

Problem 3 :

loop iteration	k	v(k)	x
1	1	2	-2
2	2	-1	2
3	3	4	-2
4	4	1	-3
5	5	-2	6
6	6	0	6

Stops because $N=6$ and now $k=7$.

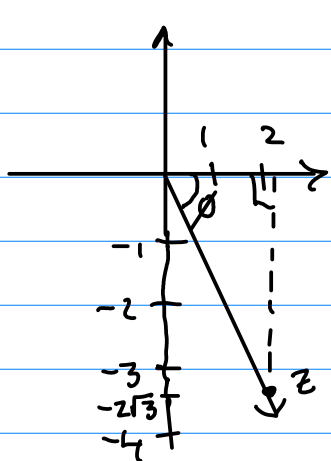
final value of k : 7

final value of x : 6

Problem 4 :

$$\begin{aligned} a) \quad \frac{w}{z} &= \frac{-2+3i}{1-4i} = \frac{-2+3i}{1-4i} \cdot \frac{1+4i}{1+4i} \\ &= \frac{-2+3i-8i+12i^2}{17} = \frac{-14-5i}{17} \end{aligned}$$

$$\begin{aligned} b) \quad z &= 2 - 2\sqrt{3}i \quad |z|^2 = (2)^2 + (-2\sqrt{3})^2 = 16 \\ r &= |z| = 4 \end{aligned}$$



$$\tan \phi = \frac{2\sqrt{3}}{2} = \sqrt{3}$$

$$\phi = \frac{\pi}{3}$$

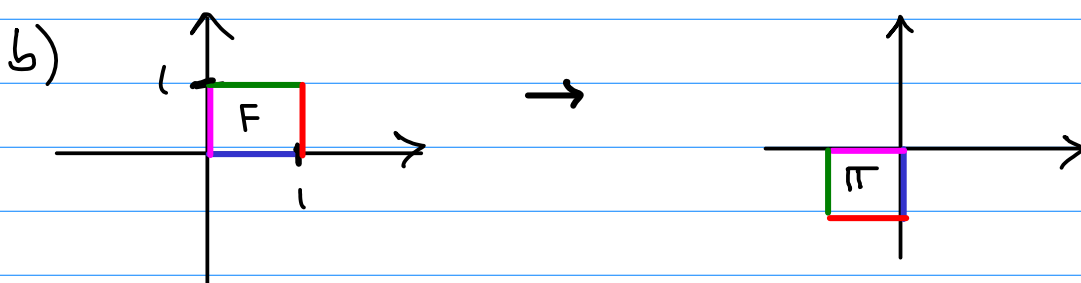
$$\theta = 2\pi - \phi = \frac{5\pi}{3}$$

$$z = 4e^{\frac{5\pi i}{3}}$$

$$\begin{aligned}
 c) \quad (2 - 2\sqrt{3}i)^{600} &= \left(4e^{\frac{5\pi}{3}i}\right)^{600} \\
 &= 4^{600} e^{\frac{5\pi}{3} \cdot 600i} = 4^{600} e^{1000\pi i} \\
 &= 4^{600} (e^{2\pi i})^{500} = 4^{600} (1)^{500} \\
 &= \boxed{4^{600}}
 \end{aligned}$$

Problem 5 :

$$a) \quad \begin{pmatrix} \cos(-\frac{\pi}{3}) & -\sin(-\frac{\pi}{3}) \\ \sin(-\frac{\pi}{3}) & \cos(-\frac{\pi}{3}) \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$$



Reflection across the line $y = -x$

c) Stretch in y by a factor of 2: $\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$

$$\text{rotation by } \pi/4: \begin{pmatrix} \cos(\frac{\pi}{4}) & -\sin(\frac{\pi}{4}) \\ \sin(\frac{\pi}{4}) & \cos(\frac{\pi}{4}) \end{pmatrix} = \begin{pmatrix} \sqrt{2}/2 & -\sqrt{2}/2 \\ \sqrt{2}/2 & \sqrt{2}/2 \end{pmatrix}$$

So what we want is the product

$$\begin{pmatrix} \sqrt{2}/2 & -\sqrt{2}/2 \\ \sqrt{2}/2 & \sqrt{2}/2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} = \boxed{\begin{pmatrix} \sqrt{2}/2 & -\sqrt{2} \\ \sqrt{2}/2 & \sqrt{2} \end{pmatrix}}$$

Problem 6: $\vec{v} - \vec{w} = \begin{pmatrix} 1 - (-3) \\ -2 - (-1) \end{pmatrix} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$

a)

$$\|\vec{v} - \vec{w}\| = \sqrt{4^2 + (-1)^2} = \sqrt{17}$$

b) $AB = \begin{pmatrix} 2 & 3 & -1 \\ 3 & 4 & 0 \end{pmatrix} \begin{pmatrix} 0 & -5 & 0 \\ -1 & 3 & 1 \\ 0 & 2 & -1 \end{pmatrix} = \begin{pmatrix} -3 & -3 & 4 \\ -4 & -3 & 4 \end{pmatrix}$

BA does not make sense

c) $AB = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} (2 \ 1 \ 0) = \begin{pmatrix} 6 & 3 & 0 \\ 2 & 1 & 0 \\ 4 & 2 & 0 \end{pmatrix}$

$$BA = (2 \ 1 \ 0) \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} = (7)$$

Problem 7:

a) $\left[\begin{array}{ccc|c} 3 & -2 & 1 & 4 \\ 1 & 3 & -1 & -3 \\ 4 & -10 & 4 & 10 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 1/11 & 0 \\ 0 & 1 & -4/11 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right]$

$$\begin{cases} x + \frac{1}{11}z = 0 \\ y - \frac{4}{11}z = 0 \\ 0 = 1 \end{cases} \quad \text{inconsistent!}$$

b) $\left[\begin{array}{ccc|c} 1 & 2 & 3 & 9 \\ 2 & 0 & -2 & -2 \\ 3 & 2 & 1 & 7 \end{array} \right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 0 \end{array} \right]$

$$\begin{cases} x = -1 \\ y = 5 \\ z = 0 \end{cases} \quad \text{unique solution}$$

c) $\left[\begin{array}{ccc|c} 3 & -2 & 4 & 7 \\ 2 & 1 & 0 & -3 \\ 2 & 8 & -9 & 2 \end{array} \right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 4/7 & 0 \\ 0 & 1 & -8/7 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right]$

$$\begin{cases} x + \frac{4}{7}z = 0 \\ y - \frac{8}{7}z = 0 \\ 0 = 1 \end{cases} \quad \text{inconsistent!}$$