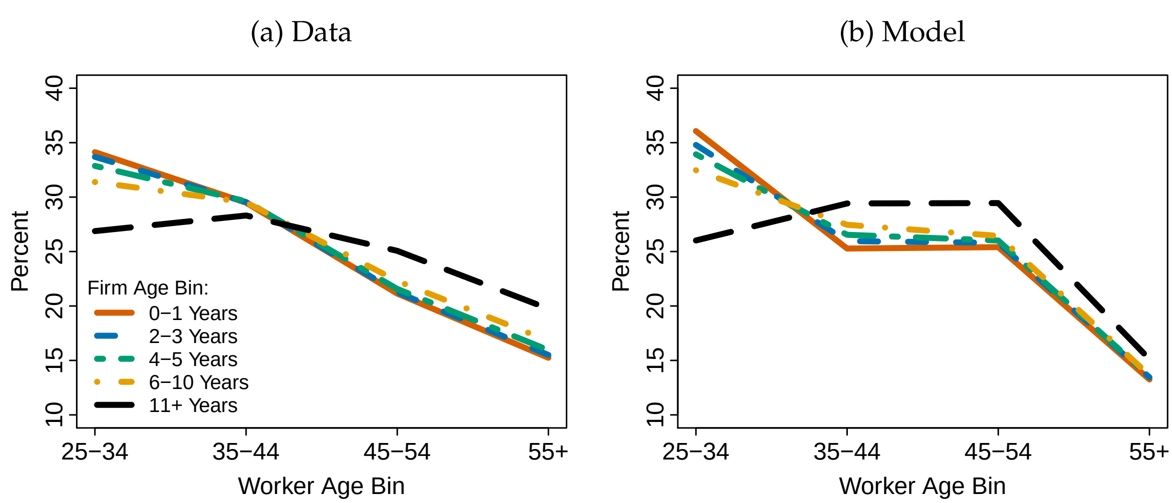
Non-Targeted Moments Though I target the overall distribution of employment across firm age, I do not directly target the distribution of employment across worker age conditional on firm age bin. As discussed in the empirical section of the paper, sorting patterns between firm age and worker age group are such that young firms play an out-sized role in young worker employment. In particular, the share of employment comprised by younger workers at younger firms is higher than that for older firms.

Figure 3: Model Fit: Targeted Moments



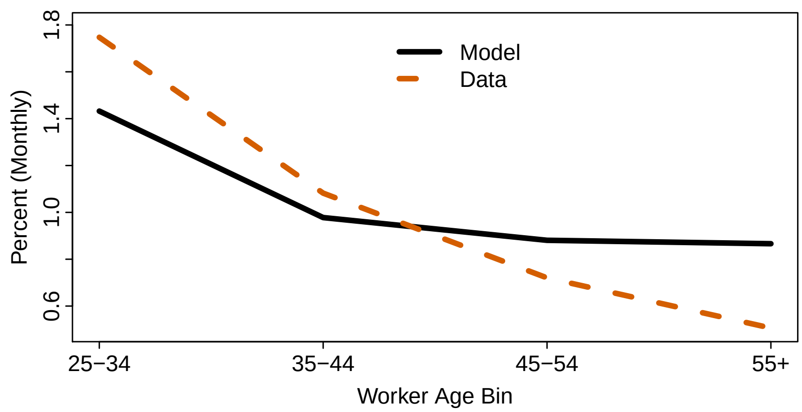
Notes: In each panel, the black solid lines show the model moments and the red dashed lines show the corresponding moments in the data. For details on moment construction, see Appendix F.

Figure 4: Model Fit: Employment Distribution Across Worker Age by Firm Age



Notes: Left panel shows the distribution of employment across workers age bins for firms of different ages in the data. Data are from the QWI. Right panel shows the distribution of employment across workers age bins for firms of different ages in the model.

Figure 5: Job-to-Job Flows



Notes: The black solid line shows the model moments and the red dashed line shows the corresponding moments in the data. Job-to-job flow rate is defined as the number of workers who directly switch jobs without an intervening spell of unemployment divided by total employment. Data are from the Census Bureau's Job-to-Job Flows database (J2J).

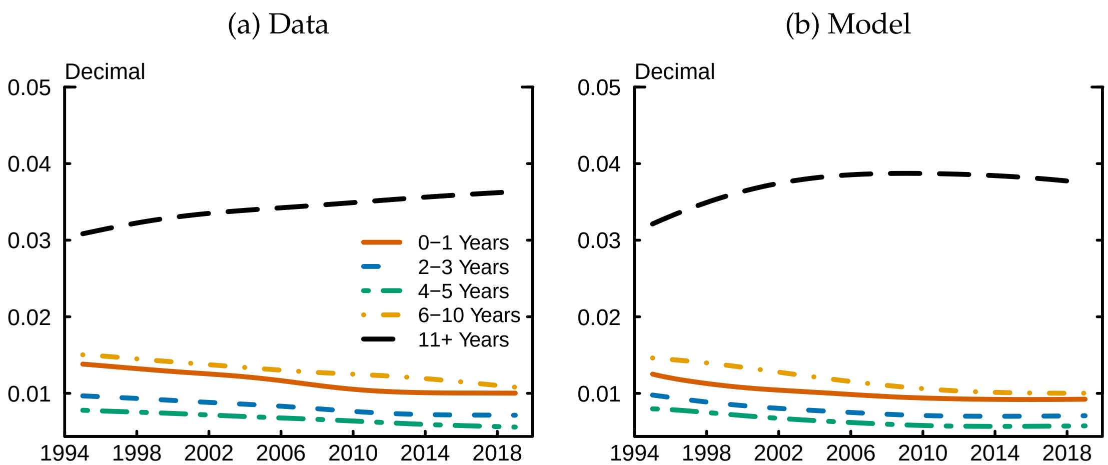
As shown in Figure 4, the model matches this feature of the data fairly well. Young firms have a higher share of younger workers and a lower share of older workers in the model. Moreover, the distribution of employment for older firms is in line with the share of old versus young workers in the overall labor force. While the average profile of the employment distribution is informed by the vacancy cost level parameters, the parameters of the match output profile help to inform the age group specific profiles. Therefore, targeting the wage profile also helps to match the sorting patterns between workers and firms in the data.

Lastly, Figure 5 shows that the model captures the shape of age profile of the job-to-job flow rate in the data. Workers at earlier stages of their careers are more likely to switch jobs as they, on average, have lower human capital (represented by the increasing profile of $p(x, y)$ over the worker life-cycle) and are employed in lower quality matches. However, as workers progress through their careers, they find better matches and therefore do not switch jobs as often.

5 Quantifying the Effects of Declining Business Dynamism

Using the calibrated model, I now simulate a decline in business dynamism in order to quantify its impacts on workers at different stages of the life-cycle. Starting from the initial steady state firm distribution in 1994, I decrease the firm entry rate as in the data and study the effects on labor market outcomes. I also allow exit rates to evolve as in the data. I first describe the calibration of the time path of the entry and exit rates below. Then,

Figure 6: Mass of Firms by Firm Age $m_t(y)$



Notes: The mass of firms by firm age bin is the ratio of the number of firms in the respective age bin to the total number of workers in the labor force. In the data, these series are constructed as follows: the number of firms by firm age bin is from the BDS. The total number of workers in the labor force is from the LFS and includes only male workers age 25 years and older. The resulting series are HP-filtered using an annual smoothing parameter.

I discuss the effects of declining dynamism on labor market outcomes in the aggregate and across cohorts of workers.

5.1 Calibrating the Law of Motion for the Mass of Firms

Equation 11 contains the law of motion for the mass of firms, which I reproduce here.

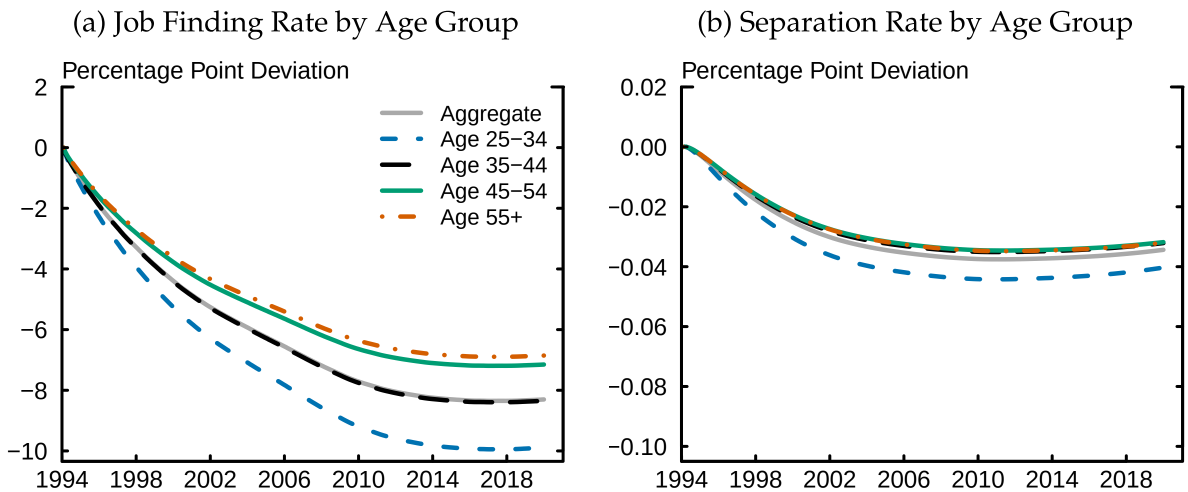
$$m_{t+1}(y') = \Pi_{y'|y} \cdot \big(1 - \zeta_{y,t}\big) m_t(y) + \gamma_t \cdot \mathbb{1}\{y' = \underline{y}\}$$

I calibrate the time path of the mass of firms by firm age bin so as to be as close to the data as possible. Then, I feed the resulting evolution of the mass of firms by firm age bin into the model and study the effects on labor market outcomes. This exercise takes *as given* the change in firm dynamics inherent in the law of motion for $m_t(y)$.

