APPLICATION COVER FORM

FEDERAL RESEARCH GRANT WRITING MENTORSHIP PROGRAM

Name: William Riley Casper Extension: _____

College: NSM Email: wcasper@fullerton.edu

Department: Mathematics Faculty Rank: Assistant Professor

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Along with this signed form, please provide the following information for consideration (12 pt. Arial font, 1" margins) in a single native PDF*:

1. A personal statement that speaks to your readiness to submit a proposal to an external funding agency, background experience and prior accomplishments. Outline your research topic and area of interest, and describe how far you are in developing a research agenda (maximum 350 words);
2. A project summary of the research project you intend to develop into a proposal, including the target federal funding opportunity and why you think the agency is a good fit for your proposed project (maximum 500 words);
3. A list of your previous experience (pre-, postdoc, or faculty) applying for research funding and/ or fellowships. Please include the following information: year proposal written/submitted, agency or sponsor, intramural or extramural, status of proposal (funded/not funded/pending);
4. An abbreviated CV (two pages, maximum).

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Please check the box and sign/date this form.*

☑ I agree to attend all required virtual cohort meetings and spend summer 2021 writing a federal grant proposal. I agree to be present and fully engaged in the program and work toward developing and submitting an external grant proposal within 12 months of completing the program.

Cohort Meeting Dates (Fridays, 9:00 a.m. – 12:00 p.m.):
January 29, 2021; February 26, 2021; March 26, 2021; April 23, 2021;

Summer Workshop: June 21-25, 2021 (Morning sessions: 9:00 a.m. – 12:00 p.m.; Afternoon sessions: Individual Writing Time)

W. Riley Casper
Faculty Signature

12/08/2020
Date

**This Application Form may be electronically signed or printed out, signed and scanned. However, the application materials must be uploaded as native .pdf files and may not be scanned.*

Personal Statement

William Riley Casper

Background. I am in my first year as an assistant professor in the Mathematics Department at California State University Fullerton. My research experience includes a three-year postdoctoral position at Louisiana State University working with Milen Yakimov and multiple research internships at Los Alamos National Lab. I have applied to multiple extramural research grants and fellowships funded by the National Science Foundation, the American Mathematical Society, and the Simons Foundation, including a successful application for an AMS-Simons Travel Grant. My research has led to publications in some of the most prestigious journals in my field, including Crelle's Journal, the American Journal of Mathematics, and Proceedings of the National Academy of Sciences.

Research Agenda. My research agenda focuses on leveraging interactions between algebra and integrable systems. An algebra is a collection A of objects like the set of whole numbers or the set of real numbers, where the operations of adding objects and multiplying objects are defined. Algebras in general are incredibly natural constructions, and show up naturally in vertually every area of mathematics, including analysis, geometry, combinatorics and number theory. Integrable systems studies systems of equations arising from physical phenomena where we can explicity solve the equations. Consequently, integrable systems connects directly to diverse beautiful phenomena in theoretical physics, such as nonlinear waves and quantum gravity. Both topics are connected together via the algebraic theory of certain nonlinear partial differential equations initiated by Krichever, Mumford, Sato, and others.

Leveraging algebraic techniques in the realm of integrable systems, my collaborators and I have been able to approach multiple problems in the related fields of spectral theory, approximation theory, and special functions from a unique perspective. So far, our work has solved the following two long-standing open problems in integrable systems and special functions

- the matrix Bochner problem – the problem of classifying systems of differential equations which are symmetric with respect to a matrix-valued inner product defined by a weight measure
- connecting bispectrality and prolate-spheroidal phenomena – showing that integral operators associated with bispectral functions satisfy the prolate-spheroidal property of commuting with a differential operator

Project Summary

William Riley Casper

Overview. The NSF DMS annually accepts proposals for research projects in algebra and number theory, with funding for research related equipment, travel expenses, conference support and summer salary for up to three years (for 2020, see NSF PD 20-1264). The following research summary is intended for pursuing funding for research through this program during the 2021 funding cycle. The project below fits with the NSF's goal of generating new knowledge, and with the specific program's target of supporting research in algebra, algebraic and arithmetic geometry, number theory, and representation theory.

Summary. In the 1830's, the scientist John Scott Russell described racing on horseback to follow strange, fast-moving water waves generated by boats in a shallow channel. Today, almost 200 years later, Russell's waves remain a topic of great interest. They are examples of solitons: nonlinear waves described by the Kadomtsev-Petviashvili (KP) equation. Incredibly, the KP equation arises in the otherwise completely unrelated topic of algebraic geometry via Shiota's famous solution of the Schottky problem. In fact, this fascinating connection between the KP equation an algebraic geometry allows us obtain explicit solutions of the KP equation, or more generally the KP hierarchy, in terms of certain special functions in algebraic geometry called theta functions.

Solutions of the KP hierarchy in general are classified by the motion of points in an infinite dimensional space, Sato's grassmannian Gr, under a natural group action. Motivated by these connections, the proposed research project will discover new connections with the KP hierarchy, expanding on recent results of the PI which fit the same theme (see Publications Related to the Proposed Project in the Brief CV). Additionally, our project aims to incorporate related undergraduate research project opportunities.

To motivate our work and highlight the potential for undergraduate collaboration, consider classic boardgame Battleship, but with "radar", where you scan the enemy fleet, giving you the sum of positions occupied by enemy ships in a specified row or column. Assuming that you've scanned your opponents fleet so that you know all the row and column sums, how do you now reconstruct the fleet? Abstractly, this problem is really about studing the linear transformation taking a battleship fleet to the collection of row and column sums.

