Math 150B Section 1
Summer 2023
Exam I
July 16, 2023
Time Limit: 1 Hour 10 Minutes

Name (Print): ____________________

Student ID: ____________________

This exam contains 6 pages (including this cover page) and 5 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books or notes on this exam. You may use a single-sided, hand-written note sheet and a basic calculator.
You are required to show your work on each problem on this exam. The following rules apply:

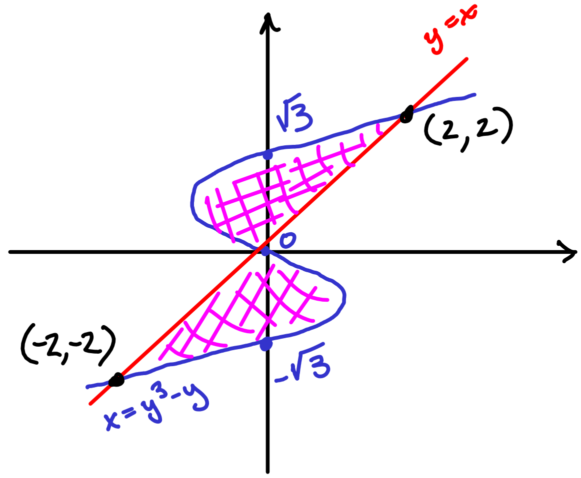
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit. This especially applies to limit calculations.
- If you need more space, use the back of the pages; clearly indicate when you have done this.
- **Box Your Answer** where appropriate, in order to clearly indicate what you consider the answer to the question to be.

Do not write in the table to the right.

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total:	50	

1. (10 points) Consider the area bounded by the curves $y = x$ and $x = y^3 - 3y$.

   (a) Draw a picture of the region bounded by the curves. Carefully label points of intersection between the curves.



   (b) Find the area of the region bounded by the curves.

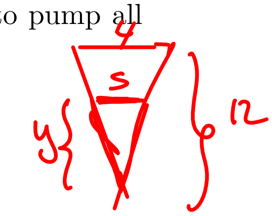
$$\int_{-2}^{0} y^3-3y-y\,dy + \int_{0}^{2} y-(y^3-3y)\,dy$$

$$= 2\int_0^2 4y-y^3\,dy = 2\left(2y^2-\frac{1}{4}y^4\right)\Big|_0^2 = 2(8-4)$$

$$= 8$$

2. (10 points)

A grain silo lies in the shape of an inverted square pyramid of base length 4 ft. and height 12 ft. as featured below. The silo is partially full of wheat, up to 8 feet in height from the tip in the bottom. Using the fact that wheat is 50 lbs/ft^3, calculate the work required to pump all the wheat through the top of the tank. Make sure to label your units!

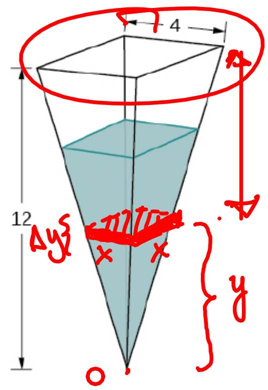


$A = \left(\frac{1}{3}y\right)^2$

$V = \left(\frac{1}{3}y\right)^2 \Delta y$

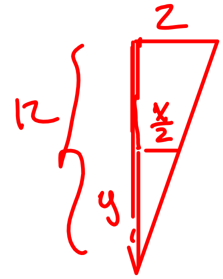
$F = 50\left(\frac{1}{3}y\right)^2 \Delta y$

$W = 50\left(\frac{1}{3}y\right)^2 (12-y)\Delta y$



$x = my + b$

$A = x^2$



Sim Δ's:

$\frac{2}{12} = \frac{x/2}{y}$

$\frac{4}{12}y = x$

$$\int_0^8 50 \frac{1}{9} y^2 (12-y)\,dy$$

$$= \frac{50}{9}\int_0^8 12y^2 - y^3\,dy = \frac{50}{9}\left(4y^3 - \frac{1}{4}y^4\right)\Big|_0^8$$

$$= \frac{50}{9}(2048-1024)$$

$$= \frac{51200}{9}$$

3. (10 points)

Evaluate each of the following integrals.