In the data, I observe: (i) exit rates by firm age bin and (ii) the ratio of the total number of firms in the economy to the total number of workers in the labor force.[15] To calibrate the law of motion for the mass of firms, I first take exit rates $\zeta_{y,t}$ directly from the data. I impute the entry rate γ_t to match the ratio of firms to the labor force $\mathcal{M}_t$ in the data. Figure 6 shows the resulting process for the mass of firms by firm age. I then feed this process into the model and study the response of the economy along the transition path.

[15] Appendix Figure B.6 plots these series. I HP-filter each series to abstract from business cycle fluctuations.

Figure 7: Effects on Labor Market Flows



5.2 Effects on the Labor Market

Figures 7 and 8 show the effects of the decline in business dynamism on key labor market variables. As the mass of firms in the economy declines, the total number of vacancies in the economy decreases. This effect can be seen by inspecting the formula for aggregate vacancies in a given time period V_t.

$$V_t = \int n_t(y) m_t(y) \, \mathrm{d}\, y$$

Aggregate vacancies are made up of two components. The first is the number of firm-level vacancies $n_t(y)$, which is pinned down by firms equating the costs and benefits of vacancy posting. Firm level vacancies are then scaled by the total number of firms in each age bin $m_t(y)$. Due to the decline in business dynamism, the mass of firms $m_t(y)$ declines for all firm types, which directly decreases V_t. The amount that V_t declines along the transition path then depends on the degree to which firm-level vacancies $n_t(y)$ respond to the drop in dynamism. This is determined by the parameters of the vacancy cost function, the expected value of a filled vacancy $J_t(y)$, and the rate at which firms contact workers μ_t.

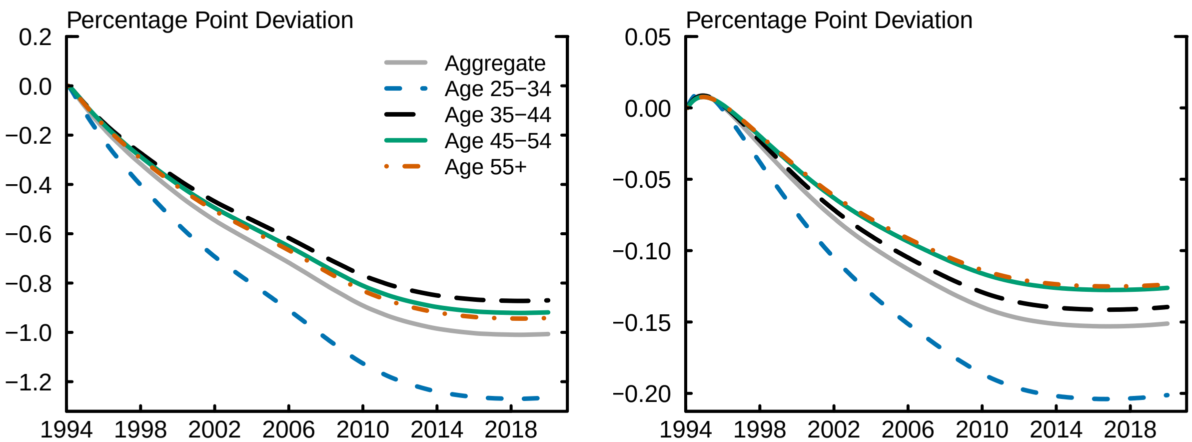
$$n_t(y) = \frac{\mu_t \cdot J_t(y)}{c_y}$$

Along the transition path, the expected value of a filled vacancy $J_t(y)$ increases because there are more unemployed workers searching for jobs. Job creation incentives in the model as captured by $J_t(y)$ are quite sensitive to changes in the stock of unemployed workers, who search with a higher intensity than employed workers (see Equation 4). In

Figure 8: Effects on Mobility and Employment

(a) Employment Rate by Age Group

(b) Job-to-Job Flow Rate by Age Group



addition, the firm contact rate μ_t does not respond along the transition path and stays at a corner solution where a firm posting vacancies will certainly contact a worker. This corner solution arises from the matching function because the total mass of firms is much smaller than the total mass of workers in the calibrated model. Therefore, the number of firm level vacancies $n_t(y)$ increases slightly along the transition path due to an increase in the expected value of posting a vacancy $J_t(y)$. However, this positive, indirect effect on V_t is not enough to offset the negative, direct effect of declining dynamism on V_t.

Figure 7 shows that the job finding rate in the economy, which is proportional to the rate at which firms contact workers $\lambda_t = \frac{\Psi(L_t,V_t)}{L_t}$, declines precipitously. With little change in separation probabilities, nonemployment rates increase for all worker age groups, with different effects for workers in different stages of their life-cycle. Hence, total employment in the economy declines due to lower overall labor demand, shown in Figure 8. Likewise, worker mobility as measured by the job-to-job flow rate also declines, with larger effects for younger worker age groups. The job-to-job flow rate also scales with the contact rate λ_t, but it is additionally influenced by the degree to which workers of different age groups are situated on high versus low rungs of the job ladder. Older workers have had more time to search for suitable matches and are on higher rungs of the job ladder.[16] They therefore switch jobs less often on average and are less exposed to the dynamism induced decline in labor demand. Consequently, the largest effects on worker mobility both in terms of movements out of unemployment and in terms of job switching are present for the youngest worker age group: 25–34 year-olds.

[16]This effect holds to a lesser extent for workers in the 55+ age bin who are nearing retirement, as total match surplus internalizes their higher exit rates.

Table 3: Quantifying the Effects of Declining Dynamism

Change: 1994–to–2019	Data	Model	Explained
Panel A: Employer Switching Rate			
Age 25–34	-0.51 pp	-0.20 pp	39.22%
Age 35–44	-0.25 pp	-0.14 pp	56.00%
Age 45–54	-0.12 pp	-0.13 pp	108.33%
Panel B: Employment–to–Population Ratio			
Age 25–34	-3.24 pp	-1.27 pp	39.20%
Age 35–44	-1.09 pp	-0.87 pp	79.82%
Age 45–54	-2.04 pp	-0.92 pp	45.10%

Notes: Data and Model columns show changes between 1994 and 2019 in percentage points (pp). Explained column displays the ratio of the Model column to the Data column, as a percent. Employer switching rate is defined as the percent of employed workers who switched employers at least once in a year. I construct this series using data from the Annual Social and Economic Supplement (ASEC) of the Current Population Survey (CPS), following the methodology of Molloy et al. (2016). Employment–to–population ratio is from the Bureau of Labor Statistics (BLS) Labor Force Statistics (LFS) database. Sample includes only male workers. Series are HP-filtered using an annual smoothing parameter.

5.3 Contribution of Declining Dynamism to Declining Mobility

I now examine the degree to which the model's predictions capture the evolution of certain labor market series in the data during the period under consideration. Table 3 shows the contribution of declining dynamism to changes in labor market outcomes.

Between 1994 and 2019, males between the ages of 25–54 experienced a decline in rates of mobility as well as overall employment. In the data, the rate at which workers switch between jobs at different firms fell by about 0.5 percentage points on a monthly basis. However, this decline was not uniform for all worker age groups. In particular, employer switching fell by more for younger worker age groups, meaning that each successive cohort of labor market entrants has faced a lower rate of employment mobility. The counterpart in the model is the job-to-job flow rate, which measures the rate at which workers switch directly between jobs at different firms. As in the data, employer switching rates for younger workers in the model decline by more in response to the decline in the firm entry rate. Moreover, the model accounts for between 35 and 110 percent of the decline in employer switching across worker age groups.[17]

The model also accounts for the fact that average employment rates declined for workers under the age of 55 between 1994 and 2019. Table 3 shows that all three age

[17] For the oldest worker age group, the business dynamism induced decline in employer switching is larger than the decline in employer switching in the data. See Table 3.

groups experienced declines in their employment-to-population ratio over this time horizon. The model counterpart of these series is the non-employment rate, as the workers in these demographic groups are highly attached to the labor market and trends in the data are likely not driven by workers dropping out of the labor force for non-economic reasons. Through the lens of the model, declining business dynamism accounts for between 35 and 80 percent of the empirical trends in the employment rate by worker age group.

