

Improper Integrals

Integrals where the function has a discontinuity or the integration interval is infinite.

Examples: $\int_0^1 \frac{1}{\sqrt{x}} dx$, $\int_2^{\infty} e^{-x} dx$, $\int_{-\infty}^{\infty} \frac{1}{1+x^2} dx$

Basic idea: we always treat ∞ as a limit

Ex: $\int_1^{\infty} \frac{1}{x^2} dx = \lim_{t \rightarrow \infty} \int_1^t \frac{1}{x^2} dx = \lim_{t \rightarrow \infty} \left(1 - \frac{1}{t}\right) = 1$

Ex: $\int_0^1 \frac{1}{\sqrt{x}} dx = \lim_{t \rightarrow 0^+} \int_t^1 \frac{1}{\sqrt{x}} dx = \lim_{t \rightarrow 0^+} 2 - 2\sqrt{t} = 2$

Ex: $\int_0^{\infty} \cos(x) dx = \lim_{t \rightarrow \infty} \int_0^t \cos(x) dx = \lim_{t \rightarrow \infty} \sin(t) \quad \text{DNE}$

DIVERGENT!