Problem 3 :

loop iteration	k	v(k)	x
1	1	2	-2
2	2	-1	2
3	3	4	-2
4	4	1	-3
5	5	-2	6
6	6	0	6

Stops because N = 6 and now k = 7.

final value of k : 7

final value of x : 6

Problem 4 :

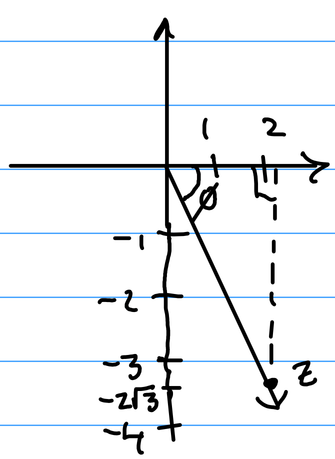
a) $\frac{w}{z} = \frac{-2+3i}{1-4i} = \frac{-2+3i}{1-4i}\,\frac{1+4i}{1+4i}$

$= \frac{-2+3i-8i+12i^2}{17} = -\frac{14}{17} - \frac{5}{17}i$

b)

$z = 2-2\sqrt{3}\,i \qquad |z|^2 = (2)^2 + (-2\sqrt{3})^2 = 16$

$r = |z| = 4$



$\tan\phi = \frac{2\sqrt{3}}{2} = \sqrt{3}$

$\phi = \frac{\pi}{3}$

$\theta = 2\pi - \phi = \frac{5\pi}{3}$

$\boxed{z = 4e^{\frac{5\pi}{3}i}}$

c) $$\left(2-2\sqrt{3}\,i\right)^{600} = \left(4e^{\frac{5\pi}{3}i}\right)^{600}$$

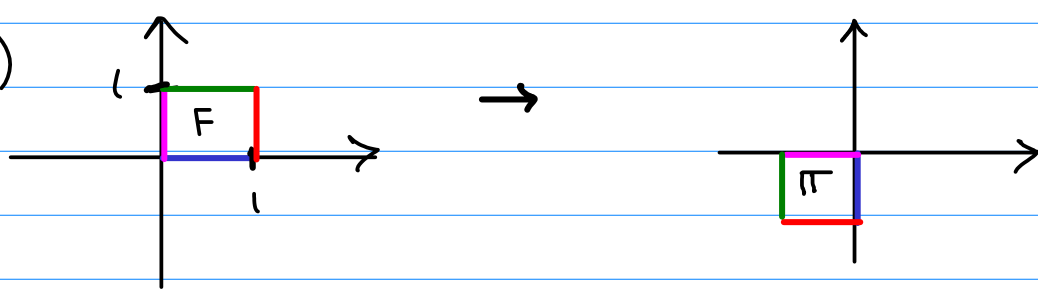
$$= 4^{600} e^{\frac{5\pi}{3}\cdot 600 i} = 4^{600} e^{1000\pi i}$$

$$= 4^{600}\left(e^{2\pi i}\right)^{500} = 4^{600}(1)^{500}$$

$$= \boxed{4^{600}}$$

Problem 5:

a) $$\begin{pmatrix} \cos\left(-\frac{\pi}{3}\right) & -\sin\left(-\frac{\pi}{3}\right) \\ \sin\left(-\frac{\pi}{3}\right) & \cos\left(-\frac{\pi}{3}\right) \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$$

b) 

Reflection across the line $y=-x$

c) Stretch in y by a factor of 2: $\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$

rotation by $\pi/4$: $\begin{pmatrix} \cos\left(\frac{\pi}{4}\right) & -\sin\left(\frac{\pi}{4}\right) \\ \sin\left(\frac{\pi}{4}\right) & \cos\left(\frac{\pi}{4}\right) \end{pmatrix} = \begin{pmatrix} \sqrt{2}/2 & -\sqrt{2}/2 \\ \sqrt{2}/2 & \sqrt{2}/2 \end{pmatrix}$

So what we want is the product

$$\begin{pmatrix} \sqrt{2}/2 & -\sqrt{2}/2 \\ \sqrt{2}/2 & \sqrt{2}/2 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} = \boxed{\begin{pmatrix} \sqrt{2}/2 & -\sqrt{2} \\ \sqrt{2}/2 & \sqrt{2} \end{pmatrix}}$$

<u>Problem 6</u>: $\vec{v}-\vec{w} = \begin{pmatrix} 1-(-3) \\ -2-(-1) \end{pmatrix} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$

a)

$$\|\vec{v}-\vec{w}\| = \sqrt{4^2+(-1)^2} = \sqrt{17}$$

b) $AB = \begin{pmatrix} 2 & 3 & -1 \\ 3 & 4 & 0 \end{pmatrix}\begin{pmatrix} 0 & -5 & 0 \\ -1 & 3 & 1 \\ 0 & 2 & -1 \end{pmatrix} = \begin{pmatrix} -3 & -3 & 4 \\ -4 & -3 & 4 \end{pmatrix}$

BA does not make sense

c) $AB = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}(2\ 1\ 0) = \begin{pmatrix} 6 & 3 & 0 \\ 2 & 1 & 0 \\ 4 & 2 & 0 \end{pmatrix}$

$BA = (2\ 1\ 0)\begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} = (7)$

<u>Problem 7</u>:

a) $\left[\begin{array}{ccc|c} 3 & -2 & 1 & 4 \\ 1 & 3 & -1 & -3 \\ 4 & -10 & 4 & 10 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & 1/11 & 0 \\ 0 & 1 & -4/11 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$

$$\begin{cases} x + \frac{1}{11}z = 0 \\ y - \frac{4}{11}z = 0 \\ 0 = 1 \end{cases}$$

inconsistent!

b) $\left[\begin{array}{ccc|c} 1 & 2 & 3 & 9 \\ 2 & 0 & -2 & -2 \\ 3 & 2 & 1 & 7 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -1 & -1 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 1 & 0 \end{array}\right]$

$$\begin{cases} x = -1+z \\ y = 5-2z \\ z = \text{free} \end{cases}$$

unique solution

c) $\left[\begin{array}{ccc|c} 3 & -2 & 4 & 7 \\ 2 & 1 & 0 & -3 \\ 2 & 8 & -8 & 2 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 4/7 & 0 \\ 0 & 1 & -8/7 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$

inconsistent!

$$\begin{cases} x + \frac{4}{7}z = 0 \\ y - \frac{8}{7}z = 0 \\ 0 = 1 \end{cases}$$