Q: For what values of x will $p_n(x) = p_{n+1}(x) = c \neq 0 \quad \forall n$.

$$x p_n(x) = a_n p_{n+1}(x) + b_n p_n(x) + c_n p_{n-1}(x)$$

$$xc = a_n c + b_n c + c_n c \qquad \forall n.$$

$$\boxed{x = a_n + b_n + c_n \qquad \forall n}$$

Hermite: $\quad a_n = \frac{1}{2}, \; b_n = 0, \; c_n = -\frac{1}{2}n$

$c = 0$: can't happen by root interlacing!

Roots have to be simple ~ roots of $p'(x)$ must be different from the roots of $p(x)$

$$\left.\begin{aligned} y' + f(x)y &= 0 \\ y(a) &= 0 \end{aligned}\right\} \text{ IVP}: \quad \left\{\begin{aligned} p(a) &= 0 \\ p'(a) &= -f(a)p(a) = 0 \end{aligned}\right.$$

$$\Rightarrow \Leftarrow.$$

Nice functions on the land of complex analysis are <u>holomorphic functions</u>.

$$\frac{1}{\sqrt{z^2-1}} \quad \text{is holomorphic on } \mathbb{C} \setminus [-1,1]$$

How do we evaluate @ complex #'s?

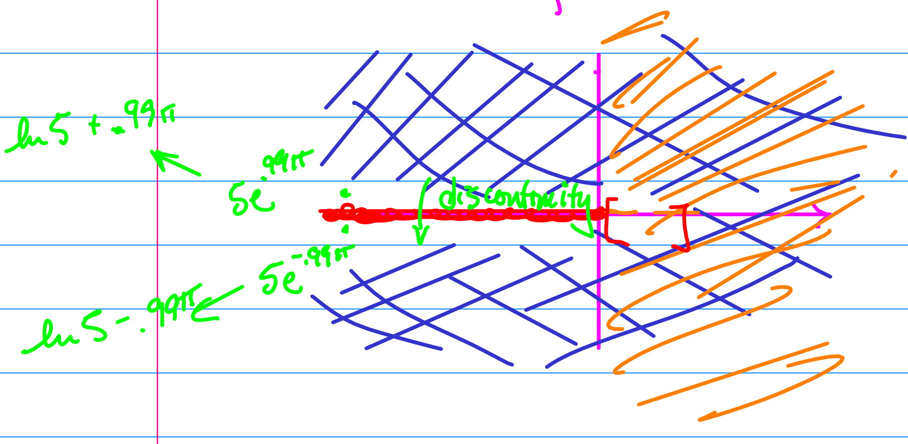
What is z^a, $a \in \mathbb{R}$?

$z^a = e^{a \ln z}$, What is $\ln z$

<u>Def</u>: The <u>principal log</u> is the function

$$\operatorname{Ln}(re^{i\theta}) = \ln(r) + i\theta, \quad \text{for } r > 0, \quad -\pi < \theta < \pi$$

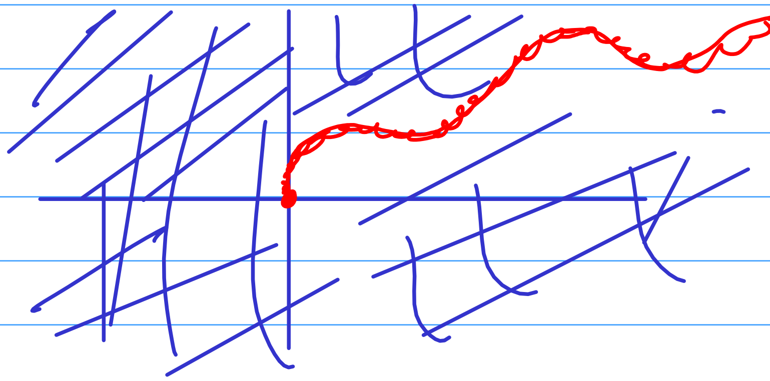
The domain of Ln is called a <u>branch cut</u>.



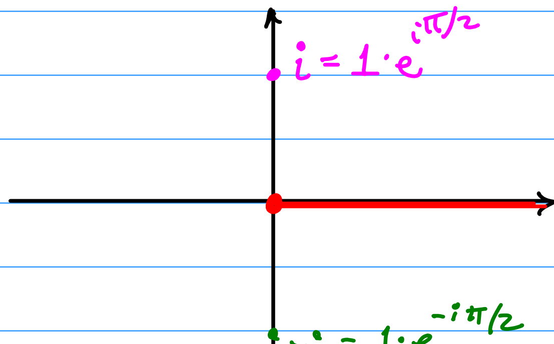
$z = re^{i\theta}$

$z^2 - 1 = r^2 e^{2i\theta} - 1$

Infinitely many different log functions, one for every branch cut!



$$\log(re^{i\theta}) = \ln r + i\theta, \quad r>0, \quad 0<\theta<2\pi$$



$\text{Ln} \neq \log$

$\text{Ln}(i) = i\frac{\pi}{2}$

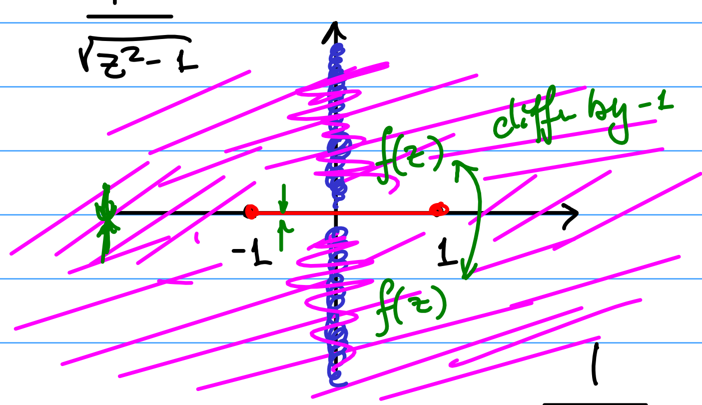
$\log(i) = i\frac{\pi}{2}$

$\text{Ln}(i) = -i\frac{\pi}{2}$

$\log(i) = i\frac{3\pi}{2}$

$$\lim_{\varepsilon\to 0} f(x+\varepsilon i) = -\lim_{\varepsilon\to 0} f(x-\varepsilon i)$$

$\dfrac{1}{\sqrt{z^2-1}}$



$$(z^2-1)^{-1/2} = e^{-\frac{1}{2}\ln(z^2-1)}$$

$$\frac{1}{\sqrt{z^2-1}} = \begin{cases} e^{-\frac{1}{2}\text{Ln}(z^2-1)}, & \text{Re}(z)>0,\ z\notin[0,1] \\ e^{-\frac{1}{2}\log(z^2-1)} & \text{Re}(z)<0,\ z\notin[-1,0] \end{cases}$$