6 Welfare Implications of Declining Business Dynamism

I now examine the consequences of the decline in business dynamism for total welfare in the economy as well as welfare for workers at different stages of their life-cycle. The most natural measure of welfare in the model would be the value function for unemployed workers $W^u(x)$. However, this is exogenously pinned down by the sequential auctions protocol, so I instead use a flow value concept of welfare. Let $\bar{w}_t$ denote the flow welfare value of employed workers, $\bar{b}_t$ denote the flow welfare value of unemployed workers, $\bar{f}_t$ denote the flow welfare value of filled vacancies, and $\bar{c}_t$ denote the flow welfare value of unfilled vacancies at time t. These objects are defined as follows:

$$\bar{w}_t \equiv \int \int e_t(x,y) w_t(x,y) \,\mathrm{d}\,x \,\mathrm{d}\,y \qquad \bar{b}_t \equiv \int u_t(x) b(x) \,\mathrm{d}\,x$$
$$\bar{f}_t \equiv \int m_t(y)(p_t(y) - w_t(y)) \,\mathrm{d}\,y \qquad \bar{c}_t \equiv \int m_t(y) \tilde{c}_t(y) \,\mathrm{d}\,y$$

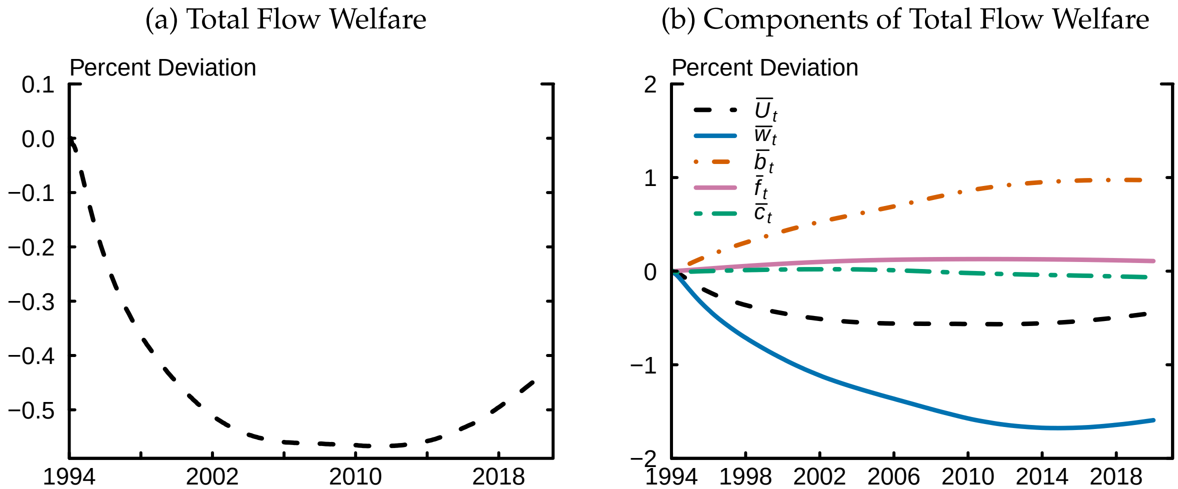
where $p_t(y) = \int p(x,y)\, e(x,y) \,\mathrm{d}\,x$ denotes total match output by firm age bin, $w_t(y) = \int w_t(x,y)\, e(x,y) \,\mathrm{d}\,x$ denotes total wages by firm age bin, $\tilde{c}_t(y) = C_y\big(\pi_v^u(y)\, n_t(y)\big)$ denotes flow vacancy posting costs by firm age bin, and $\pi_v^u(y)$ is the share of unfilled vacancies by firm age bin. Total flow welfare in the economy at time t is given by $\bar{U}_t = \bar{w}_t + \bar{b}_t + \bar{f}_t - \bar{c}_t$.

6.1 Decomposing Total Flow Welfare

Using the definition of total flow welfare, we can decompose the percentage change in welfare in the economy into each of its components. Let $dX_t = X_t - X_0$ denote a deviation of the variable X_t from its steady state level X_0. Also, let $\Delta X_t = \frac{dX_t}{X_0}$ denote a percentage deviation of the variable X_t from its steady state level X_0. Changes in total flow welfare may be decomposed as follows.

$$\Delta \bar{U}_t = \frac{d\bar{w}_t}{\bar{U}_0} + \frac{d\bar{b}_t}{\bar{U}_0} + \frac{d\bar{f}_t}{\bar{U}_0} - \frac{d\bar{c}_t}{\bar{U}_0} \tag{12}$$

Figure 9: Flow Welfare Decomposition



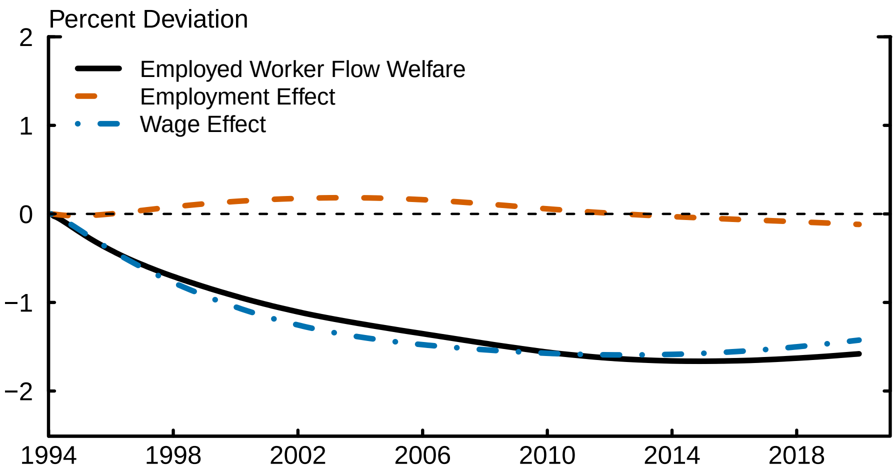
The results of this decomposition exercise are plotted in Figure 9. The components that have the largest contributions to the change in total flow welfare are employed workers welfare $\bar{w}_t$ and unemployed workers welfare $\bar{b}_t$. Along the transition path, these measures impact overall welfare in opposite directions. A decline in $\bar{w}_t$ has a negative effect on $\bar{U}_t$, while an increase in $\bar{b}_t$ has a positive effect on $\bar{U}_t$. The former effect dominates for the entirety of the transition path, so total flow welfare falls over this time horizon.

The changes in employed and unemployed worker welfare follow from the results presented in the previous section. As business dynamism falls, there is a large decline in employment and a corresponding increase in the unemployment rate for all age groups. Therefore, aggregating across a smaller (larger) number of employed (unemployed) workers results in lower (higher) overall welfare among these groups, notwithstanding changes in the flow payoffs that each group receives. For unemployed workers, these are constant along the transition path because $b(x)$ does not change over time (there are no changes in match-level output $p(x, y)$). Hence, the effects on $\bar{b}_t$ are straightforward to understand: a larger number of unemployed workers receiving the same flow benefit $b(x)$ results in overall larger $\bar{b}_t$. For employed workers, wages $w_t(x, y)$ also change along the transition path, so the effects driving the change in $\bar{w}_t$ are not as clear. I further decompose the different margins that affect $\bar{w}_t$ below.

6.2 Decomposing Employed Worker Flow Welfare

I now decompose changes in employed workers' welfare $\bar{w}_t$ into two margins. The first margin stems from changes in employment rates, while the second margin stems from changes in workers' wages. Let $e_0(x, y)$ denote match-level employment in steady state

Figure 10: Employed Worker Flow Welfare Decomposition



and let $w_0(x, y)$ denote match-level wages in steady state for matches between workers of type x and firms of type y. The percentage deviation of employed workers' welfare from steady state can be approximated (to first order) as:

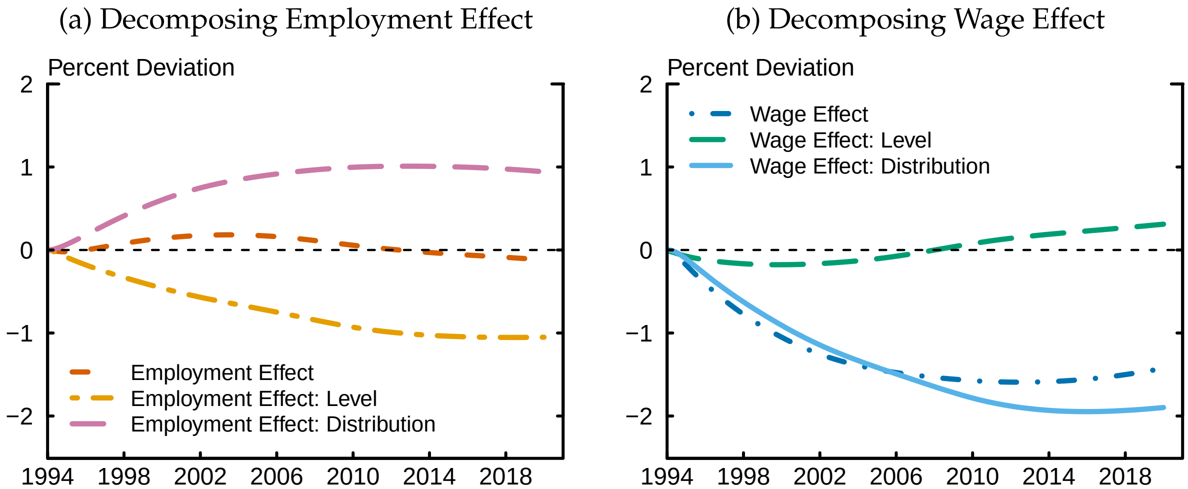
$$\Delta \bar{w}_t \approx \underbrace{\Delta\left(\int\int e_t(x,y)w_0(x,y)\,\mathrm{d}\,x\,\mathrm{d}\,y\right)}_{Employment\ Effect} + \underbrace{\Delta\left(\int\int e_0(x,y)w_t(x,y)\,\mathrm{d}\,x\,\mathrm{d}\,y\right)}_{Wage\ Effect} \tag{13}$$

Intuitively, employed worker flow welfare may change due to changes in the number of workers that are employed or to changes in the wages workers earn while employed. In the above expression, the $Employment\ Effect$ term captures the degree to which $\bar{w}_t$ changes due to changes in employment in the economy, holding match-level wages constant at their steady state value. The $Wage\ Effect$ term captures the degree to which employed worker welfare $\bar{w}_t$ changes due to changes in match-level wages, holding employment by worker age and firm age constant at their steady state values.