The linear transformation described in the previous paragraph is an example of a (discrete) integral operator with a special property called the "prolate spheroidal property", that it commutes with a (discrete) differential operator. In recent work, the PI and his collaborators established the existence of huge class of integral operators with the "prolate spheroidal property", vastly expanding the handful of examples previously known coming from spherical functions and random matrix theory. These integral operators are constructed by using certain special functions called bispectral functions, whose classification is again described by Gr, and thus linked with the KP equation. Our proposed project will leverage this connection to impose *dynamics* on prolate-spheroidal operators for the first time, with important implications to both random matrix theory and integrable systems.

Brief CV

William Riley Casper

1 Professional Preparation

University of Washington	Seattle, WA	Mathematics	Ph.D 2017
North Dakota State Unversity	Fargo, ND	Mathematics	M.S. 2010[1]
North Dakota State Unversity	Fargo, ND	Mathematics	B.S. 2010
North Dakota State Unversity	Fargo, ND	Physics	B.S. 2010

[1] all three degrees were completed simultaneously

2 Appointments

2020-Present	Assistant Professor of Mathematics, California State University, Fullerton, CA
2017-2020	Postdoctoral Researcher, Louisiana State University, Baton Rouge, LA
2011-2017	Graduate Student Instructor, UW Seattle (fall, winter, and spring)
2011-2017	Graduate Research Assistant, Los Alamos National Lab (summer)
2010-2011	Post-Bac Research Assistant, Los Alamos National Lab (year-long)
2007-2010	Teaching Assistant, North Dakota State University (year-long)

3 Publications

Publications Related to the Proposed Project

1. Casper, W. Riley and Milen Yakimov "The Matrix Bochner Problem," American Journal of Mathemtics 2020 (to appear). arXiv:1803.04405

2. Casper, W. Riley, Stefan Kolb and Milen Yakimov "Bivariate Continuous q-Hermite Polynomials and Deformed Quantum Serre Relations," Journal of Algebra and its Applications 2020 (to appear). arXiv:2002.07895

3. Casper, W. Riley, F. Alberto Grünbaum, Milen Yakimov, and Ignacio Zurrián, "Reflective prolate-spheroidal operators and the KP/KdV equations," Proc. Natl. Acad. of Sci. USA 2019, 116(37) 18310-18315. arXiv:1909.01448

4. Casper, W. Riley and Milen Yakimov, "Integral operators, bispectrality and growth of Fourier algebras," J. Reine Angew. Math (Crelle's Journal) 2019 doi:10.1515/crelle-2019-0031. arxiv:1807.09314

5. Casper, W. Riley "Elementary Examples of Solutions to Bochner's Problem for Matrix Differential Operators." Journal of Approximation Theory, 2018, 229:36-71. arxiv:1509.03674

6. Casper, W. Riley "The symmetric 2×2 hypergeometric matrix differential operators." preprint, 2019, submitted for publication. arXiv:1907.12703

Other Publications

7. Casper, W. Riley "A Connection Between Orthogonal Polynomials and Shear Instabilities in the QG Shallow Water Equations." arXiv preprint 1710.02756, 2017.

8. Casper, W. Riley and Balasubramanya Nadiga "A new spectral clustering algorithm." (submitted) arXiv preprint 1710.02756, 2017.

9. Coles, Patrick J. et al "Quantum Algorithm Implementation for Beginners." arXiv preprint 1804.03719, 2018.

10. Nadiga, Balasubramanya T., W. Riley Casper, and Philip W. Jones. "Ensemble-based global ocean data assimilation." Ocean Modelling 72 (2013): 210-230.

4 Synergistic Activities

Referee/Review Work

Ongoing	Reviewer for Zentralblatt MATH
Ongoing	Referee for Communications in Mathematical Physics, Journal of Approximation Theory, SIAM Journal on Mathematical Analysis, Studies in Applied Mathematics, and the International Electronic Journal of Geometry

Mentoring/Training

2016	Graduate student mentor for the Washington Experimental Mathematics Laboratory
2016	Graduate student mentor for the Los Alamos Summer School in Computational Physics
2015-16	Volunteer college math instructor at the Washington Corrections Center for Women (WCCW) as part of the Freedom Education Project of Puget Sound (FEPPS)
2015	Organizer for a weekly algebraic geometry seminar for students at Los Alamos National Lab, a student-run seminar discussing algebraic geometry and its scientific applications
2007-19	Tutor, teacher, grader, and mentor for diverse mathematics classes
Ongoing	Speaker multiple times for Problem Solving Seminar at CSUF
Ongoing	Research mentor for multiple undergraduate research teams at CSUF

Invited Conference Talks:

2018	ICM Satellite Conference in Cusco, Peru
2018	AMS-CMS Joint Math Conference in Shanghai, China
2018	Geometry and Physics XVI in Timisoara, Romania
2019	Orthogonal Polynomials, Special Functions and Appications (OPSFA) in Hagenberg, Austria
2019	Matrix-valued Special Functions and Integrability in Nijmegen, Netherlands
2020	13th AIMS Conference on Dynamical Systems, Differential Equations and Applications: *Algebraic and Geometric Methods in Nonlinear Differential Equations* in Atlanta, GA
2020	AMS Sectional Meeting at Tufts University, Medford, MA
2020	Orthogonal Polynomials, Special Functions, Operator Theory and Applications in Kent, UK

Previous Extramural Grant Applications

William Riley Casper

(a) 2017 NSF Postdoctoral Fellowship in Mathematics (not funded)

(b) 2018 NSF Postdoctoral Fellowship in Mathematics 2018 (not funded)

(c) 2018 AMS–Simons Foundation Mathematics Research Travel Grant (funded)

(d) 2019 NSF Research Grant in Algebra and Number Theory (not funded)

(e) 2020 NSF Research Grant in Algebra and Number Theory (pending)