Math 107 Section 2   Name (Print): _____
Spring 2022
Final Exam   Student ID: _____
May 16, 2022
Time Limit: 50 Minutes

This exam contains 6 pages (including this cover page) and 5 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books or notes on this exam. However, you may use a *basic* calculator.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.

- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit. This especially applies to limit calculations.

- If you need more space, use the back of the pages; clearly indicate when you have done this.

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total:	50	

Do not write in the table to the right.

1. (10 points)

   Consider the following lines of MATLAB code. Determine the final values of the variables k and m. Carefully show your work by filling in the missing values in the table below. Note that not all rows will necessarily be used!

```matlab
k = 13;
m = 0;

while k ~= 1
  if mod(k,2) == 1
    k = 3*k + 1;
  else
    k = k/2;
  end

  m = m + 1;
end
```

loop iteration	m	k
1	1	40
2	2	
3		
4	4	5
5		
6		
7		
8		
9		
10		
11		

- **Final k value:**
- **Final x value:**

2. (10 points)

   For each of the following, write TRUE if the statement is true or FALSE if the statement is false. No justification is required.

   (a) A homogeneous linear system of equations always has a solution.

   (b) If x is a real number satisfying $e^{ix} = 0$, then $x = 0$.

   (c) If A is any matrix and A^T is its transpose, then the matrices AA^T and $A^T A$ are the same.

   (d) The product of a complex number z with its complex conjugate $\overline{z}$ is always real.

   (e) If the RGB values of a certain pixel are $(255, 255, 255)$ then that pixel is bright white.

3. (10 points)

(a) Find the values of a for which the following matrix is invertible

$$A = \begin{bmatrix} 1 & 3 \\ a & 2 \end{bmatrix}$$

(b) Give an example of two 2×2 matrices A and B with $AB \neq BA$

(c) Calculate explicitly the value of the inverse of

$$A = \begin{pmatrix} 7 & 2 & 1 \\ 0 & 3 & -1 \\ -3 & 4 & -2 \end{pmatrix}.$$

Then use the value of the inverse to find a solution of the linear system of equations

$$\begin{cases} 7x + 2y + z & = 21 \\ 3y - z & = 5 \\ -3x + 4y - 2z & = -1 \end{cases}$$

4. (10 points) Consider the matrix

$$A = \begin{bmatrix} 5 & 2 \\ -3 & 0 \end{bmatrix}$$

(a) Calculate the eigenvalues of A by hand. Carefully show your work.

(b) For each eigenvalue of A, find all eigenvectors with that eigenvalue by hand.

5. (10 points)

   (a) Find a complex number z different from 1 satisfying $z^7 = 1$. [Hint: think about Euler's formula]

   (b) On a particularly busy day a catering business sold 85 sandwiches, 65 bags of chips, and 210 cookies for a lunch event. They observed that men each ate 2 sandwiches, 1 bag of chips, and 4 cookies; women ate 1 sandwich, 1 bag of chips, and 2 cookies; kids ate half a sandwich, a bag of chips, and 3 cookies. Set up, but do not solve, the linear system of equations described by this story problem.

   (c) Give an example of a linear system of equations with no solutions

   (d) Give an example of a linear system of equations with infinitely many solutions

   (e) Describe in your own words one application of eigenvectors and eigenvalues