Figure 10 plots this decomposition. It is clear from the figure that the $Wage\ Effect$ dominates, driving the overall decline in employed worker flow welfare. Along the transition path, employed workers receive lower wages, driven by lower between-firm poaching competition. Although the employment probability declines in response to the decline in business dynamism, the $Employment\ Effect$ plays only a small role in the decline in employed workers flow welfare.

However, examining the changes in these component masks important sorting dynamics along the transition path. To further inspect these sorting patterns, I provide an additional decomposition of the $Employment\ Effect$ and the $Wage\ Effect$ into components stemming from changes in their levels and distributions. For instance, the level

Figure 11: Level Effect vs. Distribution Effect

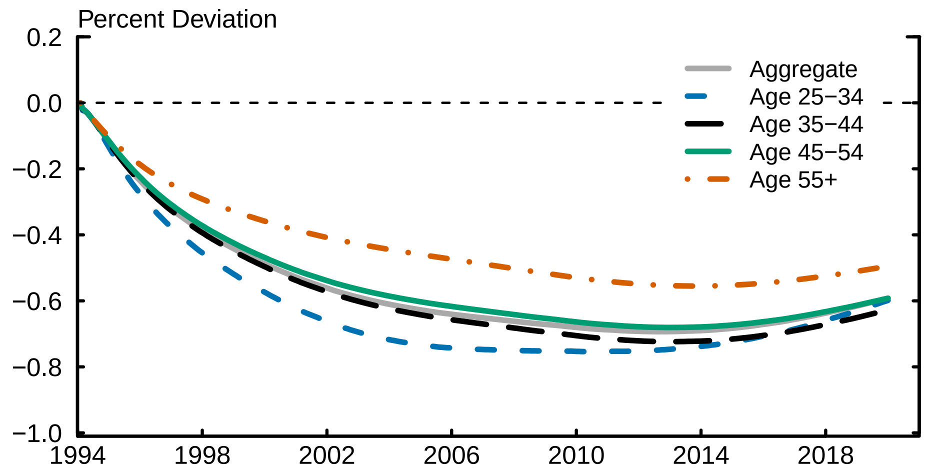


effect shows the degree to which changes in the level of employment or wages affect employed worker flow welfare. The distribution effect shows how changes in the types of matches workers sort into affects employed worker flow welfare.

Figure 11 plots this decomposition. First, we can see that the $Employment\ Effect$ is driven by offsetting changes in the level and distribution (Panel a). While overall employment falls along the transition path (level effect), workers on average sort into better matches (distribution effect), such that the match distribution shifts toward matches that pay higher wages. The former effect is present because as firm entry declines, there are fewer firms in the economy, providing fewer employment opportunities for workers. The latter effect is present because as firm entry declines, the firm age distribution shifts towards older firms that are more productive and pay higher wages.

Similar dynamics shape the evolution of the $Wage\ Effect$ (Panel b). On average, workers with a given level of the surplus share σ_t experience only small changes in their wages paid (level effect). Workers higher up on the within-match job ladder with higher surplus share all else equal have a slight decline in wages, but this is offset by a decline in wages among workers lower down on the within-match job ladder. In other words, the wage-bargaining share profile flattens within matches, on average. However, workers face a lower probability of moving up the job ladder due to the decline in business dynamism and are on average stuck in lower rungs of the job ladder (distribution effect). In other words, the match distribution shifts towards matches with lower surplus share. The net effect is that wages fall along the transition path, as workers command a lower share of the match surplus in the economy, on average.

Figure 12: Employed Worker Flow Welfare by Age Group



6.3 Welfare Changes by Worker Cohort

Lastly, I explore the welfare implications of declining business dynamism across worker age groups. As is clear from the section above, younger age groups experience larger declines in both mobility and employment rates in response to the shift in the firm age distribution. Therefore, a decline in business dynamism results in different changes in welfare for workers at different stages of the life-cycle.

Figure 12 quantifies the degree to which different worker cohorts experience different declines in flow welfare in response to a decline in business dynamism. While all age groups experience a decline in welfare, younger age groups are hit harder. Aggregate worker flow welfare falls by about 0.6 percent along the transition path, while the youngest age group of workers (25–34) experiences as much as a 0.8 percent decline in welfare. Though it recovers slightly by the end of the period under consideration, this is driven by an increase in nonemployment among young workers and therefore a larger increase in $\bar{b}_t$ for this group.

The larger decline in flow welfare for younger workers is driven by the fact that they experience larger declines in both the $Employment\ Effect$ and the $Wage\ Effect$. Along the transition path, employment levels decline by more for younger workers. Moreover, younger workers stand to benefit less from the increasing share of older, more productive firms because they sort into matches at these firms at a lower rate. Additionally, because younger workers experience larger declines in mobility rates, the decline in the distribution component of the $Wage\ Effect$ is larger for these groups, as they are unable to move out of the lower rungs on the job ladder. These effects combine to generate a larger decline in total worker flow welfare for younger cohorts.

7 Conclusion

In this paper, I assess the consequences of the recent decline in business dynamism in the United States for labor market outcomes and total welfare in the economy across different cohorts of workers. I first review several empirical patterns that suggest a link between the rate of business dynamism – the share of young relative to old firms in the economy – and labor market mobility along a worker's life-cycle. I show that in the data, there has been a decline in the rate at which new firms enter the economy, resulting in a shift of the firm age distribution towards older firms that tend to also be larger. Moreover, I show that young firms are more likely to employ younger workers in that the employment distribution of young firms is on average skewed towards young workers. This suggests that the decline in business dynamism may have disproportionately affected the labor market outcomes of more recent cohorts of workers.

Then, I set up a model of labor market sorting between heterogeneous firms and heterogeneous workers subject to search frictions in order to test this hypothesis. In the model, workers differ by the length of time since they entered the labor market and firms differ by the length of time since they entered the economy. I calibrate the model to match several features of the labor market in 1994 and then simulate a decline in business dynamism in line with the data. I find that aggregate employment declines along the transition path, leading to a decline in total welfare in the economy. However, these effects are not felt equally by all workers. Younger workers are more exposed to changes in business dynamism as they have a larger share of employment at younger firms. Mobility and employment rates decline by more for younger cohorts, leading to a decline in welfare that is more severe for these groups of workers.

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A Data Description

A.1 Quarterly Workforce Indicators (QWI)

The Longitudinal Employer Household Dynamics (LEHD) database is a linked employer-employee dataset constructed from state administrative records and maintained by the U.S. Census Bureau. While access to the underlying microdata in the LEHD is restricted, the Census Bureau publishes tabulations of the data at different levels of aggregation such as industry, geography, firm size and age, as well as worker demographics. In particular, the Census maintains the Quarterly Workforce Indicators (QWI), which contain information on hires, separations, turnover, employment growth, and earnings by industry, worker demographics, and firm age and size. The data can be downloaded from the webpage: https://lehd.ces.census.gov/data/#qwi.

A.2 Job-to-Job Flows (J2J)

To complement the QWI, the Census Bureau publishes additional detail on worker flows in the Job-to-Job Flows (J2J) database. The tabulations are similar to those in the QWI and statistics are available by firm characteristics (industry, age, and size) and by worker demographics (sex by age, sex by education, and race by ethnicity). These data contain measures of *direct* job-to-job transitions across employers and also allow to distinguish hires from other firms (poaching) from hires from the unemployment pool. They also allow to distinguish separations to another firm (job-to-job separations) from separations to nonemployment. The data can be downloaded from the webpage: https://lehd.ces.census.gov/data/#j2j.

A.3 Business Dynamics Statistics (BDS)

The Business Dynamics Statistics (BDS) datasets are maintained by the U.S. Census Bureau and contain annual measures of business dynamics such as job creation, job destruction, establishment births and deaths, and firm startups and exits. The data are available for the overall economy as well as by different establishment and firm characteristics. The BDS is derived from the Census Bureau's Longitudinal Business Database (LBD), a census of business establishments and firms in the U.S. with paid employees comprised of survey and administrative records. Data may be downloaded from https://www.census.gov/data/datasets/time-series/econ/bds/bds-datasets.html.

A.4 Current Population Survey (CPS)

Annual Social and Economic Supplement (ASEC) To construct the measure of employer switching, I use data from the Annual Social and Economic Supplement (ASEC) of the Current Population Survey (CPS). The ASEC is based on a survey of more than 75,000 U.S. households and contains detailed questions on the social and economic characteristics of each person who is a household member as of the interview date. Questions in the survey pertain to the previous calendar year.

To construct the measure of employer switching used in the paper, I use a variable in ASEC that records the responses to the following survey question: "For how many employers did (name/you) work in [year]? If more than one at the same time, only count it as one employer." Since the question asks respondents to count simultaneous employment at multiple firms as only one employer, any respondent who answers that she had more than one employer in a given year must have switched jobs between firms at some point during that year. The employer switching rate is then estimated as the number of respondents who had more than one employer divided by total employment.[18]

This approach follows Molloy et al. (2016), which is the first paper to my knowledge to construct this specific measure of employer switching. I download the variable `NUMEMPS`, which contains responses to the survey question above, from the IPUMS CPS website (Flood et al., 2022). I select wage and salary workers in the private sector who reported that they were employed or had a job during the previous calendar year. IPUMS CPS data are available at https://cps.ipums.org/cps/.

