Determine whether each of the following series converges or diverges. Carefully state what convergence test you are using.

Problem 1.

$$\sum_{n=1}^{\infty} \frac{2^{n-1}3^{n-1}}{n^n} = \frac{1}{2\cdot 3}\sum_{n=1}^{\infty}\frac{2^n 3^n}{n^n}$$

Root Test:

$$\lim_{n\to\infty} \sqrt[n]{\frac{2^n 3^n}{n^n}} = \lim_{n\to\infty} \frac{2\cdot 3}{n} = 0 < 1$$

Therefore series converges

Problem 2.

$$\sum_{n=1}^{\infty} \frac{n}{\sqrt{n!}}$$

Ratio test:

$$\lim_{n\to\infty} \frac{n+1}{\sqrt{(n+1)!}} \frac{\sqrt{n!}}{n} = \lim_{n\to\infty} \frac{n+1}{n\sqrt{n+1}} = \lim_{n\to\infty} \frac{\frac{1}{\sqrt{n}} + \frac{1}{n^{3/2}}}{\sqrt{1+\frac{1}{\sqrt{n}}}}$$

$$= \frac{0+0}{\sqrt{1+0}} = 0$$

Therefore the series converges

Problem 3.

$$\sum_{n=1}^{\infty} \tan(1/n)$$

Limit Comparison Test: $\lim_{n\to\infty} \frac{\tan(1/n)}{1/n} = \sec^2(0) = 1 > 0$

Since the harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges,

The original series must diverge.