Separable Equations

$$y' = f(y)g(x)$$

Solution: separate out x and y

$$\frac{1}{f(y)}\,dy = g(x)\,dx$$

Then integrate both sides.

Special case: autonomous equations

$$y' = f(y)$$

$$\frac{1}{f(y)}\,dy = dx \Rightarrow \int \frac{1}{f(y)}\,dy = x + C$$

Ex: Logistic equation $y' = ky(A-y)$

$$\int \frac{1}{ky(A-y)}\,dy = \int dt$$

$$\frac{1}{k}\int \frac{1/A}{y} + \frac{1/A}{A-y}\,dy = \int dt$$

$$\frac{1}{kA}\ln|y| - \ln|A-y| = t + C_1$$

$$\ln|y| - \ln|A-y| = kAt + C_2 \quad , \quad C_2 = kAC_1$$

$$\frac{y}{A-y} = C_3e^{kAt} \quad , \quad C_3 = e^{C_2}$$

$$y = A\frac{C_3e^{kAt}}{1+C_3e^{kAt}}$$

This produces a family of solutions, one for each choice of C_3.

Further examples:

$$y' = xy \Rightarrow \int \frac{1}{y}\,dy = \int x\,dx \Rightarrow \ln|y| = \frac{1}{2}x^2 + C$$

$$\Rightarrow y = e^{\frac{1}{2}x^2 + C}$$

If we take $C_1 = e^C$:

$$y = C_1 e^{\frac{1}{2}x^2}$$

This is a family of solutions called the <u>general solution</u>. If we choose a value of C_1, we get a <u>particular solution</u>.

Particular solutions arise when we want solutions satisfying some kind of initial condition, like $y(0) = 3$.

A diff equation + initial condition is called an initial value problem (IVP).

<u>Ex</u>: Solve the IVP $\quad y' = y, \quad y(0) = 2.$

$$\int \frac{1}{y}\,dy = \int 1\,dx \Rightarrow \ln|y| = x + C$$

$$\Rightarrow y = C_1 e^x \qquad C_1 = e^C$$

$$y(0) = 2 \Rightarrow 2 = C_1 e^0 \Rightarrow C_1 = 2 \qquad \boxed{y = 2e^x}$$