

Math 350 Section 1

Fall 2023

Exam I

September 19, 2023

Time Limit: 1 Hour 50 Minutes

Name (Print): _____

Student ID: _____

This exam contains 8 pages (including this cover page) and 7 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books or notes on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit.** A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit. This especially applies to limit calculations.
- If you need more space, use the back of the pages; clearly indicate when you have done this.
- **Box Your Answer** where appropriate, in order to clearly indicate what you consider the answer to the question to be.

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
Total:	70	

Do not write in the table to the right.

1. (10 points) **TRUE or FALSE!** Write TRUE if the statement is true. Otherwise, write FALSE. Your response should be in ALL CAPS. No justification is required.

(a) If a and b are nonzero numbers and $a < b$, then $1/a > 1/b$.

(b) Any set of positive integers has a minimum value

(c) The sequence $s_n = n/(2n + 3)$ is Cauchy

(d) If $A \subseteq B$ then $\sup(A) \leq \sup(B)$

(e) If a bounded sequence satisfies $s_n > 0$ for all n , then $\limsup s_n > 0$

2. (10 points)

Let F_n be the sequence of Fibonacci numbers defined by $F_1 = 1$, $F_2 = 1$, $F_3 = 2$ and for $n \geq 2$

$$F_{n+1} = F_n + F_{n-1}.$$

Use induction to prove that for all $n \in \mathbb{N}$

$$F_n F_{n+2} - F_{n+1}^2 = (-1)^{n+1}.$$

3. (10 points)

(a) Write down the definition of a lower bound of a set A

(b) Write down the definition of the infimum of a set A

(c) Determine the infimum of the set

$$A = \left\{ \frac{1}{n} : n \in \mathbb{N} \right\}.$$

(d) Prove your answer for part (c).

4. (10 points)

(a) Write down the definition of a sequence (s_n) converging to s

(b) Suppose that (s_n) is a sequence of positive numbers converging to a real number s . Prove that $(\sqrt{s_n})$ converges to \sqrt{s} .

5. (10 points)

Define a sequence (s_n) recursively by letting $s_1 = 2023$ and more generally

$$s_{n+1} = \frac{1}{2}s_n + \frac{5}{2s_n}.$$

(a) Prove that the sequence is monotone.

(b) Prove that the sequence converges.

(c) Determine the limit of the sequence.

6. (10 points)

(a) Prove that there is a sequence of rational numbers which converges to π .

(b) Prove that if (s_n) is a monotone increasing sequence which is bounded above, then s_n converges.

7. (10 points) Let (a_n) and (b_n) be bounded sequences.

(a) Define $\limsup a_n$ and $\liminf a_n$

(b) Prove

$$\limsup a_n + \liminf b_n \leq \limsup(a_n + b_n) \leq \limsup a_n + \limsup b_n$$

(c) Give an explicit example of two sequences (a_n) and (b_n) for which the above two inequalities both turn out to be equalities.