Math 150B Section 1
Summer 2023
Final Exam
August 4, 2023
Time Limit: 2 Hour 20 Minutes

Name (Print): _____

Student ID: _____

This exam contains 11 pages (including this cover page) and 10 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books or notes on this exam. You may use a double-sided, hand-written note sheet and a basic calculator.
You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit. This especially applies to limit calculations.
- If you need more space, use the back of the pages; clearly indicate when you have done this.
- **Box Your Answer** where appropriate, in order to clearly indicate what you consider the answer to the question to be.

Do not write in the table to the right.

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
Total:	100	

1. (10 points)

   (a) Set up an integral for the volume of the solid obtained by rotating the region bounded by the curves

   $$y = x^3, \quad y = 0, \quad x = 1, \quad x = 4$$

   around the y-axis, using the shell method.

   (b) Set up an integral for the volume of the solid obtained by rotating the region bounded by the curves

   $$y = 8 - x^2, \quad y = 4$$

   around the x-axis.

2. (10 points)

Consider the polar function $r = \theta^2, \quad 0 \leq \theta \leq 2\pi$

(a) Create a plot of the polar function.

(b) Set up an integral for the length of the curve above.

(c) Evalulate the integral from (b).

3. (10 points)

Calculate each of the following integrals

(a) $\int \frac{1}{\sqrt{4-(x-3)^2}} dx$

(b) $\int xe^{2x} dx$

4. (10 points)

Calculate each of the following integrals

(a) $\int \frac{4}{(x-1)^2(x+1)} dx$

(b) $\int_0^{\pi/4} \sin^8(x) \cos^3(x) dx$

5. (10 points)

A spherical tank with a radius of 3 meters is completely filled with water. How much work is required to pump the water out of a pipe which extends vertically one meter out of the top of the tank? [Remember: the density of water is 1000 kg/m^3 and gravitational acceleration is 9.8 m/s^2.

6. (10 points)

Determine carefully the limit of each of the following sequences.

(a) Find the limit of the following sequence, or show that it does not exist.

$$\lim_{n\to\infty} \frac{n^2+2}{\sqrt[3]{n^6+3n+4}}$$

(b) Suppose that we want to evaluate the limit of the sum

$$\sum_{n=2}^{\infty} \frac{1}{n\ln(n)^3}$$

with accuracy up to 4 decimal places. Calculate how many terms we should sum (you don't need to actually do the sum). [Hint: use the Integral Test estimate for the tail]

7. (10 points)

Determine if each of the following series are absolutely convergent, conditionally convergent, or divergent. Carefully justify your answer.

(a)

$$\sum_{n=0}^{\infty} \frac{5^n + 2}{4^n + 3}$$

(b)

$$\sum_{n=1}^{\infty} (-1)^n \frac{1}{2n + 3}$$

8. (10 points)

Determine if each of the following series are absolutely convergent, conditionally convergent, or divergent. Carefully justify your answer.

(a)

$$\sum_{n=0}^{\infty} \frac{2^n}{n^{3n}}$$

(b)

$$\sum_{n=1}^{\infty} \frac{n^3 + n + 1}{n^5 + 2n + 4}$$

9. (10 points)

Find the radius of convergence and the interval of convergence of the series

$$\sum_{n=1}^{\infty} \frac{x^{2n}}{n4^n}.$$

10. (10 points)

(a) Find the Taylor series of e^{-x^2} based at $x = 0$

(b) Find the Taylor series of $\int e^{-x^2} dx$