Longitudinally Linked CPS In order to construct measures of the job finding rate and the job separation rate by worker age group, I follow the procedure described in Shimer (2012) to link respondents in the CPS Basic Monthly Survey (BMS) across months. I download data from IPUMS CPS and use the unique identifier `CPSIDP` constructed by IPUMS to link individuals across surveys. I also implement additional matching criteria to ensure that individuals match on age, sex, and race characteristics.

After linking individuals across consecutive months, information on their labor market status – employed (E), unemployed (U), or not in the labor force (N) – allows me to construct job finding and job separation probabilities. The monthly job finding probability $P(UE)_t$ is defined as the fraction of unemployed individuals in month $t-1$ who are employed in month t. The monthly job separation probability $P(EU)_t$ is defined as the fraction of employed individuals in month $t-1$ who are unemployed in month t.

[18] In practice, I weight each observation using the weighting variable ASECWT provided by IPUMS CPS.

Formulas are given below.

$$P(UE)_t = \frac{\#(\text{Unemployed in month } t-1 \text{ who are Employed in month } t)}{\#(\text{Unemployed in month } t-1)}$$

$$P(EU)_t = \frac{\#(\text{Employed in month } t-1 \text{ who are Unemployed in month } t)}{\#(\text{Employed in month } t-1)}$$

The job finding and job separation rates by age group are simply constructed by applying the above formulas for the relevant age sub-sample.

A.5 Labor Force Statistics (LFS)

To construct the employment-to-population ratio and the fraction of age 55 or older workers, I use data from the U.S. Bureau of Labor Statistics (BLS) Labor Force Statistics (LFS) database. The LFS contains statistics on U.S. labor force characteristics tabulated by different demographic groups such as age, race, sex, education, and marital status. I obtain the series listed in Table A.1 from the BLS website. The fraction of age 55 or older workers is simply the number of age 55 or older workers in the civilian labor force divided by the number of age 25 or older workers in the civilian labor force. The data are available at https://www.bls.gov/cps/.

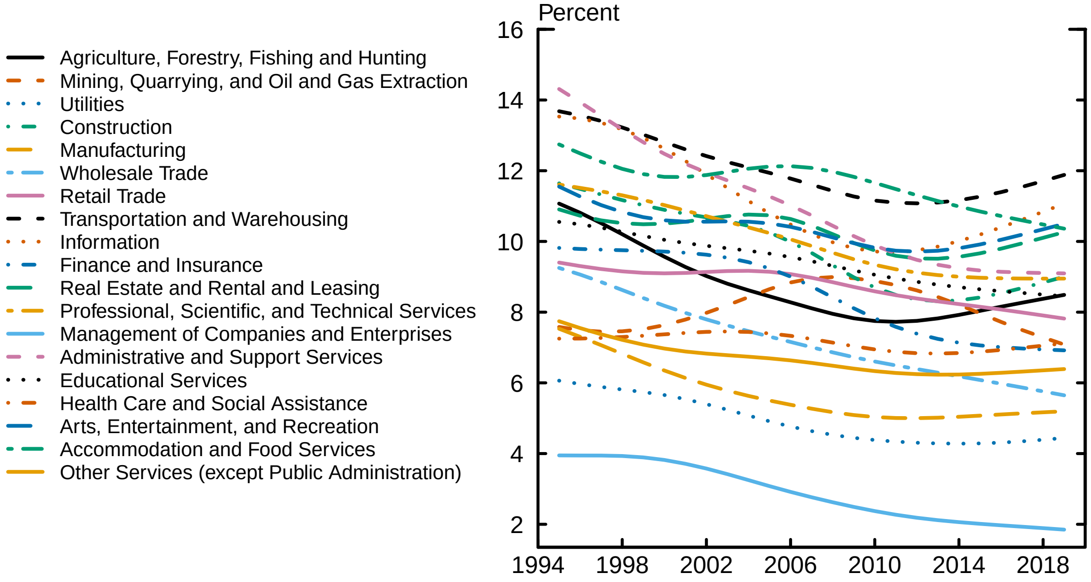
Table A.1: Variables in the LFS

Series ID	Labor Force Status	Demographic Group
LNS11000164	Civilian labor force	Men, age 25 to 34 years
LNS12300164	Employment-population ratio	Men, age 25 to 34 years
LNS11000173	Civilian labor force	Men, age 35 to 44 years
LNS12300173	Employment-population ratio	Men, age 35 to 44 years
LNS11000182	Civilian labor force	Men, age 45 to 54 years
LNS12300182	Employment-population ratio	Men, age 45 to 54 years
LNS11024231	Civilian labor force	Men, age 55 years and older
LNS12324231	Employment-population ratio	Men, age 55 years and older

Notes: Series are at the monthly frequency and are seasonally adjusted by the BLS.

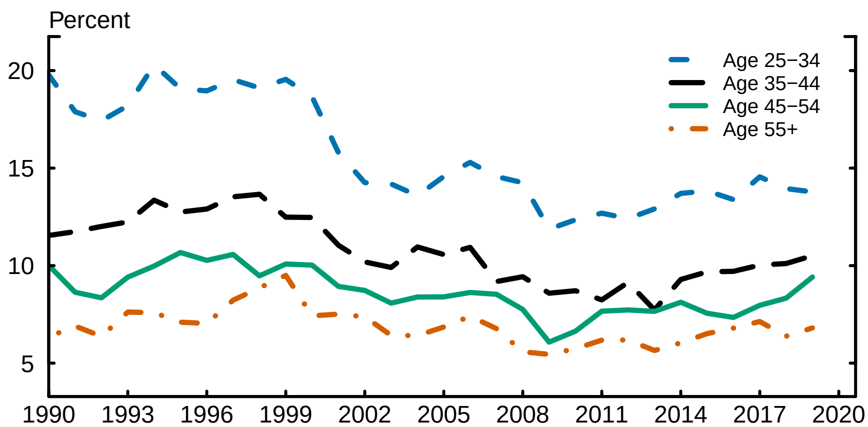
B Additional Figures

Figure B.1: Trends in Firm Entry Rate by Sector



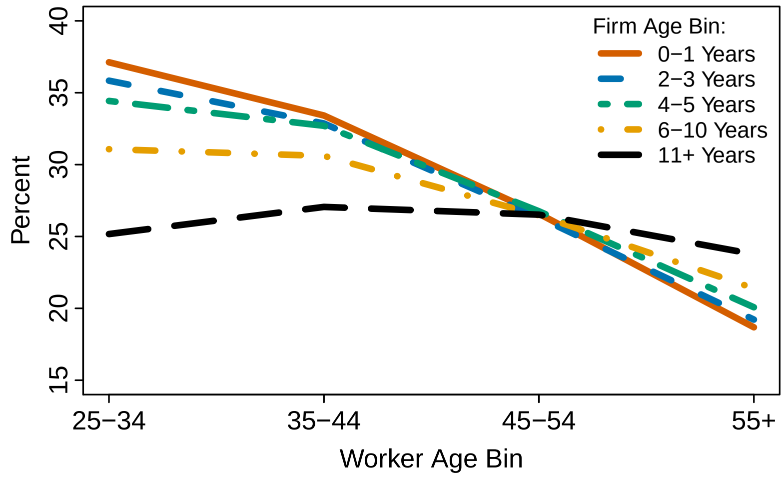
Notes: Entry rate defined as the number of age 0 firms divided by the total number of firms. Data are from the BDS. Series are HP-filtered with an annual smoothing parameter.

Figure B.2: Employer Switching Rate by Worker Age Group



Notes: Employer switching rate is defined as the percent of employed workers who switched employers at least once in a year. I construct this series using data from the ASEC supplement CPS, following the methodology of Molloy et al. (2016). The sample includes men age 25 and over employed as wage or salary workers in the private sector.

Figure B.3: Employment Distribution Across Worker Age by Firm Age, Granular Data

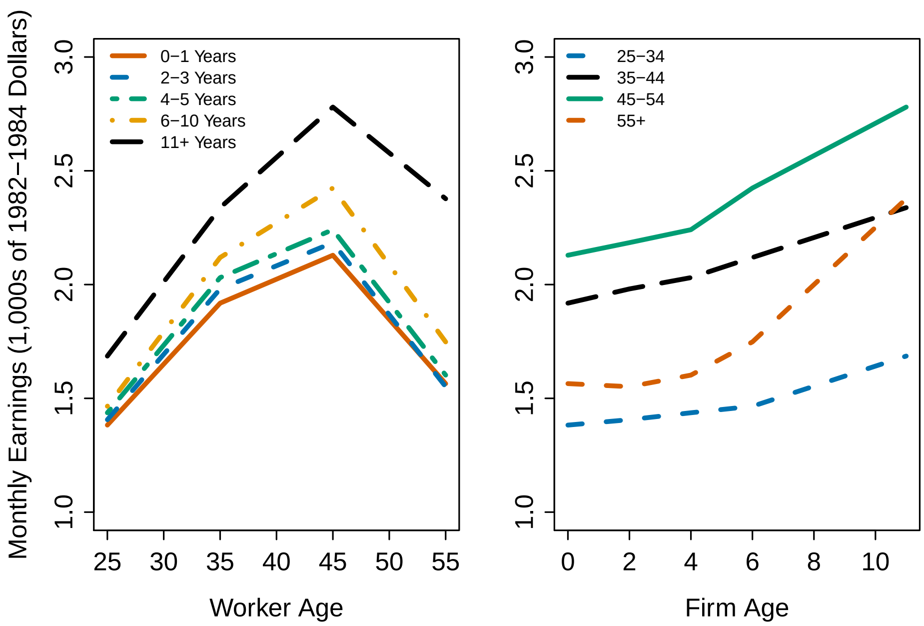


Notes: Figure shows average employment composition, in percentages, across worker age group for firms in different age groups. Data on employment by worker and firm age group are from the QWI. For all series, I include only male workers and take averages over state × industry × year cells.

