$$\vec{u} \cdot \vec{\nabla} = \sum_{k=1}^{N} u_k v_k = u_i v_i + u_2 v_2 + ... + u_n v_n$$

$$\frac{\exists x}{\hat{u}} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \end{bmatrix}, \quad \vec{\nabla} = \begin{bmatrix} \frac{2}{0} \\ \frac{1}{0} \end{bmatrix} \Rightarrow \quad \vec{u} \cdot \vec{\nabla} = 1 \cdot 2 + 2 \cdot 0 + 3 \cdot 1 = 5$$

Idea: use the dot product to multiply a vector by a matrix!

Two ways to write a matrix (A mxn matrix)

•
$$A = \begin{bmatrix} \vec{r}_1 \\ \vec{r}_2 \end{bmatrix}$$
 $\vec{r}_1, \vec{r}_2, ...$ are the rows of A

Def: We define the product of A times an nx1 column vector to to be

$$A\vec{b} = \begin{bmatrix} \vec{r}_1 \cdot \vec{b} \\ \vec{r}_2 \cdot \vec{b} \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 5 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 - 1 \\ 0 & 1 & 2 \\ 2 & 1 & 0 \\ 1 & 1 & 3 \end{bmatrix}$$

$$A\vec{b} = \begin{bmatrix} 1 \cdot (1 + 0 \cdot 0 + (-1)(-1)) \\ 0 \cdot (1 + (-1)(-1)) \\ 2 \cdot (1 + (-1)(-1)) \\ 4 \cdot (1 + (-1)(-1)) \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ 2 \\ 4 \cdot (1 + (-1)(-1)) \end{bmatrix}$$

4×1

Observation: If A is mxn and b is nx1, then Ab is mx1

CAUTION! Ab only makes sense if the number of columns of A is the same as the next of tows of b next bill by Special Case:
$$\vec{a} = [a_1 \ a_2 \ a_3 ... \ a_n]$$
 $\vec{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$

$$\vec{a}\vec{b} = \vec{a}\cdot\vec{b}$$

Product of Madrices:

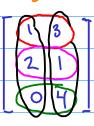
$$A = \begin{bmatrix} \vec{a}_1 \\ \vec{a}_2 \end{bmatrix}$$
 mxn matrix (each \vec{a}_1 has lungth n)

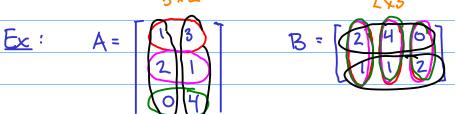
Def: The product of A and B is

$$= \begin{bmatrix} \vec{a}_{1} \cdot \vec{b}_{1} & \vec{a}_{1} \cdot \vec{b}_{2} & \vec{a}_{1} \cdot \vec{b}_{1} \\ \vec{a}_{2} \cdot \vec{b}_{1} & \vec{a}_{2} \cdot \vec{b}_{2} & \vec{a}_{2} \cdot \vec{b}_{1} \\ \vdots & \vdots & \ddots & \vdots \\ \vec{a}_{m} \cdot \vec{b}_{1} & \vec{a}_{m} \cdot \vec{b}_{2} & \vec{a}_{m} \cdot \vec{b}_{4} \end{bmatrix}$$

mx matrix

Observation: If A mxn and B nxl, Sun AB mxl Caution: For AB to make gunge, the # cols of A = # rows of B





$$AB = \begin{bmatrix} 1.2+3.1 & 1.4+3.1 & 1.0+3.2 \\ 2.2+1.1 & 2.4+1.1 & 2.0+1.2 \\ 0.2+4.1 & 0.4+4.1 & 0.0+4.2 \end{bmatrix} = \begin{bmatrix} 5 & 7 & 6 \\ 5 & 9 & 2 \\ 4 & 4 & 8 \end{bmatrix}$$

Sergio predicts AB & BA

$$\frac{Ex}{2}: A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ & 3 & 1 & 1 \\ & & & & \\ & & & \\ &$$

$$AB = \begin{bmatrix} 1.0+2.1 & 1.1+2.0 & 1.1+2.1 \\ 3.0+1.1 & 3.1+1.0 & 3.1+1.1 \\ 0.0+2.1 & 0.1+2.0 & 0.1+2.1 \\ 1.0+1.1 & 1.1+1.0 & 1.1+1.1 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 3 & 4 \\ 2 & 0 & 2 \\ 1 & 1 & 2 \end{bmatrix}$$

Properties of Matrix Multiplication A man matrix

B,C nxl matrices D lxp matrix

distributivity
$$A(B+C) = AB+AC$$

 $(B+C)D = BD+CD$

$$(B+C)D = BD + CD$$
 $c = Scalars = (AB) = A(AB)$

Some properties that don't hold:

$$\underline{\mathsf{E}_{\mathsf{X}}}: \quad \mathsf{A} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \qquad \mathsf{B} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

$$BA = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$$

AB & BA non-commutativity

$$AB = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 12 \\ 34 \end{bmatrix} = \begin{bmatrix} 34 \\ 0 & 0 \end{bmatrix}$$
 $\begin{bmatrix} 54 \\ 54 \end{bmatrix}$
 $\begin{bmatrix} 54 \\ 74 \end{bmatrix}$
 $\begin{bmatrix} 54 \\ 74 \end{bmatrix}$

$$AC = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 5 & 6 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 34 \\ 0 & 0 \end{bmatrix} \qquad B \neq C$$

Def: A Square matrix i3 a matrix w/ Mu Same # 1865 and # colums.

$$A^{2} = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A^{k} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A^{3} = AA^{2} = \begin{bmatrix} 0 & 11 \\ 0 & 01 \\ 0 & 00 \end{bmatrix} \begin{bmatrix} 0 & 01 \\ 0 & 00 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$
 even things

$$A^{4} = AA^{3} = \begin{bmatrix} 011 \\ 000 \\ 000 \end{bmatrix} \begin{bmatrix} 000 \\ 000 \\ 000 \end{bmatrix} = \begin{bmatrix} 000 \\ 000 \\ 000 \end{bmatrix}$$

$$I_m A = A$$
 and $AI_n = A$

$$\frac{E_{X}:}{2 \cdot 1} \begin{bmatrix} 1 \cdot 0 \\ 0 \cdot 1 \end{bmatrix} = \begin{bmatrix} 3 \cdot 1 + 1 \cdot 0 & 3 \cdot 0 + 1 \cdot 1 \\ 2 \cdot 1 + 1 \cdot 0 & 2 \cdot 0 + 1 \cdot 1 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 \cdot 3 + 0 \cdot 7 & 1 \cdot 1 + 0 \cdot 1 \\ 0 \cdot 3 + 1 \cdot 7 & 0 \cdot 1 + 1 \cdot 1 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$$