Problem 1. Find the length of the curve $y = \frac{2}{3}x^{3/2}$ from $x = 0$ to $x = 1$.

$y' = \sqrt{x}$

$$S = \int_0^1 \sqrt{1 + (\sqrt{x})^2}\, dx$$

$$= \int_0^1 \sqrt{1+x}\, dx \quad = \quad \int_1^2 \sqrt{u}\, du = \frac{2}{3}u^{3/2}\Big|_1^2 = \frac{2}{3}(2\sqrt{2}-1)$$

$u = 1+x$

Problem 2. For what value of A is $f(x)$ a probability density function?

$$f(x) = \begin{cases} A\sin^2(x), & 0 \le x \le \pi \\ 0 & \text{otherwise} \end{cases}$$

$$A\int_0^\pi \sin^2(x)\,dx = 1 \quad \Rightarrow \quad 1 = A\int_0^\pi \frac{1}{2} - \frac{1}{2}\cos(2x)\,dx$$

$$= A\left(\frac{1}{2}x - \frac{1}{4}\sin(2x)\right)\Big|_0^\pi$$

$$= \frac{\pi}{2}A$$

$$A = \frac{2}{\pi}$$