(a) $\int e^{2x} \sin(3x) dx$

$u = e^{2x}$, $dv = \sin(3x)dx$

$du = 2e^{2x}$ $v = -\frac{1}{3}\cos(3x)$

$-\frac{1}{3}e^{2x}\cos(3x) + \frac{2}{3}\int e^{2x}\cos(3x)dx$

$u = e^{2x}$, $dv = \cos(3x)$

$du = 2e^{2x}$, $v = \frac{1}{3}\sin(3x)$

$$-\frac{1}{3}e^{2x}\cos(3x) + \frac{2}{3}\left[\frac{1}{3}e^{2x}\sin(3x) - \frac{2}{3}\int e^{2x}\sin(3x)dx\right]$$

$$\int e^{2x}\sin(3x) = -\frac{1}{3}e^{2x}\cos(3x) + \frac{2}{9}e^{2x}\sin(3x) - \frac{4}{9}\int e^{2x}\sin(3x)dx$$

$$\int e^{2x}\sin(3x)\,dx = \frac{9}{13}\left(-\frac{1}{3}e^{2x}\cos(3x) + \frac{2}{9}e^{2x}\sin(3x)\right) + C$$

(b) $\int_0^{\pi/2} \sin^3(x)\cos^8(x)dx$

$$\int_0^{\pi/2}(1-\cos^2(x))\cos^8(x)\sin(x)dx = -\int_1^0 (1-u^2)u^8\,du = \int_0^1 u^8 - u^{10}\,du$$

$u = \cos(x)$

$du = -\sin(x)dx$

$$= \frac{1}{9} - \frac{1}{11} = \frac{2}{99}$$

4. (10 points)

Evaluate each of the following integrals.

(a) $\int \frac{3x^2}{\sqrt{1-x^2}}dx$

$x = \sin\theta$
$dx = \cos\theta \, d\theta$

$$\int \frac{3\sin^2\theta}{\cancel{\sqrt{1-\sin^2\theta}}} \cancel{\cos\theta}\, d\theta = 3\int \sin^2\theta \, d\theta$$

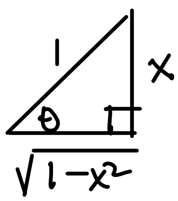
$$= 3\int \frac{1}{2} - \frac{1}{2}\cos(2\theta)\, d\theta$$

$\sin(2\theta) = 2\sin\theta\cos\theta$

$$= \frac{3}{2}\theta - \frac{3}{4}\sin(2\theta) + C$$

$\theta = \sin^{-1}(x)$

$$= \frac{3}{2}\theta - \frac{3}{2}\sin\theta\cos\theta + C$$



$\sin\theta = x$
$\cos\theta = \sqrt{1-x^2}$

$$= \frac{3}{2}\sin^{-1}(x) - \frac{3}{2}x\sqrt{1-x^2} + C$$

(b) $\int \frac{2x+3}{x^2(x+7)}dx$

$$\frac{2x+3}{x^2(x+7)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+7}$$

$$0x^2 + 2x + 3 = Ax(x+7) + B(x+7) + Cx^2$$

@ $x=0$: $3 = 7B \Rightarrow B = \frac{3}{7}$

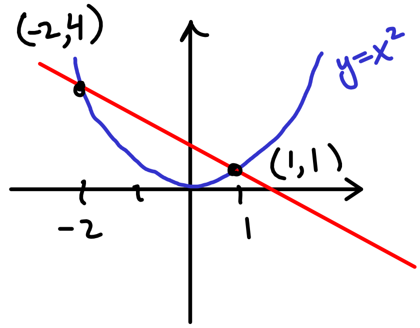
@ $x=-7$: $-11 = 49C \Rightarrow C = \frac{-11}{49}$

$A + C = 0 \Rightarrow A = -C = \frac{11}{49}$

$$\int \frac{11/49}{x} + \frac{3/7}{x^2} - \frac{11/49}{x+7}\, dx = \frac{11}{49}\ln|x| - \frac{3}{7}x^{-1} - \frac{11}{49}\ln|x+7| + C$$

5. (10 points)

(a) Sketch a graph of the region bounded by $y = x^2$ and $y = 2 - x$.



$x^2+x-2=0$

$x=-2, \quad x=1$

(b) Consider rotating the previous region around $x = 2$. Set up (but do not solve) an integral determining the volume using the Shell Method.

$$2\pi \int_{-2}^{1} \overbrace{(2-x)}^{r}\overbrace{(2-x-x^2)}^{h}\,dx$$

(c) Suppose instead that the region was rotated around the line $y = 5$. Set up (but do not solve) an integral determining the volume using the Disk/Washer Method.

$$\pi \int_{-2}^{1} (5-x^2)^2-(5-(2-x))^2\,dx$$