Figure B.4: Employment Distribution Across Worker Age by Firm Size

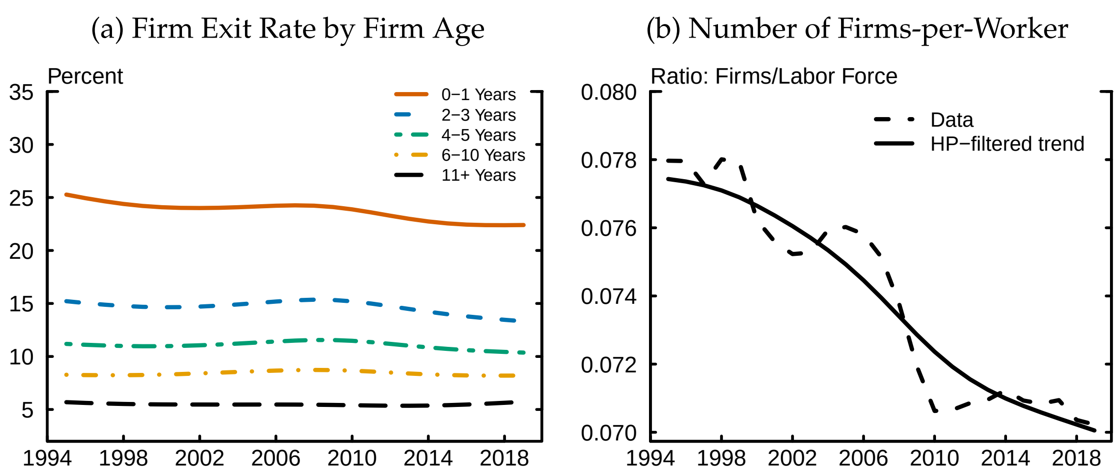
Notes: Figure shows average employment composition, in percentages, across worker age group for firms in different size groups. Data on employment by worker age and firm size group are from the QWI. For all series, I include only male workers and take averages over 1994–2019.

Figure B.5: Wage Profile by Worker Age and Firm Age



Notes: Figure shows average monthly earnings per worker, in thousands of 1982–1984 dollars, by worker and firm age group. I use the variable `earns` from the QWI database, which includes earnings of workers employed for the entire quarter. For additional details on moment construction, see Appendix F.1.

Figure B.6: Calibrating the Law of Motion for the Mass of Firms



Notes: Firm exit rate for each firm age bin is defined as the number of firm deaths in the respective age bin divided by the total number of firms in the respective age bin. Data are from the BDS. The ratio of firms/labor force is defined in the same way as the model: the total number of firms in the economy divided by the total number of male workers over the age of 25 in the labor force. Data on the total number of firms in the economy is from the BDS. Data on the labor force is from the LFS. Series are HP-filtered with an annual smoothing parameter.

C Additional Empirical Results

In this section, I conduct robustness checks of the patterns documented in Section 2 in the main text. Table 1 shows that young firms (age 0-10 years) have an employment share of young workers (age 25-44 years) about 15 percentage points higher relative to mature firms (age 11 years or older). However, these sorting patterns may be driven by worker and firm characteristics other than age.

For instance, sorting between young workers and young firms may be driven by skill and productivity differences across these groups. A large literature on labor market sorting shows that workers with higher skill levels tend to match with firms with higher productivity levels (Lise and Robin, 2017). To the extent that older workers have been able to achieve higher education levels, and education proxies for worker skill, we may expect to see a higher share of older workers at older firms. Table C.1 explores this possibility.

Table C.1: Worker and Firm Sorting Patterns by Firm Age, Worker Education Groups

	(1)	(2)	(3)	(4)
Firm Age 0–10 Years	3.443***	1.344***	0.537***	−0.478***
	(0.025)	(0.024)	(0.049)	(0.078)
Frac. Age 25–44		0.064***	0.073***	0.073***
		(0.001)	(0.001)	(0.001)
ln(Avg. Firm Size)			−0.716***	−0.580***
			(0.032)	(0.033)
Firm Age 0–10 Years × ln(Avg. Firm Size)				0.506***
				(0.030)
Year Fixed Effects	X	X	X	X
State Fixed Effects	X	X	X	X
Industry Fixed Effects	X	X	X	X
Observations	430,353	420,203	367,117	367,117
R^2	0.740	0.816	0.815	0.815
Adjusted R^2	0.740	0.816	0.815	0.815

Notes: Sample includes only male workers age 25 and over for the years 1994–2019. Frac. Age 25-44 is the fraction of a firm's workforce between the ages of 25 and 44. ln(Avg. Firm Size) is the natural logarithm of average firm size. Industry fixed effects are at the 4-digit NAICS level. Standard errors in parentheses. $^{*}p \leq 0.10$;$^{**}\, p \leq 0.05$;$^{***}\, p \leq 0.01$.

The table shows the results of regressions similar to those in main text, where instead the outcome variable is the fraction of a firm's work force with a high school education or less. From the first three columns of the table, we can see that younger firms employ, if anything, a slightly higher fraction of lower skilled workers, though the magnitude of this association reduces significantly after controlling for the fraction of young workers at

Table C.2: Worker and Firm Sorting Patterns by Firm Size, Worker Age Groups

	(1)	(2)	(3)	(4)
Firm Size < 500 Employees	2.443***	2.580***	2.309***	0.073**
	(0.041)	(0.040)	(0.035)	(0.032)
Frac. Educ. ≤ High School				0.177***
				(0.002)
Year Fixed Effects	X	X	X	X
State Fixed Effects		X	X	X
Industry Fixed Effects			X	X
Observations	425,746	425,746	425,746	388,720
R^2	0.142	0.175	0.399	0.485
Adjusted R^2	0.142	0.175	0.399	0.484

Notes: Sample includes only male workers age 25 and over for the years 1994–2019. Frac. Educ. ≤ High School is the fraction of a firm's workforce with less than or equal to a high school education. Industry fixed effects are at the 4-digit NAICS level. Standard errors in parentheses. $^{*}p \leq 0.10$;$^{**}\ p \leq 0.05$;$^{***}\ p \leq 0.01$.

a firm. Moreover, controlling for differences in firm size reveals that larger firms have a lower fraction of low skilled workers (Column (3)), but that this pattern is weaker among younger firms (Column (4)). Overall, the results are consistent with some degree of positive assortative matching between high skill workers and high productivity firms, but the firm life-cycle also plays a role; some small, yet highly productive young firms likely employ high skill workers in larger proportions. Lastly, the magnitudes of these sorting patterns are much smaller than those documented in Table 1.

To further explore the firm size dimension of worker and firm sorting patterns, I use an indicator for firm size instead of firm age as the independent variable of interest. Table C.2 displays the results. The table shows that smaller firms, on average, have a higher fraction of younger workers relative to firms with 500 employees or more. However, this pattern is largely driven by skill differences across worker age groups. Column (4) of the table shows that this association almost entirely disappears after controlling for differences in the employment share of low skill workers. Therefore, sorting on worker age and firm size likely results from the moderate degree of sorting on worker skill and firm size, as shown in Table C.1.

Lastly, Table C.3 explores worker skill and firm size sorting patterns directly. The outcome variable in this table is the fraction of workers with less than or equal to a high school education and the independent variable is an indicator for firm size instead of firm age. Here, we can see mostly clearly that even after controlling for differences in age

Table C.3: Worker and Firm Sorting Patterns by Firm Size, Worker Education Groups

	(1)	(2)	(3)	(4)
Firm Size < 500 Employees	5.589***	5.703***	4.623***	3.521***
	(0.045)	(0.043)	(0.026)	(0.021)
Frac. Age 25–44				0.084***
				(0.001)
Year Fixed Effects	X	X	X	X
State Fixed Effects		X	X	X
Industry Fixed Effects			X	X
Observations	399,297	399,297	399,297	388,720
R^2	0.040	0.095	0.686	0.770

Notes: Sample includes only male workers age 25 and over for the years 1994–2019. Frac. Age 25-44 is the fraction of a firm's workforce between the ages of 25 and 44. Industry fixed effects are at the 4-digit NAICS level. Standard errors in parentheses. $^{*}p \leq 0.10;^{**} p \leq 0.05;^{***} p \leq 0.01$.

composition across firm size categories, small firms employ a moderately higher fraction of lower skill workers. Again, these patterns are much less stable across different controls and of a much smaller magnitude than those displayed in Table 1. Therefore, I conclude that the sorting patterns between young firms and young workers are not simply masking differences in worker skill and firm productivity. Instead, the life-cycle component of employment sorting is accounted for by other forces, such as the joint dynamics of young workers and firms, or differences in where firms of different ages sit on the job ladder.

D Derivations and Proofs

To keep the notation simple, I normalize worker search intensity to 1 and abstract from retirement in the derivations below such that $\phi_x^i = \kappa_i \psi_x = 1$ and $\omega_x = 0\ \forall x$. This is without loss of generality, and the same derivations hold in the case with differences in search intensity as well as retirement rates. I also suppress the terms in the expectations operator $\mathrm{E}[\cdot]$ to conserve on notation. For unemployed workers, expectations are over values of x' and for any joint value objects, expectations are over combinations of (x', y').

