MATH 350-2 Advanced Calculus

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September 16, 2024

Outline

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Ultimate Cantor diagonalization

The **power set** $\mathcal{P}(A)$ of a set A is

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Proof.

Suppose $f : A \to \mathcal{P}(A)$ is surjective.
Consider the set

$$S = \{a \in A : a \notin f(a)\}.$$

□

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Problem

Consider the statement $x \in S$. What can you conclude?

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NOTATION:

$$\bigcup_{i=1}^{n} A_i = A_1 \cup A_2 \cup \cdots \cup A_n.$$

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- $A_i = [0, i)$

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Determine $\bigcup_{i \in I} A_i$.

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