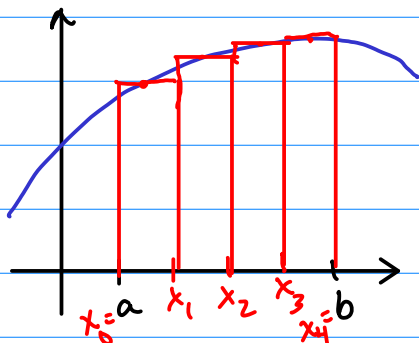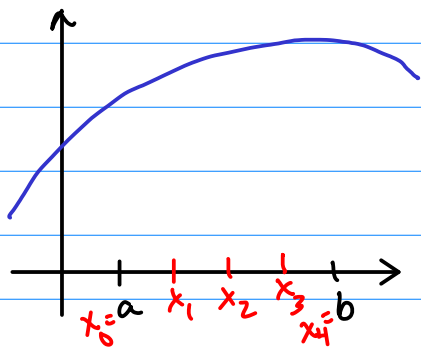# Approximating Integrals

Midpoint rule

Formula $\Delta x = \frac{b-a}{n}$

$$A_M = \Delta x \left[f\left(\frac{x_0+x_1}{2}\right) + f\left(\frac{x_1+x_2}{2}\right) + ... + f\left(\frac{x_{n-1}+x_n}{2}\right) \right]$$

Error

$$|E_M| \leq \frac{K(b-a)^3}{24\, n^2} \quad , \quad K = \max |f''(x)| \text{ on } [a,b]$$
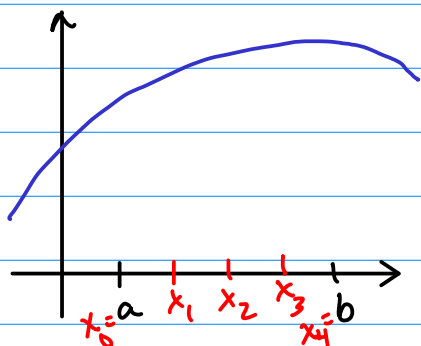
Trapezoid rule

Formula

$$A_T = \frac{1}{2}\Delta x \left[f(x_0) + 2f(x_1) + 2f(x_2) + ... + 2f(x_{n-1}) + f(x_n) \right]$$

Error

$$|E_T| \leq \frac{K(b-a)^3}{12\, n^2} \quad , \quad K = \max |f''(x)| \text{ on } [a,b]$$

Simpson's Rule

$$A_S = \frac{1}{3}\Delta x \left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + ... + 4f(x_{n-1}) + f(x_n) \right]$$

$$|E_S| \leq \frac{\tilde{K}(b-a)^5}{180\, n^4} , \quad \tilde{K} = \max |f^{(4)}(x)| \text{ on } [a,b].$$