D.1 Unemployed Worker Value Function

The assumption that workers hired out of unemployment have zero bargaining power reduces the unemployed worker's value function to: $W_t^u(x) = b(x) + \beta\,\mathrm{E}\left[W_{t+1}^u(x')\right]$.

Proof. Start with the equation for the worker's value of unemployment.

$$W_t^u(x) = b(x) + \beta\,\mathrm{E}\left[(1-\lambda_{t+1})W_{t+1}^u(x') + \lambda_{t+1}\int \max\{W_{t+1}^e(x',y'), W_{t+1}^u(x')\}\frac{v_{t+1}(y')}{V_{t+1}}dy'\right]$$

Workers hired out of unemployment have zero bargaining power and therefore receive zero surplus share. In other words, firms are able to extract the entire match surplus upon matching with an unemployed worker. Therefore, workers hired out of unemployment simply receive the value of unemployment as their continuation value when matching with a firm. This implies that $W_t^e(x,y) \equiv W_t^e(x,y,0) = W_t^u(x)$.[19] Substituting this into the equation above and reducing the expression yields the desired result.

$$\begin{aligned}
W_t^u(x) &= b(x) + \beta\,\mathrm{E}\left[(1-\lambda_{t+1})W_{t+1}^u(x') + \lambda_{t+1}\int \max\{W_{t+1}^e(x',y',0), W_{t+1}^u(x')\}\frac{v_{t+1}(y')}{V_{t+1}}dy'\right] \\
&= b(x) + \beta\,\mathrm{E}\left[(1-\lambda_{t+1})W_{t+1}^u(x') + \lambda_{t+1}\int \max\{W_{t+1}^u(x'), W_{t+1}^u(x')\}\frac{v_{t+1}(y')}{V_{t+1}}dy'\right] \\
&= b(x) + \beta\,\mathrm{E}\left[(1-\lambda_{t+1})W_{t+1}^u(x') + \lambda_{t+1}\int W_{t+1}^u(x')\frac{v_{t+1}(y')}{V_{t+1}}dy'\right] \\
&= b(x) + \beta\,\mathrm{E}\left[(1-\lambda_{t+1})W_{t+1}^u(x') + \lambda_{t+1}W_{t+1}^u(x')\right] \\
&= b(x) + \beta\,\mathrm{E}\left[W_{t+1}^u(x')\right]
\end{aligned}$$

□

[19] We can also see this by setting $\sigma_t = 0$ in the definition of the employed worker's value function written in terms of the surplus share: $W_t^e(x,y,\sigma_t) = W_t^u(x) + \sigma_t S_t(x,y)$. See below for more details.

D.2 Joint Surplus Function

The joint surplus function is defined as the joint match value net of the unemployed worker's value, $S_t(x,y) \equiv P_t(x,y) - W_t^u(x)$. As mentioned in the text, the model is block recursive such that neither the distribution of firms in the economy nor the distribution of workers across matches enters the value function for the joint surplus.

Proof. First, start with the equation for the joint match value $P_t(x,y)$.

$$\begin{aligned}
P_t(x,y) = p(x,y)& \\
+ \beta \, \mathrm{E} \, \Big[& \big(1-(1-\delta_{x,y})\mathbb{1}\{P_{t+1}(x',y') \geq W_{t+1}^u(x')\}\big) W_{t+1}^u(x') \\
& + (1-\delta_{x,y})\mathbb{1}\{P_{t+1}(x',y') \geq W_{t+1}^u(x')\} \Big((1-\lambda_{t+1}) P_{t+1}(x',y') \\
& + \lambda_{t+1} \int \max\{P_{t+1}(x',y'), W_{t+1}^e(x',y'',y')\} \frac{v_{t+1}(y'')}{V_{t+1}} \, \mathrm{d}\, y'' \Big) \Big]
\end{aligned}$$

Due to the sequential auctions framework, the continuation value in the case that an employed worker contacts another firm is independent of the worker value $W_t^e(x,y,y')$. This is because there are two cases: either the worker moves to the poaching firm y' and extracts the entire match value (net of the outside option), or the worker stays at the incumbent firm and renegotiates their surplus share upwards in accordance with the value offered by the unsuccessful poaching firm. Therefore, $P_t(x,y) \geq W_t^e(x,y,y')$. We can use this expression to reduce the match value to the equation below.

$$\begin{aligned}
P_t(x,y) = p(x,y)& \\
+ \; \beta \, \mathrm{E} \Big[& \big(1-(1-\delta_{x,y})\mathbb{1}\{P_{t+1}(x',y') \geq W_{t+1}^u(x')\}\big) W_{t+1}^u(x') \\
& + (1-\delta_{x,y})\mathbb{1}\{P_{t+1}(x',y') \geq W_{t+1}^u(x')\} P_{t+1}(x',y') \Big]
\end{aligned}$$

We then use the definition of the unemployed worker value function. As shown above, $W_t^u(x) = b(x) + \beta \, \mathrm{E} \left[W_{t+1}^u(x') \right]$. Therefore, subtracting $W_t^u(x)$ from both sides yields:

$$\begin{aligned}
P_t(x,y) - W_t^u(x) = p(x,y) - b(x) - \beta \, \mathrm{E} \left[W_{t+1}^u(x') \right]& \\
+ \; \beta \, \mathrm{E} \Big[& \big(1-(1-\delta_{x,y})\mathbb{1}\{P_{t+1}(x',y') \geq W_{t+1}^u(x')\}\big) W_{t+1}^u(x') \\
& + (1-\delta_{x,y})\mathbb{1}\{P_{t+1}(x',y') \geq W_{t+1}^u(x')\} P_{t+1}(x',y') \Big].
\end{aligned}$$

Finally, rearranging and using the definition of the joint surplus yields the desired result.

$$\begin{aligned}
P_t(x,y) - W_t^u(x) &= p(x,y) - b(x) \\
&\quad + \beta \, \mathrm{E}\Big[\Big(1 - (1-\delta_{x,y})\mathbb{1}\{P_{t+1}(x',y') \geq W_{t+1}^u(x')\}\Big)W_{t+1}^u(x') \\
&\qquad + (1-\delta_{x,y})\mathbb{1}\{P_{t+1}(x',y') \geq W_{t+1}^u(x')\}P_{t+1}(x',y') - W_{t+1}^u(x')\Big] \\
&= p(x,y) - b(x) \\
&\quad + (1-\delta_{x,y})\beta \, \mathrm{E}\Big[\mathbb{1}\{P_{t+1}(x',y') \geq W_{t+1}^u(x')\}\Big(P_{t+1}(x',y') - W_{t+1}^u(x')\Big)\Big] \\
\implies S_t(x,y) &= p(x,y) - b(x) + (1-\delta_{x,y})\beta \, \mathrm{E}\Big[\mathbb{1}\{S_{t+1}(x',y') \geq 0\}\Big)\Big(S_{t+1}(x',y')\Big)\Big] \\
&= p(x,y) - b(x) + (1-\delta_{x,y})\beta \, \mathrm{E}\Big[\max\{S_{t+1}(x',y'), 0\}\Big]
\end{aligned}$$

□

D.3 Deriving the Wage Equation

We can use the definition of the surplus share in Equation 9 to represent the worker's value of employment as a function of the surplus and the surplus share.

$$W_t^e(x,y,\sigma_t) \equiv W_t^u(x) + \sigma_t S_t(x,y)$$

From this equation, we can explicitly see that hiring from unemployment entails setting $\sigma_t = 0$. Then, if a worker employed at some firm y meets another firm y', the surplus share σ_t evolves according to the piecewise function below.

$$\sigma_t' = \begin{cases} \frac{S_t(x,y)}{S_t(x,y')} & S_t(x,y') > S_t(x,y) \\ \frac{S_t(x,y')}{S_t(x,y)} & \sigma_t S_t(x,y) < S_t(x,y') \leq S_t(x,y) \\ \sigma_t & S_t(x,y') \leq \sigma_t S_t(x,y) \end{cases}$$

Notice that this expression mirrors the function $R(\cdot)$ in the main text. In the first case, the worker is poached and moves to firm y', extracting the entire surplus $S_t(x,y)$ of her previous match at firm y. In the second case, the worker stays at firm y, but renegotiates her surplus share to the full amount of the surplus $S_t(x,y')$ at firm y'. In the third case, the offer is below her current surplus share and is therefore too low to trigger a renegotiation;

the worker simply discards the offer and stays at firm y with the same surplus share.

