Linear combinations

In linear algebra there are two key algebraic operations
- vector/matrix addition
- scalar multiplication

$$\begin{bmatrix} 1 & 3 \\ 4 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}$$ matrix addition

$$3\begin{bmatrix} 2 & 1 & 2 \\ 3 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 6 & 3 & 6 \\ 9 & 0 & 3 \end{bmatrix}$$ scalar multiplication

$$2\begin{bmatrix} 1 & 3 & 1 \\ 4 & 0 & 2 \end{bmatrix} + 3\begin{bmatrix} 0 & 1 & 2 \\ 2 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 6 & 2 \\ 8 & 0 & 4 \end{bmatrix} + \begin{bmatrix} 0 & 3 & 6 \\ 6 & 3 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 9 & 8 \\ 14 & 3 & 4 \end{bmatrix}$$ linear combination

<u>Def</u>: A <u>linear combination</u> of A and B is $aA + bB$ for some scalars a, b.

A linear combination is a <u>convex combination</u> when $a+b=1$ and $0 \le a \le 1$. (Weighted average of A and B)

<u>Ex</u>: $\frac{1}{2}A + \frac{1}{2}B$ is just the average of the matrices

$\frac{3}{4}A + \frac{1}{4}B$ is a weighted average w/ a little more A than B

Every convex combo is of the form $aA + (1-a)B$, $0 \le a \le 1$

<u>Important problem</u>: S'pose we know C is a linear combination of A and B. $C = aA + bB$.

How can we figure out a and b?

Ex: Find a, b so that

$$\begin{bmatrix} 2 \\ 3 \end{bmatrix} = a\begin{bmatrix} 1 \\ -1 \end{bmatrix} + b\begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} a \\ -a \end{bmatrix} + \begin{bmatrix} 2b \\ b \end{bmatrix}$$

$$\begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} a+2b \\ -a+b \end{bmatrix}$$

$$\left.\begin{aligned} 2 &= a + 2b \\ 3 &= -a+b \end{aligned}\right\}$$

$5 = 0 + 3b$

$5 = 3b$

$b = 5/3$

$2 = a + 2\frac{5}{3}$

$2 = a + \frac{10}{3}$

$a = -\frac{4}{3}$

$$\boxed{\begin{bmatrix} 2 \\ 3 \end{bmatrix} = -\frac{4}{3}\begin{bmatrix} 1 \\ -1 \end{bmatrix} + \frac{5}{3}\begin{bmatrix} 2 \\ 1 \end{bmatrix}}$$

More general linear combinations

$A, B, C, D, \dots$ matrices

$a, b, c, d, \dots$ scalars

$aA + bB + cC + dD + \dots$ linear combination

Quest: Is $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ a linear combination of $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ and $\begin{bmatrix} 3 & 2 \\ 1 & 0 \end{bmatrix}$?

Yes $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} = 1 \cdot \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} + 0 \cdot \begin{bmatrix} 3 & 2 \\ 1 & 0 \end{bmatrix}$

Quest: Is $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 2 \\ 1 & 0 \end{bmatrix}$ a linear combination?

YES! $(1)\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} + (-1)\begin{bmatrix} 3 & 2 \\ 1 & 0 \end{bmatrix}$

Quest: Is $\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ a linear combination of

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 2 & 3 \\ 0 & 4 \end{bmatrix}$?

NO!

$$a\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} + b\begin{bmatrix}2 & 3\\ 0 & 4\end{bmatrix} = \begin{bmatrix}a+2b & 3b\\ 0 & a+4b\end{bmatrix}$$

coefficients