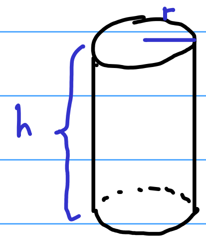
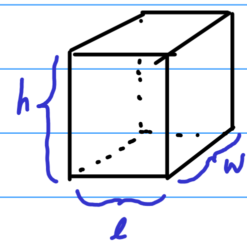
Volumes

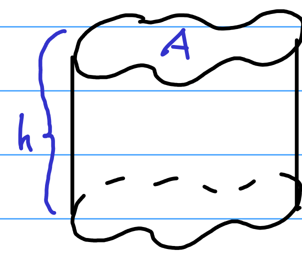
Basic idea: volumes of cylinders are easy!



$V = \pi r^2 h$



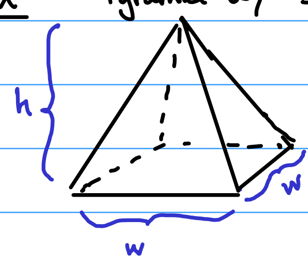
$V = \ell w h$

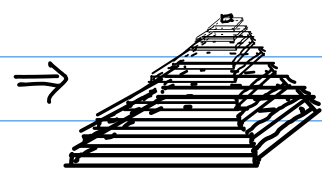


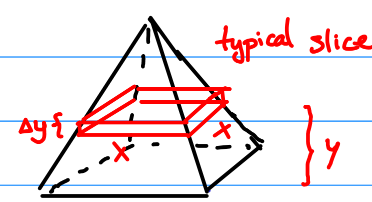
$V = A h$

We can get more complicated volumes by viewing them as limits of cylinders

Ex: Pyramid w/ square base







Similar triangles:

$$\frac{x}{h-y} = \frac{w}{h}$$

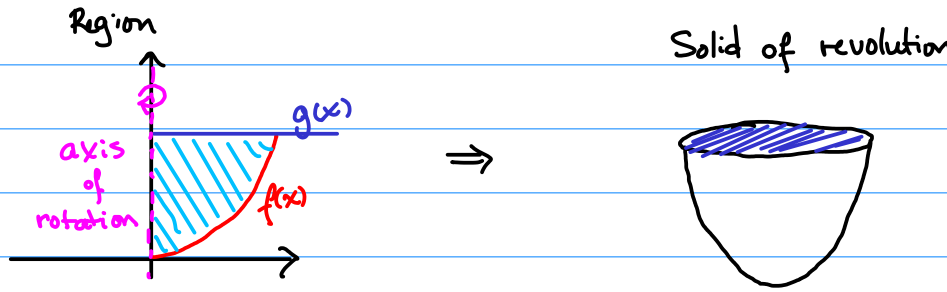
$$V \approx \sum_i \text{volume of slice } i$$

$$\approx \sum \left[\frac{w}{h}(h-y_i)\right]^2 \Delta y$$

Taking the limit:

$$V = \int_0^h \left[\frac{w}{h}(h-y)\right]^2 dy = \frac{1}{3} w^2 h$$

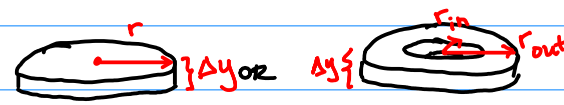
We will focus on <u>solids of revolution</u>, regions of the plane rotated around a line.



Two main methods!

- Disk/Washer Method

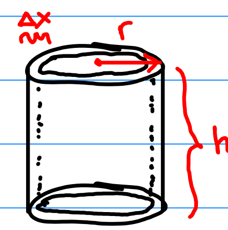
  Break up into cylinders like



$V = \pi r^2 \Delta y$ $\qquad$ $V = \pi(r_{out}^2 - r_{in}^2)$

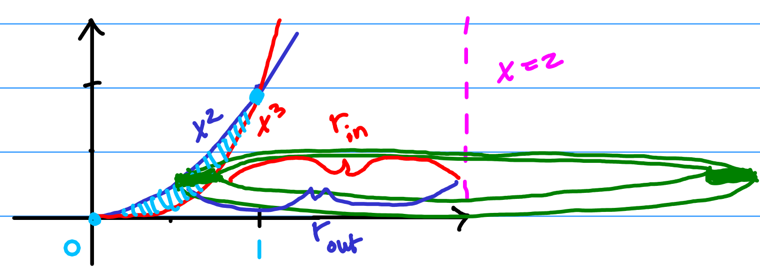
- Shell method

  Break up into cylinders like



$V = 2\pi r h$

<u>Ex</u> : Find the volume of the solid of revolution obtained by rotating the area between $y = x^2$ and $y = x^3$ around the line $x = 2$.



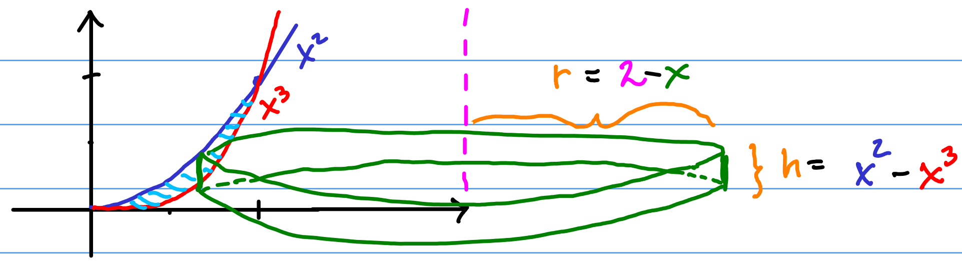
$r_{out} = 2 - \sqrt{y}$

$r_{in} = 2 - \sqrt[3]{y}$

$$V = \int \pi(r_{out}^2 - r_{in}^2)\,dy$$

$$V = \int_0^1 \pi\left((2-\sqrt{y})^2 - (2-\sqrt[3]{y})^2\right)dy = \frac{7}{30}\pi$$

Same example, but with shells:



$$V = \int 2\pi r h\, dx = \int_0^1 2\pi(2-x)(x^2-x^3)\, dx = \frac{7}{30}\pi$$

Summary:

Disk/Washer rotated around vertical: $\int \pi(r_{out}^2 - r_{in}^2)\, dy$

Disk/Washer rotated around horizontal: $\int \pi(r_{out}^2 - r_{in}^2)\, dx$

Shell rotated around vertical: $\int 2\pi r h\, dx$

Shell rotated around horizontal: $\int 2\pi r h\, dy$