Problem 1. Determine the value of the telescoping series

$$\sum_{n=1}^{\infty} \frac{2}{n(n+2)}$$

$$\frac{2}{n(n+2)} = \frac{1}{n} - \frac{1}{n+2}$$

$$S_n = \left(\frac{1}{1} - \cancel{\frac{1}{3}}\right) + \left(\frac{1}{2} - \cancel{\frac{1}{4}}\right) + \left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{5}}\right) + \left(\cancel{\frac{1}{4}} - \cancel{\frac{1}{6}}\right) + \dots + \left(\frac{1}{n} - \frac{1}{n-2}\right)$$

$$= 1 + \frac{1}{2} - \frac{1}{n-3} - \frac{1}{n-2}$$

$$\lim_{n\to\infty} S_n = 1 + \frac{1}{2} + 0 + 0$$

$$= \frac{3}{2}$$

Problem 2. Does the following sequence converge or diverge? Explain.

$$0, 1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, \dots$$

Diverges, becaues the values never settle down to either 0 or 1. They keep jumping back and forth.

Problem 3. State the Limit Comparison Test.

Suppose $\lim_{n\to\infty} \frac{a_n}{b_n} = c > 0$.

Then $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ either both converge or both diverge.