Math 150B Section 1
Summer 2023
Exam II
July 28, 2023
Time Limit: 1 Hour 10 Minutes

Name (Print): _____

Student ID: _____

This exam contains 7 pages (including this cover page) and 5 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books or notes on this exam. You may use a single-sided, hand-written note sheet and a basic calculator.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.

- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit. This especially applies to limit calculations.

- If you need more space, use the back of the pages; clearly indicate when you have done this.

- **Box Your Answer** where appropriate, in order to clearly indicate what you consider the answer to the question to be.

Do not write in the table to the right.

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total:	50	

1. (10 points)

   The national average composite ACT test score for high school seniors in the US last year was 20.8 with a standard deviation of 5.8.

   (a) Write down an integral which represents the probability that a randomly selected test-taker scored between a 30 and a 36 on their ACT?

   (b) Use the Trapezoid Rule with $n = 3$ to approximate the value of the integral in (a).

2. (10 points)

   (a) As a snowball melts, its radius decreases at a rate proportional to the square root of the radius. Write down a differential equation describing the rate of change of the radius.

   (b) Solve the initial value problem.

$$y' = 8x(1 + y^2), \quad y(1) = 0.$$

3. (10 points)

(a) Set up an integral to find the length of the parametric curve

$$x = e^t - t, \quad y = 4e^{t/2}, \quad 0 \le t \le 2.$$

(b) Find the value of the integral in part (a).

(c) Determine the slope of the line tangent to the parametric curve at the point $(e - 1, 4\sqrt{e})$.

4. (10 points)

    (a) Draw the region inside the curve $r = 9\sin(\theta)$ and outside the curve $r = 5 - \sin(\theta)$.

    (b) Use integration in polar coordinates to find the area of the region in part (a).

5. (10 points)

   (a) Calculate the limit of the following sequence

   $$a_n = \frac{2n^3 - n^2 + 3n - 4}{\sqrt{3n^6 + 2n + 4}}.$$

   If it diverges, write DIVERGES.

   (b) Express the decimal expression

   $$0.151515151515\ldots$$

   as a fraction of two whole numbers.

BONUS QUESTION (5pts):

Find the value of the integral

$$\int x^{-2} \tan^{-1}(x)\, dx.$$