

Problem 1. For which values of c is $y = c/x^2$ a solution of the differential equation

$$y'' = 3y^2.$$

$$y' = -2c/x^3$$

$$y'' = 6c/x^4$$

$$y^2 = c^2/x^4$$

$$\frac{6c}{x^4} = 3 \frac{c^2}{x^4}$$

$$\Rightarrow 2c = c^2 \Rightarrow$$

$$c = 0, 2$$

Problem 2. Determine a solution of the initial value problem

$$y' = 2y(10 - y), \quad y(0) = 1.$$

$$\int \frac{1}{y(10-y)} dy = \int 2 dx$$

$$\int \frac{1/10}{y} + \frac{1/10}{10-y} dy = \int 2 dx$$

$$\frac{1}{10} (\ln(y) - \ln(10-y)) = 2x + C_1$$

$$\ln\left(\frac{y}{10-y}\right) = 20x + C_2$$

$$\frac{y}{10-y} = C_3 e^{20x}, \quad y(0) = 1 \Rightarrow \frac{1}{9} = C_3$$

$$\frac{y}{10-y} = \frac{1}{9} e^{20x}$$

$$y = \frac{10}{9} e^{20x} - \frac{1}{9} y e^{20x}$$

$$y \left(1 + \frac{1}{9} e^{20x}\right) = \frac{10}{9} e^{20x}$$

$$y = \frac{\frac{10}{9} e^{20x}}{1 + \frac{1}{9} e^{20x}}$$