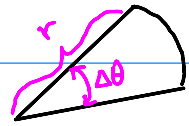
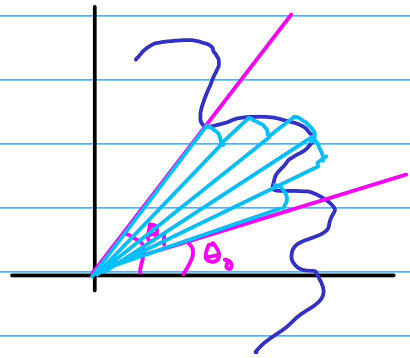
Calculus in Polar

AREAS



Pie Slice

$$A = \pi r^2 \frac{\Delta\theta}{2\pi}$$
$$= \frac{1}{2} r^2 \Delta\theta$$

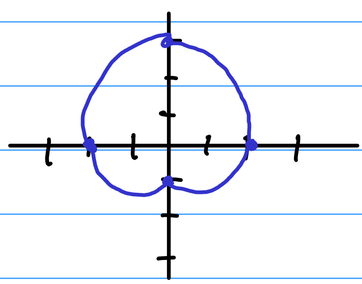


$$A \approx \sum_{i=1}^{n} \frac{1}{2} r_i^2 \Delta\theta$$

Taking the limit...

$$\boxed{A = \int_{\theta_0}^{\theta_1} \frac{1}{2} r^2 \, d\theta}$$

Ex: Area inside the cardioid $r = 2 + \sin\theta$



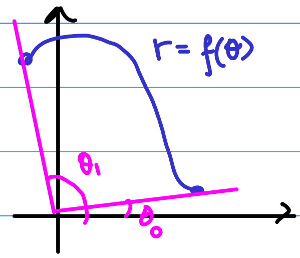
$$A = \int \frac{1}{2} r^2 \, d\theta = \int_0^{2\pi} \frac{1}{2}(2+\sin\theta)^2 \, d\theta$$

$$= \int_0^{2\pi} 2 + 2\sin\theta + \frac{1}{2}\sin^2\theta \, d\theta$$

$$= \int_0^{2\pi} \frac{9}{4} + 2\sin\theta - \frac{1}{4}\cos(2\theta) \, d\theta$$

$$= \frac{9}{4}\theta - 2\cos\theta - \frac{1}{8}\sin(2\theta) \Big|_0^{2\pi} = \frac{9}{2}\pi$$

ARC LENGTH



$$x = r\cos\theta$$
$$x = f(\theta)\cos\theta$$
$$y = r\sin\theta$$
$$y = f(\theta)\sin\theta$$

Parametric Eqn!

$$\begin{cases} x = f(\theta)\cos\theta \\ y = f(\theta)\sin\theta \end{cases}$$

$$L = \int_{\theta_0}^{\theta_1} \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2}\, d\theta$$

$$= \int_{\theta_0}^{\theta_1} \sqrt{\left(f'(\theta)\cos\theta - f(\theta)\sin\theta\right)^2 + \left(f'(\theta)\sin\theta + f(\theta)\cos\theta\right)^2}\, d\theta$$

$$\boxed{L = \int_{\theta_0}^{\theta_1} \sqrt{(f'(\theta))^2 + f(\theta)^2}\, d\theta}$$

TANGENT LINES

$$\boxed{\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{f'(\theta)\sin\theta + f(\theta)\cos\theta}{f'(\theta)\cos\theta - f(\theta)\sin\theta}}$$