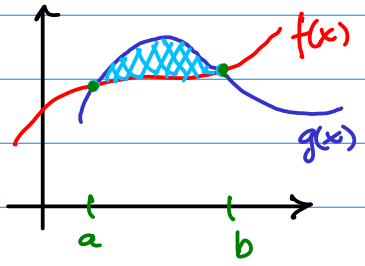
Areas between curves :

The area between curves is obtained by integrating differences of functions



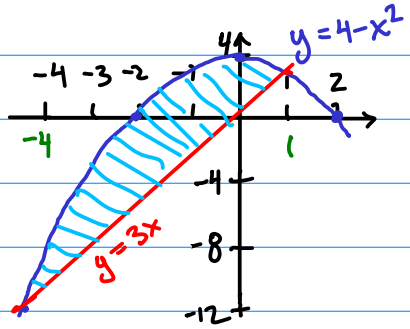
$$A = \int_a^b g(x) - f(x)\, dx$$

$$\int \text{"top curve"} - \text{"bottom curve"}\, dx$$

Ex: Calculate the area of the region bounded by $y = 4 - x^2$ and $y = 3x$

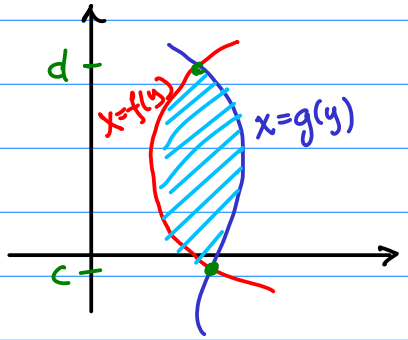
Steps:

(1) Draw a picture
(2) Set up integral
(3) Evaluate



$$A = \int_{-4}^{1} 4 - x^2 - 3x\, dx = \frac{125}{6}$$

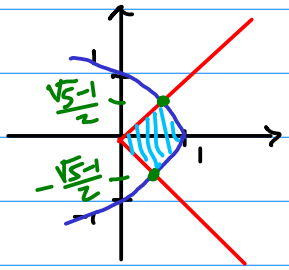
Areas sometimes need to be evaluated from left to right.



$$A = \int_c^d g(y) - f(y)\, dy$$

$$\int \text{"right curve"} - \text{"left curve"}\, dy$$

Ex: Find area of region bounded by $x + y^2 = 1$ and $x = |y|$



$$A = \int_{-\frac{\sqrt{5}-1}{2}}^{\frac{\sqrt{5}-1}{2}} 1 - y^2 - |y|\, dy = 2\int_0^{\frac{\sqrt{5}-1}{2}} 1 - y^2 - y\, dy$$

$$= \frac{1}{12}(5\sqrt{5} - 7)$$