Now, using the definition of the employed worker value $W_t^e(x,y,\sigma_t) = W_t^u(x) + \sigma_t S_t(x,y)$, we solve for a wage $w_t(x,y,\sigma_t)$ that implements this contract.

$$\begin{aligned}
W_t^e(x,y,\sigma_t) &= W_t^u(x) + \sigma_t S_t(x,y) \\
&= w_t(x,y,\sigma_t) + \beta\,\mathrm{E}\left[W_{t+1}^u(x')\right] \\
&\quad - (1-\delta_{x,y})\beta\,\mathrm{E}\left[\mathbb{1}\{S_{t+1}(x',y') \geq 0\}\Big(\lambda_{t+1}\int Q_{t+1}(x',y',\sigma_{t+1},y'')\frac{v_{t+1}(y'')}{V_{t+1}}\,\mathrm{d}\,y'' \right. \\
&\qquad\qquad \left. + (1-\lambda_{t+1})\sigma_{t+1}S_{t+1}(x',y')\Big)\right]
\end{aligned}$$

where $Q_t(x,y,\sigma_t,y')$ is defined similarly to σ_t' above and represents the surplus the worker captures due to a renegotiation. In other words, it is the second best of the three values $\sigma_t S_t(x,y)$, $S_t(x,y')$, and $S_t(x,y)$.

$$Q_t(x,y,\sigma_t,y') = \begin{cases} S_t(x,y) & S_t(x,y') > S_t(x,y) \\ S_t(x,y') & \sigma_t S_t(x,y) < S_t(x,y') \leq S_t(x,y) \\ \sigma_t S_t(x,y) & S_t(x,y') \leq \sigma_t S_t(x,y) \end{cases}$$

Next, notice that from expression for the unemployed worker's value function, we have that $\beta\,\mathrm{E}\left[W_{t+1}^u(x')\right] = W_t^u(x) - b(x)$, so we can use this to eliminate $\beta\,\mathrm{E}\left[W_{t+1}^u(x')\right]$ and $W_t^u(x)$ from the above equation. We then have

$$\begin{aligned}
\sigma_t S_t(x,y) &= w_t(x,y,\sigma_t) - b(x) \\
&\quad - (1-\delta_{x,y})\beta\,\mathrm{E}\left[\mathbb{1}\{S_{t+1}(x',y') \geq 0\}\Big(\lambda_{t+1}\int Q(x',y',\sigma_{t+1},y'')\frac{v_{t+1}(y'')}{V_{t+1}}\,\mathrm{d}\,y'' \right. \\
&\qquad\qquad \left. + (1-\lambda_{t+1})\sigma_{t+1}S_{t+1}(x',y')\Big)\right]
\end{aligned}$$

Lastly, we substitute the definition of the surplus equation into this equation and solve for $w_t(x,y,\sigma_t)$, which yields the desired result.

$$\begin{aligned}
w_t(x,y,\sigma_t) &= \sigma_t p(x,y) + (1-\sigma_t)b(x) \\
&\quad - (1-\delta_{x,y})\beta\,\mathrm{E}\left[\mathbb{1}\{S_{t+1}(x',y') \geq 0\}\cdot\lambda_{t+1}\int R_{t+1}(x',y',\sigma_{t+1},y'')\frac{v_{t+1}(y'')}{V_{t+1}}\,\mathrm{d}\,y''\right]
\end{aligned}$$

where $R_t(x,y,\sigma_t,y') \equiv Q_t(x,y,\sigma_t,y') - \sigma_t S_t(x,y)$ is defined in the main text and represents the additional surplus the worker captures due to a renegotiation.

D.4 Contract Distribution

Average wages by (x, y) pair are given by

$$w_t(x,y) = \int w_t(x,y,\sigma_t) G_t(x,y,\sigma_t) \,\mathrm{d}\,\sigma_t$$

where $w_t(x, y, \sigma_t)$ is the wage for a worker of type x employed at firm y with surplus share σ_t and $G_t(x, y, \sigma_t)$ is the distribution of σ's within all (x, y) matches. The contract distribution is defined similarly to the worker flow equations by the law of motion:

$$\begin{aligned}
G_t(x,y,\sigma_t) = \tilde{G}_t(x,y,\sigma_t)\Big[1 + \lambda_t - \lambda_t \int \frac{v_t(y')}{V_t} \mathbb{1}\{S_t(x,y') > \sigma_t S_t(x,y)\} \,\mathrm{d}\,y'\Big] \\
+ \lambda_t \int \tilde{e}_t(x,y') \frac{v_t(y')}{V_t} \mathbb{1}\{\sigma_t S_t(x,y) > S_t(x,y')\} \,\mathrm{d}\,y' \\
+ \lambda_t \, \tilde{u}_t(x) \, \frac{v_t(y)}{V_t} \, \mathbb{1}\{S_t(x,y) \geq 0\}
\end{aligned}$$

where $\tilde{G}_t(x, y, \sigma_t) = (1 - \delta_{x,y})\mathbb{1}\{S_t(x, y) \geq 0\}G_{t-1}(x, y, \sigma_{t-1})$.

E Additional Model Details

The transition matrices for worker type (worker age) and firm type (firm age) are given by the following expressions. Note that the model is set to monthly frequency.

$$\Pi_{x'|x} = \begin{bmatrix} 1-\frac{1}{120} & \frac{1}{120} & 0 & 0 \\ 0 & 1-\frac{1}{120} & \frac{1}{120} & 0 \\ 0 & 0 & 1-\frac{1}{120} & \frac{1}{120} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Pi_{y'|y} = \begin{bmatrix} 1-\frac{1}{24} & \frac{1}{24} & 0 & 0 & 0 \\ 0 & 1-\frac{1}{24} & \frac{1}{24} & 0 & 0 \\ 0 & 0 & 1-\frac{1}{24} & \frac{1}{24} & 0 \\ 0 & 0 & 0 & 1-\frac{1}{60} & \frac{1}{60} \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

E.1 Model Solution

To compute the model solution, I use standard numerical techniques to solve the value function for the joint match surplus and to find the distribution of employment across worker and firm types in steady state. Given values for $p(x,y)$, $b(x)$, and $\delta_{x,y}$, I first solve for the joint surplus function (Equation 2) by value function iteration. Then, I iterate on the worker flow equations (Equations 7 and 8) in order to solve for the steady state worker distribution, starting from an initial guess where all workers are unemployed. Each step of the iteration requires solving for aggregate search intensity (Equation 3), the value of a filled vacancy (Equation 4), and aggregate vacancies (Equation 6) in order to pin down the contact rates λ_t and μ_t. This also determines the vacancy distribution across firm types $\frac{v_t(y)}{V_t}$. Next, I solve for wages at the match level by first using Equation 10 to obtain the wage $w_t(x,y,\sigma_t)$ for any pair (x,y) and any possible surplus share $\sigma_t = \sigma_t(x,y,y')$; then, I iterate on the law of motion for the distribution of contracts across σ_t within an (x,y) pair. This allows me to compute average wages by (x,y) pair. Appendix D.4 shows the law of motion for the distribution of wage contracts. With few worker and firm types, the entire solution algorithm converges very quickly.

F Additional Calibration Details

F.1 Constructing Data Moments

Table F.1 summarizes the data moments and their sources. Below, I provide additional detail about how I construct the wage measure I target in the data.

Table F.1: Targeted Data Moments

Moment	Bins	Source
Job finding rate	Male workers age {25–34, 35–44, 45–54, 55+}	CPS
Separation rate	Male workers age {25–34, 35–44, 45–54, 55+}	CPS
Employment share	Firms age {0–1, 2–3, 4–5, 6–10, 11+}	BDS
Earnings-per-employee	Male workers age {25–34, 35–44, 45–54, 55+} × Firms age {0–1, 2–3, 4–5, 6–10, 11+}	QWI

Notes: Job finding rate is defined as the number of unemployed workers who transition into employment divided by total unemployment for each age bin. Separation rate is defined as the number of employed workers who transition into unemployment divided by total employment for each age bin. Data are from the longitudinally-linked CPS. Employment share is defined as employment in each firm age bin as a percentage of total employment. Data are from the BDS. Earnings/employee is defined as average monthly earnings, in units of 1,000s of 1982–1984 dollars. Data are from the QWI and are deflated using the CPI for All Urban Consumers.

Earnings-per-Employee In the model, there is no intensive margin of labor supply, so the concept of wages is akin to earnings. To calibrate the wage profile in the model, I target the profile of average earnings-per-employee by worker age group and firm age group in the QWI data. I use the variable `earns`, which corresponds to average monthly earnings of workers employed for the entire quarter.[20] I construct the average of this series within worker × firm age bins using the appropriate employment weights. I also average across quarters to obtain a yearly series for each worker × firm age bin.

I then deflate each resulting yearly series by the Consumer Price Index for All Urban Consumers: All Items in U.S. City Average (FRED code: CPIAUCSL). This price index measure uses the years 1982–1984 as the base years. Lastly, I HP-filter each deflated series using an annual smoothing parameter and normalize the units to thousands of dollars. Therefore, the units of my resulting average earnings measures are: thousands of 1982–1984 dollars earned per month per worker. See Figure B.5 for a plot of the earnings profile across worker and firm age bins.

[20] See the following link for variable definitions: `https://lehd.ces.census.gov/doc/QWI_101.pdf`.

F.2 Global Optimization Algorithm

Since the parameter space is fairly large and the objective function is not well behaved, I use global methods to find the parameters that minimize the distance between the model and data moments. I use a multiple restart procedure in order to select a set of candidate solutions as starting points and then run a local optimization routine from each of these starting values. The algorithm proceeds as follows:

1. Select a set of $S = 250,000$ candidate starting points using Sobol sequences.

2. Evaluate the objective function at each of these points and store the results in a vector.

3. Keep the best (i.e. lowest function value) $S^* = 1,000$ of these points.

4. Run a local optimization routine (Nelder-Mead algorithm) starting from each of these S^* points and store the resulting function values and parameter vectors.

    (a) Let f^* denote the $1 \times S^*$ vector of objective function values at the local optima corresponding to the S^* starting points.

    (b) Let θ^* denote the $N \times S^*$ matrix of parameter values at the local optima corresponding to the S^* starting points.

5. Find the lowest function value among f^* and call this $\hat{f}$; find the parameter vector in θ^* that corresponds to $\hat{f}$.

6. Let $\hat{\theta}$ denote the parameter vector that corresponds to $\hat{f}$. $\hat{\theta}$ is the global minimum.