Work

Work measures the energy required to perform a task
Units are important!

	METRIC	IMPERIAL
mass	kg	slugs
weight	Newtons	pounds
distance	meters	feet
volume	m^3	ft^3
energy	Joules	ft lbs

Primary formula: for a constant force $W = Fd$ (work, force, distance)

For non-constant forces:

$$W = \int_a^b F\,dx$$

Ex:

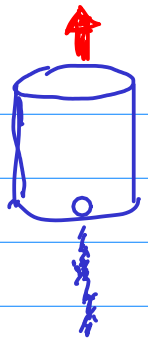
Work to lift the ball onto the table?

$W = 10 \cdot 3$ ft · lbs

$= 30$ ft · lbs

Ex: You lift a jug with a hole in the bottom 2 meters into the air. Initially there are 3 kg of water in the jug, which drains out at $\frac{1}{2}$ kg / second. If you lift the jar @ 2 m/s

How much energy?



t = time, x = height $\quad x = 2t$

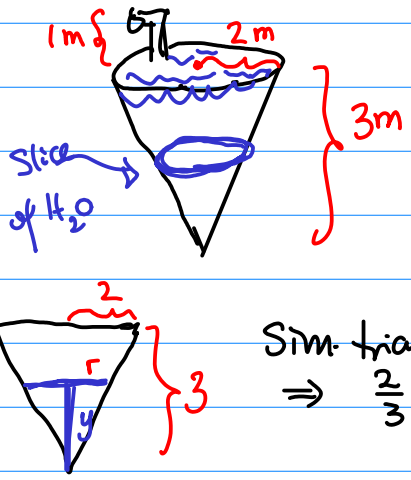
$m = 3 - \frac{1}{2}t = 3 - \frac{1}{4}x$

$g = 9.81 \text{ m/s}^2$

$F = mg = (3 - \frac{1}{4}x)g$

$W = \int_0^2 (3 - \frac{1}{4}x)\,dx = \frac{11}{2}$ J

Tank problems:



Work to drain tank?

$A_{slice} = \pi r^2 = \pi (\frac{2}{3}y)^2$

$V_{slice} = \pi (\frac{2}{3}y)^2 \Delta y$

$m_{slice} = \pi (\frac{2}{3}y)^2 \Delta y \rho$, $\rho = 1000 \text{ kg/m}^3$

Sim. triangles $\Rightarrow \frac{2}{3} = \frac{r}{y}$

$F_{slice} = \pi (\frac{2}{3}y)^2 \Delta y \rho g$

$W_{slice} = \pi (\frac{2}{3}y)^2 \Delta y \rho g (3+1-y)$

$W = \int W_{slice}\,dy = \pi \rho g \int_0^3 (\frac{2}{3}y)^2 (4-y)\,dy.$