Math 350 Section 1
Fall 2024
Exam I
September 25, 2024
Time Limit: 1 Hour 50 Minutes

Name (Print): ____________________

Student ID: ____________________

This exam contains 8 pages (including this cover page) and 7 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books or notes on this exam.
You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit. This especially applies to limit calculations.
- If you need more space, use the back of the pages; clearly indicate when you have done this.
- **Box Your Answer** where appropriate, in order to clearly indicate what you consider the answer to the question to be.

Do not write in the table to the right.

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
Total:	70	

1. (10 points) **TRUE or FALSE!** Write TRUE if the statement is true. Otherwise, write FALSE. Your response should be in ALL CAPS. No justification is required.

   (a) If a and b are nonzero numbers and $a < b$, then $1/a > 1/b$

   (b) Any non-empty set of positive integers has a minimum value

   (c) The relation $\mathcal{R}$ on $\mathbb{R}$ defined by $\mathcal{R} = \{(x, x) : x \in \mathbb{R}\}$ is an equivalence relation

   (d) If $A \subseteq B$ then $\sup(A) \leq \sup(B)$

   (e) If A is a non-empty set of real numbers with $0 < x$ for all $x \in A$, then $0 < \sup(A)$

2. (10 points)

   (a) Write down the definition of the upper bound of a set A.

   (b) Prove that the set of positive integers $\mathbb{Z}_+$ is not bounded above.

3. (10 points)

   (a) Write down the definition of the infimum of a set A

   (b) Determine the infimum of the set

$$A = \left\{ \frac{1}{n} : n \in \mathbb{N} \right\}.$$

   (c) Prove your answer for part (b).

4. (10 points)

   (a) Write down the four order axioms of the real numbers (ie. A6-A9).

   (b) Use the order axioms to prove that if x, y, and z are real numbers with $x < y$ and $z > 0$, then $xz < yz$. Carefully state each time you use one of the axioms.

5. (10 points)

Let A and B be sets and suppose $f : A \to B$ is a function.

(a) Write down the definition of f being injective.

(b) Write down the definition of f being surjective.

(c) Write down the definition of f being bijective.

(d) Prove that the function $f : \mathbb{Z}_+ \to \mathbb{Z}$ defined by

$$f(x) = \begin{cases} x/2, & x \text{ is even} \\ (1-x)/2, & x \text{ is odd} \end{cases}$$

is injective and surjective.

6. (10 points)

   Suppose that A and B are two non-empty sets of real numbers, both of which are bounded above, and let $C = \{a + b : a \in A,\ b \in B\}$.

   (a) State the Completeness Axiom

   (b) Prove that C is bounded above

   (c) Prove that $\sup(C) \leq \sup(A) + \sup(B)$

7. (10 points) Consider the family of sets $\{A_i : i \in I\}$ where the index set $I = \mathbb{Z}$ and $A_i = (i, i+1)$.

(a) Determine the value of $\bigcap_{i \in I} A_i$

(b) Carefully prove your answer in part (a) is correct.

(c) Determine the value of $\bigcup_{i \in I} A_i$

(d) Carefully prove your answer in part (c) is correct.