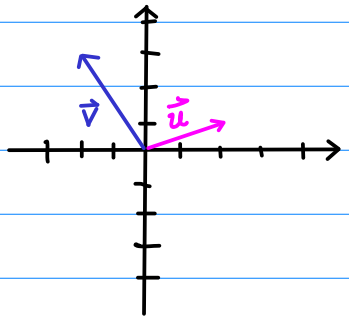
Plan for today:

1) geometry of vectors
2) exam 1 (one week from today)

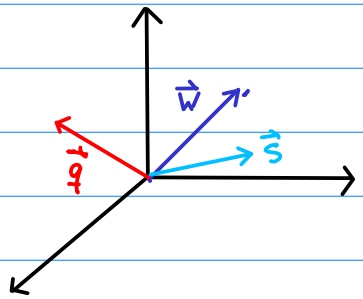
Geometry of Vectors

We can describe a vector w/ an arrow.



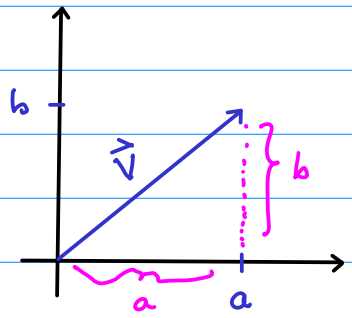
$\vec{u} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$\vec{v} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$



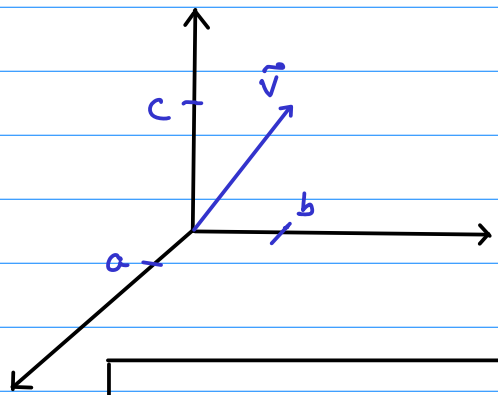
$\vec{s}$, **s**, s

Magnitude of a vector:



Magnitude of $\vec{v}$ is the length

$\|\vec{v}\| = \sqrt{a^2 + b^2}$ for $\vec{v} = \begin{pmatrix} a \\ b \end{pmatrix}$



Magnitude of $\vec{v}$ is the length

$\|\vec{v}\| = \sqrt{a^2 + b^2 + c^2}$ for $\vec{v} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$.

In general, if $\vec{v} = \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix}$, $\|\vec{v}\| = \sqrt{v_1^2 + v_2^2 + \dots + v_n^2}$

Cool property about magnitude:

$$\vec{v} = \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix}, \quad c\vec{v} = \begin{pmatrix} cv_1 \\ cv_2 \\ \vdots \\ cv_n \end{pmatrix}$$

$$\|c\vec{v}\| = \sqrt{(cv_1)^2 + (cv_2)^2 + \dots + (cv_n)^2}$$

$$= \sqrt{c^2v_1^2 + c^2v_2^2 + \dots + c^2v_n^2}$$

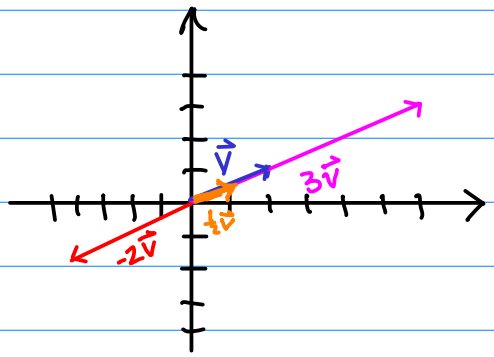
$$= \sqrt{c^2(v_1^2 + v_2^2 + \dots + v_n^2)}$$

$$= |c| \sqrt{v_1^2 + v_2^2 + \dots + v_n^2}$$

$$= |c| \cdot \|\vec{v}\|.$$

$$\boxed{\|c\vec{v}\| = |c| \cdot \|\vec{v}\|.}$$

Scalar multiplication



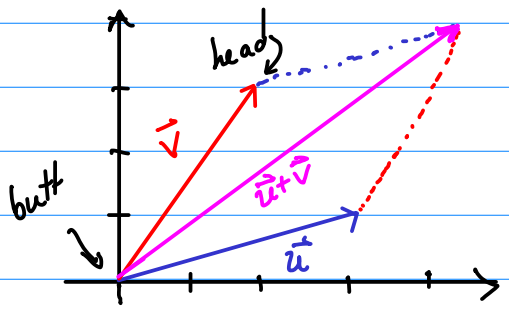
$$\vec{v} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$3\vec{v} = \begin{pmatrix} 6 \\ 3 \end{pmatrix}$$

$$-2\vec{v} = \begin{pmatrix} -4 \\ -2 \end{pmatrix}$$

$$\frac{1}{2}\vec{v} = \begin{pmatrix} 1 \\ 1/2 \end{pmatrix}$$

Vector addition

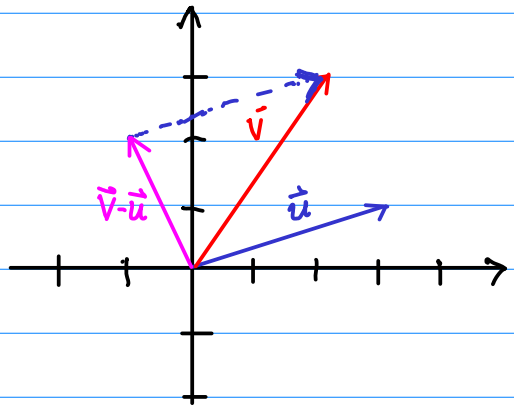


$$\vec{u} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

$$\vec{v} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

$$\vec{u}+\vec{v} = \begin{pmatrix} 5 \\ 4 \end{pmatrix}$$

Vector subtraction

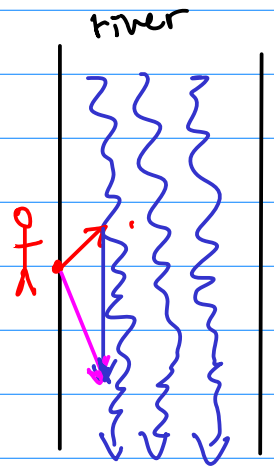


$\vec{u} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$

$\vec{v} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

$\vec{v}-\vec{u} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$

Ex:



I swim at 1 mph.

Actual velocity from combining both swimming + river pushing me downstream, use vector addition!

$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\sqrt{1^2+1^2} = \sqrt{2}$

Right direction, wrong magnitude!

7 mph South

$\begin{pmatrix} 0 \\ -7 \end{pmatrix}$

$\begin{pmatrix} x \\ x \end{pmatrix}$ magnitude = 1

$\sqrt{x^2+x^2} = 1$

$\sqrt{2x^2} = 1$

$\sqrt{2}\,x = 1$

$x = 1/\sqrt{2} = \sqrt{2}/2$

Actual combined velocity:

$$\begin{pmatrix} 0 \\ -7 \end{pmatrix} + \begin{pmatrix} \sqrt{2}/2 \\ \sqrt{2}/2 \end{pmatrix} = \boxed{\begin{pmatrix} \sqrt{2}/2 \\ \sqrt{2}/2 - 7 \end{pmatrix}}$$