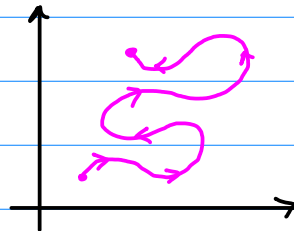
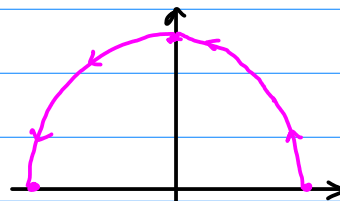


## Parametric Equations

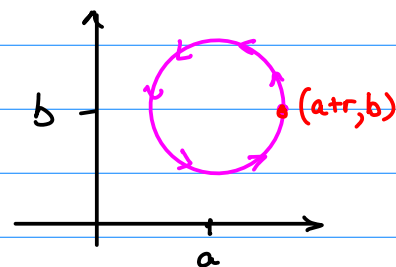
$$\begin{cases} x = f(t) \\ y = g(t) \end{cases}, \quad t_0 \leq t \leq t_1$$



Ex:  $\begin{cases} x = \cos(t) \\ y = \sin(t) \end{cases}, \quad 0 \leq t \leq \pi$

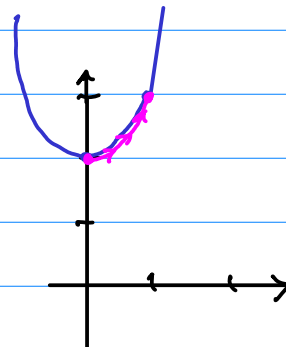


Ex:  $\begin{cases} x = r \cos t + a \\ y = r \sin t + b \end{cases}, \quad 0 \leq t \leq 2\pi$



Ex:  $\begin{cases} x = \sqrt{t} \\ y = t+2 \end{cases}, \quad 0 \leq t \leq 1$

$y = x^2 + 2$



## Calculus with parametric equations

Slope of tangent line to curve?

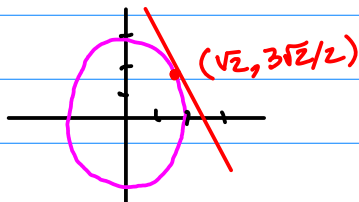
$$\frac{dy}{dt} = \frac{dy}{dx} \frac{dx}{dt} \Rightarrow \frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

Second derivative

$$\frac{d^2y}{dx^2} = \frac{d\left(\frac{dy}{dx}\right)/dt}{dx/dt}$$

Ex: Slope of tangent line to the curve  
at  $t = \frac{\pi}{4}$ .

$$\begin{cases} x = 2\cos(t) \\ y = 3\sin(t) \end{cases}, 0 \leq t \leq 2\pi$$

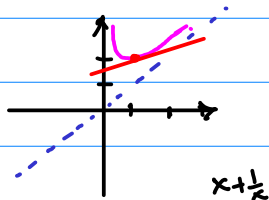


$$\frac{dx}{dt} = -2\sin(t), \quad \frac{dy}{dt} = 3\cos(t)$$

$$\frac{dy}{dx} = \frac{3\cos(t)}{-2\sin(t)} \quad @ t = \frac{\pi}{4} : \left( -\frac{3}{2} \right)$$

Ex: Find the slope of the tangent line  
at the point (1,2)

$$\begin{cases} x = e^t \\ y = e^t + e^{-t} \end{cases}, -1 \leq t \leq 1$$



$$\frac{dy}{dx} = \frac{e^t - e^{-t}}{e^t} = 1 - e^{-2t}$$

$$@ t = 0 : (1)$$

Lengths of curves:

$$S = \int_{x_0}^{x_1} \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx = \int_{t_0}^{t_1} \sqrt{1 + \left(\frac{dy/dt}{dx/dt}\right)^2} \frac{dx}{dt} dt$$

$$S = \int_{t_0}^{t_1} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

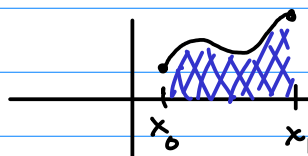
Ex: Find the length of the curve  $\begin{cases} x = t^2 \\ y = 2t \end{cases}, 0 \leq t \leq 2$

$$\begin{aligned} S &= \int_0^2 \sqrt{(2t)^2 + (2)^2} dt = 2 \int_0^2 \sqrt{1+t^2} dt \\ &= \left( t\sqrt{1+t^2} + \ln|\sqrt{1+t^2} + t| \right) \Big|_0^2 \end{aligned}$$

$$= 2\sqrt{5} + \ln(\sqrt{5}+2)$$

Area under the curve :

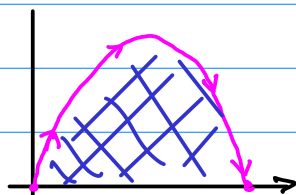
$$\begin{cases} x = f(t) \\ y = g(t) \end{cases}, t_0 \leq t \leq t_1$$



$$A = \int_{x_0}^{x_1} y \, dx = \int_{t_0}^{t_1} y \frac{dx}{dt} dt$$

$$A = \int_{t_0}^{t_1} g(t) f'(t) dt$$

Ex: Area under the curve  $\begin{cases} x = t - \sin t \\ y = 1 - \cos t \end{cases}, 0 \leq t \leq 2\pi$



$$\int_0^{2\pi} (1 - \cos t) (t - \sin t)' dt$$

$$= \int_0^{2\pi} (1 - \cos t)^2 dt = \int_0^{2\pi} 1 - 2\cos t + \cos^2 t dt$$

$$= \int_0^{2\pi} \frac{3}{2} + \frac{1}{2}\cos(2t) - 2\cos t dt = 3\pi$$

Surface Area

Rotated around x-axis:

$$\int 2\pi y \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$